CBPF-NF-022/01

N study of extreme type II superconductors in a magnetic field

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(Received March 22, 2001)

Abstract

The large N limit of an extreme type II superconductor in a magnetic field H is considered at fixed dimensionality $d = 3$. It is shown that the effective interaction remains always positive, contrary to earlier claims. However, it is shown that no fixed point is reached in the infrared if $H \neq 0$, which could be interpreted as a first-order transition. The important role of the two scales of the problem is discussed.

Pacs: 74.20.-z, 05.10Cc, 11.25.Hf

High temperature superconductors have a very large Ginzburg parameter, typically $\kappa \sim 100$. For this reason, it seems to be a good approximation to neglect magnetic thermal fluctuations in the Ginzburg-Landau (GL) model. For $\kappa \gg 1$ the Hamiltonian density of the GL model in an external magnetic field is written as

$$H = |(\nabla - ie\mathbf{A})\phi|^2 + \frac{u}{2}\left(|\phi|^2 + \frac{m^2}{u}\right)^2, \qquad (1)$$

where $\nabla \times \mathbf{A} = \mathbf{H}$ and $m^2 = a(T - T_c)$ with $a > 0$. This model Hamiltonian describes superconductors in the extreme type II limit.

Early renormalization group calculations performed by Brézin *et al.* [1] using the model (1) indicated that the phase transition is of first-order. This result has been obtained in the lowest Landau level (LLL) approximation with an $\epsilon = 6 - d$-expansion. Later, Affleck and Brézin [2] carried a large N calculation and have obtained also a first-order phase transition. The situation seems to be different from the Halperin *et al.* calculation [3] in zero field but with magnetic fluctuations. In that case the $\epsilon = 4 - d$-expansion leads to a first-order transition while a large N analysis gives a second-order transition.

A large N analysis performed by Radzihovsky [4] leads to an opposite conclusion to that of Affleck and Brézin [2]. This author obtained instead that the transition is of second-order. His analysis, however, is confined to $4 < d < 6$ while Aflleck and Brézin discuss also the interval $2 < d < 4$.

In order to solve the controversy, in this paper we revisit the problem by performing a simpler analysis with respect to the previous ones. Let us point out the main differences between the present work and the preceding ones. First, we will work directly in $d = 3$, which is the physically meaningful dimension. When only the LLL is considered, the upper critical dimension is six while the lower one is four. Nevertheless, as remarked by Affleck and Brézin [2], in large N there is no problem to consider dimensions less than four. Second, we will use the gauge $\mathbf{A} = (0, xH, 0)$ instead of the symmetric gauge $\mathbf{A} = H(-y, x, 0)$ considered in the previous works [1,2,4]. Although the symmetric gauge simplifies the renormalization group (RG) analysis in $d = 6 - \epsilon$, it will not be particularly useful in the large N three-dimensional analysis. The third point is that we will integrate out all the N components, without leaving an unintegrated field, as done in Refs. [2] and [4]. This brings some simplification to the analysis. We will see that the leading order is just the Hartree approximation considered by Lawrie [6] in his thorough study of the LLL scaling. Our main results are the following. The effective $|\phi|^4$ interaction is found to be *always positive*, in contrast to the result of Ref. [1] where a sign change is found, leading these authors to conclude that the transition will be of first-order. A positive effective interaction is also found by Radzhovsky in his large N analysis at $4 < d < 6$. However, our analysis reveal that there is also a runaway in the infrared, indicating the absence of an infrared fixed point. This behavior is not characteristic of systems exhibiting a second-order phase transition.

In the following we will assume that the external magnetic field is parallel to the z axis and that the gauge $\mathbf{A} = (0, xH, 0)$ has been chosen. We will consider the model (1) with N complex components and take the large N limit at Nu fixed. In order to treat the large N limit, we will introduce an auxiliary field σ and obtain the transformed Hamiltonian:

$$H' = |(\nabla - ie\mathbf{A})\phi|^2 + i\sigma\left(|\phi|^2 + \frac{m^2}{u}\right) + \frac{1}{2u}\sigma^2. \tag{2}$$

The new Hamiltonian H' is Gaussian in ϕ. This allows a straightforward integration of ϕ to obtain the following effective action:

$$S_{eff} = NTr\ln(-\partial^2 + 2i\omega x\partial_y + \omega^2 x^2 + i\sigma) + \int d^3r\left(\frac{m^2}{u}i\sigma + \frac{1}{2u}\sigma^2\right), \tag{3}$$

where $\omega = eH$. The leading order in $1/N$ is obtained through the minimization of S_{eff} with respect to σ. We will take σ as being uniform and given by $\sigma = -i\sigma_0$. In this way we can easily evaluate the trace of the logarithm in (3) using the eingevalues of the operator $-\partial^2 + 2i\omega x\partial_y + \omega^2 x^2 + \sigma_0$, which are the well known Landau levels. Close to the critical line $H_{c_2}(T)$ [7], the most relevant of the Landau levels is the lowest one. By doing the minimization of (3) taking only the LLL simplifies considerably the calculation. The field ϕ should be written in terms of the Landau level basis as follows:

