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Gravitational Particle Production in Spinning Cosmic String Spacetimes

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Abstract

The spontaneous loss of angular momentum of a spinning cosmic string due to particle emission is discussed. The rate of particle production between two assymptotic spacetimes: the spinning cosmic string spacetime in the infinite past and a non-spinning cosmic string spacetime in the infinite future is calculated.

Key-words: Spinning cosmic string; Particle emission.

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There is a broad class of predictions of various models in grand unified theories (GUT's). Between them, phase transitions may result in the formation of cosmic strings [1]. They are extended one dimensional infinite topological defects. The cosmic strings affect the spacetime mainly topologically, given a conical structure to the space region around the cone of the string. The conical topology may be responsible for several gravitational effects, e.g., gravitational lense [2] and particle production due to the changing gravitational field during the formation of such object [3]. There are a lot of papers studying quantum processes in a cosmic string spacetime. Of special interest for us are the following: Ref. [4] where pair production in a straight cosmic string spacetime is discussed, Ref. [5] where the rate of transition of a two-level system coupled with a scalar field in the presence of a cosmic string is analysed, and finally Ref. [6] where spinning cosmic string spacetime is investigated.

In this paper we are interested in evaluate the particle production due to the changing of the gravitational field in the situation of gradual loss of angular momentum of a spinning cosmic string. We set $\hbar = k_B = c = 1$.

Let us consider the following metric structure for the spacetime in the exterior of a straight cosmic string:

$$\mathrm{d}s^2 = \{\mathrm{d}t + \zeta(t)\mathrm{d}\varphi\}^2 - \mathrm{d}r^2 - b^2r^2\mathrm{d}\varphi^2 - \mathrm{d}z^2, \tag{1}$$

where $x^\mu = \{t, r, \varphi, z\}$ are the usual cylindrical coordinates with the range: $(0 \leq r < \infty,\ -\infty < z < \infty,\ 0 \leq \varphi \leq 2\pi)$. The constant b is called the conical parameter and is related with the deficit angle of the conical singularity by

$$b = 1 - 4\mu G, \tag{2}$$

where μ is the linear density of the string. The function $\zeta(t)$ appearing in Eq. (1) is defined by

$$\zeta(t) \equiv 2GJ\left[1 - \tanh\left(\frac{t}{t_0}\right)\right], \tag{3}$$

and the others constants are the gravitational Newton constant (G) and the angular momentum (J) of the source.

From relation (3) we can stablish the following asymptotics conditions for the metric structure:

$$\lim_{t\to+\infty}\zeta(t) \;=\; 4GJ, \tag{4}$$

$$\lim_{t\to-\infty}\zeta(t) \;=\; 0. \tag{5}$$

In the two asymptotic regions — the infinite past and infinite future — the spacetime metric structure reduces to:

$$\mathrm{d}s^2_{-\infty} \;=\; (\mathrm{d}t+4GJ\mathrm{d}\varphi)^2-\mathrm{d}r^2-b^2r^2\mathrm{d}\varphi^2-\mathrm{d}z^2, \tag{6}$$

$$\mathrm{d}s^2_{+\infty} \;=\; \mathrm{d}t^2-\mathrm{d}r^2-b^2r^2\mathrm{d}\varphi^2-\mathrm{d}z^2. \tag{7}$$

As it is well know, the above metrics represent the structure of the spacetime in the exterior region of a rotating and a non-rotating cosmic string[1], respectively.

With this picture in mind we will analyse the rate of particle produced by changing the gravitational field between the two asymptotic spacetimes. The same idea it was used in a toy model by Bernard and Duncan [7]. These authors studied a two dimensional Robertson-Walker model where the conformal scale factor has the same functional form as Eq. (3). In the two asymptotics limits, the spacetime becomes Minkowskian, and these authors were able to obtain the modes solutions of the Klein-Gordon equation in these two limits. A straighforward calculation of the Bogoliubov coefficients between the **in** and **out** modes gives the rate of particle production during the expansion of the universe.

