# An algebraic proof on the finiteness of
# Yang-Mills-Chern-Simons theory in $D = 3$

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ABSTRACT

A rigorous algebraic proof of the full finiteness in all orders of perturbation theory is given for the Yang-Mills-Chern-Simons theory in a general three-dimensional Riemannian manifold. We show the validity of a trace identity, playing the role of a local form of the Callan-Symanzik equation, in all loop orders, which yields the vanishing of the $\beta$-functions associated to the topological mass and gauge coupling constant as well as the anomalous dimensions of the fields.

Key-words: Algebraic method; Finiteness; Yang-Mills

PACS numbers: 04.62.+v, 11.10.Gh, 11.15.-q, 11.25.Db

The finiteness of the Yang-Mills-Chern-Simons (YMCS) theory [1–5] in $D = 3$ has been pursued since its evidence was first detected by one-loop order calculations [2,3], and later on up to two-loops [5]. Recently, the finiteness of the $N = 1$ super-YMCS theory [6] has been shown. A partial proof on the finiteness of $N = 2$ super-YMCS theory in the Wess-Zumino gauge is given in [7]. Since the pure Chern-Simons (CS) theory is finite to all orders in perturbation theory [8], two recent papers [9,10] have claimed the equivalence of the YMCS theory with a pure CS theory at the quantum level to argued the finiteness of YMCS up to field amplitude renormalizations.

In this letter we present a rigorous proof of the full finiteness of the YMCS theory in a general three-dimensional Riemannian manifold. The approach we propose here to quantum scale invariance of the YMCS is based on the energy-momentum (EM) tensor trace identity, playing the role of a local form of the Callan-Symanzik equation. It means exact quantum scale invariance, with vanishing $\beta$-functions and anomalous dimensions as well.

The same technique [11]) has been used to prove the full finiteness of the BF-Yang-Mills theory in $D = 3$ [12]. To give such a proof on the full quantum scale invariance of YMCS we use the algebraic renormalization method [13–15]. It is based on the BRS-formalism [13] together with the Quantum Action Principle [16], which leads to a regularization independent scheme. We think indeed that, due to the presence of the antisymmetric Levi-Civita tensor, it is difficult to establish an invariant regularization scheme without encountering problems at some or other stage of the argument.

Since we are working with an external curved dreibein, our results hold for a curved manifold, as long as its topology remains that of flat $\mathcal{R}^3$. This allows us to use the general results of renormalization theory [16,17] established in flat space.

The three-dimensional space-time is a Riemannian manifold described by a dreibein field $e_\mu^m$. The spin connection $\omega_\mu^{mn}$ depends on the dreibein due to the vanishing torsion condition. The metric tensor reads $g_{\mu\nu} = \eta_{mn} e_\mu^m e_\nu^n$, with $\eta_{mn}$ being the tangent flat space

metric. We denote by $e$ the determinant of $e^m_\mu$.

The YMCS classical action (in the Landau gauge) in a three-dimensional curved manifold reads:

$$\Sigma = \int d^3x \ \left\{ -\frac{e}{4} F^a_{\mu\nu} F^{a\mu\nu} + m\varepsilon^{\mu\nu\rho} (A^a_\mu \partial_\nu A^a_\rho + \right.$$
$$+ \frac{g}{3} f_{abc} A^a_\mu A^b_\nu A^c_\rho) - eg^{\mu\nu} (\partial_\mu b_a A^a_\nu + \partial_\mu \bar{c}_a D_\nu c^a) +$$
$$\left. + (A^{*\mu}_a s A^a_\mu + c^*_a s c^a) \right\} \quad , \tag{1}$$

where $m$ is the topological mass [1] and $g$ is the gauge coupling constant. The field strength is defined as $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$ and $c^a$, $\bar{c}^a$ and $b^a$ are the ghost, the antighost and the Lagrange multiplier fields, respectively. $A^{*\mu}_a$ and $c^*_a$ are the "antifields" (tensorial densities) coupled to the nonlinear variations of the fields $A^a_\mu$ and $c^a$ under BRS transformations $s$:

$$s A^a_\mu = -D_\mu c^a \equiv -(\partial_\mu c^a + g f_{abc} A^b_\mu c^c) \quad ,$$
$$s c^a = \frac{g}{2} f_{abc} c^b c^c \ , \quad s \bar{c}^a = b^a \ , \quad s b^a = 0 \quad . \tag{2}$$

