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DIFFRACTIVE LEPTOPRODUCTION OF SMALL MASSES IN QCD

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Abstract: In this paper we consider the process of diffraction dissociation in deeply inelastic scattering in the region of small produced mass, which we define as production of $\bar{q}\,q$-pair and $\bar{q}\,q\,G$ system in the final state. We show that the small distance contributions ($r_\perp \propto 1/Q$) to the longitudinal polarised virtual photon dominate. Formulae for the cross section using the gluon structure function are written within the framework of perturbative QCD. It is shown that the production of small masses by the transverse polarised photon is concentrated at moderate values of $r_\perp \sim 1 GeV^{-1}$, where the pQCD approach can be applied. This could be responsible for a considerable part of the diffractive production. It is shown that only $\bar{q}\,q$ pair production contribute to the diffraction dissociation at $\beta > 0.4$, the possibility to extract the value of the gluon structure function from the measurements in this kinematic region is discussed. The evolution of the DD structure function is studied, and a solution to the DD evolution equations is proposed.

Shadowing corrections are discussed for both the transverse and longitudinal polarised photon, and estimates of the different damping factors are given. The relation between diffractive production and the corrections to F_2, is alluded to.

1 Introduction

For the past decade starting from the paper of Bartels and Loewe [1] diffractive processes in deep inelastic scattering have attracted a great deal of attention, as these processes can be calculated in the framework of perturbative QCD (pQCD)[2]. These processes are of particular interest as the cross section turns out to be proportional to the square of the gluon structure function, we denote the gluon structure function by $x_P G(Q^2, x_P)$. Q^2 denotes the virtuality of the photon and x_P is equal to $\frac{Q^2+M^2}{s}$ (s is the energy squared of the reaction and M is the mass of produced hadron system). The recent renewed interest was triggered by Ryskin [3] and Brodsky et al[4], who proved that vector meson production can be calculated in leading log approximation of pQCD. The non-perturbative effects that comes fron large distances, can be factorized out in terms of the light-cone $\bar{q}q$ wave function of the produced vector meson.

The goal of this paper is to study the diffraction production (DD) of the hadron system for small values of produced mass (M). We define small masses as the production of $\bar{q}q$ -pair and $\bar{q}qG$ system in the final state of the diffraction dissociation process. We will show that

1. This process describes a significant part of the experimental DD cross section;

2. The small distance contribution ($r_\perp \propto 1/Q$) dominates the longitudinal polarised virtual photon which induces DD. This justifies using of the perturbative QCD to calculate the cross section in this case;

3. The production of small masses by transverse polarised photon are concentrated at moderate values of $r_\perp^2 \approx 1\,GeV^{-2}$. This encourages us to believe, that our estimates which are obtained in pQCD, could be responsible for a considerable part of the diffractive dissociation cross section;

4. The shadowing corrections are not negligble in the region of low x and should be taken into account.

In the next section we calculate the process of the diffraction dissociation for longitudinal and transverse polarized photons into a $\bar{q}q$ - pair. This is done in $r_\perp$ - representation, which provides a more direct way to understand at what distances the processes is viable, and provides the formalism for the calculation of the SC. Most of formulae which we obtain for transverse polarised photon have been previously derived in the momentum representation see refs.[5] [6] [7]*.

In the third section we extend the formalism suggested by Levin and Ryskin [8] and Mueller[9], to the case of the penetration of $\bar{q}q$ pair with definite value of $r_\perp$ through the proton, as well as through the nucleus. We will show that for the case of diffractive

*As we learned during Durhan workshop (Durham, Sept. 1995) Mark Wuesthoff has done the calculation for longidudinal polarized photon in his PhD thesises production of small masses by longitudinal polarised photon, we obtain an elegant closed expression for the damping factor. The damping factor is defined by:

$$D = \frac{\frac{d\sigma(\gamma^* + p \rightarrow X(M) + P)}{dt}[\ with\ SC]}{\frac{d\sigma(\gamma^* + p \rightarrow X(M) + p)}{dt}[\ without\ SC]}\ |_{t=0} \qquad (1)$$

where $t = -q_{\perp}^2$ is the momentum transferred in the reaction.

Processes at large transverse distances do not enter the formula, allowing us to calculate the SC within the framework of pQCD

We extend our formalism to diffractive dissociation by a transverse polarized photon. We discuss the large distance contribution in this case and show that the SC lead to a decrease of the typical distances that are responsible for the DD.

In the fourth section we compare the pQCD predictions with the experimental data available from HERA [10] [11]. We show that pQCD is able to account for a considerable part of the observed DD cross section, and discuss which distances contribute to the different modes present in the final state of the produced system. In all our calculations we use the GRV parametrization [13] for the gluon structure function, which is the main ingredient in our work. The GRV parametrization which starts from a low value of the photon virtuality, allows us to study the value of typical distances in the DD process.

In the fifth section we discuss the evolution equations for the DD processes.

A summary of our results and discussion are presented in the conclusions.

2 The diffractive production of $\bar{q}q$ and $\bar{q}qG$ system without shadowing corrections.

2.1 Notation:

We list below the notation that we use in this paper (see Fig.1).

1. Q^2 is the virtuality of the photon in DIS.

2. $x_P = \frac{Q^2+M^2}{s}$, where s denotes the square of the c.m. energy and M the produced hadron mass. The Bjorken x_B is given by $x_B = \frac{Q^2}{s}$.

3. $\beta = \frac{Q^2}{Q^2 + M^2} = \frac{x_B}{x_P}$

4. $a^2 = z(1-z)\,Q^2 + m_Q^2$, where z is the fraction of energy of the photon that is carried by the quark with mass m_Q.

5. $\vec{k}_{\perp}$ denotes the transverse momentum of the quark, and $\vec{r}_{\perp}$ the transverse distance between quark and antiquark.

6. $\vec{b}_\perp$ is the impact parameter of the reaction and is the variable conjugate to momentum transferred ($\vec{q}_\perp$), from the incoming proton to recoiled proton , $t = -q_\perp^2$.

7. $\vec{l}_{i\perp}$ denotes the transverse momentum of gluons, which are attached to the quark-antiquark pair (see Fig.1).

8. We use the evolution equation for the structure function in moment space. For any function $f(x)$, we define the moment $f(\omega)$ as

$$f(\omega) \;=\; \int_0^1 dx\; x^\omega\, f(x) \;\; . \tag{2}$$

Note that the moment variable ω is chosen so that the $\omega = 0$ moment, measures the number of partons, and the $\omega = 1$ moment measures their momentum. An alternative moment variable N, defined such that N $= \omega + 1$, is often found in the literature.

The x - distribution can be reconstructed by considering the inverse Mellin transform. For example, for the gluon structure function it reads

$$xG(Q^2, x) \;=\; \frac{1}{2\pi i}\int_C d\omega\; x^{-\omega}\; g(Q^2,\omega) \tag{3}$$

The contour of integration C is taken to the right of all singularities.

For the solution of the GLAP equation [14] , as well as the BFKL equation [15], one has the following form:

$$g(Q^2,\omega) \;=\; g(\omega)\, e^{\gamma(\omega) ln Q^2} \tag{4}$$

where $\gamma(\omega)$ denotes the anomalous dimension, which in the leading log $\frac{1}{x}$ approximation of pQCD, is the function of $\frac{\alpha_S}{\omega}$ and can be presented as the following series [16]:

$$\gamma(\omega) \;=\; \frac{\alpha_S N}{\pi}\cdot\frac{1}{\omega} \;+\; \frac{2\alpha_S^4 N^4 \zeta(3)}{\pi^4}\cdot\frac{1}{\omega^4} \;+\; O(\frac{\alpha_S^5}{\omega^5}) \tag{5}$$

9. Our amplitude is normalized so that

$$\frac{d\sigma}{dt} \;=\; \pi\, |f(s,t)|^2$$

and the optical theorem can be written:

$$\sigma_{tot} \;=\; 4\pi\; Im\, f(s,0)$$

The scattering amplitude in $b_\perp$-space is defined by

$$a(s,b_\perp) \;=\; \frac{1}{2\pi}\int d^2q_\perp\; e^{-i\vec{q}_\perp\cdot\vec{b}_\perp}\; f(s,t=-q_\perp^2)$$

In this representation

$$\sigma_{tot} \;=\; 2\int d^2b_\perp\; Im\; a(s,b_\perp)$$

and

$$\sigma_{el} \;=\; \int d^2b_\perp\; |\,a(s,b_\perp)|^2$$

In what follows we use the notation and normalization of Brodsky et al [4].

2.2 The cross section for the diffractive dissociation.

The cross section for the diffractive process, or for the reaction (see Fig.1):

$$\gamma^*(Q^2, x_B) \; + \; p \; \rightarrow \; X(M^2) \; + \; p \tag{6}$$

can be written in the form:

$$x_P \frac{d\sigma}{dx_P dt} \; = \; \frac{3\alpha_{em}}{8(2\pi)^2} \sum_1^{N_f} Z_f^2 \sum_{\lambda_1\lambda_2} \int_0^1 \, dz \int \frac{dk_\perp^2}{(2\pi)^2} \tag{7}$$

$$|f(\gamma^* \; + \; p \rightarrow \; q(z, k_\perp) \; + \; \bar{q}(1-z, -k_\perp \; + \; q_\perp) \; + \; p)|^2 \; (\, M^2 \; + \; Q^2 \,) \; \delta(\, M^2 \; - \; \frac{k_\perp^2}{z(1-z)} \,)$$

We have assumed in eq. (7) that the small masses are produced by the dissociation of the $\bar{q}q$ system. We will consider later, the corrections resulting from the emission of gluons. In eq. (7) λ_i denotes the polarisation of the quarks and N_f the number of quark flavours. Z_f the fraction of the charge of the electron carried by the quark with flavour f.

Integrating over z and summing over polarizations, we reduce the above equation to the form:

$$x_P \frac{d\sigma}{dx_P dt} \; = \; \frac{3\alpha_{em}}{(2\pi)^3} \sum_1^{N_f} Z_f^2 \; N_\lambda \int \; |f|^2 \, \frac{d^2 \, k_\perp}{1 \; - \; \beta} \, \frac{k_\perp^2}{M^2} \, \frac{2}{\sqrt{1 \; - \; \frac{4k_\perp^2}{M^2}}} \tag{8}$$

where $z(1-z) \; = \; \frac{k_\perp^2}{M^2}$ and $a = \sqrt{z(1-z)Q^2 + m_Q^2} = \; Q \, \frac{k_\perp}{M}$ for massless quarks. The region of integration with respect to $k_\perp$ is defined for $k_\perp^2 \; < \; \frac{M^2}{4}$.

The factor N_λ depends on the initial polarization of the photon. In the case of the longitudital polarization: $N_\lambda = 4$; for transverse polarized photon: $N_\lambda \; = \; z^2 \; + \; (1-z)^2$.

We follow refs. [9] [5] regarding normalization, and the definition of the photon wave function. The main problem is the calculation of the amplitude f.

2.3 The amplitude in $r_\perp$ representation

This approach was first formulated in ref.[8] and has been carefully developed in ref.[9]. During it's time of passage through the target, the distance $r_\perp$ between quark and anri-quark can vary by an amount $\Delta r_\perp \; \propto R \, \frac{k_\perp}{E}$. E denotes the energy of the pair, R the size of the target (see Fig.1). The quark's transverse momentum $k_\perp \; \propto \; \frac{1}{r_\perp}$. Therefore

$$\Delta r_\perp \; \propto \; R \frac{k_\perp}{E} \; \ll \; r_\perp \tag{9}$$

holds if

$$r_\perp^2 \cdot s \gg 2m \, R \tag{10}$$

where $s = 2mE$.

The above condition can be rewritten in terms of x_P as follows:

$$x_P \ll \frac{2}{(1-\beta)mR} \tag{11}$$

This means that at small values of x_P the transverse distance between the quark and antiquark is a good degree of freedom [8][9][17] and the interaction of virtual photon with the target can be written in the following form (we use the notation of ref. [4]:

$$f = \int \frac{d^2 r_\perp}{2\pi} \; \Psi^*(r_\perp, k_\perp)\; \sigma(r_\perp, q_\perp^2)\; \Psi^{\gamma^*}(Q^2, r_\perp, k_\perp) \tag{12}$$

Ψ denotes the wave function of the produced $\bar{q}q$ pair which is equal to $e^{i\vec{r}_\perp \cdot \vec{k}_\perp}$. After integrating over the azimuthal angle we have $J_0(\, k_\perp\, r_\perp\,)$ and $J_1(\, k_\perp\, r_\perp\,)$ for longitudinal and transverse polarised photon induced reaction, respectively.

Ψ^{γ^*} denotes the wave function of the longitudinally (L) or transverse (T) polarized photon, which have been given in ref [9] [5] and are equal to

$$\Psi_L(r_\perp, z) \;=\; Qz(1-z)\cdot K_0(\, a\, r_\perp\,) \;=\; Q\,\frac{k_\perp^2}{M^2}\, K_0(\, a\, r_\perp\,) \tag{13}$$

and

$$\Psi_T^{\gamma^*}(Q^2, r_\perp, k_\perp) \;=\; i\, a\, K_1(a\, r_\perp)\cdot \frac{\vec{r_\perp}}{r_\perp}\,. \tag{14}$$

We denote by $\sigma(r_\perp, q_\perp^2)$ the cross section of the $\bar{q}q$ pair with the transverse separation $r_\perp$, which scatters with transverse momentum $q_\perp$. We will discuss it in detail below, considering initially, the case where $q_\perp \,=\, 0$.

2.4 $\sigma(r_\perp, q_\perp^2)$ at $q_\perp = 0$.

The form for $\sigma(r_\perp, q_\perp^2)$ at $q_\perp$ has been found in ref. [8] (see Eq.(8) of this paper). σ can be expressed in terms of the unintegrated parton density ϕ first introduced in the BFKL papers [15] and widely used in ref. [18]. This function is clearly related to the Feynman diagrams, and to the gluon structure function, and can be calculated using the following equation:

$$\alpha_S(Q^2)\cdot xG(Q^2, x) \;=\; \int^{Q^2} dl_\perp^2\; \alpha_S(l_\perp^2)\; \phi(l_\perp^2, x) \tag{15}$$

Using the above equation we obtain the result of ref. [8], which reads

$$\sigma(r_\perp, q_\perp^2) \;=\; \frac{16 C_F}{N_c^2 - 1}\,\pi^2 \int \phi(l_\perp^2, x)\cdot \{\, 1 \,-\, e^{i\vec{l}_\perp \cdot \vec{r}_\perp}\,\}\;\cdot\; \frac{\alpha_S(l_\perp^2)}{2\pi}\cdot \frac{d^2 l_\perp}{l_\perp^2} \tag{16}$$

where $\phi \;=\; \frac{\partial xG(Q^2,x)}{\partial Q^2}$.

We calculate this integral using eqs. (3) and (4), and integrate over the azimuthal angle. Introducing a new variable $\xi = r_\perp l_\perp$ we can reduce the integral to the form:

$$\sigma(r_\perp, q_\perp^2) \;=\; \frac{16 C_F \alpha_S}{N_c^2 - 1}\, \pi^2 \int_C \frac{d\omega}{2\pi i}\, g(\omega)\, \gamma(\omega)\, (r_\perp^2)^{1-\gamma(\omega)} \int_0^\infty d\,\xi\, \frac{1 - J_0(\xi)}{(\xi)^{3-2\gamma(\omega)}} \tag{17}$$

preforming the integration over dξ (see ref.[19] **11.4.18**) we have:

$$\sigma(r_\perp, q_\perp^2) \;=\; \frac{8 C_F \alpha_S}{N_c^2 - 1}\, \pi^2 \int_C \frac{d\omega}{2\pi i}\, g(\omega)\, \gamma(\omega)\, (\frac{r_\perp^2}{4})^{1-\gamma(\omega)} \,\cdot\, \frac{\Gamma(\gamma(\omega)\, \Gamma(1-\gamma(\omega))}{(\,\Gamma(2-\gamma(\omega)\,)^2} \tag{18}$$

In the double log approximation of pQCD, $\gamma(\omega) \ll 1$, and we obtain the cross section for $N_c = 3$:

$$\sigma(r_\perp, q_\perp^2) \;=\; \frac{\alpha_S(\frac{4}{r_\perp^2})}{3}\, \pi^2\, r_\perp^2\, \left(x G^{GLAP}(\frac{4}{r_\perp^2}, x) \right) \tag{19}$$

This result coincides with the value for the cross section given in refs.[20] [21] if we neglect the factor 4 in the argument of the gluon structure function. We have checked that the Eq.(2.16) of ref. [4] also leads to the same answer, unlike the value for σ quoted in the paper (see Eq.(2.20) in ref. [4]).