In this paper we will develop a similar idea. The mathematical treatment will follows the same lines a Bernard and Duncan paper. We consider the case of a massive minimally coupled Hermitian scalar field $\phi(t,\vec{x})$ defined at all points of the 4-dimensional spacetime with line element given by eq.(1). The Klein-Gordon equation is given by:

$$\left[g^{\mu\nu}D_\mu D_\nu+M^2\right]\phi(t,\vec{x})=0, \tag{8}$$

where the symbol D_α represents the covariant derivative with respect to the metric $g_{\alpha\beta}$, and M is the mass of the quanta of the scalar field. For further reference we point out that the determinant of the metric $g_{\alpha\beta}$ for the general spacetime given by eq.(1) is:

$$\det[g_{\mu\nu}]\equiv g=-b^2r^2. \tag{9}$$

Here we are interested in studying the process of creation of particles and radiation by the gravitational field changing during the evolution of a cosmic string that looses angular

[1] By transforming the azimutal coordinate $\theta=b\varphi$ the line element reduces to

$$\mathrm{d}s^2=\mathrm{d}t^2-\mathrm{d}r^2-r^2\mathrm{d}\theta^2-\mathrm{d}z^2,$$

where the azimutal angle is defined in the interval $0\le\theta\le 2\pi b$.

momentum during the time. To mantain the particle produced in a limited region of the space we impose Dirichlet boundary conditions at $r = R$,

$$\phi(t, r, \varphi, z)|_{r=R} = 0, \tag{10}$$

and periodic boundary conditions in z with period L.

It is well know that the geometry given by Eq. (6) generate closed time-like curves (CTC). In order to circumvent this problem, we impose an additional vanishing boundary condition at $r = R_0 > 4GJ/b$. Thus, the radial coordinate has the domain $R_0 < r < R$. For a carefull study how to construct quantum field theory in a spacetime with CTC, see for instance Ref.[8]. The same problem appear in $(2+1)$ dimensional gravity since the spinning cosmic string spacetime is exactly the solution of $(2+1)$ Einstein equations of a spinning point source [9]. Deser, Jackiw and 't Hooft derived the solution to the $D = 3$ Einstein gravity with a massless spining source. The generalization for massive spining sources was obtained by Clement [10]. As we point out the solution show that in both cases the three dimensional geometry is the Minkowski space with a edge removed. In this case a non-trivial physical situation arrises. The points that we have to identify across the deleted edge differ in the time coordinate by an amount proportional to the angular momentum of the source.

In the asymptotic past — that corresponds a rotating cosmic string spacetime — the Klein-Gordon equation given by Eq. (6) reduces to the form:

$$\left[\left(1 - \frac{a^2}{b^2 r^2}\right)\frac{\partial^2}{\partial t^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) - \frac{1}{b^2 r^2}\frac{\partial^2}{\partial \varphi^2} - \frac{\partial^2}{\partial z^2} + \frac{2a}{b^2 r^2}\frac{\partial^2}{\partial t \partial \varphi} + M^2\right]\phi(t, r, \varphi, z) = 0. \tag{11}$$

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It is not difficult to find the mode solutions u_j and they are given by:

$$u_j(t, \vec{x}) = N_1 e^{-i\omega_l t} e^{ikz} e^{im\varphi} J_\mu(qr), \tag{12}$$

with

$$\mu \equiv \frac{|m + 4GJ\omega_l|}{b}, \tag{13}$$

$$q = \sqrt{\omega_l^2 - k^2 - M^2} \tag{14}$$

and

$$k = \frac{2\pi n}{L}. \tag{15}$$

We choose the constant N_1 to make the set orthonormal. Thus,

$$N_1 = (2\omega_l)^{-\frac{1}{2}} \left\{ V \left[J'_\mu(qR) \right]^2 - V_0 \left[J'_\mu(qR_0) \right]^2 \right\}^{-\frac{1}{2}}, \tag{16}$$

where we defined the 3-volumes $V \equiv b\pi LR^2$ and $V_0 \equiv b\pi LR_0^2$. The values of ω_l are determined by the vanishing boundary conditions.

The modes $u_j(t, \vec{x})$ form a basis in the space of solutions of the Klein-Gordon equation and can be used to expand the field operator in the following way:

$$\phi_{in}(\vec{x}, t) = \sum_j \left\{ \mathbf{a}_j u_j(t, \vec{x}) + \mathbf{a}_j^\dagger u_j^*(t, \vec{x}) \right\}, \tag{17}$$

where we are using a collective index $j \equiv \{l, m, n\}$.

