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Neutrino Trapping in Nonstrange Dense Stellar Matter

by

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Abstract

Neutrino trapping effects on the properties of dense stellar mater in nonstrange supranuclear regime are studied with the purpose of applying to supernovae dynamical evolution. The hadronic and leptonic compositions of stellar matter are solved in the context of the relativistic mean-field theory coupled with the β-equilibrium condition, and maintaining the charge neutrality of the stellar medium. It is shown that the matter composition depends dramatically on the confined electronic-leptonic fraction. However, the values of the isoscalar fields are almost insensitive to variations of that quantity. The softness of the equation of state and the lowering of the nuclear incompressibility values, with respect to the situation in which neutrino confinement is not considered, are the remarkable results.

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The scenario of Type-II Supernovae is strongly marked by the dynamical behavior of stellar medium under the neutrino trapping effects. During the supernova collapse phase a copious amount of neutrinos is produced by the electron capture process occuring in the compressed matter. When the density reaches values near 10^{10}–10^{11} g/cm^3, neutrinos become trapped and cannot be neglected in discussing the properties of the stellar matter equation of state. The inner core continues to collapse and supranuclear values of density are reached. A shock front is generated at the reversion of inner core collapse and the characteristics of the equation of state at this supranuclear regime determine the power of the shock front. The occurence of a successfull supernova event or its failure is defined by the power of the nascent shock wave front. The analysis of the influence of neutrino processes, its transport in dense stellar medium and its contribution to the stellar matter equation of state have followed supernovae studies since the pioneering works to nowadays [1]. The role of the neutrino trapping is also claimed to explain the so-called delayed explosion mechanism [2]. The neutrino trapping revives the degraded shock front after the core collapse reversion. Properties of trapped-neutrino matter in subnuclear regime have already been investigated [3] with the same purpose of the present study, namely the application in supernovae explosion calculations. In this work we investigate the consequences of neutrino trapping on the properties of nonstrange dense stellar matter in a supranuclear regime. The neutrinos are considered to be confined and are explicitly included into a relativistic description for the dense stellar medium. The hadronic fields are solved in a relativistic mean-field approximation where the hadronic and leptonic densities are self-consistently regulated by the constraints of β-equilibrium and charge neutrality of the matter. A similar calculation has been carried out in Ref.[4] to study stationary neutron star structure when neutrinos have already difused and left the system out. In these studies neutrinos have not been taken into account. The explicit inclusion of neutrinos is the relevant contribution of the present work to the physics of supernova collapse.

The hadronic and leptonic fields follow the prescription adopted in Ref.[4]. The Lagrangian is given by three differents sectors,

$$\mathcal{L} = \mathcal{L}_{\rm b} + \mathcal{L}_{\rm m} + \mathcal{L}_{\rm l} \,, \tag{1}$$

where $\mathcal{L}_{\rm b}$, $\mathcal{L}_{\rm m}$ and $\mathcal{L}_{\rm l}$ describe the baryonic, mesonic and leptonic sectors respectively, given by

$$\begin{aligned}
\mathcal{L}_{\rm b} &= \sum_{b=p,n} \bar\psi_b \left\{ \gamma_\mu \left[i\partial^\mu - g_\omega \omega^\mu - g_\rho \tau_{3b} \rho^\mu \right] - (m - g_\sigma \sigma) \right\} \psi_b \,, \\
\mathcal{L}_{\rm m} &= \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right) - \frac{\rm b}{3} m_n (g_\sigma \sigma)^3 - \frac{\rm c}{4} (g_\sigma \sigma)^4 \\
&\quad - \frac{1}{4}\left(\partial_\mu \omega_\nu - \partial_\nu \omega_\mu\right)^2 + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\
&\quad - \frac{1}{4}\left(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu\right)^2 + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu \,, \\
\mathcal{L}_{\rm l} &= \sum_{l=e,\mu,\nu} \bar\psi_l \left(\gamma_\mu i\partial^\mu - m_l\right) \psi_l \,.
\end{aligned}$$

The self-interaction terms for the scalar meson are the usual ones adopted to reproduce a realistic value for the nuclear incompressibility [5, 6]. For simplicity and to emphasize the relevant aspects of the problem, we consider in the baryonic sector only protons and neutrons. Hence we deal only with one coupling constant for each meson, $g_{in}{=}g_{ip}{\equiv}g_i$ $(i{=}\sigma,\omega,\rho)$, and only with one baryon mass $m_n{=}m_p{\equiv}m$. In the leptonic sector we have electrons, muons and neutrinos. We do not take into account pions in the mesonic sector. Besides, we kept only the neutral component for the charged meson ρ. It is important to remark that, for a complete description of the system in the high density regime [4], the baryonic sector should include strange baryons like Λ, Σ and Ξ besides nucleons and, consistently, also the meson sector should be completed with the inclusion of kaons.