2.5 The amplitude for small mass production at $q_\perp = 0$ by longitudinal polarised photon.

In this section we calculate the amplitude for $\bar{q}q$ production with mass M. Substituting in equation (9) our expression (14) for σ, yields.

$$f \;=\; \int r_\perp dr_\perp\, \frac{Q\, k_\perp^2}{M^2}\, K_0(\frac{Q}{M} k_\perp\, r_\perp)\, \frac{8 C_F \alpha_S}{N_c^2 - 1}\, \pi^2 \int_C \frac{d\omega}{2\pi i} \tag{20}$$
$$g(\omega)\, \gamma(\omega)\, (\frac{r_\perp^2}{4})^{1-\gamma(\omega)} \,\cdot\, \frac{\Gamma(\gamma(\omega)\, \Gamma(1-\gamma(\omega))}{(\,\Gamma(2-\gamma(\omega)\,)^2}\, J_0(k_\perp\, r_\perp)$$

Making use of well known properties of the modified Bessel functions we can perform the integration. The final general answer is

$$f \;=\; \frac{Q k_\perp^2}{M^2} \frac{8 C_F \alpha_S}{N_c^2 - 1}\, \pi^2 \int_C \frac{d\omega}{2\pi i}\, g(\omega)\, \frac{1}{a^4}\, (\frac{a^2}{\beta})^{\gamma(\omega)} \tag{21}$$

$$\Gamma(1+\gamma(\omega))\, \Gamma(1-\gamma(\omega))\, \Gamma(2-\gamma(\omega))\, \Gamma(2-\gamma(\omega)) \cdot \beta^3 \cdot {}_1F_1(\,2-\gamma(\omega), -1+\gamma(\omega), 1,\, 1-\beta\,)$$

where $a \;=\; Q\, \frac{k_\perp}{M}$.

In the case of the GLAP approach we can simplify eq. (21) considering $\gamma(\omega) \ll 1$. Neglecting $\gamma(\omega)$ with respect to 1, and using eq. (3) we get the result

$$f \;=\; -\pi^2\, \frac{8\, C_F\, \alpha_S}{N_C^2 - 1}\, \frac{Q\, \frac{k_\perp^2}{M^2}}{(\,Q^2\, \frac{k_\perp^2}{M^2}\,)^2}\, \beta^2\, x_P\, G(\,x_P, \frac{k_\perp^2}{(1-\beta)}\,)\, (\,1\, -\, 2\beta\,) \tag{22}$$

It is interesting to note that the argument of $x_P G$ is $k_\perp^2/(1-\beta)$ which means that small distances of the order of $r_\perp^2 \;\propto\; (1-\beta)/k_\perp^2$ contribute to small mass diffractive production especially at $\beta \rightarrow 1$.

2.6 The DD cross section for longitudinal polarised photon at $t = 0$.

Subtituting the amplitude in eq. (8) for the cross section, we have for three flavours and three colours:

$$x_P \frac{d\sigma}{dx_P\, dt}\,|_{t=0} = \frac{4\,\pi^2}{3} \sum Z_f^2\, \alpha_{em}\, \alpha_S^2\, \frac{1}{Q^4} \beta^3\, (1-2\beta)^2 \qquad (23)$$

$$\int_{Q_0^2}^{\frac{M^2}{4}} \frac{d\,k_\perp^2}{k_\perp^2}\, (\, x_P\, G(\, x_P, \frac{k_\perp^2}{(1-\beta)}\,)\,)^2$$

One can conclude from eq. (23) that only small distance processes contribute to the cross section. This fact manifests itself in the log integration with respect to $k_\perp^2$. Even in the double log approximation of pQCD, this integral converges and can be rewritten in the form:

$$x_P \frac{d\sigma}{dx_P\, dt}\,|_{t=0} = \alpha_{em} \sum Z_f^2 \frac{4\pi^2}{3} \frac{\alpha_S \pi}{C_A} \frac{1}{Q^4} \beta^3\, (1-2\beta)^2 \qquad (24)$$

$$\int_{x_P}^{1} \frac{d\,x'_P}{x'_P} \,(\, \frac{\partial(\, x_P\, G(\, x_P, \frac{M_\perp^2}{4\,(1-\beta)}\,)}{\partial ln(1/x'_P)}\,)^2$$

The above formula is the answer for the case of the production of two jets originating from a $\bar{q}q$- pair.

2.7 The amplitude at $q_\perp = 0$ of the DD for transverse polarised photon.

Using the formulae given in the previous subsections as well as eq. (19) and taking into account Eqs.(3) and (4) for the gluon structure function, we obtain the following expression for the amplitude f:

$$f = \int r_\perp dr_\perp\ \ a\, K_1(\, \frac{Q}{M}\, k_\perp\, r_\perp\,)\, \frac{8 C_F \alpha_S}{N_c^2 - 1}\, \pi^2 \int_C \frac{d\omega}{2\pi i} \qquad (25)$$

$$g(\omega)\ \gamma(\omega)\ (\frac{r_\perp^2}{4})^{1-\gamma(\omega)}\ \cdot\ \frac{\Gamma(\gamma(\omega)\ \Gamma(1-\gamma(\omega))}{(\,\Gamma(2-\gamma(\omega)\,)^2}\ J_1(k_\perp\, r_\perp)$$

Making use of the properties of the modified Bessel functions we perform the integration over $r_\perp$ and obtain:

$$f = \frac{8 C_F \alpha_S}{N_c^2 - 1}\, \pi^2\, k_\perp \int_C \frac{d\omega}{2\pi i}\, g(\omega)\ \frac{1}{a^4}\ (\frac{a^2}{\beta})^{\gamma(\omega)} \qquad (26)$$

$$\frac{\Gamma(1+\gamma(\omega))\,\Gamma(1-\gamma(\omega))\,\Gamma(2-\gamma(\omega))\,\Gamma(3-\gamma(\omega))}{\Gamma(2-g(\omega))\,\Gamma(2)} \cdot \beta^3 \cdot {}_1F_1(\,3-\gamma(\omega), g(\omega), 2, 1-\beta\,)$$

where $a = Q\frac{k_\perp}{M}$.

In the GLAP approach the above equation can be simplified considering $\gamma(\omega) \ll 1$, it reduces to the form:

$$f = \pi^2 \frac{16\, C_f\, \alpha_S}{N_c^2 - 1} \cdot \frac{k_\perp}{a^4} \beta^3 \cdot x_P\, G(x_P, \frac{k_\perp^2}{(1-\beta)}) \,. \qquad (27)$$

2.8 The cross section for the DD of the transverse polarised photon in $\bar{q}\,q$ pair.

Substituting the amplitude f in the general formula for the cross section (see eq. (8)) we obtain for three flavours:

$$x_P \frac{d\sigma}{dx_P\, dt}\,|_{t=0} = \alpha_{em} \frac{4\,\pi^2}{3} \alpha_S^2 \sum Z_f^2 \frac{1}{Q^2} \beta^3 (1 - \beta)^2 \qquad (28)$$

$$\int_{Q_0^2}^{\frac{M^2}{4}} \frac{d\,k_\perp^2}{k_\perp^4} \,(\, x_P\, G(\, x_P, \frac{k_\perp^2}{(1-\beta)}\,)\,)^2 \cdot \{1 - 2\frac{k_\perp^2}{M^2}\} \cdot \frac{1}{\sqrt{1 - \frac{4k_\perp^2}{M^2}}} \,.$$

2.9 The cross section for $\bar{q}\,q\,G$ production.

The emission of one additional gluon is shown in the diagrams of Fig.2. One can see two different classes of Feyman diagrams describe gluon emission. In the first, the emitted gluon does not interact with the target as shown in Fig.2a. The general way to take such gluon emission into account, is to use the evolution equations, and we will discuss this problem here. To provide a complete set of formulae, (which will enable us to compare our calculation with experimental data), we write down explicitly all formulae for the emission of one gluon, at the end of the section. The second class of diagrams describes the process in which the emitted gluon interacts with the target (see Fig.2b). These diagrams have to be calculated separately as they provide the contribution to the initial condition for the evolution equations [7].

We start with the calculation of the diagrams of Fig.2b. We would like to determine those contributions where the smallness of α_S is compensated by a large logarithm. In other words, we are looking for contributions of the order of $\alpha_S \ln(Q^2/k^2)$. Our first observation is that such a term does not exist in the case of the longitudinal polarised photon induced reaction. Indeed, this fact can easily be seen from the general structure of the diagrams of Fig. 2b, namely, they can be written in a general form:

$$x_P \frac{d\sigma}{dx_P\, dt}\,|_{t=0}\; (Fig.2b) \;\propto\; \int_0^1 dz \int_{x_P}^1 \frac{dx'}{x'} \int \frac{d^2 r_\perp}{2\pi}\, \Psi^{\gamma^*}\, \sigma(r_\perp, q_\perp^2 = 0, x')[\Psi^{\gamma^*}] \,, \qquad (29)$$

where σ is defined by (see eq. (19))

$$\sigma(r_\perp, q_\perp^2) = \frac{\alpha_S(\frac{4}{r_\perp^2})}{3}\, \pi^2\, r_\perp^2 \left(x^2 G^{(2)}(\frac{4}{r_\perp^2}, x) \right) \qquad (30)$$

with $x^2G^2(x, r_\perp^2)$ that has been introduced in ref. [22]. . The explicit expression for (x^2G^2) was derived in ref [9] and it is equal to:

$$x'^2 G^{(2)}(x', r_\perp^2) \;=\; \frac{2}{\pi^2} \int_{r_\perp^2}^{\infty} \frac{d^2 r'}{r'^4} \int_0^{\infty} db_\perp^2 \, [\, \sigma^{GG}\,]^2 \,, \tag{31}$$

where

$$\sigma^{GG}(r', x') \;=\; \frac{3\,\alpha_S}{4}\, r'^2 \,(\, x'\, G(x', \frac{4}{r'^2})\,)\,. \tag{32}$$

Substituting eq. (13) for $\Psi_L^{\gamma^*}$ in eq. (31), after the integration over z using the properties of the modified Bessel functions, we obtain

$$\sigma(r_\perp, q_\perp^2) \;\propto\; \int_{\frac{4}{Q^2}}^{\infty} \frac{dr_\perp^2}{r_\perp^4} \frac{\alpha_S(\frac{4}{r_\perp^2})}{3}\, \pi^2 \, \left(x^2 G^{(2)}(\frac{4}{r_\perp^2}, x) \right)\,. \tag{33}$$

Therefore, we have no log contribution from the $r_\perp$ integration. However, if we calculate the same integral for the transverse polarised photon, we find

$$\sigma(r_\perp, q_\perp^2) \;\propto\; \int_{\frac{4}{Q^2}}^{\infty} \frac{dr_\perp^2}{r_\perp^2} \frac{\alpha_S}{3}\, \pi^2 \, \left(x^2 G^{(2)}(\frac{4}{r_\perp^2}, x) \right) \tag{34}$$

$$\propto \int_{\frac{1}{Q^2}}^{\infty} dr'^2 \, \ln(Q^2\, r'^2) \, \left(xG(\frac{4}{r_\perp^2}, x) \right)^2\,.$$

Finally, going to the $k_\perp$ representation eq. (34) yields the result of ref. [7]:

$$x_P \frac{d\sigma_T}{dx_P dt}|_{t=0}\,(Fig.2b) \;=\; \sum_F \frac{4\,\pi^2\,\alpha_{em}\, Z_f^2}{Q^2}\, \beta \int_\beta^1 \frac{dz}{z^4} \int_{\bar{Q}_0^2}^{\frac{M^2}{4}} \frac{dk^2}{k^4} \frac{\alpha_S^3 N_c^2}{32\pi} \tag{35}$$

$$\{\, \beta^2 \,+\, (z \,-\, \beta)^2\,\}\,(1-z)^3\,(2z+1)^2\; \left(x_P\, G(x_P, k^2) \right)^2\,.$$

Taking the integral over z we obtain the answer:

$$x_P \frac{d\sigma_T}{dx_P dt}|_{t=0}\,(Fig.2b) \;=\; \sum_F \frac{4\,\pi^2\,\alpha_{em}\, Z_f^2}{Q^2} \int_{\bar{Q}_0^2}^{\frac{M^2}{4}} \frac{dk^2}{k^4} \frac{N_c^2 \alpha_S^3}{32\pi} \ln \frac{M^2}{4\,k^2} \tag{36}$$

$$\{\, \frac{(1-\beta)}{6}\, [\, 4 - 31\,\beta - 63\,\beta^2 + 50\,\beta^3 - 14\,\beta^4\,] \,-\, \ln\beta\,[\,1 + 10\,\beta - 2\,\beta^2\,]\,\}\;(\,x_P\, G(x_P, k^2)\,)^2\,.$$

We now calculate the diagrams of Fig.2a. For the transverse polarized photon, this has been done in ref. [7] and the result in our notations is:

$$x_P \frac{d\sigma_T}{dx_P dt}|_{t=0}\,(Fig.2a) \;=\; \sum_F \frac{4\,\pi^2\,\alpha_{em}\, Z_f^2}{Q^2}\, \beta \int_\beta^1 \frac{dz}{z^5} \int_{\bar{Q}_0^2}^{\frac{M^2}{4}} \frac{dk^2}{k^4} \frac{4\,\alpha_S^3}{3 N_c \pi} \ln \frac{M^2}{4\,k^2} \tag{37}$$

$$\frac{1+z^2}{1-z}\, \{\beta^2\,(z-\beta)^2 \;-\; z^6\,\beta^2 (1-\beta)^2\}\; \left(x_P\, G(x_P, k^2) \right)^2 \;=$$

$$\sum_F \frac{4\,\pi^2\,\alpha_{em}\,Z_f^2}{Q^2}\int_{\bar{Q}_0^2}^{\frac{M^2}{4}}\frac{dk^2}{k^4}\,\frac{4\,\alpha_S^3}{3N_c\pi}\,\ln\frac{M^2}{4\,k^2}$$

$$\beta\,(1-\beta)^2\,\{-2\,\beta\,\ln\beta\,+\,\frac{1}{12}\,[\,1+6\,\beta-9\,\beta^2-24\,\beta^2-6\,\beta^4-4\,\beta^5\,]\,\left(x_P\,G(x_P,k^2)\right)^2\,.$$

One can calculate the one gluon emission for the longitudinal polarised photon, using the general formula obtained in ref. [7]:

$$x_P\,\frac{d\sigma_T}{dx_P dt}|_{t=0}\,(Fig.2a)\;=\;\sum_F\,\frac{4\,\pi^2\,\alpha_{em}\,Z_f^2}{Q^2}\,\beta\int_\beta^1\frac{dz}{z} \tag{38}$$

$$\int_{\bar{Q}_0^2}^{\frac{M^2}{4}}\frac{dk^2}{k^4}\,\frac{\alpha_S^3}{32\pi}\,\ln\frac{Q^2}{k^2}\,\Phi_F^F(\frac{\beta}{z})\,\Phi_{P;T}^F(z)\,\left(x_P\,G(x_P,k^2)\right)^2$$

where Φ_F^F denotes the usual GLAP splitting function, and Φ_P^F the splitting function of the Pomeron into quark - antiquark pair. For the transverse polarisation of the photon this splitting function has been calculated previously in refs. [5] [7] and we have recalculated it in $r_\perp$ - representation (see eq. (28)):

$$\Phi_{P;T}^F\;=\;\frac{16}{N_c}\,z^2\,(1-z)^2\,. \tag{39}$$

For the longitudinal polarization we obtain the same factorized answer with one important difference, the integration over k^2 is logarithmic (see eq. (23)). However, as far as the z - dependence is concerned, the formula is the same as eq. (38) with splitting function which we calculated previously (see eq. (21)) and which is equal to:

$$\Phi_{P;L}^F\;=\;\frac{16}{N_c}\,z^3\,(1-2z)^2\,. \tag{40}$$

Finally, the answer for the one gluon emission in the case of longitudinal polarised photon is:

$$x_P\,\frac{d\sigma_L}{dx_P dt}|_{t=0}\,(Fig.2a)\;=\;\sum_F\,\frac{4\,\pi^2\,\alpha_{em}\,Z_f^2}{Q^4}\int_\beta^1\frac{dz}{z^6}\int_{\bar{Q}_0^2}^{\frac{M^2}{4}}\frac{dk^2}{k^2}\,\frac{4\,\alpha_S^3}{3\,N_c\pi}\,\ln\frac{Q^2}{k^2} \tag{41}$$

$$\beta\,\frac{1+z^2}{1-z}\,\{\beta^3\,(z-2\beta)^2\;-\;z^7\,\beta^3\,(1-2\beta)^2\}\,\left(x_P\,G(x_P,k^2)\right)^2\;=$$

$$\sum_F\,\frac{4\,\pi^2\,\alpha_{em}\,Z_f^2}{Q^4}\int_\beta^1\int_{\bar{Q}_0^2}^{\frac{M^2}{4}}\frac{dk^2}{k^2}\,\frac{4\,\alpha_S^3}{3\,N_c\pi}\,\ln\frac{Q^2}{k^2}$$

$$\{-2\,\beta^3\,\ln\beta\,(1-2\beta)^2\,+\,\frac{2}{15}\,+\,\frac{1}{6}\,\beta\,+\,\frac{2}{3}\,\beta^2\,-\,4\,\beta^3\,+\,9\,\beta^4\,+\,\frac{74}{15}\,\beta^5\,-\,\frac{19}{3}\,\beta^6\,-\,\frac{2}{3}\,\beta^7\,\}\,.$$

3 Shadowing corrections for the diffractive production of small masses.

The advantage of using the $r_\perp$ representation is not apparent in the calculation performed in the previous sections, in that we obtain the same result as had previously been obtained using momentum space calculation techniques (see for example ref. [22][18][6] [5] [7]). The above calculations are however, instructive in that they provide an explicit example of relations between different variables, used to describe deep inelastic process in different frames. In this section, where we calculate the shadowing corrections for the various processes, the $r_\perp$-representation simplifies the calculation. We will only calculate the damping factors for the penetration of $\bar{q}q$ - pair through the target, assuming that all shadowing corrections to gluon structure function have been included in the phenomenological gluon structure function which we use in the calculations. The size of the corrections due to damping in the gluon sector has been estimated in ref.[41].

