

# Discrete fields, general relativity, other possible implications and experimental evidences\*

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The physical meaning, the properties and the consequences of a discrete scalar field are discussed; limits for the validity of a mathematical description of fundamental physics in terms of continuous fields are a natural outcome of discrete fields with discrete interactions. The discrete scalar field is ultimately the gravitational field of general relativity, necessarily, and there is no place for any other fundamental scalar field, in this context. Part of the paper comprehends a more generic discussion about the nature, if continuous or discrete, of fundamental interactions. There is a critical point defined by the equivalence between the two descriptions. Discrepancies between them can be observed far away from this point as a continuous-interaction is always stronger below it and weaker above it than a discrete one. It is possible that some discrete-field manifestations have already been observed in the flat rotation curves of galaxies and in the apparent anomalous acceleration of the Pioneer spacecrafts. The existence of a critical point is equivalent to the introduction of an effective-acceleration scale which may put Milgrom's MOND on a more solid physical basis. Contact is also made, on passing, with inflation in cosmological theories and with Tsallis generalized one-parameter statistics which is regarded as proper for discrete-interaction systems. The validity of Boltzmann statistics is then reduced to idealized asymptotic states which, rigorously, are reachable only after an infinite number of internal interactions. Tsallis parameter is then a measure of how close a system is from its idealized asymptotic state.

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## I. INTRODUCTION

Although it is considered that a scalar field has not been observed in nature as a fundamental field its use as such is very frequent in the modern literature, particularly in elementary particles, field theory and cosmology. Here we will apply to the scalar field the concepts and results developed in the reference [1], referred here as the paper I, where the concept of a discrete field was introduced and its wave equation and its Green's function discussed. The standard field and its formalism, which for a distinction, we always append the qualification continuous, are retrieved from an integration over the discrete-field parameters. Remarkable in the discrete field is that it has none of the problems that plague the continuous one so that the meaning and origin of these problems can be left exposed on the passage from the discrete to the continuous formalism [1]. Although the motivations for the introduction of a generic discrete field in paper I have been made on pure physical grounds of causality, a deeper discussion about its physical interpretation have been left for subsequent papers on specific fields. This discussion will be retaken here with the simplest structure of a field, the scalar one. It would be a too easy posture to see the discrete field as just an ancillary mathematical construct devoid of any physical meaning, a vision that could be re-enforced with the discrete field as a pointlike signal. The idea of a pointlike field may sound weird at a first sight but this represents the same symmetry of quantum field theory where fields and sources are equally treated as quantized fields. Here they are seen from a reversed classical perspective. Besides, pointlike object is not a novelty in physics and one of the major motivations of the string theory is of avoiding [2] infinities and acausalities in the fields produced by point sources; problems that do not exist for the discrete field, according to the reference [3].

This paper is structured in the following way. Section II, on the sake of a brief review of the mathematical definition of discrete fields, is a recipe on how to pass from a continuous to a discrete field formalism, and vice-versa. The discrete scalar field, its wave equation, its Lagrangian and its energy tensor are discussed in Section III. The paper major contribution begins in Section IV that discusses the consequences of discrete interactions for the mathematical description of the physical world. Then it gains generality as the discussions leaves the specificity of scalar interactions widening to the universality of all fundamental interactions. Calculus (integration and differentiation) which is based on the opposite idea of smoothness and continuity, has its full validity for describing dynamics restricted then to a very efficient approximation in the case of a high density of interaction points, such that the concept of acceleration as a continuous change of velocity may be introduced in an effective physical description of fundamental interactions. This seems to be an answer to the Wigner's pondering [4] about the reasons behind the unexpected effectiveness of mathematics on the physical description of the world. It is argued in Section V, after the results of the Section IV, that the scalar field must necessarily describe the gravitational interaction of general relativity whose character of a second-rank tensor is assured by the way the scalar field is attached to the definition of the metric tensor. After decoding

the physical meaning of the scalar-field sources one is led to the unavoidable conclusion that there is no place, in this context, for the existence of any other fundamental scalar field. This has deeper theoretical and observational implications, discussed in Section VI, where the possibility that consequences of discrete gravity have already been observed is considered. This would set experimental limits on the validity of general relativity as an effective field theory. Contact is made, on passing, with inflationary cosmology and with the Tsallis's statistics. The paper ends with some concluding remarks in Section VII.

## II. FROM CONTINUOUS TO DISCRETE

For a concise introduction of the discrete-field concept it is convenient to replace the Minkowski spacetime flat geometry by a conical projective one in an embedding (3+2) flat spacetime:

$$\{x \in R^4\} \Rightarrow \{x, x^5 \in R^5 | (x^5)^2 + x^2 = 0\}, \quad (1)$$

where  $x \equiv (\vec{x}, t)$  and  $x^2 \equiv \eta_{\mu\nu} x^\mu x^\nu = |\vec{x}|^2 - t^2$ . So a change  $\Delta x^5$  on the fifth coordinate, allowed by the constraint  $(\Delta x^5)^2 + (\Delta x)^2 = 0$ , is a Lorentz scalar that can be interpreted as a change  $\Delta\tau$  on the proper-time of a physical object propagating across an interval  $\Delta x$ :  $\Delta x^5 = \Delta\tau = \pm\sqrt{(\Delta t)^2 - (\Delta\vec{x})^2}$ .

The constraint

$$(\tau - \tau_0)^2 + (x - x_0)^2 = 0 \quad (2)$$

defines a double hypercone with vertex at  $(x_0, \tau_0)$ , whilst

$$(\tau - \tau_0) + f_\mu (x - x_0)^\mu = 0 \quad (3)$$

defines a family of hyperplanes tangent to the double hypercone and labelled by their normal<sup>1</sup>  $f_\mu$ , a constant four-vector. The intersection of the double hypercone with a hyperplane defines its  $f$ -generator tangent to  $f^\mu$  ( $f^\mu := \eta^{\mu\nu} f_\nu$ ). A discrete field is a field defined with support on this intersection (extended causality) in contraposition [1] to the continuous field, defined with support on a hypercone (local causality):

$$\phi_f(x, \tau) := \phi(x, \tau) \Big|_{\substack{\Delta\tau + f_\mu \Delta x^\mu = 0 \\ \Delta\tau^2 + \Delta x^2 = 0}} := \phi|_f. \quad (4)$$

The symbol  $|_f$  is a short notation for the double constraint in the middle term of Eq. (4). The constraint (4) induces the directional derivative (along the fibre  $f$ , the hypercone  $f$ -generator)

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<sup>1</sup>The Eq. (3) can be written in  $R^5$  as  $f_M \Delta x^M = 0$ ,  $M = 1, 2, 3, 4, 5$  with  $f_M = (f_\mu, 1)$

$$\nabla_\mu \phi_f(x, \tau) := (\partial_\mu - f_\mu \partial_\tau) \phi_f(x, \tau). \quad (5)$$

An action for a discrete scalar field is

$$S_f = \int d^5x \left\{ \frac{1}{2} \eta^{\mu\nu} \nabla_\mu \phi_f(x, \tau) \nabla_\nu \phi_f(x, \tau) - \phi_f(x, \tau) \rho(x, \tau) \right\}, \quad (6)$$

where  $d^5x = d^4x d\tau$ , and  $\rho(x, \tau)$  is the source for the scalar field. There can be no mass term in a discrete-field Lagrangian because it would imply on a hidden breaking of the Lorentz symmetry with non-propagating discrete solutions of the field equations. In other words no physical object could be described by such a Lagrangian with an explicit mass term. Nevertheless, as discussed in paper I, the action (6) still describes both, massive and massless fields. The mass of a massive discrete field is implicit on its propagation with a non-constant proper-time. Eq. (6) is a scale-free

action expressing the (1+1)-dynamics of a discrete field, massive or not, on a fibre  $f$ ; a mass term would break its conformal symmetry [1].

