Gravitational Space Experiments Near Libration Points on Board a Satellite

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Abstract

A new method of determining the gravitational constant G, possible equivalence principle violation (measured by the Eotvos parameter η) and the hypothetic fifth force parameters (α and Λ) on board a drag-free Earth's satellite is suggested: to follow the motion of a light body ("particle") near its libration point (L_1 or L_2) situated near a ball of about $500 kg$ mass. The particle trajectories are studied analytically up to second order deflections from the libration points. Estimations show that the present-day uncertainties in G and η, and also α for $\Lambda \sim 1\ m$ can be reduced by at least two orders of magnitude.

Key-words: Space experiment; Measurement of G.

1 Introduction

Constants in any physical theory characterize the stability properties of matter. With the progress of science some theories substitute others, so new constants appear, some relations between them are being established. So, the number of fundamental constants is changing. At present such choice seems more preferable: c, $\hbar$, e, m_e, the Weinberg angle Θ_w, Cabbibo angle Θ_c, QCD cut-off parameter Λ_{QCD}, G, Hubble constant H, mean density of the Universe, cosmological constant Λ. They may be classified as universal, as constants of interactions, and as constants of elementary constituences of matter[1].

The knowledge of constants has not only a fundamental meaning but also the metrological one. Modern system of standards is based mainly on stable physical phenomena. So the stability of constants plays a crucial role. As all physical laws were established and checked during last 2-3 centuries in experiments on the Earth and its surroundings, i.e. at a rather short space and time intervals in comparison with the radius and age of the Universe the possibility of slow variations of constants cannot be excluded a priori.

The problem of the gravitational constant G stability is a part of a very much developing field, called gravitational-relativistic metrology. It appeared due to the growth of a measuring technique precision, spread of measurements over large scales and tendency to the unification of fundamental physical interaction (see[7]).

Absolute value measurements of G. There are several laboratory determinations of G with precisions of 10^{-3} and only 4 at the level of 10^{-4}. They are (in $10^{-11} \cdot m^3 kg^{-1} s^{-2}$);
1. Facy, Pontikis, 1972 – 6,6714± 0,0006
2. Sagitov et al., 1979 – 6,6745± 0,0008
3. Luther, Towler, 1982 – 6,6726± 0,0005
4. Karagioz, 1988 – 6,6731± 0,0004

From this table it is seen that the first three contradict each other (they do not overlap within their accuracies). So the fourth experiment is in accordance with the third.

The official CODATA value of 1986

$$G = (6,67259 \pm 0,00085) \cdot 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$$

is based on the Luther and Towler determination. One should make a conclusion that he problem is still open and we need further experiments on the absolute value of G. Many groups are preparing them using different types of technique, among them are Karagioz (Russia).

There exist also some satellite determinations of G (namely $G \cdot M_{earth}$) at the level of 10^{-8} and several geophysical determinations in mines. The last give much higher G values than the laboratory ones.

The precise knowledge of G is necessary for the evaluation of mass of the Earth, planets, their mean density and in the end for the construction of Earth models; transition from mechanical to electromagnetic units and back; evaluation of other constant through relations between them given by unified theories; finding new possible types of interactions and geophysical effects.

Among the fundamental physical constants the gravitational constant G is known with the least accuracy: according to CODATA, the error is about 10^{-4}, while the other constants are known up to 10^{-6} or better[1] . Despite the repeated suggestions of laboratory

G measurements at the level of 10^{-5}, not a single group in the world succeeded in penetrating beyond 10^{-4}. Moreover, three of the four best absolute G determinations are at variance with each other at their accuracy levels[2].

In ref. 3 a new method of determing G and other gravitational interaction parameters by tracking the motion of two bodies on board a drag-free Earth's satellite, using the horseshoe type trajectories, was suggested: the lighter body ("particle"), moving along a lower orbit than the heavier one ("shepherd"), overtakes it and, due to their interaction, passes to a higher orbit and begins to lag behind (the Satellite Energy Exchange, or SEE method). In space, unlike an earthborne laboratory, one avoids the environmental influence difficult to account for and the conditions can be created when a particle is not subject to forces much greater than those of its interaction with the shepherd: the tidal forces in the Earth's field are of the same order. However, not a single analytic solution has been found for the SEE method, even in a highly idealized problem setting-up and thus all the calculations are numerical.