3.1 The $b_\perp$ dependance of the amplitude.

To estimate the shadowing corrections, we need to know the profile function of the amplitude in impact parameter space, which means that we need to know the amplitude not only at $q_\perp$=0, but also at all values of the momentum transfer. The gluon structure function is weakly dependent on $q_\perp$ at small values of $q_\perp$, both in double log approximation (see detail discussion in ref. [18]), and in the BFKL approach (see ref.[24] [25]). Therefore, the leading $q_\perp$-dependence comes from the form factor of the $\bar{q}q$ pair with the transverse separation $r_\perp$, and the form factor of the target (proton).

As the form factor cannot be treated theoretically in pQCD, and we will assume an exponential parametrization for it, namely

$$F_p(q_\perp^2) \;=\; e^{-\frac{B}{4}q_\perp^2}$$

The slope B can be extracted from the experimental data on hadron-hadron collisions, provided we take the Pomeron slope α'=0. Namely, $B\,=\,B_{el}^{pp}(\alpha'\,=\,0)$, where B_{el} is the slope in the differential cross section of proton-proton collision. For the numerical estimates we use information from phenomenological sources to extract the value of B (see ref.[31]).

It turns out that the value of B defined in a such way, is very close to the one obtained from the proton electromagnetic form factor.

The form factor of the $\bar{q}q$ pair with the transverse separation $r_\perp$ is equal to

$$F_{\bar{q}q}(q_\perp^2) \;=\; \Psi_{\bar{q}q}^{i}\left(\frac{(\vec{k}_{1\perp}\;-\;\vec{k}_{2\perp})\cdot\vec{r}_\perp}{2}\right)\;\cdot\;\Psi_{\bar{q}q}^{f*}\left(\frac{(\vec{k'}_{1\perp}\;-\;\vec{k'}_{2\perp})\cdot\vec{r}_\perp}{2}\right) \qquad (42)$$

where k_i (k_i') denote the momentum of the quark i before and after collision. Each of the wave functions is given by the exponent and simple sum of different attachment of gluon lines to quark lines. Hence,

$$F_{\bar{q}q}(q_\perp^2) \;=\; e^{i\frac{\vec{q}_\perp \cdot \vec{r}_\perp}{2}} \;\cdot\; \{\, 1 \;-\; e^{i\vec{l}_\perp \cdot \vec{r}_\perp} \,\} \tag{43}$$

The last factor is absorbed in the expression for the cross section, while the firstfactor gives the $q_\perp$ dependence of the $\bar{q}q$ form factor, which after integration over the azimuthal angle is

$$F_{\bar{q}q}(q_\perp^2) \;=\; J_0(\frac{q_\perp \; r_\perp}{2}) \tag{44}$$

To proceed with the calculation we require the profile function in $b_\perp$ space

$$S(b_\perp^2) \;=\; \frac{1}{4\pi^2} \int d^2 q_\perp \; e^{i\vec{b}_\perp \cdot \vec{q}_\perp} \; F_p(q_\perp^2)\; F_{\bar{q}q}(q_\perp^2) \tag{45}$$

The explicit calculation gives

$$S(b_\perp^2) \;=\; \frac{1}{\pi B}\, I_0\left(\frac{b_\perp \; r_\perp}{B}\right)\; e^{-\frac{b_\perp^2 \,+\, \frac{r_\perp^2}{4}}{B}} \tag{46}$$

To simplify the calculation we replace the above function by

$$S(b_\perp^2) \;=\; \frac{1}{\pi B'}\, e^{-\frac{b_\perp^2}{B'}} \tag{47}$$

where

$$B' \;=\; B\left(1 \;+\; \frac{r_\perp^2}{4B}\right) \;\approx\; B\left(1 \;+\; \frac{1}{a^2\, B}\right)$$

Equation (29) has the same value for the radius in the expansion at small $b_\perp^2$ as eq. (81). For sufficiently large values of a, we consider $a^2 B \;\gg\; 1$, and neglect the second term in our calculation.

3.2 Penetration of $\bar{q}q$-pair through the target.

To calculate the shadowing correction we follow the procedure suggested in refs. [8][9]. Namely, we replace $\sigma(r_\perp, q_\perp^2 = 0)$ in eq. (12) by

$$\sigma^{SC}(r_\perp) \;=\; 2 \int d^2 b_\perp \;\left(1 \;-\; e^{-\frac{1}{2}\,\sigma(r_\perp, q_\perp^2 = 0)\; S(b_\perp^2)}\right) \tag{48}$$

The above formula is the solution of the s-channel unitarity constrain:

$$2\; Im\, a(s, b_\perp) \;=\; |a(s, b_\perp)|^2 \;+\; G_{in}(s, b_\perp) \tag{49}$$

where a denotes the elastic amlitude for $\bar{q}q$ pair with the transverse separation $r_\perp$, while G_{in} is the contribution of the all inelastic processes. The inelastic cross section is equal to

$$\sigma_{in} \;=\; \int d^2\, b_\perp \; G_{in}(s, b_\perp) \;=\; \int d^2 b_\perp \;\left(1 \;-\; e^{-\,\sigma(r_\perp, q_\perp^2 = 0)\; S(b_\perp^2)}\right) \tag{50}$$

Eq. (45) is based on the physical assumption that the structure of the final state, is the uniform parton distribution that follows from the QCD evolution equation. We neglect, in particular the contribution to the inelastic final state of all diffraction dissociation processes, with large rapidity gap. For example, "fan" diagrams that give the most important contribution are neglected [18]), as well as the diffraction dissociation in the region of small mass, which cannot be presented as the decomposition of the $\bar{q}q$ wave function.

From the point of view of the Feyman diagrams, eq. (50) sums all the diagrams of Figs. 1 and 2 -type in which the $\bar{q}q$ -pair rescatters with the target and exchanges many "ladder" diagrams, each of which can be represented by the gluon structure function. This sum has been performed by Mueller [9], and we shall perform the calculation for the case of the diffractive production.

3.3 Damping factor for diffractive production of small masses for longitudinal polarised photon.

Substituting σ^{SC} of eq. (50) in eq. (12) and using σ in the form of eq. (19), allows us to estimate the general term of the expansion with respect to power of σ. It has the form:

$$f^n = C\,\frac{k_\perp^2}{2Q}\cdot\int\frac{d^2 r_\perp}{\pi}\,K_0(a\,r_\perp)\frac{(-1)^{n-1}}{n!}\cdot\left(\frac{\alpha_S\,4C_F\pi^2}{N_c^2-1}\right)^n \tag{51}$$

$$\prod_i^n\int_{C_i}\frac{d\omega_i\,e^{\sum\omega_i\ln(1/x_P)}}{2\pi i}\,g_i(\omega_i)\cdot\frac{\Gamma(1+\gamma(\omega_i))\,\Gamma(1-\gamma(\omega_i))}{(\Gamma(2-\gamma(\omega_i))^2}$$

$$\cdot\left(\frac{r_\perp^2}{4}\right)^{n-\sum_i^n\gamma(\omega_i)}\quad J_0(k_\perp\,r_\perp)\,,\int d^2b_\perp\,S^n(b_\perp^2)$$

We replaced all numerical coefficients by the factor C.

Carrying out the integrals over $r_\perp$ and $b_\perp$ we obtain the expression:

$$f^n = C\frac{1}{2Q}\cdot\frac{(-1)^{n-1}}{n\,n!}\cdot\left(\frac{\alpha_S\,4\,C_F\pi}{B'(N_c^2-1)}\right)^n \tag{52}$$

$$\prod_i^n\int_{C_i}\frac{d\omega_i\,e^{\sum\omega_i ln(1/x_P)}}{2\pi i}\,g_i(\omega_i)\cdot\frac{\Gamma(1+\gamma(\omega_i))\,\Gamma(1-\gamma(\omega_i))}{(\Gamma(2-\gamma(\omega_i))^2}$$

$$(\frac{a^2}{\beta})^{\sum\gamma(\omega_i)}\;\Gamma^2(1+n-\sum_i^n\gamma(\omega_i))\,\left(\frac{\beta^2}{a^2}\right)^n\;{}_1F_1(1+n-\sum\gamma(\omega_i),-n+\sum\gamma(\omega_i),1,\,1-\beta)$$

Neglecting γ and taking $1-\beta\ll 1$ we derive a very simple formula in the double log approximation of pQCD $\gamma(\omega_i)\ll 1$. Taking the integral over ω_i yields

$$f^n = C\,B'\,\frac{\beta^2}{2\,Q}\cdot\frac{(-1)^{n-1}n!}{n} \tag{53}$$

$$\cdot \left(\frac{C_F \pi}{B'(N_c^2-1)} \, \frac{4\beta}{a^2} \, \alpha_S \, x_P G(\frac{a^2}{\beta}, x_P) \right)^n$$

Finally for f we have

$$f \;=\; C\; B' \sum_{n=1}^{\infty} \frac{\beta^2}{2\,Q} \, (n-1)! \tag{54}$$

$$\left(\frac{C_F \pi}{B'(N_c^2-1)} \, \frac{4\beta}{a^2} \, \alpha_S \, x_P G(\frac{a^2}{\beta}, x_P) \right)^n$$

The above series can be summed ito give the analytic function E_1,namely

$$f \;=\; C\; B' \, \frac{\beta^2}{2\;Q} \, E_1(\frac{1}{\kappa}) \cdot e^{\frac{1}{\kappa}} \tag{55}$$

where (for N_c= 3)

$$\kappa_q \;=\; \frac{2}{3} \cdot \frac{\alpha_S \pi\, \beta}{B'\; a^2} \cdot x_P G(\frac{a^2}{\beta}, x_P) \;=\; \frac{2}{3} \cdot \frac{\alpha_S \pi\, (1-\beta)}{B'\; k_\perp^2} \cdot x_P G(\frac{k_\perp^2}{(1-\beta)}, x_P) \tag{56}$$

Using the above equation we obtain the following expression for the damping factor

$$D_L^2 \;=\; \frac{\int_{Q_0^2}^{\frac{M^2}{4}} k_\perp^2 \, d\,k_\perp^2 \; E_1^2(\frac{1}{\kappa_q}) \cdot e^{\frac{2}{\kappa_q}}}{\int_{Q_0^2}^{\frac{M^2}{4}} k_\perp^2 \, d\,k_\perp^2 \; \kappa_q^2} \tag{57}$$

The behaviour of the damping factor for small and large κ_q can be found by using the well known property of E_1. Namely, for $\kappa_q \ll 1$, $D \to 1$, while at $\kappa_q \gg 1$ the damping factor vanishes as

$$D_L^2 \;\propto\; \frac{\int_{Q_0^2}^{\frac{M^2}{4}} k_\perp^2 \, d\,k_\perp^2 \ln^2 \kappa_q}{\int_{Q_0^2}^{\frac{M^2}{4}} k_\perp^2 \, d\,k_\perp^2 \, \kappa_q^2} \,.$$

The behaviour of the damping factor as function of β and Q^2 is given in Fig.3. One can see that the value of the damping factor reaches 0.5 in the region of small β and Q^2. It means that the SC should be included even for the longitudinally polarised photon, where the smallest distances of the order of $\frac{1}{Q}$ contributes.

3.4 The damping factor for DD for transverse polarized photon.

Repeating all steps of section 3.3, we derive the formula for the damping factor for transverse polarized photon. Substituting σ^{SC} of eq. (50) in eq. (12) we obtain for the general term of the expansion :

$$f^n \;=\; C\; a \cdot \int \frac{d^2\, r_\perp}{\pi} \, K_1(a\, r_\perp) \frac{(-1)^{n-1}}{n!} \cdot \left(\frac{\alpha_S \, 4 C_F \pi^2}{N_c^2 - 1} \right)^n \tag{58}$$

$$\prod_i^n \int_{C_i} \frac{d\omega_i\, e^{\sum \omega_i \ln(1/x_P)}}{2\pi i}\, g_i(\omega_i)\ \cdot\ \frac{\Gamma(1+\gamma(\omega_i))\,\Gamma(1-\gamma(\omega_i))}{(\Gamma(2-\gamma(\omega_i))^2}$$

$$\cdot \left(\frac{r_\perp^2}{4}\right)^{n - \sum_i^n \gamma(\omega_i)} J_1(k_\perp\, r_\perp)\ ,\ \int d^2 b_\perp\ S^n(b_\perp^2)$$

C stands for all numerical coefficients.

After integration over $r_\perp$ and $b_\perp$ we have:

$$f^n\ =\ C\beta\frac{k_\perp}{a^2}\ \cdot\ \frac{(-1)^{n-1}}{n\ n!}\ \cdot\ \left(\frac{\alpha_S\, 4\, C_F \pi}{B'(N_c^2-1)}\right)^n \tag{59}$$

$$\prod_i^n \int_{C_i} \frac{d\omega_i\, e^{\sum \omega_i ln(1/x_P)}}{2\pi i}\, g_i(\omega_i)\ \cdot\ \frac{\Gamma(1+\gamma(\omega_i))\,\Gamma(1-\gamma(\omega_i))}{(\Gamma(2-\gamma(\omega_i))^2}\, (\frac{a^2}{\beta^2})^{\sum\gamma(\omega_i)} \left(\frac{\beta^2}{a}\right)^n$$

$$\Gamma(2+n-\sum_i^n \gamma(\omega_i))\ \Gamma(1+n-\sum_i^n \gamma(\omega_i))\ \ {}_1F_1(2+n-\sum\gamma(\omega_i), 1-n+\sum\gamma(\omega_i), 2,\ 1\ -\ \beta\ )$$

Again we derive a very simple formula in the double log approximation of pQCD $\gamma(\omega_i) \ll 1$, neglecting γ as well as considering $1-\beta \ll 1$. Taking the integral over ω_i we have

$$f^n = C\ B'\ \frac{\beta k_\perp}{a^2}\ \cdot\ \frac{(-1)^{n-1} n!(n+1)}{n} \tag{60}$$

$$\cdot \left(\frac{C_F\pi}{B'(N_c^2-1)}\ \frac{4\beta}{a^2}\ \alpha_S\ x_P G(\frac{a^2}{\beta}, x_P)\right)^n$$

Finally for f we have to answer

$$f\ =\ C\ B'\ \sum_{n=1}^{\infty} \frac{\beta k_\perp}{a^2}\ [(n-1)!\ +\ n!] \tag{61}$$

$$\left(\frac{C_F\pi}{B'(N_c^2-1)}\ \frac{4\beta^2}{a^2}\ \alpha_S\ x_P G(\frac{a^2}{\beta^2}, x_P)\right)^n$$

The above series can written in terms of the analytic function E_1,namely

$$f\ =\ C\ B'\ \frac{\beta k_\perp}{2\,a^2}\ \{1\ +\ \left(1\ -\ \frac{1}{\kappa_q}\right)\ E_1(\frac{1}{\kappa})\ \cdot\ e^{\frac{1}{\kappa}}\ \} \tag{62}$$

where κ_q is given by eq. (56).