Then the field equation and the tensor energy for a discrete field are, respectively,

$$\eta^{\mu\nu} \nabla_\mu \nabla_\nu \phi_f(x, \tau) = \rho(x, \tau), \quad (7)$$

$$T_f^{\mu\nu} = \nabla^\mu \phi_f \nabla^\nu \phi_f - \frac{1}{2} \eta^{\mu\nu} \nabla^\alpha \phi_f \nabla_\alpha \phi_f. \quad (8)$$

They must be compared to the standard expressions for the continuous field:

$$(\eta^{\mu\nu} \partial_\mu \partial_\nu - m^2) \phi(x) = \rho(x), \quad (9)$$

$$T^{\mu\nu}(x) = \partial^\mu \phi \partial^\nu \phi - \frac{1}{2} \eta^{\mu\nu} \partial^\alpha \phi \partial_\alpha \phi \quad (10)$$

which can be obtained from the action

$$S = \int d^4x \left\{ \frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi_f \partial_\nu \phi_f - \frac{m^2}{2} \phi^2 - \phi(x) \rho(x) \right\}, \quad (11)$$

So, the passage from a continuous to a discrete field formalism can be summarized in the following schematic recipe (the arrows indicate replacements):

$$\begin{cases} \{x\} \Rightarrow \{x, x^5\}; \\ \phi(x) \Rightarrow \phi(x, \tau)|_f; \\ \partial_\mu \Rightarrow \nabla_\mu, \end{cases} \quad (12)$$

accompanied by a dropping of the mass term from the Lagrangian. Moreover a discrete field requires a discrete source [1]. A continuous  $\rho(x)$  is replaced by a discrete set of pointlike sources  $\rho(x, \tau)$ . Any apparent continuity is reduced to a question of scale in the observation.  $\rho(x, \tau)$  is, like  $\phi_f(x, \tau)$ , a discrete field defined on a hypercone generator too, which just for simplicity, is not being considered here. This is a symmetry between fields

and sources: they are all discrete fields, and the current density of one is the source of the other.

Reversely, in the passage from discrete to continuous, the continuous field and its field equations are recuperated in terms of effective average fields smeared over the hypercone

$$\Phi(x, \tau) = \frac{1}{2\pi} \int d^4 f \delta(f^2) \Phi_f(x, \tau). \quad (13)$$

This passage provokes the appearing of the mass term and the breaking of the conformal symmetry of the action (6). This has been explicitly proved, for both the massive and the massless fields, in the reference [1].

### III. THE DISCRETE SCALAR FIELD

Comparing the actions of Eqs. (6) and (11) one should observe that the first one contains explicit manifestations only of the constraint (3) through the use of the directional derivatives (5), but not of the constraint (2). This one is only dynamically introduced through the solutions of the field equation, like it happens also (local causality) in the standard formalism of continuous fields [6]. As a matter of fact all the information contained in the new action (6) can be incorporated in the old action (11), without its mass term, with the simple inclusion of the constraint (3)

$$S_P = \int d^4 x d\tau \delta(\Delta\tau + f \cdot \Delta x) \left\{ \frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \phi(x, \tau) \rho(x, \tau) \right\}, \quad (14)$$

as the very restriction to the hyperplane (3) by itself implies on the whole recipe (12).  $P$  in Eq. (14) stands for any generic fixed point, the local hypercone vertex:  $P = (x_0, \tau_0)$ ,  $\Delta\tau = \tau - \tau_0$  and  $\Delta x = x - x_0$ . Local causality, dynamically implemented through the field equations, imply that the field propagates on a hypercone (the lightcone, if a massless field) with vertex on  $P$ , which is an event on the world line of  $\rho(x, \tau)$ . The constraint (3) included in this action (14) further restricts the field to the fiber  $f$ , expressing an extended concept of causality [1,5].

Whereas there is no restriction on  $\rho(x)$  for a continuous field, for a discrete one, as already mentioned, it must be a discrete set of point sources. A continuously extended source would not be consistent as it would produce a continuous field. The source of a discrete scalar field is given by

$$\rho(x, t_x = t_z) = q(\tau_z) \delta^{(3)}(\vec{x} - \vec{z}(\tau_z)) \delta(\tau_x - \tau_z), \quad (15)$$

where  $z(\tau)$  is its world line parameterized by its proper time  $\tau$ ;  $q(\tau)$  is the scalar charge whose physical meaning will be made clear later. The sub-indices in  $t$  and  $\tau$  specify the respective events  $x$ ,  $y$  and  $z$ . That  $t_x$  must be equal to  $t_z$  on the left-hand side of Eq. (15) is a consequence of the deltas on its right-hand side and of the constraint (2). Initially, it

is assumed that both  $\dot{q} \equiv \frac{dq}{d\tau}$  and  $\ddot{q} \equiv \frac{d\dot{q}}{d\tau}$  exist and that they may be non null. The field eq. (7) is solved by

$$\phi_f(x, \tau) = \int d^5y G_f(x - y, \tau_x - \tau_y) \rho(y, \tau_y) \quad (16)$$

with

$$\eta^{\mu\nu} \nabla_\mu \nabla_\nu G(x, \tau) = \delta^{(5)}(x) = \delta(\tau) \delta^{(4)}(x). \quad (17)$$

The discrete Green's function associated to the Klein-Gordon operator is given [1] by

$$G_f(x, \tau) = \frac{1}{2} \theta(bf^4 t) \theta(b\tau) \delta(\tau + f \cdot x), \quad \vec{x}_T = 0, \quad (18)$$

where  $b = \pm 1$ , and  $\theta(x)$  is the Heaviside function,  $\theta(x \geq 0) = 1$  and  $\theta(x < 0) = 0$ . The labels <sub>L</sub> and <sub>T</sub> are used as an indication of, respectively, longitudinal and transversal with respect to the space part of  $f$ :  $\vec{f} \cdot \vec{x}_T = 0$  and  $x_L = \frac{f \cdot \vec{x}}{|f|}$ .

Remarkably  $G_f(x, \tau)$  does not depend on anything outside its support, the fibre  $f$ , as stressed by the append  $\vec{x}_T = 0$ . One could retroactively use this knowledge in the action (6) for rewriting it as

$$S_f = \int d^5x \delta^{(2)}(\vec{x}_T) \left\{ \frac{1}{2} \eta^{\mu\nu} \nabla_\mu \phi_f \nabla_\nu \phi_f - \phi_f(x, \tau) \rho(x, \tau) \right\}, \quad (19)$$

just for underlining that the fibre  $f$  induces a conformally invariant (1+1) theory of massive and massless fields, embedded in a (3+1) theory, as generically discussed in paper I. Actually, the factor  $\delta^{(2)}(\vec{x}_T)$  is an output of the actions (6) or (14) (it is not necessary to put it in there by hand) and it can never be incorporated as a factor in the definition (18) of  $G_f(x, \tau)$ , except under an integration sign as in Eqs. (16) and (19).