We suggest a method preserving all the advantages of the SEE method but, in addition, admitting an approximate analytic trajectory description. Namely, we suggest that the particle move near one of its equilibrium points in the joint field of the Earth and the shepherd (the libration points, whose positions are the only known analytic solutions of the general three-body problem[4]): one can find approximate analytic solutions to the equations of motion near those points. Thus, compared with Ref. 3 it is easier to choose optimum trajectories for tracking and some additional estimates are available. In addition, for such trajectories the drag-free satellite capsule can be smaller (about 1.5 m instead of 20 m) and have a spherical shape instead of a cylindrical one; such a configuration is more favourable from both physical and economic viewpoints. A comparative shortcoming is that we lose the possibility of studying hypothetic inverse square law violations (the fifth force[5–8]) at distances greater than 1 meter. In other respects the experimental strategy and error analysis can be close to the SEE method.

2 Equations of Motion

Let us consider a particle ($m \approx 100\ g$) moving closely to the shepherd ($M \approx 500\ kg$) at a circular orbit around the Earth with the orbital radius a_0. It should be noted that with these mass values we cannot neglect m compared with M and thus abandon the frames of the "restricted" three-body problem (where the third body is considered to be negligibly light) but instead we have a new small parameter, the ratio s/a_0 where s is the center-to-center distance between the shepherd and the particle.

Let us introduce the shepherd's comoving coordinates with the origin at its center:

x, backward along the tangent of the shepherd's orbit;

y, along the radius drawn from the Earth center through the shepherd's center;

z, perpendicularly to the orbital plane.

The particle equations of motion in the joint Newtonian gravitational fields of the Earth and the shepherd in the quadratic approximation with respect to s/a_0 are

$$\ddot{x} - 2\omega\,\dot{y} - 3Bxy/a_0^4 + Ax/s^3 = 0\,, \tag{1a}$$

$$\ddot{y} + 2\omega\,\dot{x} - 3\omega^2 y - (3B/2a_0^4)(x^2 - 2y^2 + z^2) + Ay/s^3 = 0\,, \tag{1b}$$

$$\ddot{z} + \omega^2 z - 3Bz/a_0^4 + Az/s^3 = 0\,, \tag{1c}$$

where $A = G(M + m)$, $B = GM_E$, $\omega = (GM_E/a_0^3)^{1/2}$ is the orbital frequency (M_E is the Earth's mass) and $s = |\,\overline{s}\,|$, $\overline{s} = (x,\, y,\, z)$.

The particle equilibrium points (libration points) are determined from (1) under the conditions $\dot{\overline{s}} = \ddot{\overline{s}} = 0$, whence

$$x = z = 0 \qquad y = y_0[1 + |\,y_0\,|\,/3a_0 + 0(y_0^2/a_0^2)]\,; \qquad y_0 = \pm(A/3B)^{1/3}\,. \tag{2}$$

In the linear approximation

$$x = z = 0\,, \qquad y = y_0 = \pm a_0[G(M + m)/(3GM_E)]^{1/3}\,. \tag{3}$$

These are the two libration points close to the shepherd, denoted $L_{1,2}$ ($y_0 > 0$ for L_1). Although an exact solution for the libration points exists for any values of masses, for our purpose here it is sufficient to restrict ourselves to the linear approximation (3). Indeed, if $a_0 = 8000km$ (the orbital height $H \approx 1600km$), $M = 500kg$, $m = 100g$, then $|\,y_0\,| \approx 24.3cm$, so that $|\,s/a_0\,| \approx 3.10^{-8}$ and the second-order corrrection $|\,s^2/a_0\,| \sim 10^{-6}cm$, a value close to the possible measurement error; it must be taken into account in an actual experiment but can be neglected at the stage of a tentative study aimed at strategy choice.

Notably in order that $L_{1,2}$ be sufficiently far from the shepherd surface, the ball itself must be fabricated from a very dense material: e.g., a $500kg$ tungsten ball radius is about $18.4cm$ while that of copper one is about $23.8cm$.