Using the above equation we have for the damping factor

$$D_T^2\ =\ \frac{\int_{Q_0^2}^{\frac{M^2}{4}}\ d k_\perp^2 \{1\ +\ \left(1\ -\ \frac{1}{\kappa_q}\right)\ E_1(\frac{1}{\kappa})\ \cdot\ e^{\frac{1}{\kappa}}\}^2}{4\ \int_{Q_0^2}^{\frac{M^2}{4}}\ d k_\perp^2\ \kappa_q^2}\ . \tag{63}$$

Fig. 4 shows the dependence of D_T^2 on β and Q^2. It is interesting to notice that the value of D_T^2 is small at small β which reflects the well known fact that the SC are big for the diffractive dissociation in the system with large mass [18].

3.5 The relationship between the SC, $F_2(x_B, Q^2)$ and the DD processes.

The simplest relation between the corrections to F_2 and cross section of the diffractive dissociation can be derived directly from AGK cutting rules [23] (see ref. [7]) and it reads:

$$\frac{\Delta F_2(x_B, Q^2))}{F_2(x_B, Q^2} \;=\; \frac{\sigma^{DD}}{\sigma_{tot}}\,, \tag{64}$$

where

$$F_2 \;=\; F_2^{GLAP} \;-\; \Delta F_2\,. \tag{65}$$

However, eq. (64) is only valid when the diffractive dissociation cross section is small. The notion of what is meant by "small" in diffractive production is not unique, since DD is small in two cases: i) when the kinematic region between partons is very small, this can be dealt with in a perturbative way, or ii) in the case when the interaction is so strong that we have scattering off a black disc. As we have shown the damping factors for the DD turns out to be rather large. It is therefore necessary to reconsider the simple relation in eq. (64).

We generalise eq. (64) by calculating the SC for F_2 due to penetration of the $\bar{q}q$ pair through the nucleon. As we have mentioned above, we adopt the approach throughout the paper, that the SC in gluon sector have been taken into account in the phenomenological set of structure functions, that we use. The expression for the F_2 including SC for $\bar{q}q$ pair has been derived by Mueller [9] and in our notation has the following form

$$F_2(x, Q^2) \;=\; \sum_f Z_f^2 \int_0^1 dz \int \frac{d^2 r_\perp}{2\pi}\, \Psi_T^{\gamma^*}(Q^2, r_\perp, z)\, \sigma^{SC}(r_\perp)\, [\Psi_T^{\gamma^*}(Q^2, r_\perp, z)]^*\,. \tag{66}$$

As was shown in ref.[9], whithin the LLA of pQCD which we use throughout this paper, we can safely replace $\Psi_T \Psi_T^*$ by $\frac{1}{r_\perp^4}$ after integration over z in eq. (66). Mueller's formula finally reads (for $N_c = N_f = 3$)

$$F_2(x, Q^2) \;=\; \sum_f Z_f^2 \frac{6}{\pi^2} \int \frac{d^2 b_\perp}{\pi} \int_{\frac{4}{Q^2}}^{\infty} \frac{d^2 r_\perp}{\pi} \frac{1}{r_\perp^4} \{\, 1 \;-\; \exp(-\frac{1}{2}\sigma(r_\perp, q_\perp^2 = 0)\, S(b_\perp^2))\,\}\,, \tag{67}$$

Adopting the same procedure of integration as for eq. (57) and eq. (63) we derive the formula for elastic damping factor which we define by:

$$D_{el}^2 \;=\; \frac{|\Delta F_2|}{|F_2^{NBA}|}\,, \tag{68}$$

where F_2^{NBA} denotes the correction to F_2 due to two Pomeron exchange or in other words F_2^{NBA} is the next order to the Born approximation of eq. (67). The formula for D_{el}^2 is:

$$D_{el}^2 \;=\; \frac{|(1 + \kappa_q)(E_1(\kappa_q) + \ln \kappa_q + C) \;+\; 1 \;-\; e^{-\kappa_q} \;-\; 2\,\kappa_q|}{\frac{\kappa_q^2}{4}}\,, \tag{69}$$

where κ_q is defined by eq. (56) at $\frac{a^2}{\beta} = Q^2$. In Fig.5 D^2_{el} is plotted for different values of Q^2. It is interesting to note that D^2_{el} turns out to be very close to 1 in a wide range of x_P because $D^2_{el} = 1 - \frac{\kappa_q}{9}$ at small value of κ_q.

Using D^2_{el} and $D^2_T(D^2_L)$ we can rewrite eq. (64) in more general form, namely

$$\frac{\Delta F_2(Q^2,x)}{F_2(Q^2,x)} = \tag{70}$$

$$- \frac{1}{\sigma_{tot}(Q^2,x)} \cdot \int_0^1 d\beta \frac{d\sigma_T^{DD}(Q^2,x,\beta)}{dx_P} \cdot \frac{D^2_{el}}{D^2_T} - \frac{1}{\sigma_{tot}(Q^2,x)} \cdot \int_0^1 d\beta \frac{d\sigma_L^{DD}(Q^2,x,\beta)}{dx_P} \cdot \frac{D^2_{el}}{D^2_L}.$$

The ratio $\frac{D^2_{el}}{D^2_T}$ can be rather large, and reaches a value of about 2 at low x in the HERA kinematic region.

4 Numerics and results.

4.1 What distances are essential in the DD processes?

The first question that we wish to address, is what are the distances which are important in the DD processes?

1. In the case of the longitudinal polarised photon, the answer is obvious, just from eq. (23). Indeed, in all calculations that lead to eq. (22) and eq. (23) the typical distances are $r_\perp \propto \frac{1}{a} = \frac{M}{Q\,k_\perp}$. Since the integral over $k_\perp$ in eq. (23) is logarithmic, we conclude that $k_\perp \approx M/2$, from which we estimate that the dominant distances are $r_\perp \approx \frac{2}{Q}$. Hence, for inclusive production initiated by a longitudinal polarised photon we can safely utilize the pQCD approach. In Fig.6 we plot our prediction for $\beta = 0.8$. In spite of the $1/Q^2$ suppression we obtain a large value for the cross section. This suggests an alternate way to extract the value of the gluon structure function, which is similar to vector meson production [6] [4], but has several advantages: (i) The cross section is larger than that for vector meson production; (ii) The prediction is independent of the form of the nonperturbative wavefunction of the produced vector mesons, which is an inherent difficultly in making a definite prediction for the cross section of DD for vector meson production [41] [42]. (iii) The seperation of the longitudinal polarised photon from the transverse polarised one, is not difficult, as we have not found any contamination fron transverse polarised photon induced reaction, for the events with large values of β (practically for $\beta > 0.7$).

2. In the case of the transverse polarised photon, the value of the distances for which the pQCD calculation of the DD cross section is valid, depends crucially on the behaviour of the gluon structure fiunction at relatively small values of the photon virtuality. Indeed, the cross section of eq. (28) has an extra k^2 in the dominator, and the integral on first sight appears to be infrared divergent. This is not so, as the gluon structure function should be proportional to k^2/μ^2 at small values of k^2. This fact is a direct consequence

of the gauge invariance of QCD. However, there is a danger that at very small values of k^2 (μ^2 is about the size of the confiniment radius) in the nonperturbative QCD region we still could have divergencies. We know that μ depends on x, and tends to be large at very small x [18][9]. The question arises what is the situation in the kinematic region at HERA ? In an attempt to answer this question we adopt the following strategy. We believe that available parametrizations of the deep inelastic structure functions (GRV[13] , MRS [26], CTEQ [27]) describe the behaviour of the gluon structure function in the region of relatively small virtualities. The common features of all these parametrization is the fact that the behaviour $xG(x,k^2) \propto k^2$ starts at virtualities which are sufficiently large in the small x region. For larger x and at larger values of k^2, $xG(x,k^2)$ starts to be proportional to k ($xG(x,k^2) \propto k/\mu$). For example, at $Q^2 = 2.5GeV^2$ in the GRV parametrization $xG(x,k^2) \propto k/\mu$ at $x < 3.10^{-3}$, while in the MRS(A) this happens at $x < 10^{-2}$. Relying on the available sets of deep inelastic structure function (which contain all accumulated experimental information on the subject), then even for the transverse polarized photon, the typical distances in the DD processes are not larger than $r_\perp \approx 1GeV^{-1}$. Hence, we can apply the pQCD approach in this case as well.

In Fig.7 we plot the integrand of eq. (28)

$$I(\beta,Q^2,x_P) \;=\; \frac{\left(x_P G(x_P,\frac{k_\perp^2}{1-\beta})\right)^2}{k_\perp^2}\cdot\{1\,-\,2\,\frac{k_\perp^2}{M^2}\}\cdot\frac{1}{\sqrt{1\,-\,\frac{4\,k_\perp^2}{M^2}}}$$

as a function of x_P and Q^2 at fixed β, which illustrates this claim. We wish to emphasise that the integrand can be viewed as the product of two factors: $(x_P G(x_P,\frac{k_\perp^2}{1-\beta}))^2/k_\perp^2$ and the kinematic factor $[\,1-2\,\frac{k_\perp^2}{M^2}\,]\cdot\frac{1}{\sqrt{1-\frac{4k_\perp^2}{M^2}}}$. The first factor has a maximum at $k_\perp^2 \approx 1\,GeV^2$ for all available parametrization of the gluon structure functions. The origin of this maximum is very simple and can easily be seen if one uses the semiclassical parametrisation of the solution to the evolution equations in the form:

$$x_P\, G(x_P,k^2) \;\rightarrow\; (\frac{1}{x_P})^{\omega(x_P,k^2)}\;(k^2)^{\gamma(x_P,k^2)} \quad at \;\; x_P \;\rightarrow\; 0\;,$$

where both ω and γ are smooth functions of $\ln(1/x_P)$ and $\ln k^2$. The common feature of the all parametrisatios for the gluon structure function is the fact that $\gamma \;>\; \frac{1}{2}$ at $x_P \;\leq\; 10^{-3}$. Such a behaviour manifest itself in a maximum of the first factor at $k^2 \approx 1GeV^2$. It should be stressed that the argument of the gluon structure function in eq. (28), namely $k_\perp^2/(1-\beta)$ leads to substantial increase of the typical transverse momentum in the integral, or in other words in a decrease of the typical value of the distances essential for the diffractive $\bar{q}\,q$ production. The kinematic factor also tends to increase at large value of k^2.

Finally, we would like to repeat our statement that the DD cross section for both longitudinal and transverse polarised photon occur at small distances. This fact is crucial for justifing the use of perturbative QCD for the calculation of the DD processes.

4.2 The x_P dependence, factorization and "Pomeron structure function".

In Fig.8 we plot the x_P dependance of calculated

$$x_P\, F_2^{DD(3)}(Q^2,\beta,x_P) \;=\; \frac{Q^2}{4\,\pi^2\,\alpha_{em}} \int dt\; x_P\,\frac{d\sigma}{dx_P dt} \;=\; \frac{Q^2}{4\,B\,\pi^2\,\alpha_{em}}\; x_P\,\frac{d^2\sigma}{dx_P dt}|_{t=0}\;, \quad (71)$$

where B is the slope of the DD cross section. We take $B = 4.5 GeV^{-2}$ in accord with the the prelimenary experimental data from HERA (see ref. [28]). The value of B has not yet been measured with good accuracy. The above value of B coincide with the slope that has been extracted from the high energy phenomenology of "soft" processes (see, for example, ref. [31]).

We wish to stress that in all our comparisons with the experimental data, including Fig.8, we include the SC, which have been calculated in the previous section. This means that the formulae for the cross sections, obtained in section 2, are multiplied by the damping factor D^2_L or D^2_T, calculated in section 3.

We now concentrate on x_P dependence of $F_2^{DD(3)}$. Based on the Ingelman-Schlein hypothesis of the Pomeron structure function, $F_2^{DD(3)}$ can be written in the factorised form, namely

$$F_2^{DD(3)} \;=\; f(x_P)\; F_2^P(Q^2,\beta)\;, \quad (72)$$

where F_2^P is the Pomeron structure function and $f(x_P)$ is the Pomeron flux factor which is proportional to $f(x_P) \;\propto\; (\frac{1}{x_P})^n$ if the Pomeron is a simple Regge pole. n is intimately related to the intercept of the Pomeron trajectory ($\alpha_P \;=\; \alpha_p(0) \,+\, \alpha'_P|t|, \alpha_P(0) \;=\; 1 + \epsilon$) , namely $n \;=\; 2\,\alpha_P(0) \;-\; 1 \;=\; 1 \;+\; 2\,\epsilon$.

Our approach is orthogonal to the IS one. We do not expect eq. (72) to scale, and if we use the parametrisation suggested in eq. (72), and fit the x_P dependence in the form

$$F_2^{DD(3)} \;=\; (\frac{1}{x_P})^n\; F_2^P(Q^2,\beta) \quad (73)$$

we expect n to depend on both β and Q^2.

We also show in Fig.9 $n(Q^2,\beta)$ which we get from our calculation. We would like to point out that the value of n is much bigger ($n \;\approx\; 0.55$) than that was observed experimentally ($n \approx 0.2$) (see ref. [29]), this arises from the steep behaviour of the gluon structure function in the GRV parametrisation. Notice, however, that the new ZEUS data [12] gives the value of $n \;\approx\; 0.4$, which is much closer to our result.

Direct comparison with the experimental data given in Fig.8 shows that our calculation is able to describe the experimental data at $x_P \;\approx\; 10^{-3}$, but it substancially overshoots the experimental data for smaller values of x_P. We think this behaviour is an artifact of the GRV parametrisation, in the leading order of perturbative QCD.

One can see from Fig.9 where the ratio of the leading order GRV gluon structure to the next to leading order one ($x_P G(x_P,Q^2)^{LO}/x_P G(x_P,Q^2)^{NLO}$) is plotted that the use of $x_P G(x_P, Q^2)^{NLO}$ instead of $x_P G(x_P, Q^2)^{LO}$ suppresses the value of the diffractive cross section at small x_P. It is interesting to note that the effective power n reduces to the value $n \approx 0.3$ which is much closer to the experimental one, especially to the new ZEUS data [12].

The second source of suppression at low x_P comes from the energy behaviour of the slope B, where $B = B_0 + 2\alpha'_P \ln(1/x_P)$ in the case of the Reggeon approach to the structure of the Pomeron, where the Pomeron trajectory is $\alpha_P(t) = 1 + \epsilon + \alpha'_P t$.

Nevertheless, the fact that the perturbative QCD calculation predicts a larger cross section than experimental one, lends support to our statement in the previous subsection, namely, that at small distances, where we can trust the perturbative QCD approach, the calculation of the diffractive production of small mass, appears to work.

How the x_P dependence is affected by the different parametrization of the structure function is still an open question, we plan on clarifying this in our further publications.

4.3 $F_2^{DD(2)}$.

In Fig.10 we display the calculated values for $F_2^{DD(2)}$ defined as

$$F_2^{DD(2)} = \int_{x_{P\,max}}^{x_{P\,min}} dx_P \; F_2^{DD(3)} \tag{74}$$

where the values of $x_{P\,min}$ and $x_{P\,max}$ are taken to have the same values as in the relevant experiment. We also indicate the contribition of different subprocesses in the diffractive production.

In Fig.11 we compare the calculated values with the relevant experimental data.

From these figures we are able to conclude:

1.The pQCD calculation provides a fair description of the experimental data;

2. The contribution of the longitudinal polarised photon is important at large $\beta > 0.7$;

3. The process with an emission of the extra gluon gives rise to a sizeble contribution for $\beta < 0.4$ amd should be taken into account in more consistent manner by solving the evolution equation for DD processes (see the next section);

4. For $\beta > 0.4$ the emission of an extra gluon provides only a small contribution, so one can attempt to extract the gluon structure function by measuring $F_2^{DD(2)}$ at large β.

4.4 The transverse momentum spectra.

Our formulae (see Eqs.(23),(28),(36),(37) and (41)) can be used for more detailed analysis of DD events. In particular, we can describe the transverse momentum spectra of produced parton jets in DD. Basically these spectra are given by the integrand of our formulae, as the $k_\perp$ which appears is just the transverse momentum of the produced jet.

In Fig.12 the ratio

$$R = \frac{F_2^{DD(2)}(k_\perp^2 > k_0^2)}{F_2^{DD(2)}} \tag{75}$$

is presented with different values of k_0^2. The physical meaning of this ratio is that it indicates the fraction of the all DD events possessing transverse momenta larger than k_0. One can see from Fig.13 that we expect a fairly large fraction of the events with big transverse momenta, this is in agreeement with the new H1 data (see ref.[30]).