Then one could, just formally, use

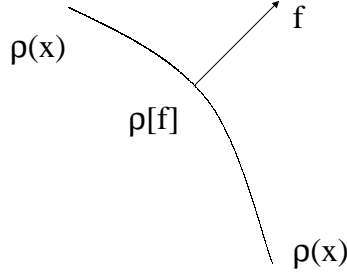
$$\rho_{[f]}(x - z, \tau_x - \tau_z) = q(\tau) \delta(\tau_x - \tau_z) \delta(t_x - t_z) \delta(x_L - z_L), \quad (20)$$

where  $\rho_{[f]}$  represents

the source density  $\rho$  stripped of its explicit  $\vec{x}_T$ -dependence, for reducing the action to

$$S_f = \int d\tau_x dt_x dx_L \left\{ \frac{1}{2} \eta^{\mu\nu} \nabla_\mu \phi_f \nabla_\nu \phi_f - \phi_f(x, \tau) \rho_{[f]}(x, \tau) \right\}, \quad (21)$$

by just omitting the irrelevant transversal coordinates. Eq. (6) then, after its output Eq. (18), is formally equivalent to Eq. (21). But we should observe that this is no more than a formal expression once  $\rho_{[f]}$  then represents just an event, the intersection of the worldline of  $\rho(x)$ , whose support is not  $f$ , with the fibre  $f$ , support of  $\phi_f(x)$ . See the Figure 1.



**Fig. 1.** The meaning of  $\rho[f]$ : the value of  $\rho(x)$  at the specific point defined by the intersection of the worldline of  $\rho(x)$ , whose support is not  $f$ , with the fibre  $f$ , support of  $\phi_f(x)$ .

The solutions from Eq. (9), with  $m = 0$ , for a point source are well known massless spherical waves propagating (forwards or backwards in time) on a lightcone in contradistinction to the solutions (18) that are, massive or massless point signals propagating always forwards in time on a straight line, a generator of the hypercone (2). Being massive or massless is determined by  $\tau$  being constant or not, as discussed in paper I. For a massive field, its mass and its timelike four velocity are hidden behind a lightlike  $f$  and a non-constant  $\tau$ ; they become explicit only after the passage from discrete to continuous fields. But as it will be made clear in Section V, there is no point on considering a massive discrete scalar field because any discrete scalar field must be associated to the gravitational field of general relativity. So massive discrete scalar fields will not be considered here any further. With  $b = +1$  and  $f^4 \geq 1$  which implies an emitted field, one has from Eqs. (18) and (15) that

$$\begin{aligned} \phi_f(x, \tau_x) &= \int d^5 y \theta(t_x - t_y) \theta(\tau_x - \tau_y) \delta[\tau_x - \tau_y + f \cdot (x - y)] q(\tau_z) \delta^4(y - z) = \\ &= \int d\tau_y \theta(t_x - t_y) \theta(\tau_x - \tau_y) \delta[\tau_x - \tau_y + f \cdot (x - y)] q(\tau_z), \end{aligned} \quad (22)$$

where an extra factor 2 accounts for a change of normalization with respect to Eq. (18) due to the exclusion of the annihilated field (which corresponds in Eq. (13) to the integration over the future lightcone). Then,

$$\phi_f(x_L, \vec{x}_T = \vec{z}_T, t_x, \tau_x = \tau_z) = \theta(t_x - t_z) \theta(\tau_x - \tau_z) q(\tau_z) \Big|_{f \cdot (x-z)=0} \quad (23)$$

or for short, just

$$\phi_f(x, \tau) = q(\tau) \theta(t) \theta(\tau) \Big|_f. \quad (24)$$

$\nabla\theta(t)$  and  $\nabla\theta(\tau)$  do not contribute [1] to  $\nabla\phi_f$ , except at  $x = z(\tau)$ , as a further consequence of the field constraints. So, for  $t > 0$  and (therefore)  $\tau \geq 0$  one can write just

$$\phi_f(x, \tau_x) = q(\tau_z)\Big|_f \quad (25)$$

$$\nabla_\nu\phi_f = -f_\nu\dot{q}\Big|_f \quad (26)$$

With Eq. (26) in Eq. (8) one has

$$T_f^{\mu\nu}(x, \tau_x) = f^\mu f^\nu \dot{q}^2\Big|_f \quad (27)$$

The field four-momentum, given by  $\int T^{\mu\nu}n_\nu d\sigma$  for a continuous field, is reduced, thanks to the field pointlike character and to its independence from the transversal coordinates, to

$$p_f^\mu = T_f^{\mu\nu}n_\nu = f^\mu \dot{q}^2\Big|_f \quad (28)$$

where  $n$  is a spacelike four vector [1] such that  $n.f = 1$ . The conservation of the energy-momentum content of  $\phi_f$  is assured then just by  $f$  being lightlike,  $f^2 = 0$ ,

$$\nabla_\mu T_f^{\mu\nu} = -2f_\mu f^\mu f^\nu \dot{q}\ddot{q}\Big|_f = 0. \quad (29)$$

It is justified naming  $\phi_f$  a discrete field because although being a field it is not null at just one space point at a time; but it is not a distribution, a Dirac delta function, as it is everywhere and always finite. Its differentiability, in the sense of having space and time derivatives, is however assured by its dependence on  $\tau$ , a known continuous spacetime function. It is indeed a new concept of field, a very peculiar one, discrete and differentiable; it is just a finite pointlike spacetime deformation projected on a null direction, with a well defined and everywhere conserved energy-momentum. It is this discreteness in a field that allows the union of wave-like and particle-like properties in a same physical object (wave-particle duality); besides this implies [13] finiteness and no spurious degree of freedom (uniqueness of solutions).

#### IV. DISCRETE PHYSICS

According to Eq. (25), the field  $\phi_f$  is given, essentially, by the charge at its retarded time, i.e. the amount of scalar charge at  $z$ , the event of its creation. It has a physical meaning, in the sense of having an energy-momentum content, when and only when  $\dot{q} \neq 0$ . So, the emission or the absorption of a scalar field is, respectively, consequence or cause of a change in the amount of scalar charge on its source. This is so because emitting or absorbing a scalar field requires a change in the state of its source which is so



poor of structure that has nothing else to change but itself, and this is fundamental for determining the scalar-charge nature. The picture becomes clearer after recalling that we are dealing with discrete field and discrete interactions which implies that the change in the state of a field source occurs at isolated events.  $q(\tau)$  is not a continuous function:

$$q(\tau) := \sum_i q_{\tau_{i+1}} \bar{\theta}(\tau_{i+1} - \tau) \bar{\theta}(\tau - \tau_i), \quad (30)$$

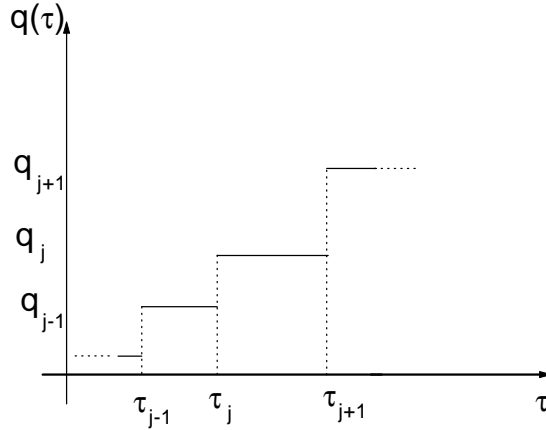
where

$$\bar{\theta}(x) = \begin{cases} 1, & \text{if } x > 0; \\ 1/2, & \text{if } x = 0; \\ 0, & \text{if } x < 0, \end{cases} \quad (31)$$

and the index  $i$  labels the interaction points on the source worldline,  $i = 1, 2, 3 \dots$ . For a given  $\tau$  only one, or at most two terms contribute to the sum in Eq. (30)

$$q(\tau) = \begin{cases} q_{\tau_j}, & \text{if } \tau_j < \tau < \tau_{j+1}; \\ \frac{q_{\tau_{j-1}} + q_{\tau_j}}{2}, & \text{if } \tau = \tau_j; \\ q_{\tau_{j-1}}, & \text{if } \tau_{j-1} < \tau < \tau_j, \end{cases} \quad (32)$$

as indicated in the graph of the Figure 2.