$$\phi(\mathbf{r}) = \sum_n \int \frac{dp_y}{2\pi} \int \frac{dp_z}{2\pi} \hat{\phi}_{n,p_y,p_z} \chi_{n,p_y,p_z}(\mathbf{r}), \tag{4}$$

where $\chi_{n,p_y,p_z}(\mathbf{r})$ are the Landau level eigenfunctions given by

$$\chi_{n,p_y,p_z}(\mathbf{r}) = \frac{1}{\sqrt{2^n n!}}\left(\frac{\omega}{\pi}\right)^{1/4} e^{i(p_z z + p_y y)} e^{-\omega(x - p_y/\omega)^2/2} H_n\left(\sqrt{\omega}x - \frac{p_y}{\sqrt{\omega}}\right), \tag{5}$$

with energy eigenvalues $E_n(p_z) = p_z^2 + (2n+1)\omega + m^2$ and where H_n are the Hermite polynomials. The LLL approximation correspond to taking only the $n = 0$ eingenfunction. By minimizing Eq. (3) with respect to σ_0 we obtaine the gap equation:

$$(\omega + \sigma_0)(\sigma_0 - m^2)^2 - \frac{N^2\omega^2 u^2}{16} = 0. \tag{6}$$

The critical field is obtained from Eq. (6) by setting $\sigma_0 = 0$. The result is

$$H_{c_2}(T) = \frac{16a^2(T_c - T)^2}{N^2 e u^2}. \tag{7}$$

Since $H_{c_2} \sim (T_c - T)^{2\nu}$, we obtain the critical exponent $\nu = 1$, in agreement with Ref. [6].

Let us calculate the quadratic fluctuations in σ. This will allow us to obtain the σ propagator which corresponds to the effective $|\phi|^4$ coupling. In order to perform this calculation, we will substitute in Eq. (3) $i\sigma = \sigma_0 + i\delta\sigma$, where $i\delta\sigma$ is a small fluctuation around σ_0. Thus, up to quadratic order in $\delta\sigma$, the effective action is

$$S_{eff} = S_{eff}^{(0)} + \frac{1}{2u}\int d^3r \int d^3r' [\delta^3(\mathbf{r} - \mathbf{r}') + Nug_0(\mathbf{r}, \mathbf{r}')g_0(\mathbf{r}', \mathbf{r})]\delta\sigma(\mathbf{r})\delta\sigma(\mathbf{r}') + (h.o.t.), \tag{8}$$

where S_{eff}^0 corresponds to the saddle point solution and $g_0(\mathbf{r}, \mathbf{r}')$ is the LLL Green function of the operator $-\partial^2 + 2i\omega x\partial_y + \omega^2 x^2 + \sigma_0$. Thus, the effective $|\phi|^4$ interaction is given in momentum space by

$$U_\sigma(\mathbf{p}) = \frac{u}{1 + \frac{Nu\omega}{2\pi}\frac{e^{-\frac{1}{2\omega}(p_x^2 + p_y^2)}}{\sqrt{\sigma_0 + \omega}[p_z^2 + 4(\sigma_0 + \omega)]}}. \tag{9}$$

Before proceeding, it is useful for the sake of clarity to compare the above effective interaction with the one obtained from the well known large N solution of the ϕ^4 theory [5]. In that case, the effective interaction V_σ is given in $d = 3$ by

$$V_\sigma(|\mathbf{p}|) = \frac{u}{1 + \frac{Nu}{4\pi\sqrt{\mathbf{p}^2}}\arctan\left(\sqrt{\frac{\mathbf{p}^2}{4\tilde{\sigma}_0}}\right)}, \tag{10}$$

where $\tilde{\sigma}_0 = \xi^{-2}$. As the critical point is approached, $\tilde{\sigma}_0 \to 0$ and the denominator of Eq. (10) becomes $1 + Nu/(8|\mathbf{p}|)$. By writing $\mu = |\mathbf{p}|$ and defining the dimensionless coupling $g = V_\sigma(\mu; \tilde{\sigma}_0 = 0)/\mu$, we can obtain easily the beta function $\beta(g) = \mu\partial g/\partial\mu$:

$$\beta(g) = g\left(\frac{Ng}{8} - 1\right). \tag{11}$$

This beta function has an infrared stable fixed point $g_* = 8/N$. This fixed point can be obtained directly from Eq. (10) by taking the following limit:

$$\lim_{\mu\to 0}\lim_{\tilde{\sigma}_0\to 0}\frac{V_\sigma(\mu)}{\mu} = g_*. \tag{12}$$

Alternatively, we could set $\mathbf{p} = 0$ in Eq. (10) to obtain $V_\sigma(0) = u/(1 + Nu/8\pi\tilde{\sigma}_0^{1/2})$ and take the scale as being $\bar{\mu} = \pi\tilde{\sigma}_0^{1/2}$. By defining the coupling constant as $\bar{g} = V_\sigma(0)/\bar{\mu}$, we obtain the beta function for this coupling with the same functional form as Eq. (11). This argument shows that scaling holds in the large N solution of the ϕ^4 theory.