The mean-field approximation consists of replacing all baryon currents in the Euler-Lagrange equations obtained from Eq.(1) by their ground-state expectation values. The baryon ground state consists of a degenerated Hartree state, constructed by filling states with quantum numbers $(\vec{k}, \tau_3)$ up to the Fermi level k_{F_b} with solutions of the field equations for the baryons in which meson fields are replaced by their mean values [7]. In a system with spherical symmetry the mean value of the spatial components of the vector meson fields vanishes and the resulting equations read

$$\left[\gamma^{\mu} i\partial_{\mu} - g_{\omega}\gamma^{0}\omega_{0} - g_{\rho}\gamma^{0}\tau_{3b}\rho_{0} - (m - g_{\sigma})\right]\psi_{b} \quad = \quad 0\,, \tag{2}$$

$$m_{\sigma}^{2}\sigma \quad = \quad -g_{\sigma}\mathrm{b}m\,(g_{\sigma}\sigma)^{2} - g_{\sigma}\mathrm{c}\,(g_{\sigma}\sigma)^{3} + g_{\sigma}\sum_{b=p,n}\bar{\psi}_{b}\psi_{b}\,, \tag{3}$$

$$m_{\omega}^{2}\omega^{0} \quad = \quad g_{\omega}\sum_{b=p,n}\psi_{b}^{\dagger}\psi_{b}\,, \tag{4}$$

$$m_{\rho}^{2}\rho^{0} \quad = \quad g_{\rho}\sum_{b=p,n}\tau_{3b}\psi_{b}^{\dagger}\psi_{b}\,. \tag{5}$$

These are nonlinear coupled equations for the values of the mean fields. The model has six external parameters, namely k_{F_n}, k_{F_p}, k_{F_e}, k_{F_μ}, $k_{F_{\nu_e}}$ and $k_{F_{\nu_\mu}}$, related to the respective particle density ρ through $\rho{=}\gamma k_F^3/6\pi^2$. For the degeneracy factor, γ, we use $\gamma{=}2$ in the case of massive particles and $\gamma{=}1$ for neutrinos. We can reduce the number of independent parameters by using the constraints of β-equilibrium and, in the case of neutral matter, charge neutrality of the medium. The baryon density ρ_b, and the electronic-leptonic fraction $Y_{l_e} = (\rho_e + \rho_{\nu_e})/\rho_b$, are the independent parameters in our calculation. Although baryons and leptons are not coupled by explicit vertex in the Lagrangian, the coupling between them is taken into account through the constraint of β-equilibrium.

The weak processes considered here are

$$n + \nu_e \longleftrightarrow p + e\,, \tag{6}$$

and

$$e \longleftrightarrow \mu + \bar{\nu}_{\mu} + \nu_e\,. \tag{7}$$

For $T{=}0$ the equilibrium is established when the fermions occupy their lowest energy states up to energies to satisfy the balance equation for the chemical potentials

$$\mu_n + \mu_{\nu_e} = \mu_p + \mu_e\,, \tag{8}$$

and

$$\mu_e = \mu_{\mu} + \mu_{\bar{\nu}_{\mu}} + \mu_{\nu_e}\,. \tag{9}$$

From Eq.(8) we can see that, for $T{=}0$, the direct neutron decay $n \leftrightarrow p + e + \bar{\nu}_e$ cannot coexists with Eq.(6), which describes the relevant weak process in supernovae collapse.

The chemical potentials are given by

$$\mu_i \quad \equiv \quad \frac{\partial\varepsilon}{\partial\rho_i}\,, \qquad i = n, p, e, \mu, \nu_e, \bar{\nu}_{\mu}\,, \tag{10}$$

$$\mu_n \quad = \quad (\lambda_{\omega} - \lambda_{\rho})\,\rho_p + (\lambda_{\omega} + \lambda_{\rho})\,\rho_n + \sqrt{k_{F_n}^2 + m^{*2}}\,, \tag{11}$$

$$\mu_p \quad = \quad (\lambda_{\omega} + \lambda_{\rho})\,\rho_p + (\lambda_{\omega} - \lambda_{\rho})\,\rho_n + \sqrt{k_{F_p}^2 + m^{*2}}\,, \tag{12}$$

$$\mu_{e,\mu} \quad = \quad \sqrt{k_{F_{e,\mu}}^2 + {m_{e,\mu}}^2}\,, \tag{13}$$

$$\mu_{\nu_e,\bar{\nu}_{\mu}} \quad = \quad k_{F_{\nu_e,\bar{\nu}_{\mu}}}\,, \tag{14}$$

where

$$m^{*} \quad = \quad m - g_{\sigma}\sigma\,, \qquad \lambda_i = (g_i/m_i)^2\,, \quad i = \sigma, \omega, \rho\,, \tag{15}$$

and ε is the energy density. Charge neutrality is preserved demanding that the charged particle densities are related by

$$\rho_e + \rho_{\mu} = \rho_p\,. \tag{16}$$

In order to explore the effects of the neutrino trapping for a given total density of baryons

$$\rho_b = \rho_p + \rho_n \ , \tag{17}$$

and a fixed electronic-leptonic fraction

$$Y_{l_e} = \frac{\rho_e + \rho_{\nu_e}}{\rho_b} \equiv Y_e + Y_{\nu_e} \ , \tag{18}$$

we solve the Euler-Lagrange equations for the baryonic, mesonic and leptonic fields obtained from the Lagrangian (1) with the constraints (8), (9) and (16), by using the second set of coupling constants from Table II of Ref.[8]. We note that the muonic-leptonic fraction Y_{l_μ}=$(\rho_\mu - \rho_{\bar{\nu}_\mu})/\rho_b$ is zero from Eq.(7).

Particle populations in the case where neutrinos are absent are shown in Fig.1. Note that when the Fermi momentum of the electrons reaches the muon mass, the muon levels begin to be populated and, due to charge neutrality expressed in Eq.(16), the proton-electron degenerated curves are splitted into two different ones. The threshold for muon production occurs just below normal nuclear density. In Fig.2 we see how this threshold is displaced to higher densities, $\rho \approx 2\rho_\circ$, when neutrinos are trapped in the system for a typical Y_{l_e}=0.3 final collapse value. This is due to the fact that electronic neutrinos produced in Eq.(7) are Pauli blocked. In this case more neutrons must decay in order to preserve the value of Y_{l_e} and the matter becomes richer in protons, so the total isospin of the system is greater for the neutrino-free case (Fig.1) than for the Y_l=0.3 case (Fig.2).