The creation and anihilation operators $\mathbf{a}_j^\dagger$ and $\mathbf{a}_j$ satisfies the usual comutation relations:

$$[\mathbf{a}_j, \mathbf{a}_j^\dagger] = \delta_{j,j'}, \tag{18}$$

and the in-vacuum state is defined by

$$\mathbf{a}_j |0, in >= 0 \quad \forall\, j. \tag{19}$$

We can follow the same lines to canonical quantize the field in the infinite future. The Klein-Gordon equation in the non-rotating cosmic string spacetime given by eq.(7) reads

$$\left[\frac{\partial^2}{\partial t^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} \right) - \frac{1}{b^2 r^2}\frac{\partial^2}{\partial \varphi^2} - \frac{\partial^2}{\partial z^2} + M^2 \right] \phi(t, r, \varphi, z) = 0. \tag{20}$$

The out modes solutions of the Klein-Gordon equation also form a complete set and can be used to expand the field operator as:

$$\phi_{out}(t, \vec{x}) = \sum_j \left\{ \mathbf{b}_j v_j(\vec{x}, t) + \mathbf{b}_j^\dagger v_j^*(t, \vec{x}) \right\}, \tag{21}$$

where the modes v_j is defined by:

$$v_j(t, \vec{x}) = N_2 e^{-i\Omega_l t} e^{ikz} e^{im\varphi} J_\nu(\bar{q} r), \tag{22}$$

with

$$\nu \;\equiv\; \frac{|m|}{b}, \tag{23}$$

$$\bar{q} \;=\; \sqrt{\Omega_l^2 - k^2 - M^2}. \tag{24}$$

Choosing the constant N_2 in order to make the set of modes $\{v_j, v_j^*\}$ orthonormal, results:

$$N_2 = (2\Omega_l)^{-\frac{1}{2}} \left\{ V \left[J'_\nu(qR) \right]^2 - V_0 \left[J'_\nu(qR_0) \right]^2 \right\}^{-\frac{1}{2}}. \tag{25}$$

Similarly the creation and anihilation operators $\mathbf{b}_j^\dagger$ and $\mathbf{b}_j$ satisfies the usual comutation relation:

$$[\mathbf{b}_j, \mathbf{b}_j^\dagger] = \delta_{j,j'}, \tag{26}$$

and the vacuum state in the out-spacetime, is defined by

$$\mathbf{b}_j|0, out >= 0 \;\; \forall\; j. \tag{27}$$

Following Parker we will calculate the rate of particle production between two asymptotic spacetimes discussed above: the spinning cosmic string spacetime in the infinite past and a unspinning cosmic string spacetime in the infinite future.

A important point is that in our model we have not to due with the problems of junction conditions since there is no sudden approximation here. The metric evolves continuously between both asymptotics states. The angular momentum of the spinning cosmic string is lost by particle emission processes. The fundamental quantity we have to calculate is the Bogoliubov coefficients between the modes in the non-rotating and rotating cosmic string spacetime. The average number of in-particles in the modes (l, m, n) produced by this process is given by:

$$< in, 0|b_j^\dagger b_j|0, in >= \sum_i |\beta_{ij}|^2 . \tag{28}$$

Using the definition of the Bogoliubov coefficients β_{ij} given by

$$\beta_{jj'} = -(u_j, v_{j'}^\dagger) \tag{29}$$

we have

$$\beta_{jj'} = -2\pi b L(\Omega_l + \omega_{l'}) N_1 N_2 \xi(R, R_0) \delta_{m,m'} \delta_{n,n'} \tag{30}$$

where

$$\xi(R, R_0) \equiv \int_{R_0}^{R} \mathrm{d}r\, r\, J_\mu(qr)\, J_\nu(\bar{q}r). \tag{31}$$

Substituting (30) in Eq. (28) and use the definitions of the normalization constants N_1, N_2, the average number of particles in the modes (l, m, n) produced is:

$$\begin{aligned} < in, 0|b_j^\dagger b_j|0, in > &= \pi b^2 L^2 \left[\left(\frac{\Omega_l}{\omega_{l'}} \right)^{\frac{1}{2}} + [\left(\frac{\omega_l}{\Omega_{l'}} \right)^{\frac{1}{2}} \right]^2 \\ &\times \left\{ \left[V J_\mu'(qR)^2 + V_0 J_\mu'(qR_0)^2 \right] \left[V J_\nu'(\bar{q}R)^2 + V_0 J_\nu'(\bar{q}R_0)^2 \right] \right\}^{-1} \end{aligned} \quad (32)$$