**Fig. 2.** Discrete changes on a discrete scalar charge along its worldline. A discrete scalar charge is so poor of structure that there is nothing else to change but itself. There is change in the state of a scalar source only at the interaction points on its worldline which is labelled by its proper time. If only the (discrete) interaction points are relevant the proper time may be treated as a discrete variable. In the limit of a worldline densely packed of interaction points a continuous graph is a good approximation.

The change in the state of the scalar source is not null only at the (discrete) interaction points and so, rigorously, it cannot be defined as a time derivative, as there is no continuous variation, just a sudden finite change. The naive use of

$$\dot{q} = q(\tau)\delta(\tau - \tau_z), \quad (33)$$

would be just an insistence on an inappropriate continuous formalism, besides artificially introducing infinities where there is none. It means that one must replace time derivatives by finite differences

$$\dot{q}(\tau) \Rightarrow \begin{cases} \Delta q_{\tau_j} & \text{if } \tau = \tau_j; \\ 0 & \text{if } \tau \neq \tau_j, \end{cases} \quad (34)$$

and a proper-time integration by a sum over the interaction points on the charge. The existence and meaning of any physical property that corresponds to a time derivative must be reconsidered at this fundamental level. Velocity ( $v$ ) exists as a piecewise smoothly continuous function (discontinuous at the interaction points). Acceleration ( $a$ ) and derivative concepts like force ( $F$ ), power ( $P$ ), etc rigorously do not exist. We must deal with finite differences, respectively, the sudden changes of velocity ( $v$ ), momentum ( $p$ ) and energy ( $E$ ):

$$\begin{cases} a \Rightarrow \Delta v \\ F \Rightarrow \Delta p \\ P \Rightarrow \Delta E \end{cases} \quad (35)$$

The observability of an interaction discreteness is in fact controlled by the ratio ( $\frac{\Delta q_j}{\Delta \tau_j}$ ) of the two parameters  $\Delta q_j$  and  $\Delta \tau_j$  shown in the Figure 2, as the validity of an approximative continuous description of fundamental interactions requires the existence of

$$\dot{q}_j = \frac{\Delta q_j \rightarrow 0}{\Delta \tau_j \rightarrow 0} \neq 0, \quad (36)$$

which is interpreted as a time derivative of  $q(\tau)$ , taken as a smooth continuous function of  $\tau$ . But actually

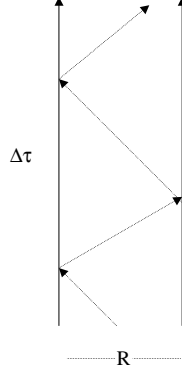
$$\Delta X_j \rightarrow 0 \quad (37)$$

has the meaning that both discrete changes,  $\Delta q_j$  and  $\Delta \tau_j$ , are smaller than their respective experimental thresholds of detectability, which, of course, is existing-technology dependent. For two-body interactions  $\Delta \tau_j$  is twice the flying time between them and is then proportional to their space separation,

$$\Delta \tau_j = \frac{2R}{c}.$$

See the Figure 3. For a large number of interacting bodies  $\Delta \tau_j$  is a statistical average time-interval between two consecutive interaction events on one body worldline. It decreases

with the number of participants, and therefore, in the case of gravitational interaction, with the masses of the macroscopic interacting bodies.

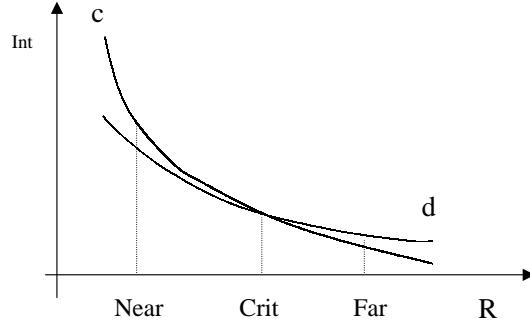


**Fig. 3.** Discrete two-body interactions.  $\Delta\tau_j$  is the time interval between two consecutive interaction events on a worldline.

$\Delta q_j$  is interaction dependent. It defines the interaction symmetry. Therefore,  $\frac{\Delta q_j}{\Delta\tau_j}$  would diverge if  $\Delta\tau_j$ , but not  $\Delta q_j$ , would satisfy Eq. (37), and it would unduly<sup>2</sup> be null in the case of only  $\Delta q_j$ , but not  $\Delta\tau_j$ , satisfying it. The interaction strength is described by the limit of  $\frac{\Delta q_j}{\Delta\tau_j}$  (as a time derivative of  $q(\tau)$ ) for a continuous interaction, and by both independent parameters  $\Delta q_j$  and  $\Delta\tau_j$  for a discrete one. For a discrete interaction the ratio  $\frac{\Delta q_j}{\Delta\tau_j}$  has no special meaning. Both results, infinity and an undue zero, evince the existence of two demarcating points, a near and a far one, signaling the inadequacy of the approximative continuous-interaction description. The two points delimit the range of the ratio-parameter  $\frac{\Delta q_j}{\Delta\tau_j}$  where there is no observationally detectable difference between a discrete and a continuous interaction. This defines the domain of validity of a continuous field as an effective physical description. A continuous field is then stronger below the near point and weaker above the far one than its corresponding discrete field. Outside the range delimited by these points a discrete-interaction description must be used. This is schematically represented in Figure 4 that superposes, with two graphs  $q \times R$ , both the continuous and the discrete descriptions of a given interaction. For the sake of simplicity, the discrete description is also represented by a smooth and continuous curve. The region delimited by the two curves and the demarcating points is, by definition, not resolved with the present technology. The two demarcating points, near and far, represent

<sup>2</sup>Because the actual interaction is not null.

the experimental resolution thresholds of the two descriptions. They are, by definition, dependent of the existing technology but there is, inside this region, a critical point of absolute equality of the two descriptions, defined by the two-curve crossing, which is technology independent. The existence of this critical point sets a scale for the interaction strength in terms of an effective time derivative of  $q(\tau)$ . The discrete field formalism, we remind, being conformally symmetric [1], is scale free.



**Fig. 4.** Two descriptions for a same interaction: continuous (cc) and discrete (dd). For convenience the discrete one is approximated by a smoothly continuous curve. The near and the far demarcating points delimit the thresholds of existing technology for resolving the two curves. The critical point, defined by the two-curve crossing is technologically independent and represents a fundamental scale for the interaction intensity in terms of an effective derivative of  $q(\tau)$ .

The two curves are just, respectively discrete and continuous, representations of a given generic interaction. We are interested on their asymptotic regions where, in principle, discrepancies between them can be detected. An interaction where  $\Delta\tau_j$  but not  $\Delta q_j$  goes to zero with the distance  $R$ , diverges in the continuous description as  $\frac{\Delta q_j}{\Delta\tau_j}$  goes to infinity whereas it remains finite in the discrete one. In the discrete description the interaction is always finite, no matter how strong. It has been discussed in [3,5,7] for both the gravity and the electromagnetic field. The inconsistencies of the continuous fields, made explicit through divergences and causality violations, disappear with the discreteness, with the existence of a non null lapse of time between two consecutive interaction points, or in other words, with the recognition that each interaction point is an isolated event.