The action (1) is also invariant under diffeomorphisms

$$\delta^{(\varepsilon)}_{\text{diff}} \Phi = \mathcal{L}_\varepsilon \Phi \quad , \tag{3}$$

where $\Phi = \left( A^a_\mu, e^m_\mu, c^a, b^a, \bar{c}^a, A^{*\mu}, c^*_a \right)$ and $\mathcal{L}_\varepsilon$ the Lie derivative along the infinitesimal vector field $\varepsilon^\mu$; and under infinitesimal local Lorentz transformations

$$\delta^{(\lambda)}_{\text{Lorentz}} \Phi = \frac{1}{2} \lambda_{mn} \Omega^{mn} \Phi \quad , \quad \Phi = \text{any field} \quad , \tag{4}$$

with $\Omega^{[mn]}$ acting on $\Phi$ as a Lorentz matrix in the appropriate representation.

The BRS invariance of the action is expressed in a functional way by the Slavnov-Taylor (ST) identity

$$\mathcal{S}(\Sigma) = \int d^3x \left( \frac{\delta\Sigma}{\delta A^{*\mu}_a} \frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta c^*_a} \frac{\delta\Sigma}{\delta c^a} + b^a \frac{\delta\Sigma}{\delta \bar{c}^a} \right) = 0 \tag{5}$$

where the corresponding linearized ST operator reads

$$\mathcal{B}_\Sigma = \int d^3x \left( \frac{\delta\Sigma}{\delta A_a^{*\mu}} \frac{\delta}{\delta A_\mu^a} + \frac{\delta\Sigma}{\delta A_\mu^a} \frac{\delta}{\delta A_a^{*\mu}} + \frac{\delta\Sigma}{\delta c_a^*} \frac{\delta}{\delta c^a} + \right.$$
$$\left. + \frac{\delta\Sigma}{\delta c^a} \frac{\delta}{\delta c_a^*} + b^a \frac{\delta\Sigma}{\delta \bar{c}^a} \right) \quad . \tag{6}$$

The operators $\mathcal{S}$ and $\mathcal{B}$ obey the following nilpotency identities: $\mathcal{B}_\mathcal{F} \, \mathcal{S}(\mathcal{F}) = 0 \; \forall \mathcal{F}$, and $(\mathcal{B}_\mathcal{F})^2 = 0$ if $\mathcal{S}(\mathcal{F}) = 0$. In particular, since the action $\Sigma$ obeys the ST identity (5), we have the nilpotency property $(\mathcal{B}_\Sigma)^2 = 0$.

In addition to the ST identity (5), the action (1) satisfies the constraints: the Landau gauge condition

$$\frac{\delta\Sigma}{\delta b_a} = \partial_\mu \left( e g^{\mu\nu} A_\nu^a \right) \; ; \tag{7}$$

and the "antighost equation" (in the Landau gauge [8])

$$\bar{\mathcal{G}}^a \Sigma = \int d^3x \left( \frac{\delta}{\delta c^a} + g f^{abc} \bar{c}_b \frac{\delta}{\delta b^c} \right) \Sigma = \Delta_{\mathrm{cl}}^a \; ; \tag{8}$$

with $\Delta_{\mathrm{cl}}^a = g \int d^3x f^{abc} (A_b^{*\mu} A_{c\mu} - c_b^* c_c)$. Note that the right-hand side of (8) being linear in the quantum fields, will not be submitted to renormalization.

The Ward (W) identities for the diffeomorphisms (3) and the local Lorentz transformations (4) read:

$$\mathcal{W}_X \Sigma = \int d^3x \sum_{\mathrm{all\ fields}} \delta_X \Phi \frac{\delta\Sigma}{\delta\Phi} = 0 \; , \tag{9}$$

where $X = (\mathrm{diff}, \mathrm{Lorentz})$.

Commuting (5) and (7) we obtain

$$\mathcal{G}^a \Sigma = \frac{\delta\Sigma}{\delta \bar{c}_a} + \partial_\mu \left( e g^{\mu\nu} \frac{\delta\Sigma}{\delta A_a^{*\nu}} \right) = 0 \; , \tag{10}$$

which is the "ghost equation" [15]. It implies that the theory depends on the field $\bar{c}_a$ and on the antifield $A_a^{*\mu}$ through the combination $\hat{A}_a^{*\mu} = A_a^{*\mu} + e g^{\mu\nu} \partial_\nu \bar{c}_a$.