The scalar field σ is much less sensitive to the values of Y_{l_e}, as we see from Fig.3. The vector field ω is proportional to the total baryonic density and it does not depend on the values of Y_{l_e}. Thus the isoscalar mesonic sector is almost insensitive to the electronic-leptonic fraction.

In order to obtain the equation of state we calculate the pressure p and the energy density ε in a standard procedure

$$p = \frac{1}{3}\langle T^{ii}\rangle \ , \qquad \varepsilon = \langle T^{00}\rangle \ ,$$

with the energy-momentum tensor $T^{\mu\nu}$ constructed from the Lagrangian (1). The sound velocity and the adiabatic index Γ are defined as

$$v_s = \left(\frac{\partial p}{\partial \varepsilon}\right)^{1/2} \ , \qquad \Gamma = \frac{\partial \log p}{\partial \log \varepsilon} \ .$$

In supernova studies to determine the sonic point of the hydrodynamical evolution of the medium it is necessary to have the sound velocity as a function of density. In Fig.4 we show the influence of the neutrino trapping on the sound velocity.

In Fig.5 we give the adiabatic index Γ as a function of the baryonic density in order to construct a polytropic-equivalent form for the equation of state ($p = k\rho^{\Gamma(\rho)}$).

In Fig. 6 we show the effect of neutrino trapping on the equation of state. When neutrinos are taken into account their contribution to the pressure stiffes the equation of state at low densities. This is due to the fact that neutrino trapping pushes the threshold for muon production to higher densities, as we discussed above. At high densities this effect is reverted.

An important element of the equation of state is the nuclear incompressibility K defined as

$$K = 9\frac{\partial p}{\partial \rho}(\rho_\circ). \tag{19}$$

This quantity gives an idea of how stiff the equation of state is at the saturation density $\rho_\circ$. In Fig.7 we show K value as a function of Y_{l_e} (solid curve), compared to the case where neutrinos are absent (dashed curve). In the neutrino trapped regime the incompressibility is a decreasing function of the electronic-leptonic fraction. We can see for the region $Y_{l_e}\geq 0.25$, which is the relevant region for the supernova problem, how the presence of neutrinos in the system lowers the value of K below the *free*-case value. The point in the solid curve corresponds to the value Y_{l_e}=0.3 ($K \approx 195$ MeV) and must be compared with the *free*-case value of K=219 MeV. Remember that both curves are obtained with the same set of coupling constants, they differ only in the number of degrees of freedom: with and without neutrinos.

In this work we investigate the consequences of neutrino trapping on the properties of nonstrange nuclear matter in a relativistic mean-field theory. The neutrino degree of freedom is taken into account in a fully consistent way. The weak interaction between the hadronic and leptonic sector is simulated via the constraint of β-equilibrium. We find that the system composition depends dramatically on the leptonic-electronic fraction Y_{l_e} values, while the isoscalar meson fields are relatively insensitive to this quantity. In the low density regime, which is the relevant region for the supernovae core bounce, the equation of state becomes stiffer when neutrinos are included. At higher densities the situation is reverted. The nuclear incompressibility is smaller in the case of neutrino trapped matter than in the case where neutrinos are absent. It is important to stress here that the so-called prompt explosion mechanism [9] needs, to get explosion events, an incompressibility smaller than those extracted from heavy ions collision studies. Neutrino trapping effects in the supranuclear regime of stellar matter may be relevant in order to ensure compatibility between both the supernova calculations and the heavy ion collision results. Hydrodynamical calculations of supernovae core bounce with neutrino-trapped matter are currently under progress.

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Figure 1. Relative particle populations as functions of baryon density (in units of the saturation nuclear density ρ_0=0.15 baryons/fm^3) for the case where neutrinos are absent.

Figure 2. Relative particle populations as functions of baryon density (in units of ρ_0) for the value of the electronic leptonic fraction Y_{l_e}=0.3, which is a typical value in supernovae collapse.

Figure 3. The effective mass of baryons in units of the bare mass as a function of baryon density (in units of ρ_0). Notice that the three curves in the figure, corresponding to Y_{l_e}=0.2, 0.3 and the neutrino-free case, almost coincide.

Figure 4. The sound velocity (in units of the speed of light) as a function of the baryon density (in units of ρ_0) for different values of Y_{l_e}, compared to the neutrino-free case.

Figure 5. The adiabatic index Γ as a function of the baryon density (in units of ρ_0) for different values of Y_{l_e}, compared to the neutrino-free case.

Figure 6. The equation of state for some values of Y_{l_e}, compared to the neutrino-free case.

Figure 7. The nuclear incompresibility K at ρ_b=ρ_0 as a function of Y_{l_e}. The curve labeled *free* corresponds to the case where neutrinos are absent. The point in the solid curve corresponds to the value Y_{l_e}=0.3.