In the far asymptote, for an interaction with

$$\Delta q_j \geq \text{const} > 0, \quad (38)$$

$\dot{q}$  in the continuous description goes to zero as  $R$  (and therefore  $\Delta\tau_j$ ) goes to infinity whereas the discrete one tends to a finite and constant value. It just becomes more and more intermittent but not necessarily goes to zero.

At very large distances where  $\Delta\tau_j$  becomes detectable the field asymptotic limit should reveal its discrete nature. Actually this possibility is spoiled, in the case of a matter-polarizing field like the electromagnetic one, by the shielding effect: The field is canceled before  $\Delta\tau_j$  grows to the point of detectability. This, of course, does not happen to gravity and so effects of this expected discreteness must be observed but this discussion will be deferred to Section VI.

Careful observation at both small and large distances for these cases should reveal that the strength of the actual interaction ( $\Delta q_j$ ), respectively, grows and decreases at a smaller rate than the theoretical prediction from a continuous interaction. When observed, in a context of continuous interactions, these effects may require the use of regularization and renormalization techniques or may give origin to various misleading interpretations like the existence of new forms of fundamental continuous interactions or of strange and yet to be observed form of matter, for example. Calculus (integration and differentiation) in a discrete-interaction context becomes useless for a rigorous description of fundamental physical processes. But in practice such a detailed strictly discrete calculus is not always necessary and in some cases may not even be feasible. What effectively counts is the scale determined by  $\Delta\tau_j$ , the time interval between two consecutive interaction events, face the accuracy of the measuring apparatus. The question is if  $\Delta\tau_j$  is large enough to be detectable, or how accurate is the measuring apparatus used to detect it. The density of interaction points on the world line of a given point charge is proportional to the number of point charges with which it interacts. Let one consider the most favorable case of a system made of just two point charges. As the argument is supposedly valid for all fundamental interactions one can take the hydrogen atom in its ground state for consideration, treating the proton as if it were also a fundamental point particle. The order of scale of  $\Delta\tau_j$  for an electron in the ground state of a hydrogen atom is given then by the Bohr radius divided by the speed of light

$$\Delta\tau_j \sim 10^{-18}s$$

which corresponds to a number of  $\frac{\pi}{\alpha} \sim 400$  interactions per period ( $\alpha$  is the fine-structure constant) or  $\sim 10^{10}$  interactions/cm. So, the electron worldline is so densely packed with interaction events that one can, in an effectively good description for most of the cases, replace the graph of the Figure 2 by a continuously smooth curve. The validity of calculus in physics is then fully reestablished in the interval between the two demarcating points as a consequence of the limitations of the measuring apparatus. The Wigner's questions [4] about the unexpected effectiveness of mathematics in the physical description of the world is recalled. The answer lies on the huge number of point sources in interaction (a sufficient condition), the large value of the speed of light and the small (in a manly scale) size of atomic and subatomic systems, which indirectly is a consequence of  $h$ , the Planck constant.

Even in these situations where  $\Delta\tau_j$  may not be measurable, at least with the present technology, the discrete formalism is justified not for replacing the continuous one where

it is best, which is confirmed by high precision experiments [23,20] but mostly for defining and understanding its limitations. There are, besides this very generic justification, many instances of one-interaction-event phenomena, like the Compton effect, particle decay, radiation emission from bound-state systems, etc, where discrete interactions are the natural and the more appropriate approach. These are, of course, all examples of quantum phenomena, but primarily because quantum here implies discreteness.

### A. Discrete-continuous transition

It would be interesting to have a framework where this change from continuous to discrete interaction and vice-versa could be formally realized in a simple and direct way. One can deal with them considering the behaviour under a derivative operator of  $\bar{\theta}(\tau)$  which is the mathematical description of the interaction discreteness. Then one must require that, symbolically

$$\frac{\partial}{\partial \tau} \bar{\theta}(\tau - \tau_i) := \delta_{\tau \tau_i}, \quad (39)$$

with  $\delta_{\tau \tau_i}$  the Kronecker delta

$$\delta_{\tau \tau_i} = \begin{cases} 1, & \text{if } \tau = \tau_i; \\ 0, & \text{if } \tau \neq \tau_i, \end{cases} \quad (40)$$

with the meaning that at the points where the left-hand side of Eq. (39) is not null, which are the only relevant ones,  $\tau$  must be treated as a discrete variable and that the operator  $\frac{\partial}{\partial \tau}$  must be seen as (or replaced by) just a sudden increment  $\Delta$  and not as the limit of the quotient of two increments.

Then with such a convention one has from Eq. (30) that

$$\nabla_\nu q(\tau) \Big|_f = -f_\nu \sum_i q_{\tau_i} \{ \bar{\theta}(\tau_{i+1} - \tau) \delta_{\tau \tau_i} - \delta_{\tau_{i+1} \tau} \bar{\theta}(\tau - \tau_i) \} := -f_\nu \dot{q}(\tau), \quad (41)$$

which implies that  $\dot{q}(\tau)$  is null when  $z(\tau)$  is not a point of interaction on the charge world line. For such an interaction point  $\tau_j$  one has

$$\dot{q}(\tau_j) = q_{\tau_j} \bar{\theta}(\tau_{j+1} - \tau_j) - q_{\tau_{j-1}} \bar{\theta}(\tau_j - \tau_{j-1}) = q_{\tau_j} - q_{\tau_{j-1}} \quad (42)$$

or, generically

$$\dot{q}(\tau) = \begin{cases} \Delta q_i = q_{\tau_i} - q_{\tau_{i-1}} & \text{for } \tau = \tau_i; \\ 0 & \text{for } \tau \neq \tau_i, \end{cases} \quad (43)$$

and, from the middle term of Eq. (41)

$$\nabla_{\sigma}^f \nabla_{\nu}^f q(\tau) = -2f_{\sigma} f_{\nu} \sum_i q_{\tau_i} \delta_{\tau\tau_i} \delta_{\tau_{i+1}\tau} = 0. \quad (44)$$

In Eq. (41)  $i$  labels the vertices and only these points on the world line contribute. That is why one has to define Eq. (39). In a limit where a summation over  $i$  may be approximated by a time integration the Kronecker delta may be replaced by a Dirac delta function and then one may have Eq. (33) as a good operational approximation to Eq. (43).

Therefore we understand Eqs.(25,26) as meaning, respectively

$$\phi_f(x) = q(\tau)\Big|_f = \begin{cases} q_{\tau_{j+1}}\Big|_f & \text{if } \tau_j < \tau_{ret} < \tau_{j+1} \\ \frac{q_{\tau_{j+1}} + q_{\tau_j}}{2}\Big|_f & \text{if } \tau_{ret} = \tau_j \end{cases} \quad (45)$$

and

$$\nabla_{\mu} \phi_f(x) = -f_{\mu} \Delta q(\tau)\Big|_f = \begin{cases} -f_{\mu} (q_{\tau_{j+1}} - q_{\tau_j})\Big|_f & \text{if } \tau_{ret} = \tau_j \\ 0 & \text{if } \tau_{ret} \neq \tau_j \end{cases} \quad (46)$$

The field  $\phi_f(x, \tau)$  is just like an instantaneous picture of its source at its retarded time; a travelling picture. If  $z(\tau_{ret})$  is not a point of change in the source's state,  $\phi_f(x)$  is not endowed with a physical meaning as its energy tensor is null. A physical discrete field always corresponds to a sudden change in its source's state at its retarded time. If there is no change the field is not real, in the sense of having zero energy and zero momentum. Having no physical attribute it corresponds to a pure "gauge field" of the continuous formalism.