In Fig.13 we plot the calculated value

$$x_P F_2^{DD(4)} = \frac{Q^2}{4\pi^2 B\,\alpha_{em}} \frac{x_P\, d^3\sigma}{dx_P dt dk_\perp^2} |_{t=0} ,$$

where $k_\perp$ denotes the transverse momentum of the jet. The comparison with the experimental data is also quite qood, at least, the $k_\perp$ distribution reproduces the main qualitative feature of the experimental data [30], namely, smoother behaviour than $\frac{1}{k_\perp^4}$ at small values of $k_\perp^2$, and $\frac{1}{k_\perp^4}$ behaviour for larger $k_\perp$. On the other hand, we predict a sharp drop at large $k_\perp$, which has not seen in the data. Perhaps, the emission of two gluons starts to be important at such large values of transverse momenta.

4.5 Matching with the vector meson diffraction.

The vector meson production corresponds to the kinematic region of $\beta \to 1$, namely

$$\beta = \frac{Q^2}{Q^2 + M_V^2} \to 1 - \frac{M_V^2}{Q^2} , \tag{76}$$

where M_V^2 is the mass of produced vector meson. In this kinematic region we can rewrite eq. (23) and eq. (28) in the form:

$$M^2 \frac{d\sigma^L}{dM^2 dt} |_{t=0} = \tag{77}$$

$$\frac{4\pi^2 \alpha_{em}}{3} \sum Z_f^2\, \alpha_S^2 \frac{M_V^2}{Q^6} \int_{Q^2_{min}}^{\frac{M_V^2}{4}} \frac{d k_\perp^2}{k_\perp^2} \frac{1}{\sqrt{1 - \frac{4k_\perp^2}{M_V^2}}} \left(x_P\, G(x_P, \frac{k^2\, Q^2}{M_V^2}) \right)^2 .$$

$$M^2 \frac{d\sigma^T}{dM^2 dt} |_{t=0} = \tag{78}$$

$$\frac{4\,\pi^2\,\alpha_{em}}{3}\sum Z_f^2\,\alpha_S^2\,\frac{M_V^6}{Q^8}\int_{Q^2_{min}}^{\frac{M_V^2}{4}}\frac{d\,k_\perp^2}{k_\perp^4}\,\frac{1}{\sqrt{1\,-\,\frac{4k_\perp^2}{M_V^2}}}\{1\,-\,2\,\frac{k_\perp^2}{M_V^2}\}\,\left(x_P\,G(x_P,\frac{k^2\,Q^2}{M_V^2})\right)^2\,.$$

These formulae give the cross sections for production of all hadrons with mass M_V and they reproduce the main features of the exclusive vector meson production (see refs. [3] [4]). It is interesting to calculate the cross sections in the region of $M_V^2 = m_\rho^2$ where we do not expect any other mesons, besides the ρ-meson to be produced. In the double log approximation of the perturbative QCD, we can put the argument of the gluon structure function equal to Q^2. Evaluating the remaining integrals we obtain:

$$\frac{\sigma^L}{\sigma^T}\;=\;\frac{Q^2\,Q^2_{min}\,ln\frac{M_V^2}{4\,Q^2_{min}}}{M_V^4}\,. \tag{79}$$

The value we choose for Q^2_{min}, depends on how strong our belief in hadron - parton duality is. If we believe that the hadron - parton duality can be used at large distances, we can choose the low limit of integration from the condition that the argument of the gluon structure function in eq. (77) and in eq. (78) is bigger than the intitial vituality in the GRV parametrization, namely $Q^2_{min}Q^2/M_V^2 = Q_0^2$ or $Q^2_{min} = Q_0^2M_V^2/Q^2$. For such a value of Q^2_{min} we have $\sigma^l/\sigma^T = \frac{Q_0^2}{M_V^2}\ln\frac{4\,Q^2}{Q_0^2}$.

The experience from e^+e- annihilation teaches us that Q^2_{min} could be very small, about m_π^2 [35]. In this case, we still have the Q^2 rise of the ratio, but the coefficient is very small, namely $\sigma^L/\sigma^T = 0.125Q^2$.

In Fig.14 we plot σ^L and σ^T, using eq. (77) and eq. (78) with cutoff $Q^2_{min} = m_\pi^2$.

Concluding this section, we would like to reiterate our claim that the simple pQCD calculation provides a reliable basis for the discussion of the origin of the DD in DIS. It also suggests a the new way to determine the gluon structure function. This new measurement can be done in two unambigous way:

1. the measurement of $F_2^{DD(2)}$ at $\beta > 0.7$, allows us to extract the longitudinal polarised photon structure function which is only sensitive to small distances of the order $r_\perp \approx 1/Q$.

2. the measurement of $F_2^{DD(2)}$ for $k_\perp > k_0 \geq 1GeV^2$. In this case only distances $r_\perp \leq 1/k_0$ contribute to our formulae, and we have reliable pQCD predictions for the fraction of DD events.

5 Evolution equations for the DD.

In this section we discuss the evolution equations for the DD structure functions that have been introduced in the previous section. The evolution equations for the transverse polarised photon have been proposed and derived in pQCD in ref.[7], and we will comment on them later. We first discuss the evolution equations for the longitudinal polarised photon, which have not been formulated previously, these provide an interesting theoretical insight into the structure of the evolution of exclusive processes in QCD.

5.2 Evolution of the DD structure function (greneral consideration).

In the general case we can rewrite eq. (24) in the form

$$F_L^{DD} = \frac{\alpha_S^2}{N_c}\int_{Q_0^2}^{Q^2(1-\beta)}\frac{d\,k_\perp^2}{k_\perp^2}\int_\beta^1 dz\frac{\beta}{z}\,(\Sigma_S^S(\frac{\beta}{z},\ln\frac{Q^2}{k_\perp^2})\;\Phi_{P;L}^F(\,x_P\,G(\,x_P,\frac{k_\perp^2}{(1-\beta)}\,))^2 \quad (85)$$

where Σ denotes the singlet quark and antiquark structure function $(q(x,Q^2)+\bar{q}(x,Q^2))$ and the Pomeron splitting function Φ_L^F has been defined above (see eq. (40)). It should be stressed that only Σ_S^S enters the evolution equation in the leading order of pQCD.

Differentiating the above equation we obtain the evolution equations for the monents in the form:

$$\frac{df_{int;L}^{DD;S}(\omega,Q^2)}{d\ln Q^2} = \quad (86)$$

$$P_S^S(\omega)\;\cdot\;f_{int;L}^{DD;S}(\omega,Q^2)\;+\;P_G^S(\omega)\;\cdot\;f_{int,L}^{DD;G}(\omega,Q^2)\;+\;\frac{\alpha_S^2}{N_c}f^{G^2}(\omega,Q^2)\;,$$

where P are the kernels of the GLAP evolution equations. We obtain the ordinary GLAP evolution equations with an inhomogenious term. This equation has the solution:

$$f_{int}^{DD}(\omega,Q^2) = \quad (87)$$

$$\frac{\alpha_S^2}{N_c}\,f^{G^2}(\omega,Q_0^2)\,[\,\frac{1}{\gamma_2(\omega)\,-\,\gamma^+(\omega)}+\frac{1}{g_2(\omega)\,-\,\gamma^-(\omega)}\,]\cdot e^{\gamma_2(\omega)\ln\frac{Q^2}{Q_0^2}}$$

$$+\;f_{int}^{DD}(\omega,Q_0^2)\;\cdot\;\{\,\frac{-\gamma^-}{\gamma+(\omega)\,-\,\gamma^-(\omega)}e^{\gamma^+(\omega)\,\ln\frac{Q^2}{Q_0^2}}+\frac{\gamma^+}{\gamma+(\omega)\,-\,\gamma^-(\omega)}e^{\gamma^-(\omega)\,\ln\frac{Q^2}{Q_0^2}}\}$$

where the functions $f_{int}^{DD}(\omega,Q_0^2)$ and $F^{G^2}(\omega,Q_0^2)$ denote the intial conditions for the evolution equation, and describes large distance physics. It is obvious that the second term of eq. (87) is the solution of the GLAP equation for fixed α_S, which we have assumed in order to simplify the solution. One can find this form of the solution in refs. [33] [34] and in references contained therein. $\gamma^{+,-}$ denotes the eigenvalues of the matrix of anomalous dimensions and in the leading order of pQCD they are equal to:

$$\gamma^\pm = \frac{1}{2}\{P_F^F\;+\;P_G^G\;\pm\;\sqrt{(\,P_F^F\;-\;P_G^G\,)^2\;+\;4P_G^FP_F^G}\,\} \quad (88)$$

where

$$P_F^F = \frac{C_F\alpha_S}{2\pi}\{\frac{3}{2}-\frac{1}{\omega+1}-\frac{1}{\omega+2}-2\,[\psi(\omega+1)\,-\,\psi(1)]\,\}\;;$$

$$P_G^G = \frac{\alpha_S}{2\pi}\{2C_A\,[\frac{11}{12}+\frac{1}{\omega}-\frac{2}{\omega+1}+\frac{1}{\omega+2}-\frac{1}{\omega+3}-\psi(\omega+1)+\psi(1)\,]\,-\,\frac{2T_RN_f}{3}\}\;;$$

$$P_F^G = \frac{C_F\alpha_S}{2\,\pi}\,\{\frac{2}{\omega}-\frac{2}{\omega+1}+\frac{1}{\omega+2}\} \quad (89)$$

$$P_G^F = \frac{2T_R N_f \alpha_S}{2\pi} \{ \frac{2}{\omega+3} - \frac{2}{\omega+2} + \frac{1}{\omega+1} \} .$$

It should be stressed that in pQCD only γ^+ has a singularity in the second term at small ω, and $\gamma^+ \ll g_2(\omega)$ at $\omega \rightarrow 0$. This means that only the first term survives at small x.

We do not claim that this result is valid at small virtualities, where pQCD arguments are not applicable. The value of the diffractive dissociation at small vituality of the photon ($Q^2 = Q_0^2$) which can be measured experimentally, defines only the specific sum of singularities, namely:

$$f_{int;L}^{DD;S}(x_B, Q_0^2) = \qquad (90)$$

$$\frac{1}{2\pi i} \int_C d\omega \{ \frac{\alpha_S^2}{N_c} f^{G^2}(\omega, Q_0^2) [\frac{1}{\gamma_2(\omega) - \gamma^+(\omega)} + \frac{1}{\gamma_2(\omega) - \gamma^-(\omega)}] + f_L^{DD;S}(\omega, Q_0^2) \}$$

The fact that these two terms have different anomalous dimensions, suggests that one might be able to distinguish between them using their x_P dependance. The second term is proportional to $(x_P G(x_P, Q_0^2))^2$ for the unintegrated F^{DD}, but there is no theoretical argument why such a behaviour should be valid for a small value of Q_0. Phenomenologically, we associate these two contributions with so called "soft" and "hard" Pomeron. This terminology just reflects our hope that they would have different x_P behaviour, namely, assuming that their behaviour can be approximately parameterized as $x_P^{2\epsilon}$, we expect for the "soft" (second) term $\epsilon_{soft} \sim 0.08$ - 0.1 while for the "hard" one (first term in eq. (87)) $\epsilon_{hard} \sim 0.2$-0.5. It should be stressed that the problem of the separation of these two contributions is beyond the scope of pQCD, and has to be tackled using a nonperturbative approach. As a guide to phenomenological applications we would like to mention that the Ingelman and Schlein approach based on the so called Pomeron structure function, means that only the "soft" contribution survives, while the "hard" Pomeron approach deals only with the first tem in eq. (87). Returning to F^{DD} (see eq. (85)) we can write the evolution equation at fixed x_P:

$$\frac{\partial\ F_L^{DD;S}(\beta, x_P, Q^2)}{\partial\ \ln Q^2} = \qquad (91)$$

$$P_F^F(\omega)\ F_L^{DD;S}(\beta, x_P, Q^2) + P_G^F(\omega)\ F_L^{DD;G}(\beta, x_P, Q^2)\ \frac{\alpha_S^2}{N_c}\ \beta^2 (1-2\beta)^2\ (x_P\, G(x_P, \frac{Q^2}{4\beta}))^2$$

In moment space with respect to $\ln(1/\beta)$, the equation reduces to the form:

$$\frac{d\, f_L^{DD;S}(\omega, x_P, Q^2)}{d \ln Q^2} = \qquad (92)$$

$$P_S^S(\omega)\, f_L^{DD;S}(\omega, x_P, Q^2) + P_G^S(\omega)\, f_L^{DD;G}(\omega, x_P, Q^2) + t(\Omega) \left(x_P\, G(x_P, Q^2) \right)^2 ,$$

where

$$T_{G;L}^F(\omega) = \frac{1}{\omega+2} - \frac{4}{\omega+3} + \frac{4}{\omega+4} \rightarrow |_{\omega\rightarrow 0} \frac{1}{6} .$$

The general solution of the above equation is

$$f_{int}^{DD}(\omega, Q^2) = \qquad (93)$$

$$\frac{\alpha_S^2}{N_c}\,\{\,f_{int}^{DD}(\omega,Q_0^2)\;\cdot\;\{\frac{-\gamma^-(\omega)}{\gamma+(\omega)\;-\;\gamma^-(\omega)}e^{\gamma^+(\omega)\ln\frac{Q^2}{Q_0^2}}\;+\;\frac{\gamma^+(\omega)}{\gamma+(\omega)\;-\;\gamma^-(\omega)}e^{\gamma^-(\omega)\ln\frac{Q^2}{Q_0^2}}\}$$

$$+\,[\frac{1}{\gamma_2(\omega)\;-\;\gamma^+(\omega)}\;+\;\frac{1}{g_2(\omega)\;-\;\gamma^-(\omega)}]\cdot\int_{Q_0^2}^{Q^2}\frac{dQ'^2}{Q'^2}\,T(\omega)\,\left(x_P\,G(x_P,Q^2)\right)^2\}$$

$$=\;f_{SOFT}^{DD}+f_{HARD}^{DD}\;.$$

The first term F_{SOFT}^{DD} is associated with soft diffraction, and it requires the phenomenological input $f^{DD}(\omega,x_P,Q_0^2)$. We choose for this input :

$$f^{DD}(\omega,x_P,Q_0^2)\;=\;T(\omega)\,(\,x_P\,G(x_P,Q'^2)\,)^2\;. \tag{94}$$

We still need to fix the value of Q_0 we should take for our initial condition. There is no reason to assume the same value of Q_0 in the DD and DIS. We choose the value of Q_0^2 for the DD case to be $1-2GeV^2$. In doing so we can use pQCD to evaluate the two terms appearing in eq. (93), this leads to eq. (94) for the first term.

In β - representation the intial condition looks simple, namely

$$F_L^{DD}(\beta,x_P,Q_0^2)\;=\;\frac{\alpha_S^2}{N_c}\beta^2\,(1\;-\;2\beta)^2\;\left(x_P\,G(x_P,\frac{Q_0^2}{4\beta})\right)^2\;. \tag{95}$$

This is plotted in Fig.15 for different values of x_P.

5.3 Evolution equation for transverse polarized photon initiated DD.

The evolution equations for the transverse polarised photon has been given in ref. [7]. Introducing the structure functions $q_P(x,Q^2)$ and $G_P(x,Q^2)$ which denote the quark and gluon density in the Pomeron, we can rewrite the evolution equations in the form (see ref.[7] for details):

$$\frac{\partial\,\Sigma_P(x,Q^2)}{\partial\ln Q^2}\;= \tag{96}$$

$$\int_\beta^1\frac{dz}{z}\,\{P_S^S(z)\,\Sigma_P(\frac{\beta}{z},Q^2)\;+\;P_G^S(z)\,G_P(\frac{\beta}{z},Q^2)\}\;+\;\frac{1}{Q^2}\,\Phi_{P;T}^q(\beta)\,\left(x_P\,G(x_P,Q_0^2)\right)^2\;;$$

$$\frac{\partial\,G_P(x,Q^2)}{\partial\ln Q^2}\;=\;\int_\beta^1\frac{dz}{z}$$

$$\{P_S^G(z)\,\Sigma_P(\frac{\beta}{z},(1-\beta)Q^2)\;+\;P_G^G(z)\,G_P(\frac{\beta}{z},(1-\beta)Q^2)\}$$

$$+\;\frac{1}{Q^2}\,\Phi_{P;T}^G(\beta)\,\left(x_P\,G(x_P,Q_0^2)\right)^2\;,$$

where $\Sigma\;=q(x,Q^2)\;+\;\bar{q}(x,Q^2)$.