In the case of the effective interaction U_σ, the problem is more subtle. Let us take first $p_x = p_y = 0$ and choose $\mu = |p_z|$. Remember that the critical point corresponds to $\sigma_0 = 0$. Thus, in contrast to the pure ϕ^4 case, being at the critical point does not mean a massless propagator. If we take the analogous limit of Eq.(12) in Eq. (9), we obtain

$$\lim_{\mu\to 0}\lim_{\sigma_0\to 0}\frac{U_\sigma(p_x = 0, p_y = 0, |p_z| = \mu)}{\mu} \to \infty. \tag{13}$$

The behavior (13) implies a runaway of the defined coupling constant in the infrared. Thus, there is no evidence for a fixed point using the above scaling, which is analogous to the scaling defined by Eq. (12) in the case of the ϕ^4 theory. However, we must remember that when $\sigma_0 = 0$, $\omega = \omega_{c_2} = eH_{c_2}(T)$, and therefore there is still one scale left in the limit (13). Thus, we can define a coupling $\tilde{g} = U_\sigma(0;\sigma_0 = 0)/\tilde{\mu}$ where $\tilde{\mu} = \pi\omega_{c_2}^{1/2}$. It is then straightforward to obtain the beta function for the coupling $\tilde{g}$ as $\beta(\tilde{g})$, where $\beta(x)$ is the function given by Eq. (11). Now we have obtained an infrared stable fixed point but this should not be a surprise since this fixed point is reached when $\tilde{\mu} \to 0$ which means $T \to T_c$. Since our theory is not rotation invariant in momentum space, we can consider a third situation where we define the coupling constant as $\tilde{g} = U(\mu = |\mathbf{p}_\perp|, p_z = 0; \sigma_0 = 0)/\mu$, where $\mathbf{p}_\perp^2 = p_x^2 + p_y^2$. Now the scale μ is associated to the degeneracy of the LLL. We obtain the following flow equation:

$$\mu\frac{\partial\tilde{g}}{\partial\mu} = -\tilde{g} + \frac{N}{8\pi}\frac{\mu^3}{\omega_{c_2}^{3/2}}\exp\left(-\frac{\mu^2}{2\omega_{c_2}}\right)\tilde{g}^2. \tag{14}$$

For a given fixed temperature, no nontrivial fixed point is reached from Eq. (14). If $\mu/\omega_{c_2} = c$, where c is a nonuniversal constant, we can rescale the coupling $\tilde{g} \to$

$c\exp(-c^2/2)\tilde{g}/\pi$ to obtain once more the beta function $\beta(\tilde{g})$. Thus, the scaling where the coupling constant is defined at $\sigma_0 = 0$ through $\mathbf{p} = 0$ and $\mu = \omega_{c_2}$ is equivalent to the scaling where $p_z = 0$ and $\mu = |\mathbf{p}_\perp| = c\omega_{c_2}^{1/2}$. We could choose alternatively $\mathbf{p}_\perp = 0$ and $\mu = |p_z| = c\omega_{c_2}^{1/2}$, or $\mu = |p_z| = |\mathbf{p}_\perp| = c\omega_{c_2}^{1/2}$ and obtain the same result. Thus, a second-order fixed point is obtained only if the scaling $\mu \sim \omega_{c_2}^{1/2}$ holds. Therefore, we find no evidence for a second-order phase transition over the line H_{c_2}, except for the point $T = T_c$ in the phase diagram in the TH-plane. However, it is difficult to conclude that the phase transition is of first-order as one crosses over the H_{c_2} line. The point is that we have two scales in this problem and the propagator for $\sigma_0 = 0$ becomes critical only for $T = T_c$. Note that the effective interaction is always positive, in contrast to the conclusions of Refs. [1,2]. The effective interaction obtained for arbitrary $4 < d < 6$ in Ref. [4] is similar to ours and is also always positive. This fact suggests that the mechanism for the first-order phase transition, if it takes place, is more subtle and does not rely on a simple sign change of the effective $|\phi|^4$ interaction.

We must be aware that the phase diagram may be even more complicated. As pointed out in Ref. [8], the vortex fluid phase would be constituted by two phases separated by a second-order line terminating in a tricritical point. This scenario is confirmed by recent Monte Carlo simulations [9]. Roughly speaking, the theoretical argument leading to such a phase diagram is the same that shows that the Halperin *et al.* [3] first-order scenario breaks down in the type II regime [10]. It consists of a duality argument in the lattice superconductor that allows to construct a field theory of vortex lines [11]. In this duality context, the magnetic field plays the role of a charge [8].

Acnowledgements

APCM would like to thank the hospitality of the Centre de Physique Théorique of the Ecole Polytechnique, where part of this work has been done, and acknowledges the finacial support of the brazilian agency CNPq. CdC would like to thank the hospitality of the Centro Brasileiro de Pesquisas Físicas and the finacial support of the brazilian agency CAPES during his stay. The work of FSN is supported by the Alexander von Humboldt foundation.

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