## V. SCALAR FIELD AND GENERAL RELATIVITY

It takes an external agent to cause a change  $\Delta q$  on the charge  $q$  of a scalar source; a positive  $\Delta q$  means that a scalar field  $\phi_f(x, \tau)$  has been, say, absorbed whereas a negative one means then an emission. Therefore, a discrete scalar field carries itself a charge  $\Delta q$  and can, consequently, interact with other charge carriers and be a source or a sink for other discrete scalar fields. It carries a bit of its very source, a scalar charge; it is an abelian charged field. On the other hand a new look at equations (28) and (43) reveals that  $(\Delta q_j)^2$  describes the energy-momentum content of the field. So, the source of a discrete scalar field is any physical object endowed with energy which corresponds then to the scalar charge. Energy, of course, is a component of a four-vector and not a Lorentz scalar. Its four-vector character comes from the  $f^{\mu}$  factor in Eq. (28): the energy of  $\phi_f(x, \tau)$  is the fourth component of the current of its squared scalar charge. The scalar charge conservation is therefore assured by and reduced to the conservation of energy and momentum given by Eq. (29). Considering the relativistic mass-energy relation this

implies that the discrete scalar field satisfies the Principle of Equivalence and that all physical objects interact with the scalar field through its energy-tensor. This is a form of the Principle of Universality of gravitational interaction, introduced by Moshinski [25]. So,  $\phi_f(x, \tau)$  must necessarily be connected to the gravitational field. Having necessarily energy for source implies on an important consequence of uniqueness, of excluding the existence of any other distinct fundamental discrete scalar field as it must necessarily be taken as the gravitational field<sup>3</sup>. Moreover, as energy is not a scalar, the symmetry between discrete fields and sources, both taken as fundamental fields, implies also that there should be no fundamental scalar source representing an elementary field; it must be a scalar function of a non-scalar fundamental field, like the trace of an energy tensor, for example. This lets then explicit a known symmetry of nature: the four fundamental interactions are described by gauge fields having vector currents for sources ( $j = qv$ , as they are pointlike sources), including gravity since the energy tensor is just a current of its charge, the four-vector momentum. So, this symmetry is not broken with gravity being a second-rank tensor field.

This possible physical interpretation is compatible with the General Theory of Relativity, according to the work done in the references [7,26], where a discrete gravitational field defined by

$$g_{\mu\nu}^f(x) = \eta_{\mu\nu} - \chi f_\mu f_\nu \phi_f(x, \tau), \quad (47)$$

as a point deformation in a Minkowski spacetime, propagating on a null direction  $f$ , upon an integration on  $f$ , in the sense of Eq. (26), reproduces the standard continuous solutions. That gravity be either totally [8] or partially [19,20] described by a scalar (continuous) field is an old idea [10–12], but Eq. (47) implies on regarding gravity as being ultimately described by a discrete scalar field in a metric theory. With the metric in this form the Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \chi T_{\mu\nu} \quad (48)$$

is reduced [7] to

$$f_\mu f_\nu \eta^{\alpha\beta} \nabla_\alpha \nabla_\beta \phi_f(x, \tau) = \chi T_{\mu\nu}, \quad (49)$$

as the gauge condition used in [7]

$$f^\mu \nabla_\mu \phi_f(x, \tau) = 0 \quad (50)$$

becomes an identity after Eq. (26), as  $f^2 = 0$ .

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<sup>3</sup>There would be no point on assuming that a same charge could be the source of two or more distinct fields with the same characteristics



Inherent to discrete fields, irrespective of their tensor or spinor character, is the implicit conservation of their sources as a consequence of their (discrete fields) very definition<sup>4</sup>. This is discussed in Section V of paper I. So, whereas  $T^{\mu\nu}{}_{;\mu} = 0$  is assured by the symmetry of the Einstein tensor on the left-hand side of Eq. (48), in Eq. (49) it is just a consequence (see Eq.(29)) of Eq. (26). This symmetry of the Einstein tensor is in this way similar to the one of the Maxwell tensor that assures charge conservation in the standard continuous-field formalism but that is a consequence of extended causality (discrete-field definition) and Lorentz symmetry [13] in a discrete-field approach.

The Eq. (47) reminds an old derivation [24] of the field equations of general relativity by consistent re-iteration of

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \chi h(x)_{\mu\nu}, \quad (51)$$

as solution from a gauge invariant wave equation for the field  $g_{\mu\nu}(x)$  in a Minkowski spacetime. The non-linearity of the Einstein's equations comes from contribution to  $g_{\mu\nu}(x)$  from all terms of higher orders in  $h_{\mu\nu}$ . Therefore, the results obtained in the reference [7] imply that if  $h_{\mu\nu}$  is ultimately a discrete scalar field

$$h_{\mu\nu} = f_{\mu} f_{\nu} \phi_f(x, \tau),$$

there is no higher order contribution essentially because  $f^2 = 0$ . A discrete field has no self-interaction, a consequence of its definition (4) and that is explicitly exhibited in its Green's function (18). Discrete fields are solutions from linear equations. Whereas this is true for  $g_{\mu\nu}^f$  of Eq. (47) it is not for its  $f$ -averaged  $g_{\mu\nu}$  of Eq. (51). The non-linearity of general relativity appears here then as a consequence of the averaging process of Eq. (13) that effectively smears the discrete field over the

lightcone, erasing all the information contained in  $f$ . The interested reader is addressed to the references [7] and [26].

On the other hand the energy tensor in Eq. (49) must be traceless, also a consequence of  $f^2 = 0$ . This reminds an old known problem in standard field theory that comes when a massless theory is taken as the ( $m \rightarrow 0$ )–limit of a massive-field theory [15–19], but for a discrete field, in contradistinction, a traceless tensor does not necessarily mean a massless source [1]. The wave equation (49) must be preceded by some careful qualifications, however. A discrete field requires a discrete source. The source in Eq. (49) must be treated as a discrete set of point sources  $T_{\mu\nu}^f(x, \tau)$  for which  $f^{\mu} T_{\mu\nu}^f(x, \tau) = 0$ . This implies that there is no exterior solution for a discrete gravitational field, only vacuum solutions. Any interior continuous solution must be seen then as an approximation for a densely packed set of point sources. From the discrete vacuum solution of Eq. (49) one can, in principle, with an integration over its  $f$ -parameters, obtain any continuous vacuum

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<sup>4</sup>Schematically:  $j^{\mu} = qv^{\mu} \Rightarrow \nabla_{\mu} j^{\mu} = -qa^{\mu} f_{\mu} \equiv 0$  as  $a \cdot f \equiv 0$ , according to Eq. (22) of [1].

solution of an imposed chosen symmetry<sup>5</sup> [7]. This justifies, up to a certain point, not regarding the right-hand side of Eq. (47) as just the first two terms of a series of possible contributions from higher rank tensors. Even for a massive point-source, however, being itself a discrete field,  $T_{\mu\nu}^f$  cannot be expressed in terms of its mass and of its actual four-velocity  $v$ . A traceless  $T_{\mu\nu}^f(x, \tau)$  with  $f^\mu T_{\mu\nu}^f(x, \tau) = 0$  does not necessarily represent a massless source nor  $f$  represents its four-velocity, as discussed in Section V of paper I.

The geometrical description of gravity as the curvature of a pseudo-Riemannian space-time has its validity, in the range of the ratio parameter (36) limited by the two demarcating points of the Figure 4, always assured as an absolutely good approximation due to the high density of interaction points in any real measurement, as discussed in the previous section.