3 Linearized Equations

Let us study possible particle motion near the libration point L_1 ($y_0 < 0$) or L_2 ($y_0 > 0$) in the linear approximation in $\Delta s/s_0$, where $s_0 = |\,y_0\,|$, $\Delta\overline{s} = (x,\ \xi,\, z)$ and $\xi = y - y_0$. Eqs. (1) lead to the following linear equations with constant coefficients:

$$\ddot{x} - 2\omega\,\dot{\xi} + 3\omega^2 x = 0\,, \tag{4a}$$

$$\ddot{\xi} + 2\omega\,\dot{x} - 9\omega^2 \xi = 0\,, \tag{4b}$$

$$\ddot{z} + 4\omega^2 z = 0 \tag{4c}$$

The third equation is unbound from the other ones and yields oscillations in the z direction with double orbital frequency. As for $(4a, b)$, seeking solutions in the form

$$\Delta\overline{s} = \begin{pmatrix} x \\ \xi \end{pmatrix} = \begin{pmatrix} x_0 \\ \xi_0 \end{pmatrix} e^{p\omega t} \tag{5}$$

one gets the characteristic equation

$$p^4 - 2p^2 - 27 = 0 \ , \tag{6}$$

whence

$$\begin{aligned} p_{1,2} &= \pm(1 + 2\sqrt{7})^{1/2} \approx \pm 2,5083 \ ; \\ p_{3,4} &= \pm i(-1 + 2\sqrt{7})^{1/2} \approx \pm 2,0715 i \ . \end{aligned} \tag{7}$$

Moreover, (4a) gives

$$\xi_0 / x_0 = (p^2 + 3)/(2p) = c_i \ , \quad i = 1,\, 2,\, 3,\, 4 \ , \tag{8}$$

where c_i are found by substituting p_i from (7). Thus the general solution of (4a,b) can be written in the form

$$\Delta \overline{s} = \left(\begin{array}{c} x \\ \xi \end{array} \right) = \sum_{i=1}^{4} k_i \left(\begin{array}{c} 1 \\ c_1 \end{array} \right) e^{p_i \omega t} \ , \tag{9}$$

where k_i are arbitrary constants, while the approximate values of c_i are

$$c_{1,2} = \pm 1,852 \ , \qquad c_{3,4} = \pm ic = \pm 0,3117 i \ . \tag{10}$$

The constants k_i can be related to arbitrary initial data at $t = 0$:

$$x(0) = x_0 \ , \quad \xi(0) = \xi_0 \ , \quad \dot{x}\ (0) = u \ , \quad \dot{\xi}\ (0) = v \ ; \tag{11}$$

with (11) Eq. (9) leads to two pairs of linear algebraic equations with respect to $k_\pm = k_1 \pm k_2$, $1_\pm = k_3 \pm k_4$; solving them, we arrive at the final form of the solution expressed in terms of the initial data:

$$x(t) = k_+ ch\, kt + k_- sh\, kt + 1_+ cos\, \tilde{\omega} t + 1\, sin \tilde{\omega} t \ , \tag{12a}$$

$$\xi(t) = c_1 (k_- ch\ kt + k_+ sh\ kt) + c(1 \cos \tilde{\omega} t + 1_+ sin\ \tilde{\omega} t) \ ; \tag{12b}$$

$$\begin{aligned} k_+ &= 0,122\ x_0 + 0,189\ v/\omega \ , \\ k_- &= 0,678\ \xi_0 - 0,102\ u/\omega \ , \\ 1_+ &= 0,878\ x_0 - 0,189\ v/\omega \ , \\ 1 &= -0,821\ \xi_0 + 0,6064\ u/\omega \ ; \end{aligned} \tag{13}$$

$$\begin{aligned} k &= p_1 \omega = 2,5083\, \omega \, ; \qquad c_1 = 1,852 \, ; \\ \tilde{\omega} &= -ip_3 \omega = 2,0715\, \omega \, ; \quad\ c = 0,3117 \, . \end{aligned} \tag{14}$$