Let us sumarize the results obtained in the paper. We discussed particle production by lost of angular momentum in a spinning cosmic string spacetime. To circumvent the problem of CTC's we assume a cosmic string with a radius fixed. Moreover, to avoid the problem of square of distribution we following Parker's arguments impossing vanishing boundary conditions a cylinder with finite radius R.

A possible continuation of this paper is to formulate the energy conservation law, that is to show if there is a balance between the total energy of the particles creation and the energy associated with loss of angular momentum. This can be done comparing the vacuum stress-tensor of the massive field in the spinning and non-spinning cosmic string spacetime. The calculation for a massless conformally coupled scalar field in the non-spinning cosmic string spacetime has been done by many authors [11]. The same calculation in the spinning cosmic string spacetime has been done by Matsas [12]. As far as we know the renormalized stress tensor of a massive minimally coupled scalar field has not been investigated in the literature. The evaluation of such quantite is fundamental to investigate the model taken into acount the back reation problem. Note that the source of Einstein's equations that generates the line element in the non asymptotic limit in unknown. Actually, the particle production must be included in the energy momentum tensor of the source and together with the matter energy momentum tensor satisfy the semi-classical Eintein's equations. The calculation of the renormalized energy momentum tensor of the massive scalar field and the corresponding energy balance in under investigation.

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References

[1] T.W. Kibble, J.Phys. **A 9**, 1387 (1976); A. D. Linde, Rep.Prog.Phys. **42**, 390 (1979).

[2] A. Vilenkin, Phys.Rep. **121**, 263 (1985).

[3] L. Parker, Phys.Rev.Lett. **59**, 1369 (1987); G. Mendell and W.A. Hiscock, Phys.Rev. **D 40**, 282 (1989).

[4] D. Harari and V.D. Skarzhinsky, Phys.Lett **B 240**, 322 (1990); J. Audretsch and A. Economou, Phys.Rev. **D 44**, 980 (1991); J.Audretsch, A.Economou and D.Tsoubelis, Phys.Rev.D **45**, 1803 (1992).

[5] P.C.W. Davies and V. Sahini, Class.Q.Grav. **5**, 1 (1988); B.F. Svaiter and N.F. Svaiter, Class.Q.Grav. **11**, 347 (1994); L. Iliadakis, U. Jasper and J. Audretsch, Phys.Rev. **D 51**, 2591 (1995).

[6] P.O. Mazur, Phys.Rev.Lett. **57**, 929 (1986); D. Harari and A.P. Polychronakos, Phys.Rev. **D 38**, 3320 (1988), B.Jensen and H. H.Soleng, Phys.Rev.D **45**, 3528 (1992), C.Doran, A.Lasenby and S.Gull, Phys.Rev.D **54**, 6021 (1996).

[7] C. Bernard and A. Duncan, Ann.Phys. **107**, 201 (1977).

[8] D. G. Boulware, Phys.Rev.D, **46**, 4421 (1992).

[9] S. Deser, R. Jackiw and G. T'Hoft Annals of Phys. **152**, 220 (1984); ibid **153**, 405 (1984); J.R. Gott III and M. Alpert, Gen.Rel.Grav. **16**, 243 (1984); S. Giddings, J. Abhott and K. Kuchar, Gen.Rel.Grav. **16**, 751 (1984).

[10] G.Clement, Int.Jour.Theor.Phys. **24**, 267 (1985).

[11] T.M.Helliwell and D.A.Konkowski, Phys.Rev.D **34**, 1918 (1986), B.Linet, Phys.Rev.D **35**, 536 (1987), W.A.Hiscock, Phys. Lett.B **188**, 317 (1987), J.S.Dowker, Phys.Rev.D **36**, 3095 (1987), P.C.W.Davies and V.Sahni, Class.Q.Grav. **5**, 1 (1988).

[12] G.E.A.Matsas. Phys.Rev.D **42**, 2927 (1990).