Moreover, the action (1) is invariant under the rigid gauge transformations, given by the W identity

$$\mathcal{W}_{\text{rigid}}^a \Sigma = \int d^3 x \sum_{\phi = A, c, \bar{c}, b, A^*, c^*} f^{abc} \phi_b \frac{\delta}{\delta \phi^c} = 0 \quad , \tag{11}$$

by anticommuting (5) and (8).

In order to give a proof of the renormalizability of (1), we have to show that all constraints defining the classical theory also hold at the quantum level, *i.e.* that we can construct a renormalized vertex functional $\Gamma = \Sigma + \mathcal{O}(\hbar)$, obeying the same constraints and coinciding with the classical action at order zero in $\hbar$.

The first point to be checked is the power-counting renormalizability. The ultraviolet dimension, as well as the ghost number and the Weyl dimension of all fields and antifields are collected in Table I.

In order to explicitly find the possible renormalizations and anomalies of the theory, we can use the following result [7]: the degree of divergence of a 1-particle irreducible Feynman graph $\gamma$ is given by

$$d(\gamma) = 3 - \sum_{\tilde{\Phi} = \Phi, g} d_{\tilde{\Phi}} N_{\tilde{\Phi}} \quad , \qquad \text{with} \qquad d_g = \frac{1}{2} \quad . \tag{12}$$

Here $N_\Phi$ is the number of external lines of $\gamma$ corresponding to the field $\Phi$, $d_\Phi$ is the dimension of $\Phi$ as given in Table I, and $N_g$ is the power of the coupling constant $g$ in the integral corresponding to the diagram $\gamma$. In order to apply the known results on the quantum action principle [16] to the present situation, we have considered $g$ as an external field of dimension $\frac{1}{2}$.

Thus, including the dimension of $g$ into the calculation, we may state that the dimension of the counterterms of the action is bounded by 3. However, since they are generated by loop graphs, they are of order 2 in $g$ at least. This means that, not taking now into account the dimension of $g$, we can conclude that their real dimension is bounded by 2. The same holds for the possible breakings of the ST identity.

The second point to be discussed concerns about the functional identities to be obeyed by the vertex functional $\Gamma$. The gauge condition (7), antighost equation (8), ghost equation (10) as well as rigid gauge invariance (11) can be easily shown to hold at all orders,

*i.e.* are not anomalous [15]. The validity to all orders of the W identities of diffeomorphisms and local Lorentz will be assumed in the following: the absence of anomalies for them has been proved in [19,20] for the class of manifolds we are considering here.

It remains now to show the possibility of implementing the ST identity (5) for the vertex functional $\Gamma$. As it is well known [15], this amounts to study the cohomology of the nilpotent operator $\mathcal{B}_\Sigma$, defined by (6), in the space of local integrated functionals $\Delta$ of the fields involved in the theory. The cohomology classes of $\mathcal{B}_\Sigma$ are defined such that $\Delta$ and $\Delta + \mathcal{B}_\Sigma \hat{\Delta}$ belong to the same equivalence class. The set of these classes is called the cohomology group $\mathcal{H}^p(\mathcal{B}_\Sigma) = \mathcal{Z}^p(\mathcal{B}_\Sigma)/\mathcal{Q}^p(\mathcal{B}_\Sigma)$; $\mathcal{Z}^p(\mathcal{B}_\Sigma)$ being the space of cocycles (the nontrivial part of the general solution) and $\mathcal{Q}^p(\mathcal{B}_\Sigma)$ being the space of coboundaries (BRS-variation) both of ghost number $p$. The cohomological group $\mathcal{H}^0(\mathcal{B}_\Sigma)$ constitutes the non-trivial invariants of the theory, *i.e.* the arbitrary invariant counterterms we can add to the action at each order of perturbation theory which correspond to the renormalization of the physical parameters (coupling constants and masses), whereas $\mathcal{Q}^0(\mathcal{B}_\Sigma)$ represents the non-physical renormalizations (field amplitudes). On the other hand, $\mathcal{H}^1(\mathcal{B}_\Sigma)$ is related to the possible anomalies.

In the both cases, $\mathcal{H}^0$ and $\mathcal{H}^1$, the super-renormalizability by power-counting restricts the dimension of the integrand of $\Delta$ to 2. Moreover, the constraints (7–11), valid now for the vertex functional $\Gamma$, imply for $\Delta$ the conditions

$$\frac{\delta}{\delta b_a}\Delta = \int d^3x \frac{\delta}{\delta c^a}\Delta = \mathcal{G}^a\Delta = \mathcal{W}_X\Delta = 0 \ , \tag{13}$$

where $X = (\text{diff}, \text{Lorentz}, \text{rigid})$.