An arbitrary solution (12) can be presented as a superposition of hyperbolic (the first two terms, hereafter marked by an overbar) and elliptic (the remaining two terms, to be marked by a tilde):

$$x(t) = \overline{x}(t) + \tilde{x}(t), \quad \xi(t) = \overline{\xi}(t) + \tilde{\xi}(t).$$

4 Elliptic and Hyberbolic Trajectories

Consider the elliptic component. Excluding t from $\tilde{x}(t)$ and $\tilde{\xi}(t)$, we obtain the trajectory equation

$$\tilde{x}/\tilde{a}^2 + \tilde{\xi}^2/\tilde{b}^2 = 1; \quad \tilde{a}^2 = 1^2 + 1_+^2, \quad \tilde{b} = \tilde{a}c \approx 0,31\tilde{a}. \tag{15}$$

It is an ellipse whose major semiaxis lies along the x axis while the minor one equals ≈ 0.31 of the major one. Calculating the velocities, we see that the motion is clockwise. The frequency of these revolutions is independent of the ellipse size (as long as our linear approximation is valid): $\tilde{\omega} \approx 2,07\omega$.

In particular, if one requires that the motion be purely elliptic, i.e., $k_+ = k_- = 0$, the initial positions and velocities are connected by the relations:

$$u/\omega \approx 6,65\ \xi_0\ ; \qquad v/\omega \approx -0,6455\ x_0. \tag{16}$$

Since the initial instant is arbitrary, this is the position-velocity relation at any instant of purely elliptic motion.

The very fact that such closed equilibrium trajectories exist near the unstable equilibrium points $L_{1,2}$, is of interest. Certainly these trajectories are also unstable: any violation of (16) would mean hyperbolic component appearance, leading to infinite motion.

Now let us consider the hyperbolic component. The path equation is obtained similarly to (15) from $\overline{x}(t)$ and $\overline{\xi}(t)$:

$$x^2 - \xi^2/c_1^2 = k_+^2 - k_-^2, \qquad c_1 \approx 1,852. \tag{17}$$

The hyperbole semiaxes $\overline{a}$ and $\overline{b}$ (see Fig. 1) are

$$\overline{a}^2 = \mid k_+^2 - k_-^2 \mid, \qquad \overline{b} = c_1\overline{a}. \tag{18}$$

Thus for $\mid k_+ \mid \ > \ \mid k_- \mid$ the particle travels along the right or left branch, and with the opposite inequality along the upper or lower one. If $\mid k_+ \mid = \mid k_- \mid$, the particle moves along one of the straight lines $\xi = \pm c_1 x$; the travel directions are indicated in Fig. 1 by arrows.

Purely hyperbolic motion takes place under the conditions $1 = 1_+ = 0$, i.e., if the position and the velocity are related by

$$u/\omega \approx 1,354\ \xi_0; \qquad v/\omega \approx 4,676\ x_0. \tag{19}$$

The hyperbole in the case of purely hyperbolic motion has its center at one of the libration points, L_1 or L_2.

5 Some Estimates

Let us estimate particle travel times for different trajectories. For an elliptic trajectory the revolution period is $T = 2\pi/\tilde{\omega} \approx 2\pi/2,0715\omega$ where the orbital frequency is $\omega \approx 0,9 \cdot 10^{-3}/c$ for $a_0 = 7900km$ (the orbital height $H \approx 1500km$). Consequently,

$$T \approx 3370\ s \sim 1\ hour$$

for any size of the ellipse.

Turning to the hyperbolic trajectories, let us first consider, e.g., the right one with the starting point $x = \bar{a},\ \ \xi = 0$, then

$$k_- = 0, \qquad \xi = c_1 \overline{a} sh\ kt,$$
$$t = (1/k) Arsh(\xi / c_1 \overline{a}) \approx (1/2,\ 51\omega)\ ln\ (2\xi/1,\ 85\overline{a}). \tag{20}$$

(assuming $\xi \gg \overline{a}$). Thus for $\overline{a} = 10^{-2} cm$, $\xi = 5cm$

$$t \approx 2800s$$

It takes about 2800 seconds to cover the arc from $\xi_0 = 5cm$ to the "turning point" if it is $0.1mm$ far from one of the libration points. The time t depends logarithmically on the hit exactness.