On inspection we note that the structure of the equations is the same as for the longitudinal polarised photon, namely, eq. (96), and is a normal GLAP evolution equation with a inhomogenious term. P are the kernels of the GLAP equation while Φ denote the Pomeron splitting functions:

$$\Phi^{S}_{P;T} \;=\; \frac{\alpha_S}{N_c}\, z^2\,(1-z)^2\;; \tag{97}$$

$$\Phi^{G}_{P;T} \;=\; \frac{\alpha_S N_c^2}{2(N_c^2-1)}\,\frac{1}{z}\,(1-z)^2\,(1+2z)^2\;.$$

The set of eq. (96) can be solved in the moment representation using the standard technique (see refs. [33] [34] and references therein). The main difference in comparison with the lomgitudinal polarised photon, is the fact that the inhomogenious term has extra $1/Q^2$ suppression, provided that we choose the appropriate initial condition for eq. (96). The contribution of this term in the evolution equation is consequently small. we have a sufficiently small contribution of this term in the evolution. To understand the structure of the result we write the solution of eq. (96) in the region of small β ($\omega \to 0$), where the emission of extra gluons and quarks could be important (as have been seen in section 3). In the region of small ω we can neglect γ^- (see for example ref. [34]) and the solution has a simple form:

$$\Sigma_P(\omega,Q^2) \;=\; \tag{98}$$

$$,\{\,\Sigma_P(\omega,Q_0^2)\,\frac{\gamma^-(\omega)}{\gamma^+(\omega)\,-\,\gamma^-(\omega)}\;+\;G_P(\omega,Q_0^2)\frac{2N_f P^S_G(\omega)}{\gamma^+(\omega)\,-\,\gamma^-(\omega)}\}\,\cdot\,e^{\gamma^+(\omega)\,\ln(Q^2/Q_0^2)}$$

$$+\,,\int^{Q^2}_{Q_0^2}\frac{dQ'^2}{Q'^4}\,\frac{\Phi^S_{P;T}(\omega)\,+\,\Phi^G_P(\omega)}{\tilde{\gamma}_2(\omega)\,-\,\gamma^+(\omega)}(\,x_P\,G(x_P,Q'^2)\,)^2\;,$$

where $\tilde{g}_2(\omega)\;\to\;\frac{4N_c\alpha_S}{\pi\omega}-1$ at $\omega\;\to\;0$ and $\Phi_P(\omega)$ is the ω image of the Pomeron splitting functions of eq. (97). They are of the form:

$$\Phi^{S}_{P;T} \;=\; \frac{\alpha_S}{N_c}\{\frac{1}{\omega+2}-\frac{2}{\omega+3}+\frac{1}{\omega+4}\}; \tag{99}$$

$$\Phi^{G}_{P;T} \;=\; \frac{\alpha_S N_c^2}{2(N_c^2-1)}\{\frac{1}{\omega}+\frac{2}{\omega+1}-\frac{3}{\omega+2}-\frac{3}{\omega+3}-\frac{4}{\omega+4}+\frac{4}{\omega+5}\}\;.$$

To fix the two initial moments $\Sigma_P(\omega,Q_0^2)$ and $G_P(\omega,Q_0^2)$ which enter the solution of one phenomenological term f^{DD}_{SOFT} at low β, we need to choose a phenomenological input. We use Reggeon phenomenology to fix this term, [36] rewriting it in the form:

$$f^{DD}_{SOFT}(\omega,x_P,Q_0^2) \;=\; (\,\frac{1}{x_P}\,)^{2\varepsilon_{soft}}\;f^{DD}_{SOFT}(\omega,Q_0^2) \tag{100}$$

where $F^{DD}(\omega,Q_0^2)$ is the ω -image of $G_{PRP}\;\beta^{\Delta_R(0)}(1-\beta)^n$. $\Delta_R(0)\;=\;\alpha_R(0)-1$ where $\alpha_R(0)$ is the intercept of the secondary Reggeon trajectory ($\alpha_R(0)\;\sim\;0.5$). The power n is not defined in Reggeon phenomenology and we only know that $n>1$. G_{PRP} denotes the Pomeron - Reggeon vertex (see for example ref. [36]).

The second term is the hard diffractive contribution, which is calculated theoretically.

There is an alternate way to implement the initial condition, i.e. using the result of our calculation in low mass region. Namely, we showed in our calculation for small mass production, that the typical virtuality is sufficiently large . Therefore, we can choose the value of the initial virtuality Q_0^2 for the DD process to be about $1-2GeV^2$ and calculate f^{DD}_{SOFT} in pQCD. In this case

$$\Sigma_P(\omega,Q_0^2) \;=\; \frac{1}{Q_0^2}\,\Phi^S_{P;T}(\omega)\;(\,x_P\,G(x_P,Q'^2)\,)^2$$

and

$$G_P(\omega,Q_0^2) \;=\; \Phi^G_P(\omega)\,(\,x_P\,G(x_P,Q'^2)\,)^2$$

.

This input corresponds the initial distributions in the β - representation:

$$\Sigma_P(\beta,x_P,Q_0^2) \;= \tag{101}$$

$$\frac{1}{Q_0^2}\,\Phi^S_{P;T}(\beta)\,\left(x_P\,G(x_P,Q_0^2)\right)^2\;;\quad G_P(\beta,x_P,Q_0^2) \;=\; \frac{1}{Q_0^2}\,\Phi^G_{P;T}(\beta)\,\left(x_P\,G(x_P,Q_0^2)\right)^2\;.$$

In Fig.16 we plot this distributions at different values of x_P at two values of Q_0^2 ($Q_0^2 = 1Gev^2$ and $Q_0^2 = 2GeV^2$).

We wish to emphasis two points. One, we propose to separate the soft and hard diffraction processes in a way which conforms with the factorization theorem (see ref. [37]) for the integrated diffractive structure function. Although, there is no proof that the factorization theorem holds for the case of diffractive contribution, in deep inelastic scattering (see ref. [38]), all known contributions which violate the factorization theorem turn out to be small (see discussion above and ref. [39] [32] [40]).

The second point is the value of the scale Q_0. There is no reason to assume the same value of Q_0, taken in the evolution equation for the deep inelastic structure function. Depending on the choice of the scale Q_0^2, we can start the evolution with very small values of Q_0 and use Reggeon phenomenology as an initial condition for the DD structure function. Or, we can start with sufficently large value of Q_0 in the DD processes, and reconstruct the initial distributions using pQCD calculations. We followed the second alternative, and our calculations for small masses supports this strategy.

6 Conclusions.

The diffraction dissociation processes in DIS provide a new window to collective phenomena in the parton cascades since they originate from parton - parton interactions, and vanish if there is no interaction between partons. However, the DD processes possess an inherent awkwardness in that it is difficult to predict what is small or large in DD. Indeed, in two extreme limits for small and for large (“black disc”) parton - parton interactions the cross section of the DD turns out to be small. In this paper an attempt is made to discuss the DD process within a pQCD framework.

Our main results look are:

1. The DD cross section originates at small distances ($r_\perp$ <0.2 fm) even for the transverse polarised photon. This fact justifies our use of pQCD for DD processes.

2. For β $>$ 0.4 only production of $\bar{q}q$ pairs contribute to the cross section of DD. This suggests a method to extract the value of the gluon structure function from the DD measurements in this kinematic region.

3. For β >0.7 the longitudinal polarised photon provides the dominant contibution to DD processes. The DD of longitudinal photon is one of the most promising processes for pQCD calculations, since only small distances ($r_\perp$ $\propto$ $1/Q$) contibute to this process. Therefore, the measurement of DD at such big values of β allows one to extract the value of the gluon structure function which has all advantages of the vector meson production in DD (see, for example, ref. [4]) but without any of the uncertainties present due to our poor knowledge of the hadronic wave function (see refs. [42] [43]).

4. The general approach to the evolution equation for DD has been formulated and solutions to the evolution equations have been found. The main difference between the evolution equations for the DD and the GLAP evolution equations for the deep inelastic structure functions, is the appearance of the inhomogenious term in the DD equations. The form of this term and its influence on the solution has been discussed.

5. The formulae for the SC has been obtained, and the damping factors have been calculated. The main result here is, that SC are important in the case of DD, and the value of the cross section for the DD processes allows us to estimate the corrections to the deep inelastic structure function (F_2) which turns out to be sufficiently large, namely, $\frac{\Delta F_2}{F_2} \approx 2\,\frac{\sigma^{DD}}{\sigma_{tot}}$.

We consider our paper as a first step in the understanding of the DD as a new source of information about collective phenomena in deep inelastic scattering in the region of small x. We hope this paper will give some impetus to treat DD as a "hard"" process, within the solid theoretical framework of pQCD.

Acknowledgements: While completing this paper our attention was drawn to two recent papers [44] [45] which deal with some of the topics covered in this paper. E.M. Levin wishes to thank LAFEX-CBPF for kind hospitality and CNPq, Brazil for partial financial support.

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Figure Captions.

Fig.1 : The diffractive production of quark-antiquark pair.

Fig.2: The extra gluon emission in the diffractive production.

Fig.3: The damping factor for longitudinally polarised photon.

Fig.4: The damping factor for transversely polarised photon.

Fig.5: The damping factor for $F_2(x, Q^2)$.

Fig.6: The DD by longitudinally and transversely polarised photon at $\beta = 0.8$.

Fig.7: The integrand of Eq.(28).

Fig.8: $x_P\, F_2^{DD(3)}$ versus x_P.

Fig.9: Effective power $n(\beta, Q^2)$ and ratio $\frac{x_P G(x_P,Q^2)^{LO}}{x_P G(x_P,Q^2)^{NLO}}$ in the GRV paramerization.

Fig.10: Different contributions to $F_2^{DD(2)}$.

Fig.11: Comparison of $F_2^{DD(2)}$ with the HERA experimental data.

Fig.12: Ratio R (Eq.(75) for different values of cutoff k_0.

Fig.13: The cross section for ρ production (in arbitrary units) for Eqs.(77) and (78).

Fig.14: The transverse momentum distribution for the jet in the DD.

Fig.15: The initial parton distribution (Eq.(91)) for the DD evolution equation (longitudinally polarised photon).

Fig.16: The initial parton distributions (Eq.(97)) for the DD evolution equations (transversally polarised photon).

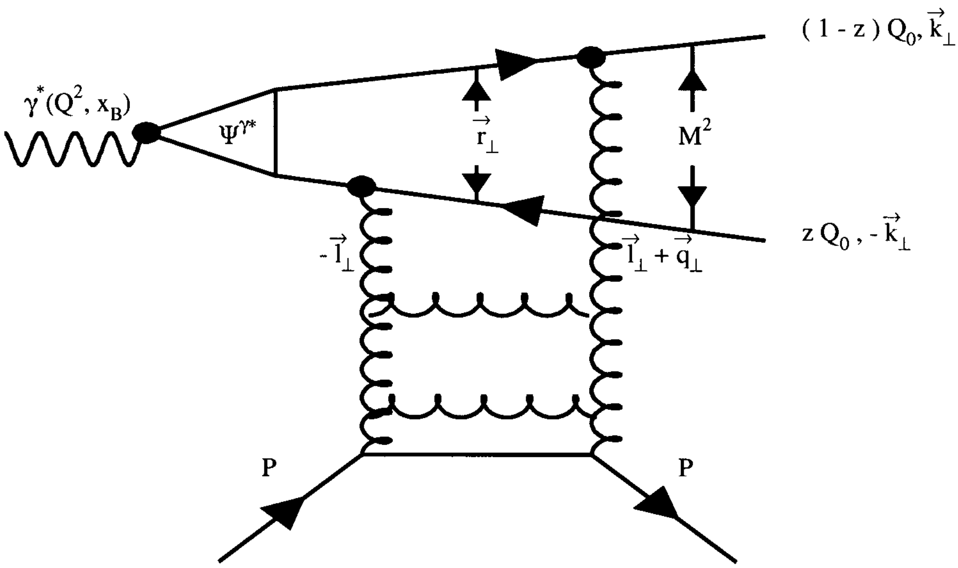


Figure 1: The diffractive production of quark - antiquark pair.

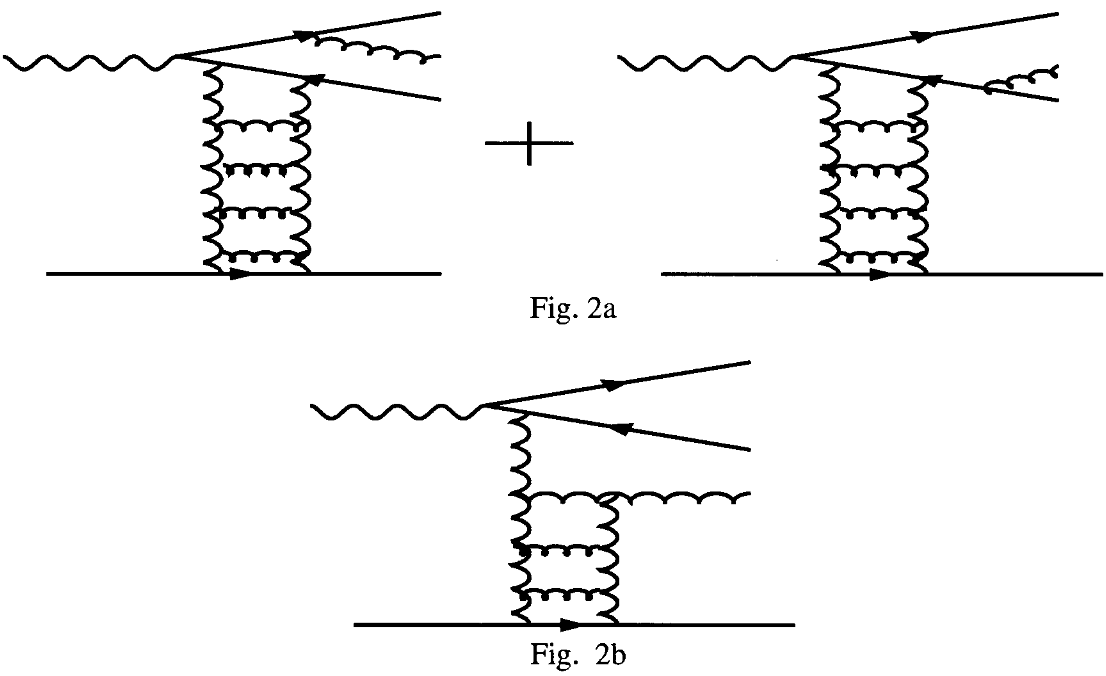


Figure 2: The extra gluon emission in the diffractive production.

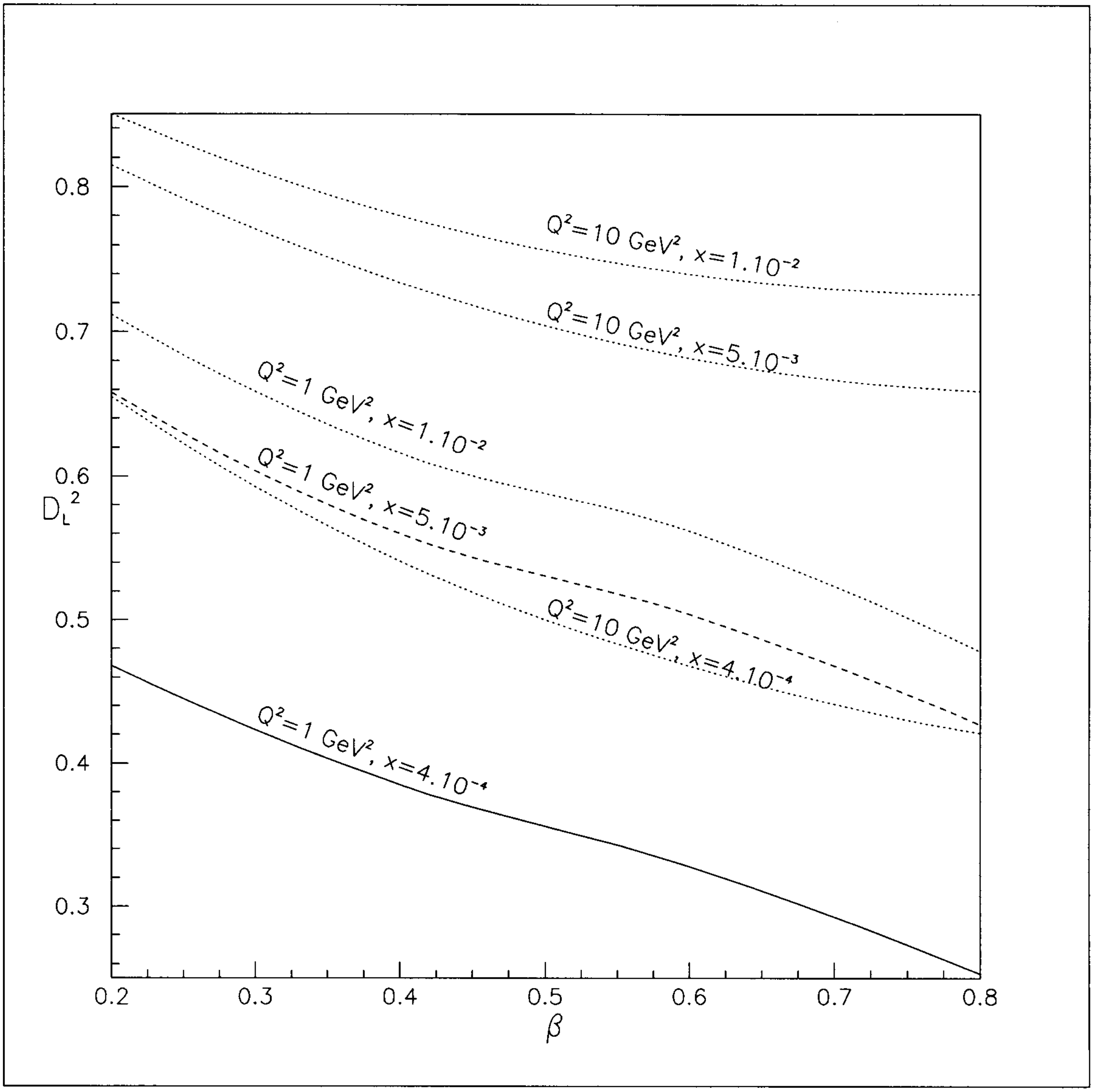


Figure 3: The damping factor for longitudinally polarised photon.