Fig. 1

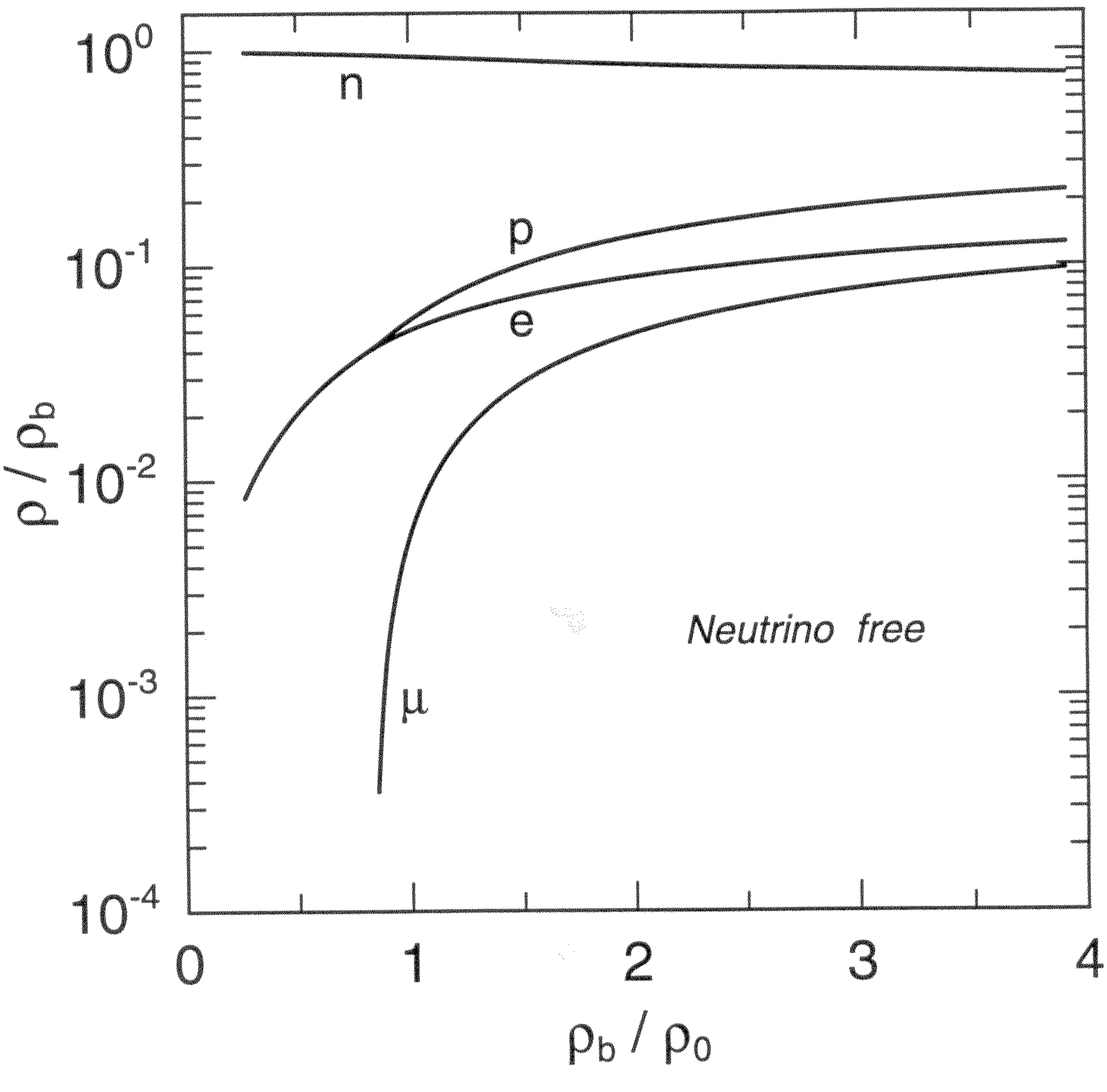

10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
ρ / ρ_b
n
p
e
μ
Neutrino free
0
1
2
3
4
ρ_b / ρ_0