For the upper hyperbolic branch we obtain in a similar way: the travel time from $x(0) = 0,\ \ \xi(0) = \bar{b}$ to a point with given x is

$$t \approx (1/2,\ 51\omega)\ ln\ (3, 7x/\overline{b}), \tag{21}$$

so that for, e.g., for $\bar{b} = 10^{-2} cm$ and $x = 3cm$

$$t \approx 3100c.$$

The extreme case of hyperbolic motion is a straight trajectory hitting exactly L_1 or L_2, for which the travel times tend to infinity; however, at any realistic deflections from $L_{1,2}$ we return to the scale $t \gtrsim 1000c$.

Thus, in all the cases the characteristic travel times are of the order of 1 hour, quite sufficient for precision measurements.

Let us now estimate the possible particle trajectory sensitivity to the values of G. Let the particle move along the upper hyperbolic branch with a given "impact parameter" $\bar{b}$ from a libration point (see Fig. 1), between the points with $x = -x_0$ and $x = +x_0$, at an orbit $3000km$ high. By (21) the travel time is

$$t \approx (1142s)\ ln\ (3, 7x_0/\overline{b}) \quad for \quad x_0 \gg \overline{b}.$$

The libration point locations are determined by (3). Therefore a variation ΔG provided that $a_0,\ \ M,\ \ m$, and GM_E are unchanged, leads to a variation of y_0 such that

$$\Delta y_0 / y_0 = \Delta G / 3G,$$

Thus, for instance, if $\Delta G/G = 10^{-6}$, then $\Delta y_0 \approx 10^{-5} cm$; the distance $\bar{b}$ is changed by the same value, implying the t variation of

$$\Delta t \approx -(1142s) \Delta y / \overline{b}.$$

For $\bar{b} = 0,01cm$ this gives an easily measured interval of $\approx -1,14s$.

It should be pointed out that the particle velocity is sufficient for fixing the instants when $x = \pm x_0$ up to at most $10^{-3}s$ if the coordinate measurement error is, as assumed before, $\delta l = 10^{-6} cm$.

Even this tentative estimate shows that with such $\delta 1$ the constant G can be measured no worse than up to 10^{-6}.

6 Second-Order Deflections from $L_{1,2}$

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To find out, on the one hand, how close are the above first-order solutions to real trajectories at given separations from the libration points $L_{1,2}$ and, on the other hand, in which way are the idealized elliptic and hyperbolic trajectories actually distorted, it is helpful to consider second-order (quadratic) deflections from $L_{1,2}$ as corrections to the first-order ones.

The second-order deflections $\underline{x}(t)$, $\underline{\xi}(t)$, $\underline{z}(t)$ obey the equations

$$\begin{aligned} \ddot{\underline{x}} - 2\omega \dot{\underline{\xi}} + 3\omega^2 \underline{x} &= 2Qx\xi, \qquad Q = 9\omega^2/(2y_0); \\ \ddot{\underline{\xi}} + 2\omega \dot{\underline{x}} - 9\omega^2 \underline{\xi} &= Q(-2\xi^2 + x^2 + z^2); \\ \ddot{\underline{z}} + 4\omega^2 \underline{z} &= 2Q\xi z \ , \end{aligned} \tag{22}$$

where $x,\ \xi,\ z$ are the first-order quantities (12). Note that y_0 (see (3)) and Q are positive for L_2 and negative for L_1.

Generally there is rather a big variety of second-order deflection behaviors for different first-order trajectories and second-order initial data, in particular, the z deflections are now coupled to those in the orbital plane. Here we would like to be restricted to the simplest elliptic and hyperbolic trajectories along with the simplest initial data $\underline{x}(t)$, $\underline{\xi}(t)$ and $\underline{z}(t)$.