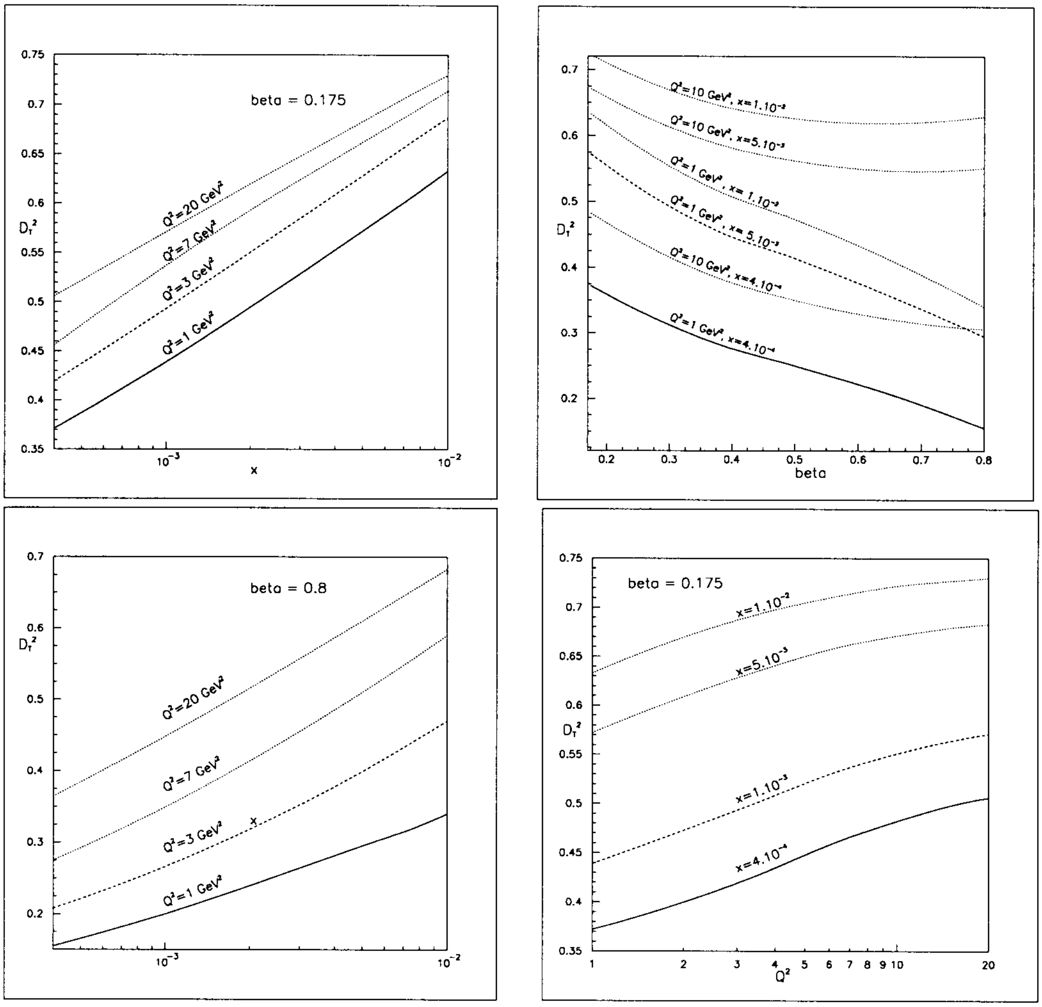


Figure 4: The damping factor for the transversely polarised photon.

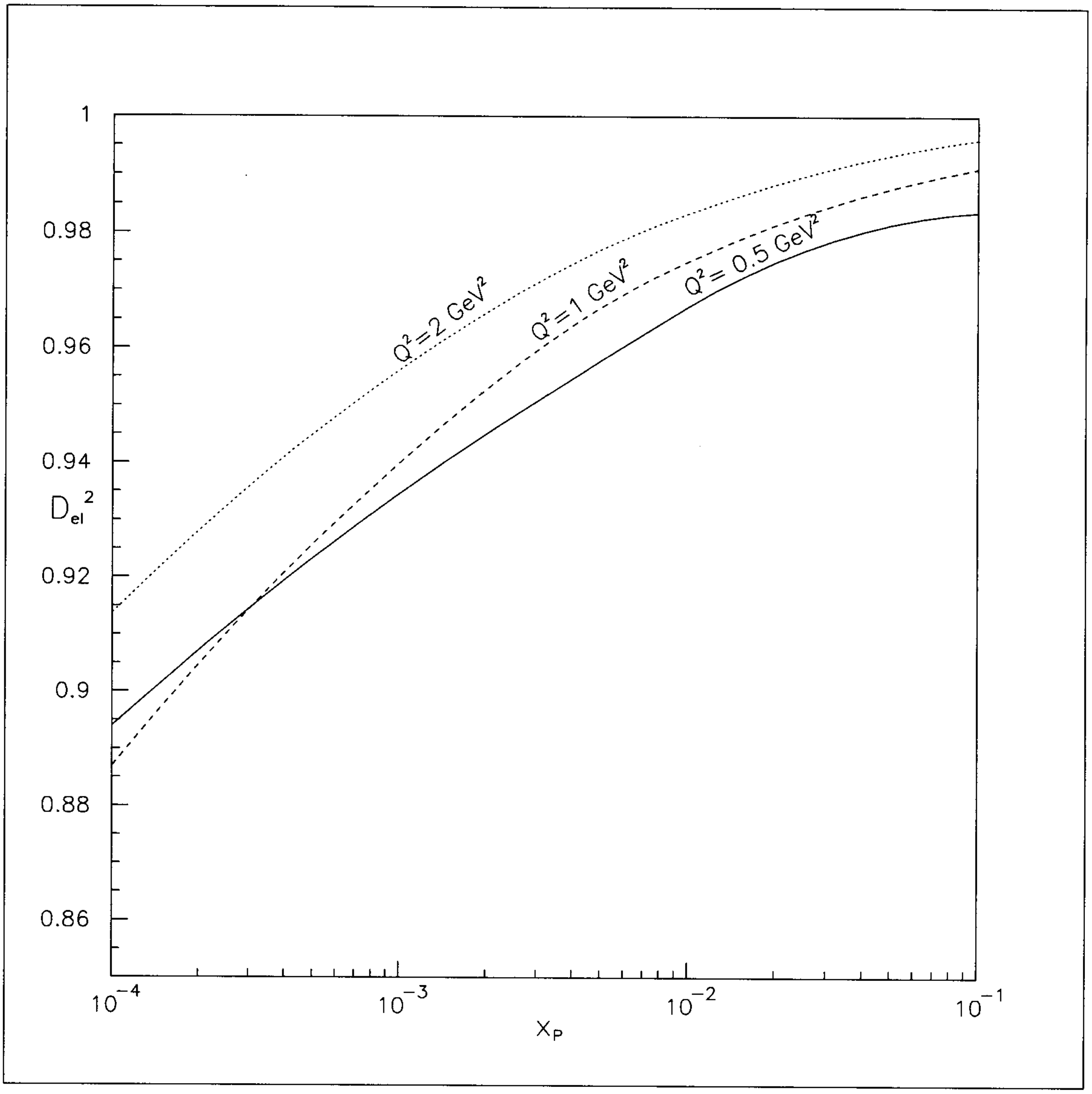


Figure 5: The damping factor (D_{el}^2) for $F_2(x, Q^2)$.

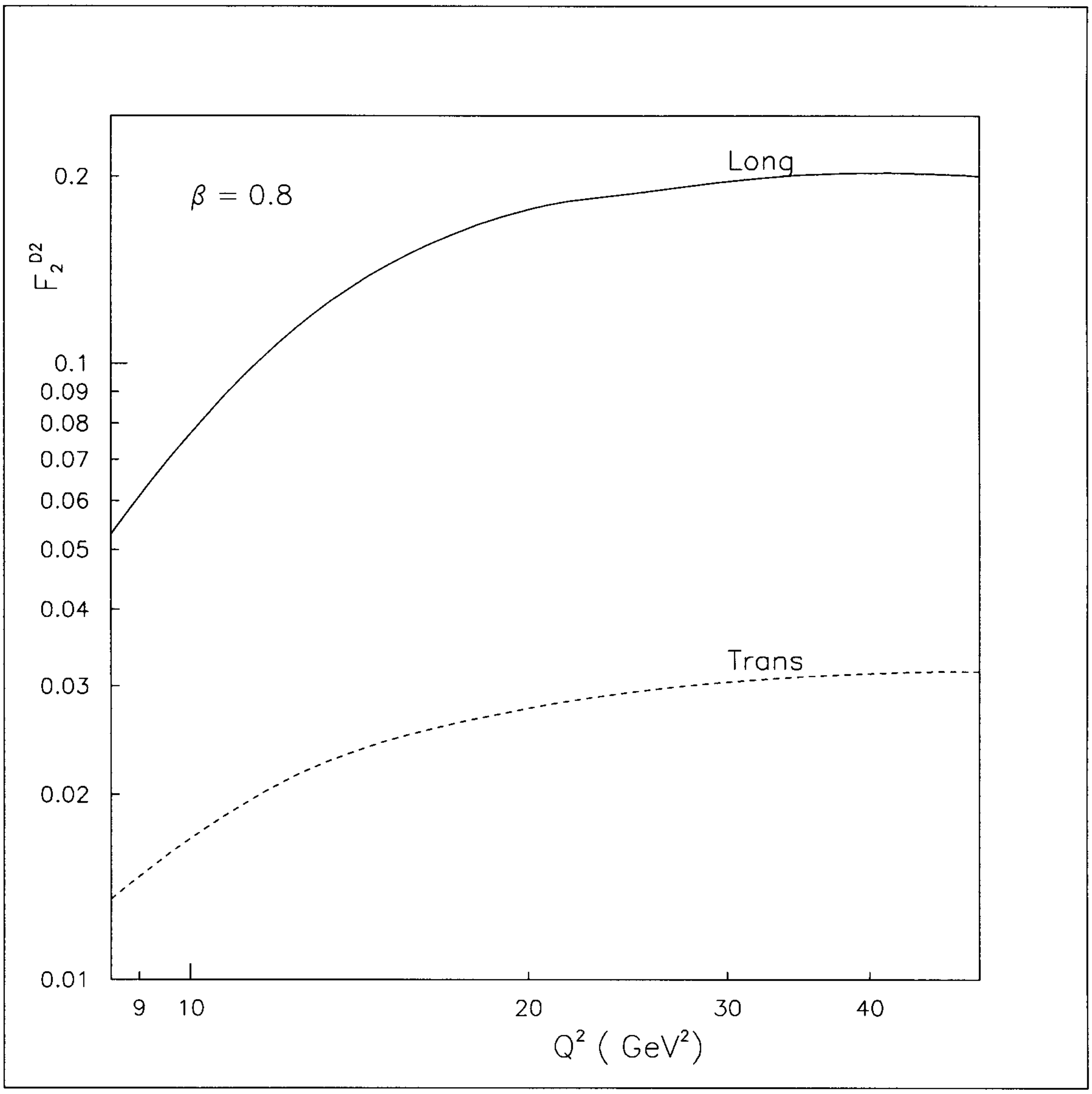


Figure 6: The DD by longitudinally and transversely polarised photon at $\beta = 0.8$.

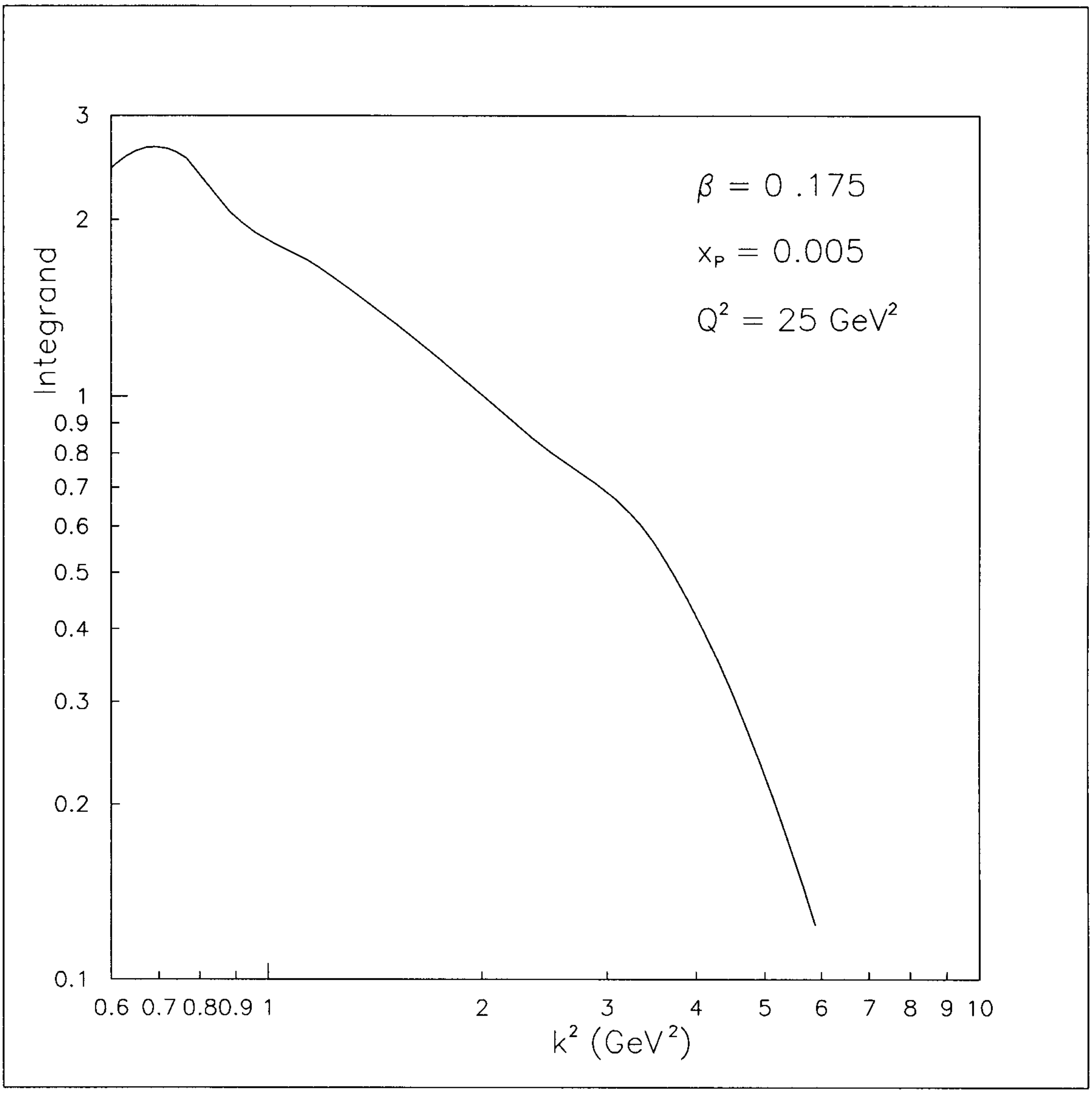


Figure 7: The integrand of Eq.(28).