It has been proven in quite generality [20,21] that in such a gauge theory the cohomology in the sector of ghost number 1 is independent of the external fields (antifields). We can thus restrict the field dependence of $\Delta$ to $A_\mu^a$ and $c^a$, with the dependence on $c^a$ being through its derivatives due to the second of the constraints (13).

Beginning with the anomalies, we know [19,21] that, in three dimensions, the cohomology in this sector is empty, up to possible terms in the Abelian ghosts. However, they

can be seen, by using the arguments of [22], not to contribute to the anomaly, due to their freedom or soft coupling. We thus conclude to the absence of gauge anomaly, hence to the validity of the ST identity (5) to all orders for the vertex functional $\Gamma$.

Going now to the sector of ghost number 0, $i.e.$ looking for the arbitrary invariant counterterms which can be freely added to the action at each order. According to the above discussion the counterterm is at least of order $g^2$. Thus, the most general expression for the nontrivial part of $\Delta$ reads

$$\Delta_{\text{phys.}} = z_m m \frac{\partial}{\partial m} \Sigma \quad , \tag{14}$$

where $z_m$ is an arbitrary parameter. Eq.(14) shows that, $a$ $priori$, only the parameter $m$ can get radiative corrections. This means that the $\beta_g$-function related to the gauge coupling constant $g$ is vanishing to all orders of perturbation theory, and the anomalous dimensions of the fields as well. This concludes the proof of the renormalizability of the theory: all functional identities hold without anomaly and the renormalizations might only affect the CS coupling, $i.e.$ the topological mass $m$. But the latter turns out to be not renormalized, too. We shall indeed show that its corresponding $\beta_m$-function vanishes at all orders, which yields the full finiteness of the YMCS theory in a three-dimensional Riemmanian manifold.

Now, a precise study on the quantum scaling properties of the YMCS theory demands a local version of the Callan-Symanzik equation. Its local form arises from the "trace identity". It will be useful to exploit the fact that the integrand of the CS action is not gauge invariant, in spite of its integral be. This strong constraint upon the quantum insertions, together with the others, will guarantee that no insertions survive at all, therefore, as a consequence, the vanishing of the topological mass $\beta_m$-function. Above all, let us introduce the EM tensor, defined as the following tensorial quantum insertion:

$$\Theta_\nu{}^\mu \cdot \Gamma = e^{-1} e_\nu{}^m \frac{\delta \Gamma}{\delta e_\mu{}^m} \quad . \tag{15}$$

The integral of the trace of the tensor $\Theta_\nu{}^\mu$

$$\int d^3x \; e \; \Theta_\mu{}^\mu = \int d^3x \; e_\mu{}^m \frac{\delta\Sigma}{\delta e_\mu{}^m} \equiv \mathcal{N}_e \Sigma \tag{16}$$

follows from the identity

$$\mathcal{N}_e \Sigma = \left[ \sum_{\text{all fields}} d_W(\Phi) \mathcal{N}_\Phi + m\partial_m + \frac{1}{2} g\partial_g \right] \Sigma \;, \tag{17}$$

where the operators $\mathcal{N}_\Phi = \int d^3x \; \Phi\frac{\delta}{\delta\Phi}$ are the counting operators and $d_W(\Phi)$ the Weyl dimension (see Table I) of the field $\Phi$. It should be noticed that (17) is nothing else than the W identity for the rigid Weyl symmetry [23].

The trace $\Theta_\mu{}^\mu \cdot \Gamma$ turns out to be vanishing up to total derivatives and dimensionful couplings, in the classical approximation, due to the field equations, which means that (15) is the improved EM tensor. It is easy to check that from the classical action the following equation holds

$$w\Sigma \equiv \left[ e_\mu{}^m \frac{\delta}{\delta e_\mu{}^m} - \sum_{\text{all fields}} d_W(\Phi)\Phi\frac{\delta}{\delta\Phi} \right] \Sigma = \Lambda \;, \tag{18}$$

with $\Lambda$ being $\mathcal{B}_\Sigma$-invariant. It should be pointed out that $\Lambda$ is the effect of the breaking scale invariance caused by the dimensionful couplings. In fact, it is a soft breaking, since its dimension is lower than 3 (the dimensions of $m$ and $g$ are not taken into account)