Thus for elliptic trajectories of the form

$$x(t) = 1_+ cos\tilde{\omega}t, \quad \xi(t) = -c1_+ sin\tilde{\omega}t, \quad z(t) = 0 \tag{23}$$

($\tilde{\omega} \approx 2.07\omega$, $c \approx 0.31$) the second-order deflections are

$$\underline{x}(t) = \underline{x}_s sin\ 2\tilde{\omega}t, \quad \underline{\xi}(t) = \underline{\xi}_0 + \underline{\xi}_c cos\ 2\tilde{\omega}t, \quad \underline{z} = 0, \tag{24}$$

distorting the path as shown in Fig. 2. The amplitude values are

$$\underline{\xi}_0 \approx -0.403\ 1_+^2/y_0; \quad \underline{x}_s \approx -0.026\ 1_+^2/y_0; \quad \underline{\xi}_c \approx -0.214\ 1_+^2/y_0. \tag{25}$$

Specifically, for an ellipse with major semiaxis $1_+ = 3cm$ at the orbital height of $3000km$ (so that $y_0 \approx 29cm$ for L_2)

$$\underline{\xi}_0 \approx -0.125cm; \quad \underline{x}_s \approx -0.008cm; \quad \underline{\xi}_c \approx -0.066cm$$

(near L_1 these quantities have the opposite sign).

Similarly for the upper hyperbolic trajectory near L_2

$$x(t) = k_- sh\, kt, \quad \xi(t) = c_1 k_- ch\, kt, \quad z(t) = 0 \tag{26}$$

($k \approx 2.51\omega$, $c_1 \approx 1.85$) the second-order quantities are

$$\underline{x} = \underline{x}_s sh\, 2kt, \quad \underline{\xi} = \underline{\xi}_0 + \underline{\xi}_c ch\, 2kt, \quad \underline{z} = 0 \tag{27}$$

with the coefficients

$$\underline{\xi}_0 \approx 1.15\ \xi_0^2/y_0; \quad \underline{x}_s \approx -0.082\ \xi_0^2/y_0; \underline{\xi}_c \approx -0.423\ \xi_0^2/y_0\ , \tag{28}$$

where ξ_0 is the hyperbole vertical semiaxis. The coefficient ξ_0^2/y_0 is rather small: even for a "very roughly aimed" trajectory with $\xi_0 = 1cm$ we have $\xi_0^2/y_0 \approx 0.0345cm$ (though, due to the exponential character of (27), the second-order deflections grow rapidly with t).

We can conclude that in situations of interest the second-order deflections are small as compared with the first order ones, so that the latter can be successfully used for planning the experiment.

7 Equivalence Principle Violation

The (weak) equivalence principle (EP) is violated if bodies of different chemical composition experience different accelerations in the same gravity field, as measured by the Eotvos parameter $\eta = 2(a_1 - a_2)/(a_1 + a_2)$. By the modern terrestrial experimental data, $|\ \eta\ | \lesssim 10^{-12}$.[7,9] A possible EP violation for the bodies M and m under consideration is described by just including the term $-\eta G M_E/R^2$ in the r.h.s. of (1b). If $\eta \sim 10^{-13}$ (an order of magnitude less than that ruled out by Ref. 9), the above additional acceleration is about $7 \cdot 10^{-11} cm/s^2$ for the orbital height $H \approx 1600km$ and causes libration point displacements of about $10^{-5}cm$, a quantity measurable in the experiment discussed. Moreover, possible particle trajectory displacements $\delta 1$ due to the EP violating force can be of the order of $3 \cdot 10^{-4}cm$ for the period of about 3000 seconds, since this force acts in a constant (radial) direction, so that the displacement can be estimated by the uniform-acceleration formula $\delta 1 = at^2/2$. Thus hopefully in this experiment it is possible to improve the accuracy of η determination by two orders of magnitude.

8 Tentative Error Analysis

So far we have been discussing the highly idealized situation of a perfectly spherical shepherd at a perfectly circular orbit in the Earth's spherically symmetric gravitational field. To assess the viability of the experimental conception it is necessary to examine different sources of influence and error. A next step is to decide to which extent they should be taken into account in an ultimate theoretical model to be used in the actual experiment, or what requirements should be met by the experimental equipment. Some (most evident) estimates are presented in Table 1 in terms of particle displacements $\delta 1$ resulting from accelerations of different origin, as obtained from the majorizing formula $\delta 1 = at^2/2$ for the measurement time of 3000 seconds. The value of $\delta 1$ is of primary interest since it is the particle position that is to be actually measured. For defeniteness we assumed $M = 500kg$, $m = 100g$, $a_0 = 8000km$ and (in tidal acceleration estimates) $s = 50cm$.