## VI. THEORY AND OBSERVATIONS: POSSIBLE LINKS

In this section we want to make some brief comments on some possible theoretical and observational evidences of direct consequences of interaction discreteness, particularly in gravity. The comparison between the discrete and the continuous description of an interaction leads to the existence of the critical point and of experimental thresholds (near and far) for resolving the interactions, as shown in the Figure 4. The interior segment, between these two values, defines the domain of validity of the continuous-interaction approximation where the polygonal worldline of the sources are so densely packed of interaction points that they can be effectively replaced by smoothly continuous curves and the concept of acceleration and of spacetime curvature at a point on the worldline make sense. We are not proposing, it is worth emphasizing, the replacement of general relativity in its domain of validity by a discrete scalar field theory of gravity, and similar statements should be assumed for other field theories. The point is that in this domain, i.e. for  $\frac{\Delta q_j}{\Delta \tau_j}$  between the two demarcating points, it cannot make, by definition, any experimentally detectable difference. Considering the small strength of its coupling the gravitational interaction is irrelevant for physical systems involving relatively few fundamental elements. Even a gravitational Aharanov-Bohm-like experiment [21] would require the gravitational field of a macroscopically large object, like the Earth. The sufficient condition for a high density of interaction points is assured and justifies continuous descriptions of gravity, of which general relativity seems to be the best proposal [20]. Moreover the undetectability of discrete gravity in this region is tantamount to the unobservability of the Minkowski spacetime. At this level of approximation the Minkowski spacetime becomes the local tangent space of an effective curved space-time and  $f$  a generator of the local hypercone in its tangent space. This would lead to full general relativity in accordance to a general

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<sup>5</sup>From the superposition of the discrete fields of a spherical distribution of massless dust one retrieves the Vaydia metric [22].

uniqueness result [9,27] that any metric theory with field equations linear in second derivatives of the metric, without higher-order derivatives in the field equations, satisfying the Newtonian limit for weak fields and without any prior geometry must be exactly Einstein gravity itself. This reminds again the already mentioned [24] derivation of general relativity from flat spacetime but now with the distinctive aspect that the effective Riemannian spacetime comes not from a consistency requirement but as an approximation validated by the limitation of our experimental capacity, which can always be improved, be placed on more stringent limits, but never be totally eliminated.

On the other hand, outside this region, i.e. below or above the thresholds, the discrepancies between a discrete and a continuous interaction cannot be overlooked. This casts doubts on the results about asymptotic fields and their singularities of any continuous-field theory. By the way, considering that the discrete field is weaker than the continuous one in the origin neighborhoods we can suggest or expect that the discrete field may give an explanation to inflation or at least alleviate its need in cosmological theories.

### A. Discrete Newtonian potentials

The evolution of a system through a sequence of  $n$  discrete interactions is described by a series involving combinatorials of  $n$ , i.e.  $n(n-1)(n-2)\dots$ . This is a natural consequence of discrete interactions: power series replacing continuous functions obtained from integrations of differential equations. The evolution of any system is given in terms of power series. A continuous interaction, irrespective of its duration, would always be equivalent to an infinite  $n$ . This is the meaning of a conservative potential and this is why a continuous interaction invariably has problems with infinities. Just for the sake of illustrating this very important point let us, anticipating some results<sup>6</sup>, consider the much simpler case of a radial motion with a non-relativistic axially symmetric interaction (a logarithmic effective potential, an effective inversely-proportional-to-the-distance acceleration). This symmetry implies that the change in speed at each interaction is a (very small) constant  $\Delta$ . For initial conditions taken, right after an interaction event, as

$$r(t_0) = r_0;$$

$$v(t_0) = v_0,$$

the next interaction will occur at

$$t_1 = t_0 + \Delta t_0 = t_0 + \alpha r_0,$$

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<sup>6</sup>This will be presented with details elsewhere. Its anticipation here is just for the sake of illuminating the arguments.

where  $\alpha$  is also a very small constant, and

$$v(t_1) = v_1 = v_0 - \Delta;$$

$$r(t_1) = r_0 + v_0 \alpha r_0 = (1 + \alpha v_0) r_0,$$

as there is free propagation between any two consecutive interactions. Therefore, for the  $n^{\text{th}}$  interaction

$$r_n = r_{n-1} + v_{n-1} \Delta t_{n-1} = (1 + \alpha v_{n-1}) r_{n-1} = r_0 \prod_{i=0}^{n-1} (1 + \alpha v_i), \quad (52)$$

with

$$v_i = v_0 - i \Delta. \quad (53)$$

Then, from Eq. (52),

$$\frac{r_n}{r_0} = 1 + \alpha \sum_{i_1=0}^{n-1} v_{i_1} + \alpha^2 \sum_{i_1=0}^{n-1} \sum_{i_2=i_1+1}^{n-1} v_{i_1} v_{i_2} \dots + \alpha^{n-1} \left( \sum_{i_1=0}^{n-1} \sum_{i_2=i_1+1}^{n-1} \dots \sum_{i_{n-1}=i_{n-2}+1}^{n-1} \right) v_{i_1} v_{i_2} \dots v_{i_{n-1}}, \quad (54)$$

a finite series that with the use of Eq. (53) exhibits the following structure

$$\begin{aligned} \frac{r_n}{r_0} = & 1 + \alpha \left( \binom{n}{1} v_0 - \binom{n}{2} \Delta \right) + \alpha^2 \left[ v_0^2 \binom{n}{2} - v_0 \Delta \left( -3 \binom{n}{3} + 2 \binom{n}{2} \binom{n}{1} - 2 \binom{n}{2} \right) \right. \\ & \left. + \Delta^2 \left( -3 \binom{n}{4} + \binom{n}{2} \binom{n}{2} - 4 \binom{n}{3} - \binom{n}{2} \right) \right] + \mathcal{O}(\alpha^3). \end{aligned} \quad (55)$$

If  $n \gg 1$ , by considering just the largest contribution from each term in this finite series we have

$$\frac{r_n}{r_0} = 1 + \alpha n \left( v_0 - \frac{n \Delta}{2} \right) + \frac{\alpha^2 n^2}{2} \left( v_0 - \frac{n \Delta}{2} \right)^2 + \mathcal{O}(\alpha^3), \quad (56)$$

or

$$\frac{r_n}{r_0} = \sum_{k=0}^{n-1} \frac{1}{k!} \left[ \alpha n \left( v_0 - \frac{n \Delta}{2} \right) \right]^k. \quad (57)$$

From Eq. (53), we have

$$n = \frac{v_0 - v_n}{\Delta},$$

which in Eq. (57) produces

$$\frac{r_n}{r_0} = \sum_{k=0}^{n-1} \frac{1}{k!} \left[ \frac{\alpha (v_0^2 - v_n^2)}{\Delta} \right]^k. \quad (58)$$

The bigger is  $n$  the better this finite series can be approximated by an exponential

$$\frac{r_n}{r_0} \approx \exp\left(\frac{\alpha (v_0^2 - v_n^2)}{\Delta}\right), \quad (59)$$

which can be re-written as

$$\frac{v_0^2}{2} + \frac{\Delta}{\alpha} \ln r_0 \approx \frac{v_n^2}{2} + \frac{\Delta}{\alpha} \ln r_n = \text{const.} \quad (60)$$

This is energy conservation with an effective potential energy given by

$$U(r) = \frac{\Delta}{\alpha} \ln r. \quad (61)$$

Then, for the gravitational interaction we identify the constants as

$$GM = \frac{\Delta}{\alpha},$$

where  $M$  is the central mass. An infinite  $n$  would make the right-hand side of Eq. (61) to be an exact expression (in the corresponding classical, non-relativistic limit) for the effective potential energy but as  $n$  may at most be a huge but finite number this represents just the sum of the largest contribution from each term in this series. In other words, the right-hand side of Eq. (61) is just an effective expression with a large but limited domain of validity due to neglecting the smaller terms in the combinatorials. So, remarkable here is not only the appearing of the Newtonian potential as an effective field but also its asymptotic character: Energy is conserved at each interaction but the exact mathematical expression of the potential energy is given by Eq.(61) only after an infinite number of interactions.