Fig. 2

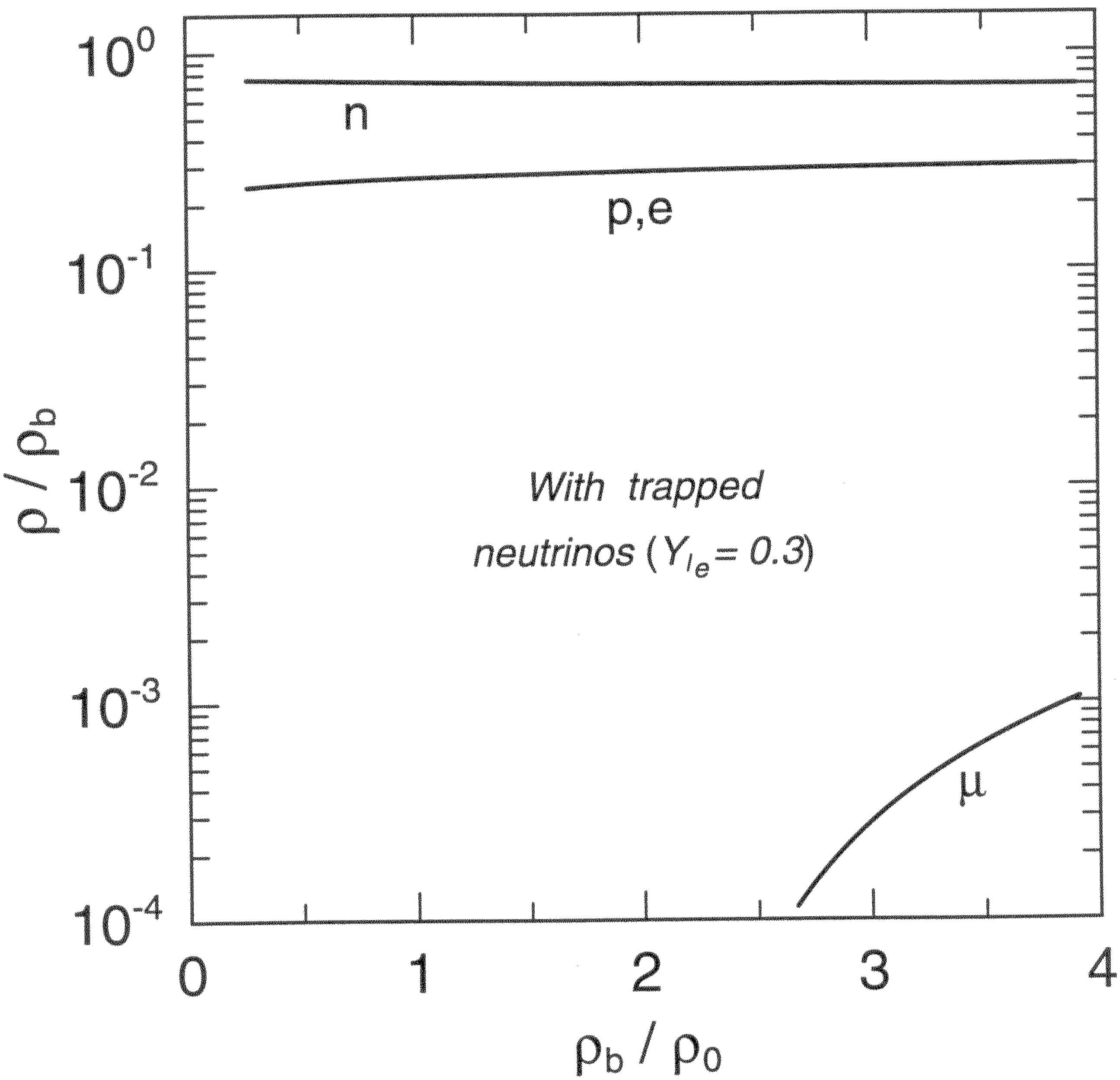

10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
ρ / ρ_b
n
p,e
With trapped
neutrinos (Y_{l_e} = 0.3)
μ
0
1
2
3
4
ρ_b / ρ_0

Fig. 3

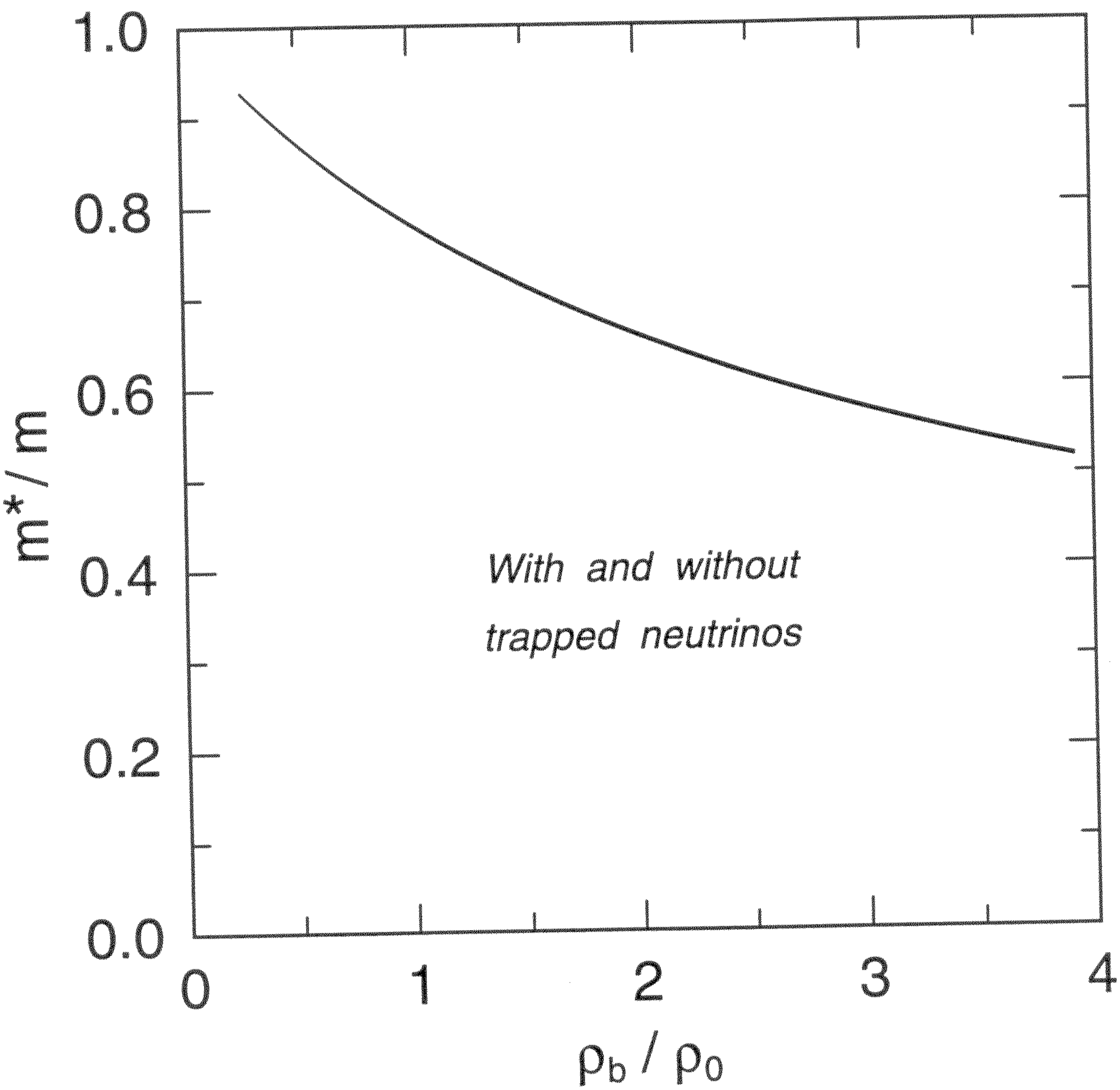

1.0
0.8
0.6
0.4
0.2
0.0
m*/ m
0
1
2
3
4
ρb / ρ0
With and without
trapped neutrinos