Clearly the factors 1, 3, 4 and 7 are significant, the factors 4 and 6 are not, while the remaining ones require further analysis.

Table 1: Estimated effects on particle motion in a drag-free satellite

Factors	Accelarations (cm/s^2)	Maximum displacements for $t = 3000s(cm)$
1. Terrestrial quadrupole tides	$\leq 5.10^{-10}$	$\leq 3.10^{-3}$
2. Higher geopotential harmonics	$\leq 10^{-13}$	$\leq 10^{-6}$
3. Solar tides	$\sim 3.10^{-12}$	$\sim 10^{-5}$
4. Lunar tides	$\sim 10^{-11}$	$\sim 10^{-4}$
5. Lunar nonsphericity	$\sim 3.10^{-19}$	$\sim 10^{-12}$
6. Jovian tides	$\sim 3.10^{-17}$	$\sim 10^{-10}$
7. Relativistic tidal effects	$\sim 10^{-13}$	$\sim 4.10^{-7}$
8. Uncertainty of shepherd's orbit ($\Delta R = 1cm$)	$\sim 3.10^{-13}$	$\sim 10^{-6}$
9. Possible EP violation ($\eta = 10^{-13}$)	$\sim 7.10^{-11}$	$\sim 3.10^{-4}$

Effects changing the satellite orbit are not included since the actual orbit is assumed to be known with a certain accuracy from radar or laser measurements. However, the corresponding (possibly systematic) error implies tidal acceleration uncertainty as reflected in line 8 of the table. It can be concluded that the orbit uncertainty is a key factor for the experiment viability since a better accuracy than that to $\Delta R \sim 1cm$ is not expected in the coming years and even 1 cm is questionable.

A tentative conclusion is that G and η can be measured two orders of magnitude better as compared with the present-day accuracy. However, for better studying the capabilities of the proposed methodology, it is necessary to carry out a thorough computer analysis of particle motions in a satellite at realistic orbits, taking into account the inevitable orbit eccentricity, its tilt with respect to the Earth's axis, the three-dimensional character of particle trajectories inside the capsule, etc. Error analysis can be carried out along the lines of Ref. 10 dedicated to the SEE method[3]. In particular, it is helpful to carry out computer simulation of the measurement process by inserting a Gaussian noise to the coordinate values of a precalculated ("true") trajectory to simulate measurement error. The work is in progress.

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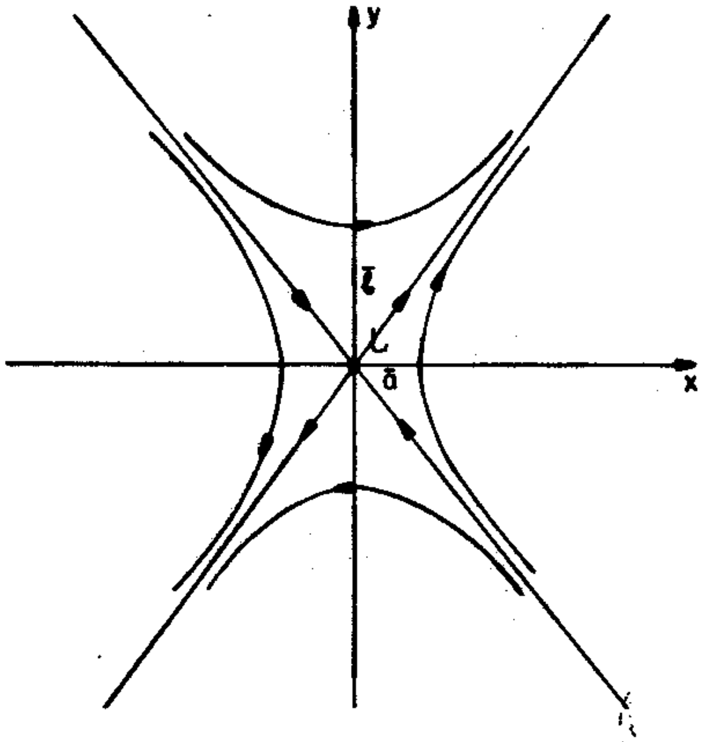