## B. The essential question

The essential question that is posed now is which is the true nature of fundamental interactions: Continuous or discrete? This must be an experimentally based decision but there are some arguments favoring<sup>7</sup> the discrete case:

Continuous interactions are plagued by infinities and causality problems. They are inherent to the continuous hypothesis. The discrete interaction is free of them and can profitably reproduce the entire continuous formalism in terms of effective continuous interactions. The continuous case is contained in the discrete one. The immediate profits are the many ad hoc features of continuous fields but that are natural consequences of either a discrete field or from the discrete-to-continuous passage. The following subsection considers further implications of interaction discreteness.

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<sup>7</sup>The reasons have been detailed on the references [1,3,5,7,13,14,26]. Parts of the old ones may have been superseded by the more recent ones.

### C. Boltzmann and Tsallis Statistics

With discrete interactions we, rigorously, do not have differential equations nor integrations. The evolution of any system is done through sudden and discrete finite differences that are just superimposed. Between two consecutive interaction points every point like component just moves freely on straight lines. All exact physical statements are expressed as finite power series involving those combinatorials. This is a general statement in the sense that any physical system, even a macroscopic one, composed by an immense number of point like fundamental elements has its states, its conservation laws, its evolution, its statistical distributions, etc. described in terms of power functions. This is so because there are no exact smoothly continuous solutions but segments of straight lines or as an idealized limit which should be attainable only after an infinite number of steps. An infinite number does not exist, and infinity is just an idealized concept of a limit, of an unreachable boundary. Being so, the world is surprisingly simpler and our standard vision of it is richer of such idealized, unreachable concepts than we had previously conceded. A whole paraphernalia of mathematical tools, so useful in physics - differential equations, integrations, differential geometry, topology, just for citing a few - and so many familiar and daily used mathematical functions like sine, exponentials, harmonic and coulombian potentials, circles, ellipses, etc., etc., do not belong to the realm of the physical world; they are just unreachable, idealized limiting boundaries as much as an ideal gas and a macroscopic reversible process.

This supports the generalized one-parameter power function definition of entropy introduced in 1988 by Tsallis [35], which provides a power-law distribution of probabilities. The number of its application to the most diverse systems has, since then, steadily and rapidly increased [36]. It is reduced to Boltzmann statistics when its parameter is equal to unity. This parameter is then a measure of how close the system is from its idealized asymptotic state, that rigorously, is reachable only after an infinite number of interactions. It is a proper statistics for a world made of discretely interacting point like objects. The Boltzmann statistics, as any mathematical formulation for physics, based on continuous interactions, is displaced, according to this viewpoint, to these idealized boundaries. But, of course, an immense  $n$ , in most cases, is an excellent approximation to infinity. The extensive applicability of Tsallis statistics on the most diverse real systems may be an indication of the true nature of the world, if continuous or absolutely discrete.

### D. Possible experimental evidences

On the observational side we note that for the asymptotic region above the critical point the continuous asymptotically null fields are replaced by discrete interactions that become more and more intermittent with the distance, but do not necessarily go to zero. This may be detectable for the gravitational field as it does not have shielding effects although it requires huge masses for detecting very weak gravitational fields and huge

distances for producing a detectable  $\Delta\tau_j$ ; both conditions found at and above galactic scales. Therefore, a right place for checking for signs of discreteness may be the rotational dynamics of galaxies which is essentially given by

$$\frac{GMm}{R^2} = \frac{mv^2}{R}, \quad (62)$$

so that the orbital velocities of galaxies would be expected to be inversely proportional to the square root of the radial distance from the central mass. But both sides of this equation are heavily dependent on the assumption of a continuous interaction. The Newtonian field is a consequence, in a discrete interaction context, of a large frequency of interaction points and, therefore, of a small  $\Delta\tau_j$ . This is explicitly shown in [1,7,14]. The centripetal force is an expression of inertia in a circular motion but for discrete interaction the circle is replaced by a polygon as the body freely moves on a straight line between two consecutive interaction events. Let us consider a polygon circumscribed on a circle of radius  $R$ . Then

$$\frac{v}{c} = \frac{\Delta x}{2R} \sim \frac{2\pi R}{n2R} = \frac{\pi}{n}, \quad (63)$$

where  $c$  is the speed of light,  $n$  is the (enormous) number of interaction events (the number of vertices) that, may depend on  $v$ , but not on  $R$ . Then the orbital velocity becomes independent<sup>8</sup> of  $R$  after the critical point.

So, flat rotation curve is something very natural in a discrete-field context! It is therefore a real possibility that the critical point for gravity has already been detected in the flat rotation curves of galaxies [28]. The flatness feature of a rotation curve of a galaxy, as remarked by Milgrom [29], is determined not by its central mass  $M$  alone nor just by the distance  $R$  but by the acceleration which is equivalent to the ratio-parameter (36) as  $\Delta q_j$  for gravity corresponds to a change of speed. Therefore the existence of the critical point in the continuous/discrete physical description justifies the introduction of a new fundamental scale for the interaction strength in terms of an effective acceleration. This may put Milgrom's MOND [29] on a more sound physical basis. The actually prevailing wisdom that a flat rotation curves is the (ad hoc) indication of some strange, ubiquitous but still to be detected cold dark matter is not free of problems and is far from being unanimous [29–32,34,33].

Another possible evidence of discrepancy that must be considered is the apparent anomalous, weak, long-range acceleration observed in the Pioneer 10/11, Galileo, and Ulysses data [35]. Due to their spin-stabilization and to the great distance (30 to 67 AU) from the Sun the spacecrafts are excellent for dynamical astronomy studies as they permit precise acceleration estimation to the level of  $10^{-10} \text{ cm/s}^2$ . The detected anomalous acceleration comes from the second largest contribution from those mentioned  $n$ -combinatorials.

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<sup>8</sup>Another way of seeing it is that both  $\Delta x_j$  and  $\Delta\tau_j$  are proportional to  $R$ .

Eq. (52) is, of course, not valid for circular motion, and so there is no second largest contributions and, therefore, no Pioneer effect on planetary orbits [36]. Both cases, the rotation curves and the spacecraft dynamics, in the context of discrete interactions, will be discussed with details elsewhere.

## VII. CONCLUSIONS

The thesis that fundamental interactions are discrete is being developed. If this is the case there is no really compelling reason for excluding gravity from such a unifying idea. The knowledge of a supposedly true discrete character of all fundamental interactions is a permanent reminder of the limits of a continuous approximate description. The idea of an essential continuity of any physical interaction allows unlimited speculations that will always go beyond any level of possible experimental verifications which brings then the risk of not being able of distinguishing the reign of possibly experimentally-grounded scientific

research from plain philosophical speculation or even just fiction. Regardless the possibility that some of its consequences have already been experimentally detected, a discrete gravitational interaction, even in the range where it is not experimentally detectable, still for a long time to come, may just make sense of existing theories for delimiting their domain of validity as it has historically happened with all new discreteness introduced in the past, like the ideas of molecules, atomic transitions, and quarks, for example.

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