Fig. 4

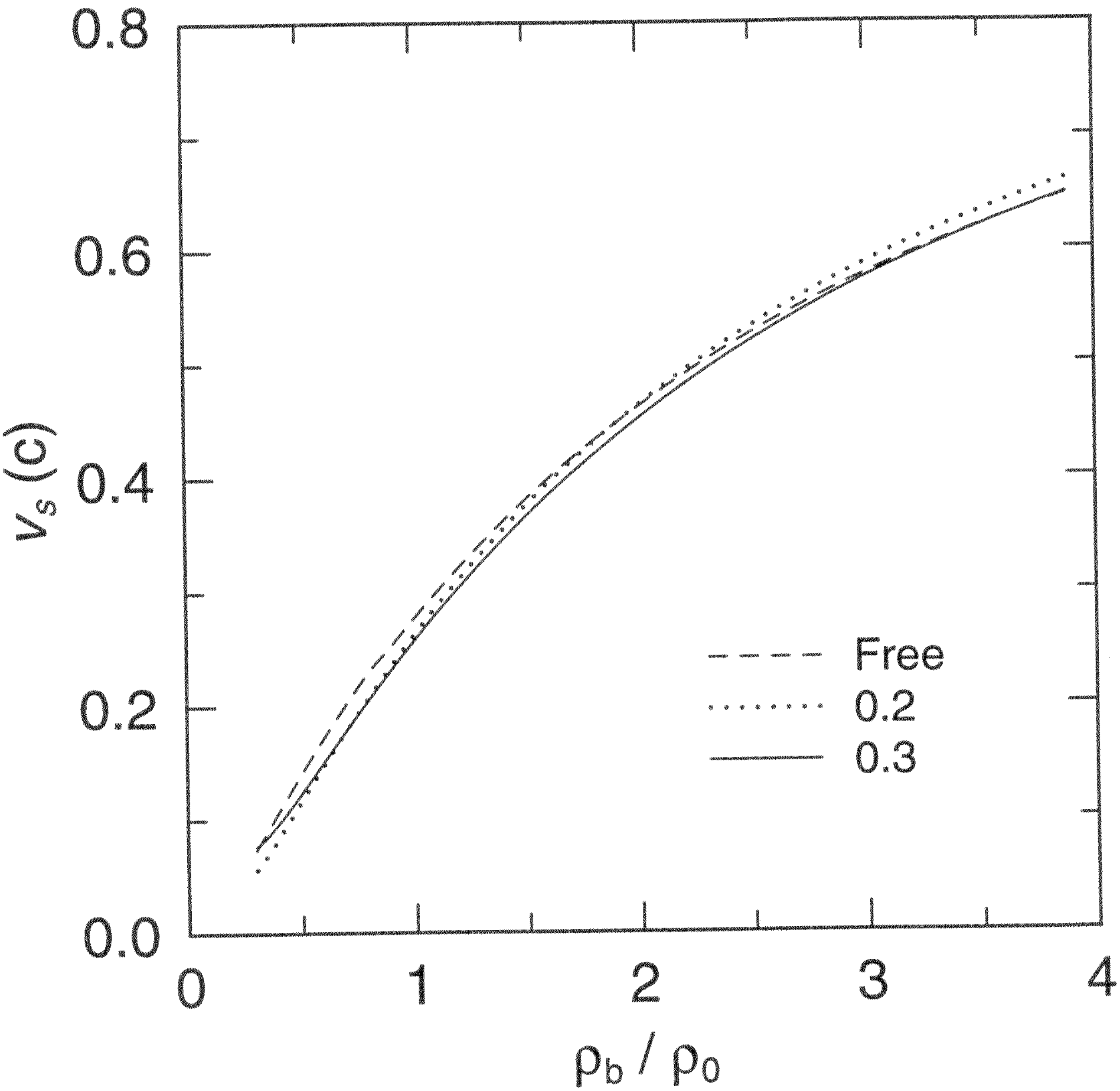


Fig. 5

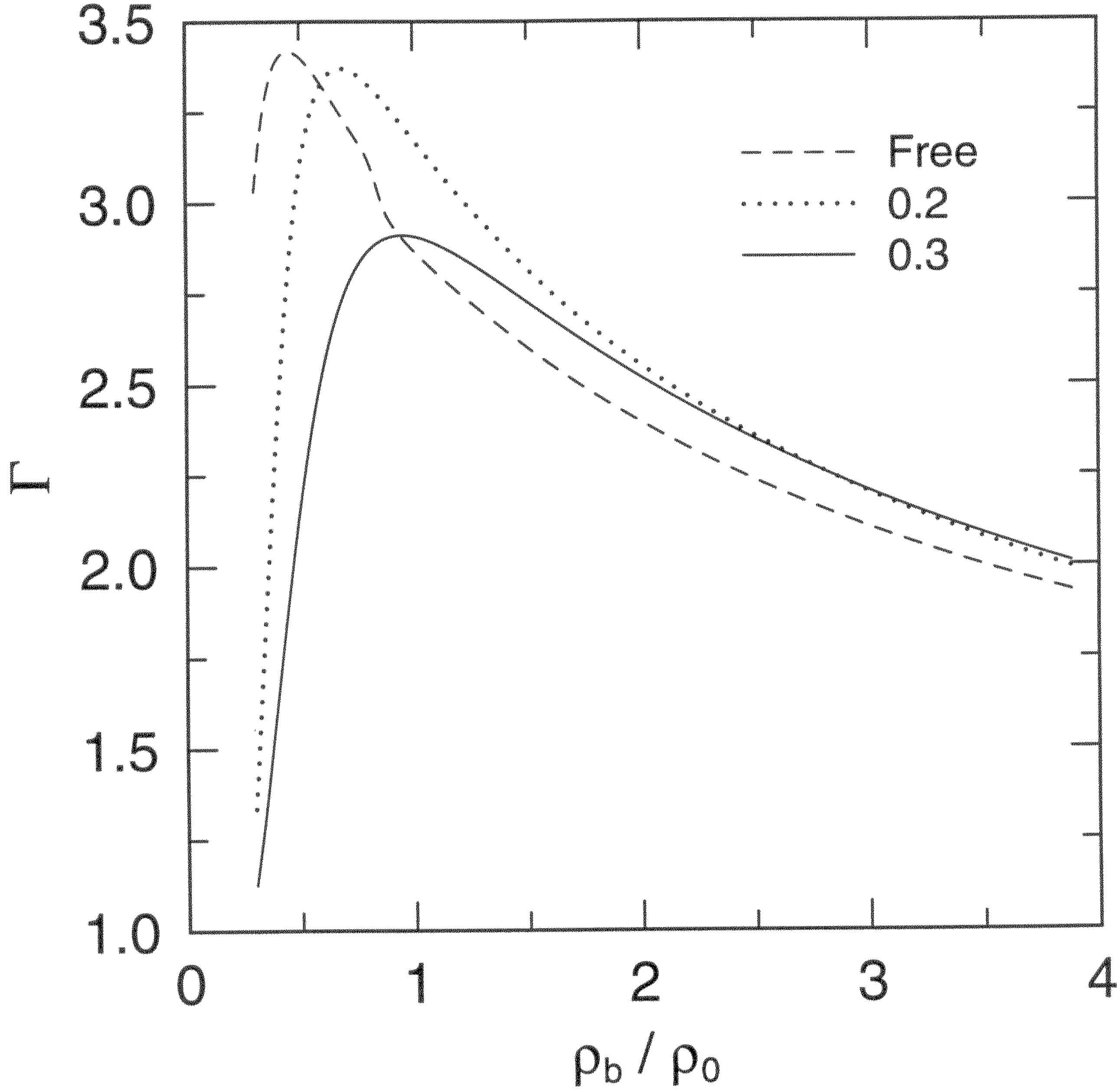


Fig. 6