FIG. 1

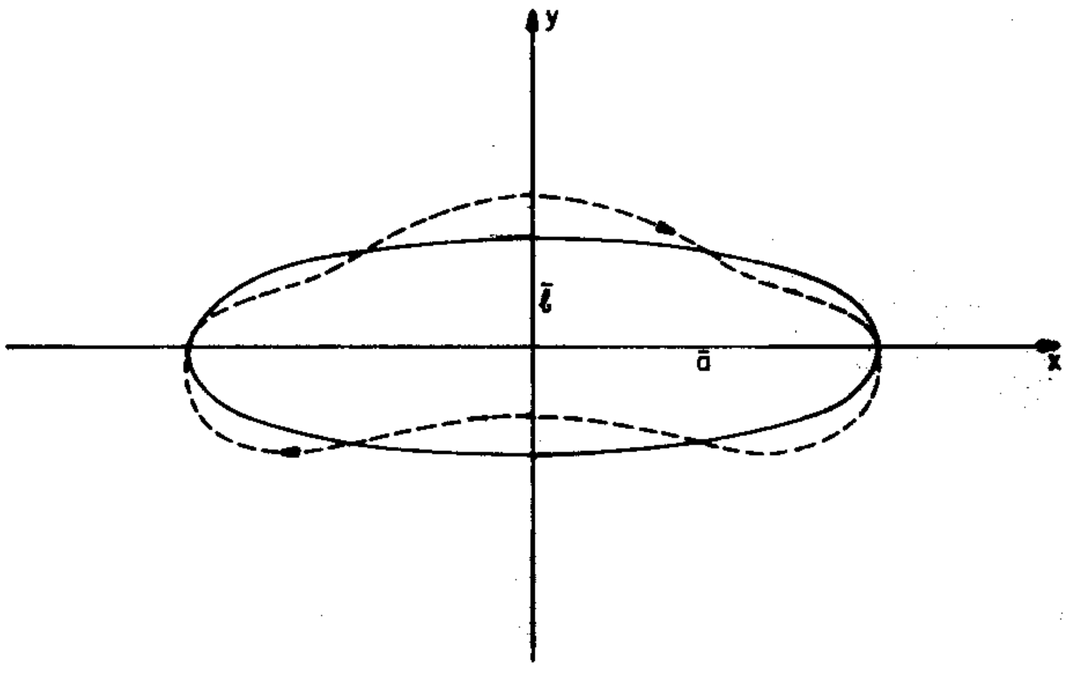

FIG. 2

References

[1] K.P. Staniukovich and V.N. Melnikov. Hydrodynamics, Fields and Constants in Gravitation Theory. Energoatomizdat, Moscow 1983 (in Russian).

[2] V.N. Melnikov. In: Itogi Nauki i Tekhniki. Klassicheskaya Teoriya Polia i Teoriya Gravitatsii. V.1. Gravitatsiya i Kosmologiya. VINITI, Moscow 1991 (in Russian). P. 49.

[3] A. Sanders and W.E. Deeds, Phys. Rev. D. 46, 489 (1992).

[4] G.N. Duboshin. Celestial Mechanics. Analytic and Qualitative Methods. Nauka, Moscow 1964.

[5] V. de Sabatta, V.N. Melnikov and P.I. Pronin, Progr. Theor. Phys. 88, 623 (1992).

[6] V.N. Melnikov and A.G. Radynov, Metrologiya, No. 11, p.3 (1990).

[7] V.N. Melnikov. In: Gravitational Measurements, Fundamental Metrology and Constants. Eds.: V. de Sabatta and V.N. Melnikov. Kluwer Acad. Publ. Holland, 1988.

[8] M. Novello and E. Elbaz. Fortschr. Phys. 40, 7, 651 (1992).

[9] V.B. Braginsky and V.I. Panov, Sov. Phys. JETP 34, 463 (1972).

[10] K.A. Bronnikov, V.N. Melnikov et al., Izmeritelnaya Tekhnika, No. 8, 9, 10 (1993).

[11] O.B. Karagioz, V.N. Melnikov et al., Izmeritelnaya Tekhnika, No. 10 (1993).