To promote the trace identity (18) to the quantum level, we first note that the following conditons for the insertion $w\Gamma$ hold

$$\mathcal{B}_\Gamma w(x)\Gamma = 0 \;, \quad \bar{\mathcal{G}}^a w(x)\Gamma = \frac{1}{2}\frac{\delta\Gamma}{\delta c_a(x)} \;,$$
$$\frac{\delta}{\delta b_a(y)} w(x)\Gamma = -\frac{3}{2}\partial_\mu^x \delta(x-y)(eg^{\mu\nu}A_\nu^a)(y) \;, \tag{19}$$
$$\mathcal{G}^a(y) w(x)\Gamma = \frac{3}{2}\partial_\mu^x \delta(x-y)\left( eg^{\mu\nu}\frac{\delta\Gamma}{\delta A_a^{*\nu}} \right)(y) \;,$$

where we use again the fact that the constraints (7), (8) and (10) can be maintained at the quantum level [15].

The quantum version of (18) is written as

$$w\Gamma = \Lambda \cdot \Gamma + \Delta \cdot \Gamma \;, \tag{20}$$

where $\Lambda \cdot \Gamma$ is some quantum extension of the classical insertion $\Lambda$, subjected to the same constraints (19) as $w\Gamma$ (see [12]). It follows that the insertion $\Delta \cdot \Gamma$ defined by (20) obeys the homogeneous constraints

$$\mathcal{B}_\Gamma[\Delta \cdot \Gamma] = \frac{\delta}{\delta b_a}[\Delta \cdot \Gamma] = \bar{\mathcal{G}}^a[\Delta \cdot \Gamma] = \mathcal{G}^a[\Delta \cdot \Gamma] = 0 \tag{21}$$

beyond the conditions of invariance or covariance under $\mathcal{W}_{\text{diff}}$, $\mathcal{W}_{\text{Lorentz}}$ and $\mathcal{W}_{\text{rigid}}$.

By power-counting the insertion $\Delta \cdot \Gamma$ has dimension 3, but being an effect of the radiative corrections, it possesses a factor $g^2$ at least, and thus its effective dimension is at most 2. It turns out that there is no insertion obeying all these constraints, the power-counting selects the CS Lagrangian, but the latter is not BRS invariant. Therefore, $\Delta \cdot \Gamma = 0$: there is no radiative correction to the insertion $\Lambda \cdot \Gamma$ describing the breaking of scale invariance. It follows that (20) becomes

$$e \, \Theta_\mu{}^\mu \cdot \Gamma = \sum_{\text{all fields}} d_W(\Phi)\Phi\frac{\delta\Gamma}{\delta\Phi} + \Lambda \cdot \Gamma \quad . \tag{22}$$

This local trace identity leads to a Callan-Symanzik equation (see Section 6 of [11]):

$$\left(m\partial_m + \frac{1}{2}g\partial_g\right)\Gamma = \int d^3x \, \Lambda \cdot \Gamma \quad , \tag{23}$$

where no radiative effect contributes, that results in the vanishing $\beta$-functions associated to the parameters $g$ $(\beta_g)$ and $m$ $(\beta_m)$ as well as the anomalous dimensions of the fields. The scale invariance remains affected only by the soft breaking $\Lambda$. We have thus shown that there is no renormalization at all: the Yang-Mills-Chern-Simons theory in $D = 3$ is UV finite.

In conclusion, the method we have presented here has been allowed us to give a rigorous proof based on general theorems of renormalization theory on the full finiteness of the YMCS theory in a three-dimensional Riemannian manifold at all orders in perturbation theory. Also, this method turns out possible the identification of the real causes that are from behind the finiteness of the YMCS theory.

|       | $A_\mu$ | $b$ | $c$ | $\overline{c}$ | $A^{*\mu}$ | $c^*$ | $g$ |
|-------|---------|-----|-----|---------------|-----------|-------|-----|
| $d$   | $1/2$   | $3/2$ | $-1/2$ | $3/2$ | $5/2$ | $7/2$ | $1/2$ |
| $\Phi\Pi$ | $0$ | $0$ | $1$ | $-1$ | $-1$ | $-2$ | $0$ |
| $d_W$ | $-1/2$  | $3/2$ | $-1/2$ | $3/2$ | $1/2$ | $1/2$ | $1/2$ |

TABLE I. Ultraviolet dimension $d$, ghost number $\Phi\Pi$ and Weyl dimension $d_W$.

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