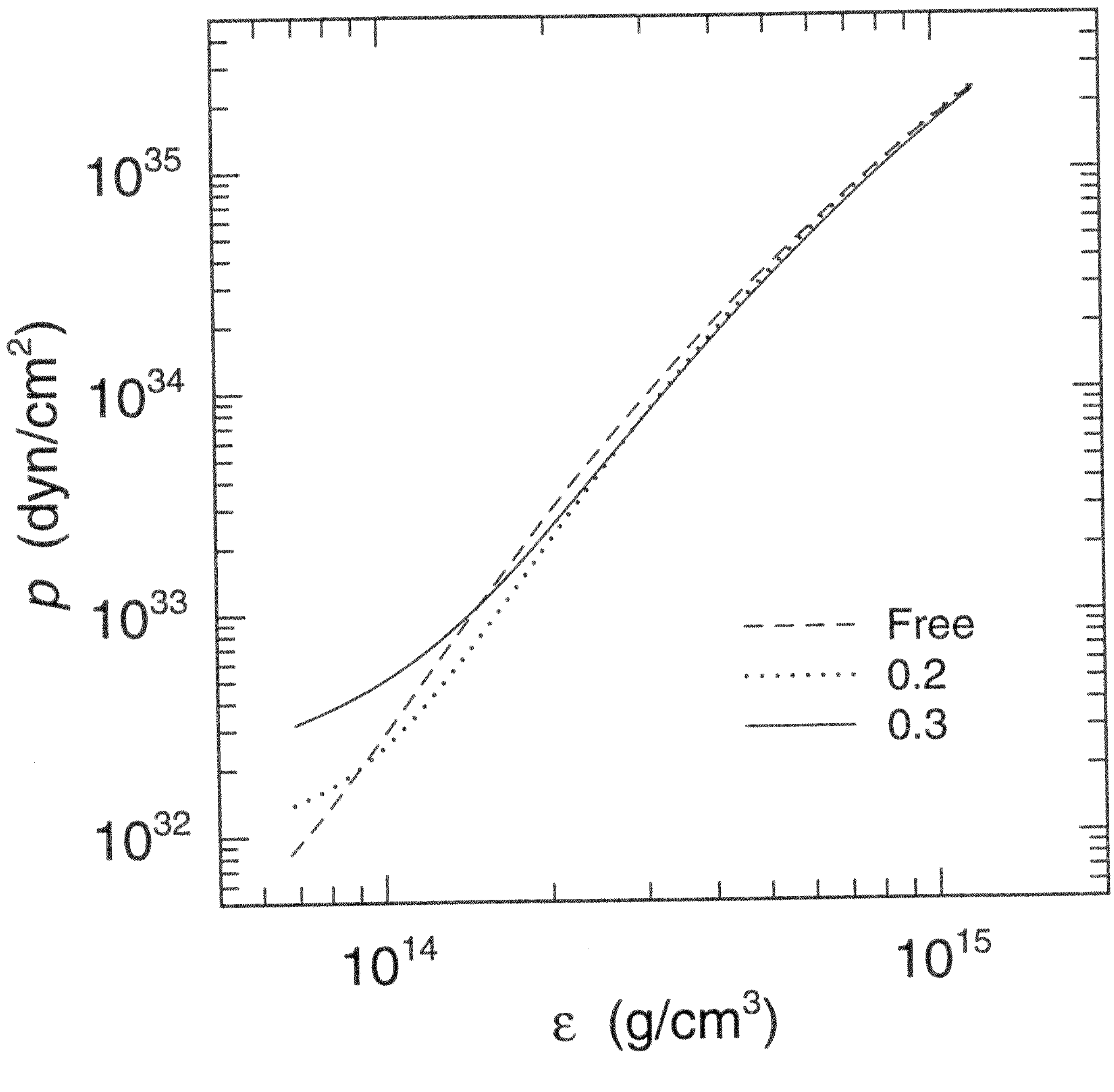
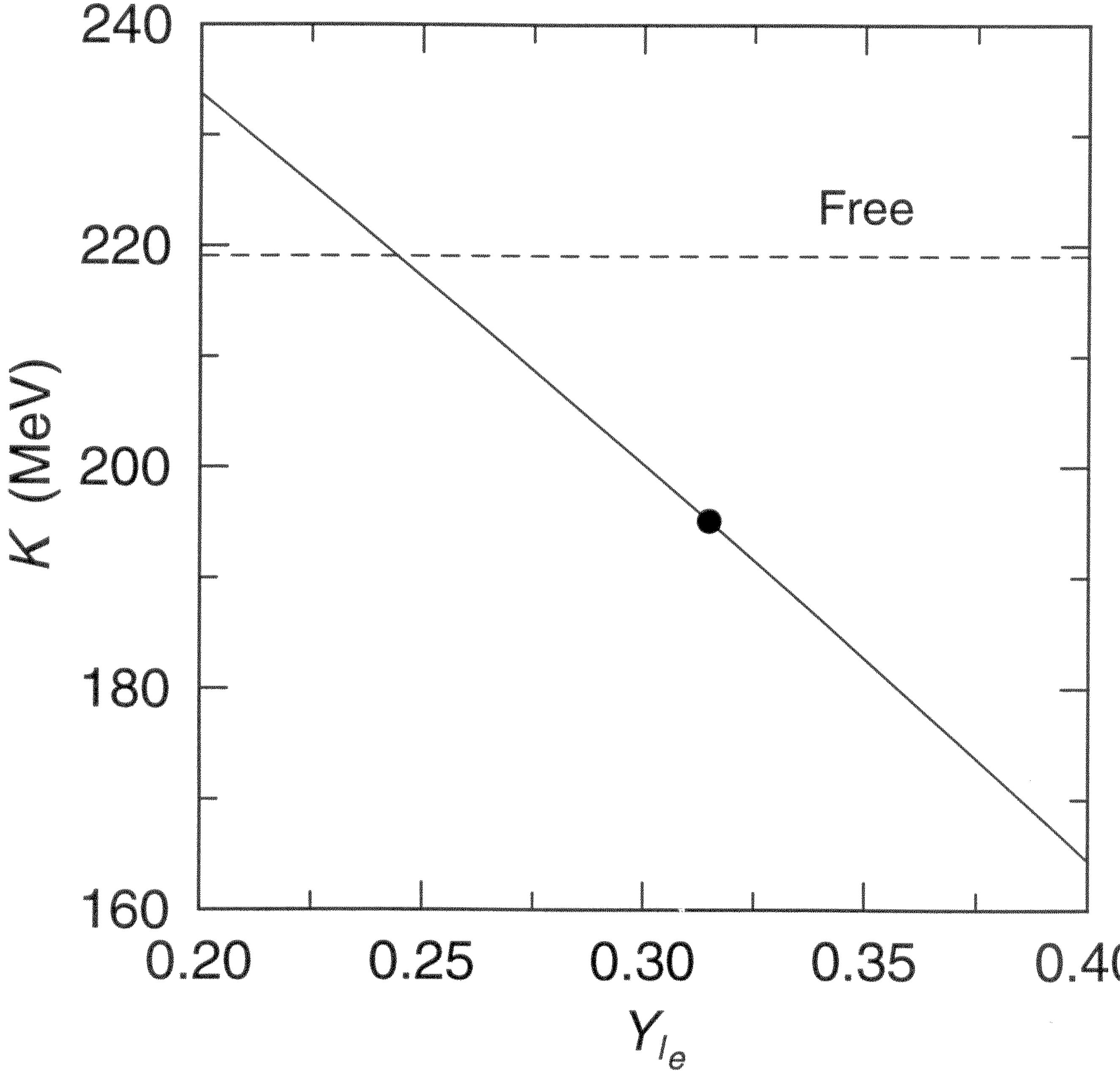

Free
0.2
0.3
10^{35}
10^{34}
10^{33}
10^{32}
p (dyn/cm^2)
10^{14}
10^{15}
ε (g/cm^3)

Fig. 7



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