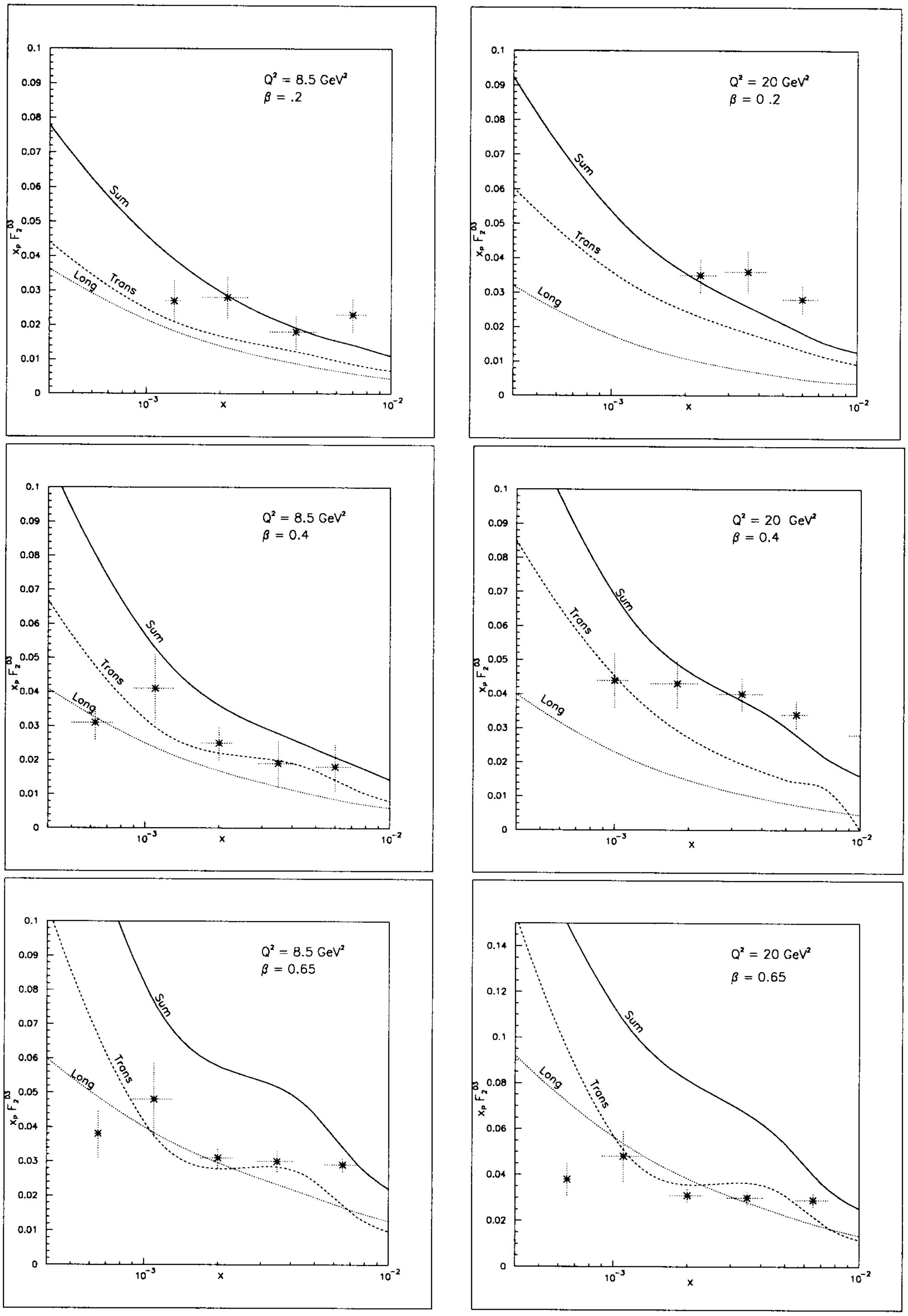


Figure 8: $x_P\, F_2^{DD(3)}$ versus x_P.

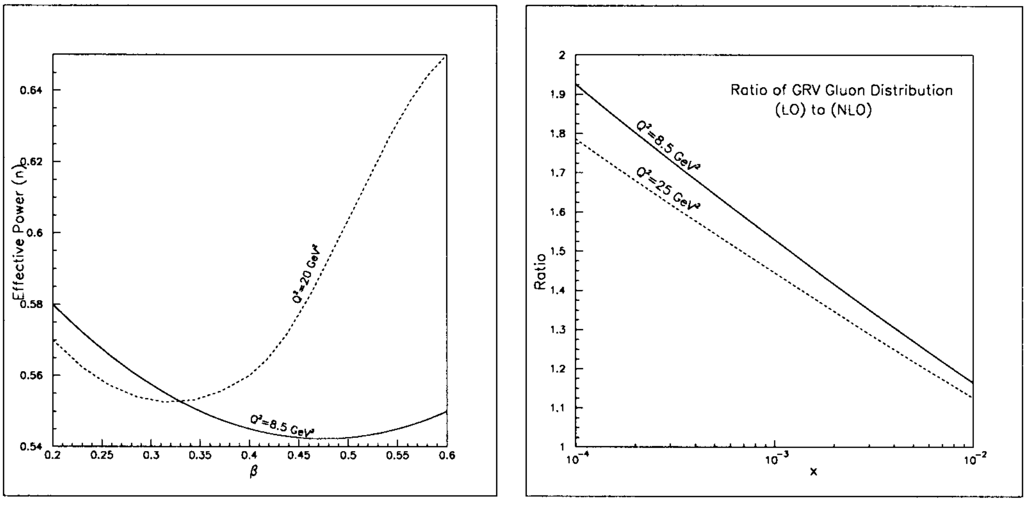


Figure 9: Effective power $n(\beta, Q^2)$ and ratio $\frac{xG(x,Q^2)^{LO}}{xG(x,Q^2)^{NLO}}$ in the GRV parametrization.

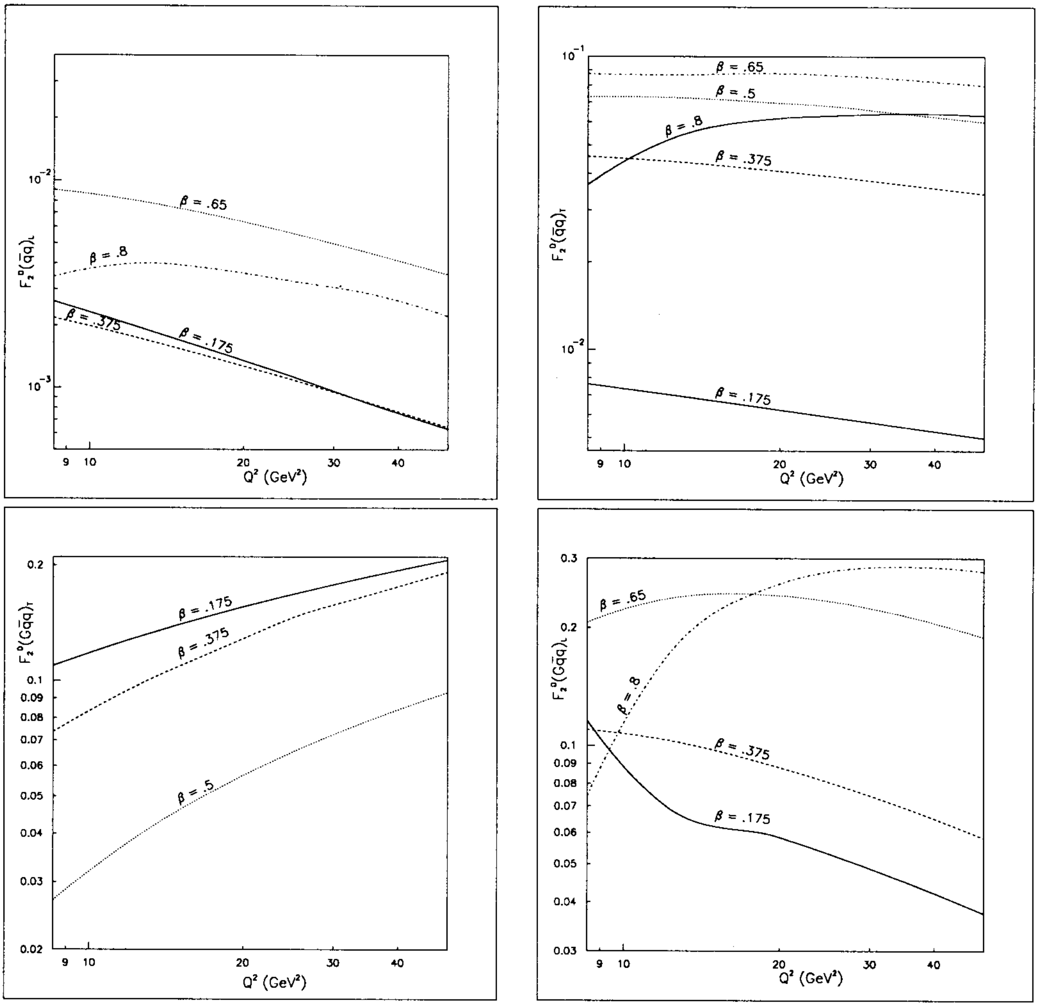


Figure 10: Different contributions to $F_2^{DD(2)}$.

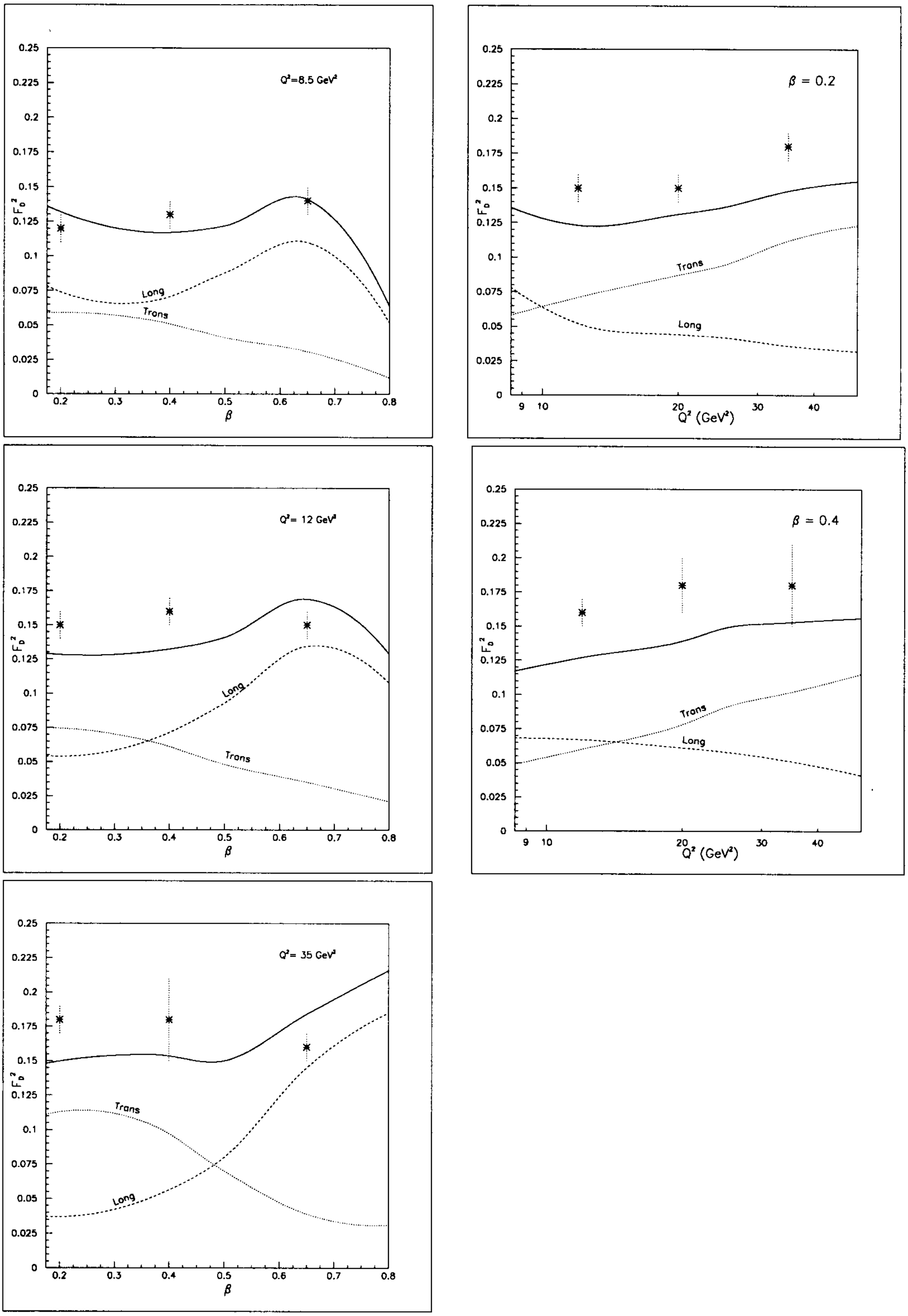


Figure 11: Comparison of $F_2^{DD(2)}$ with the HERA experimental data.

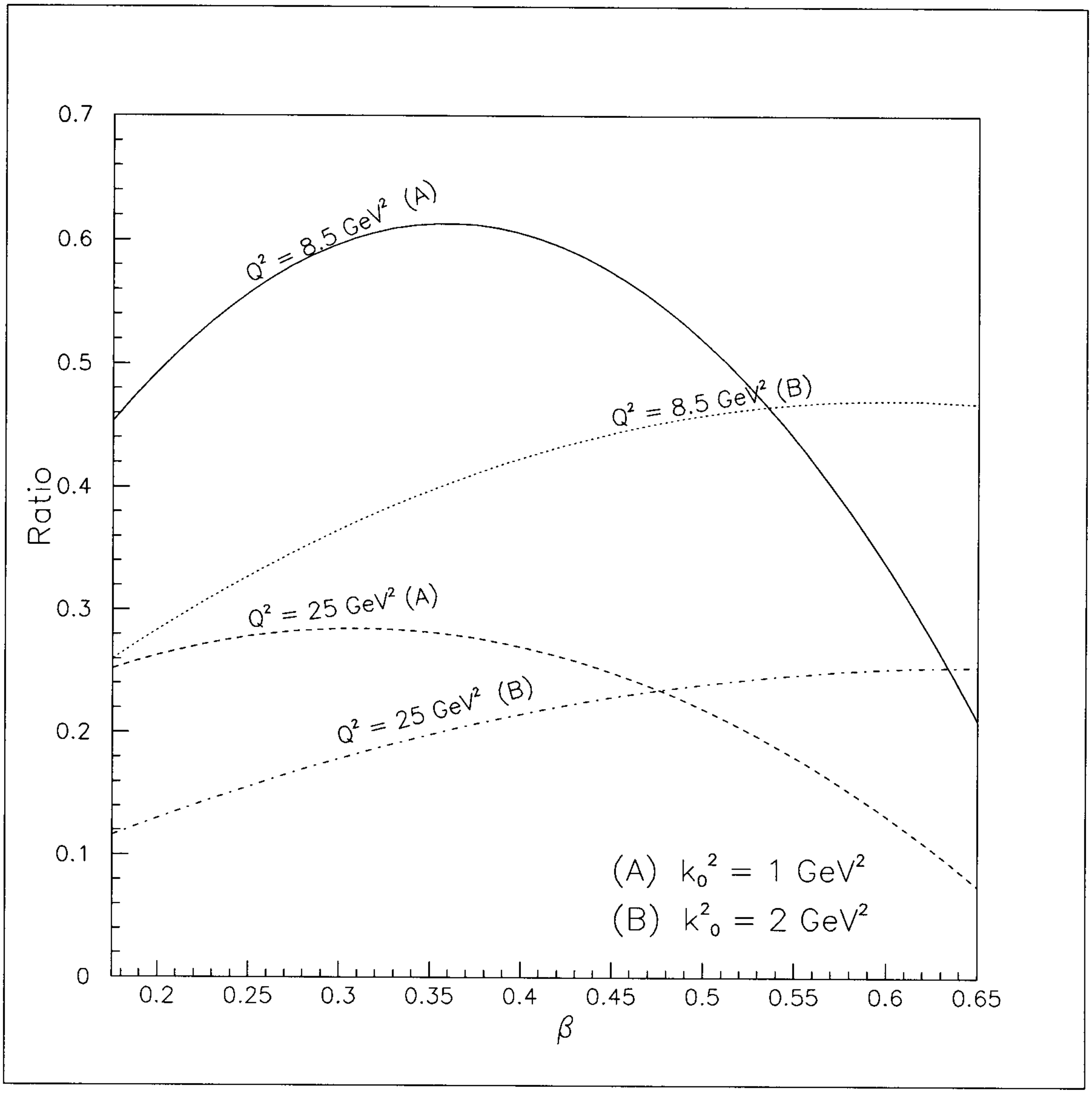


Figure 12: Ratio R (Eq.(75)) for different values of k_0.

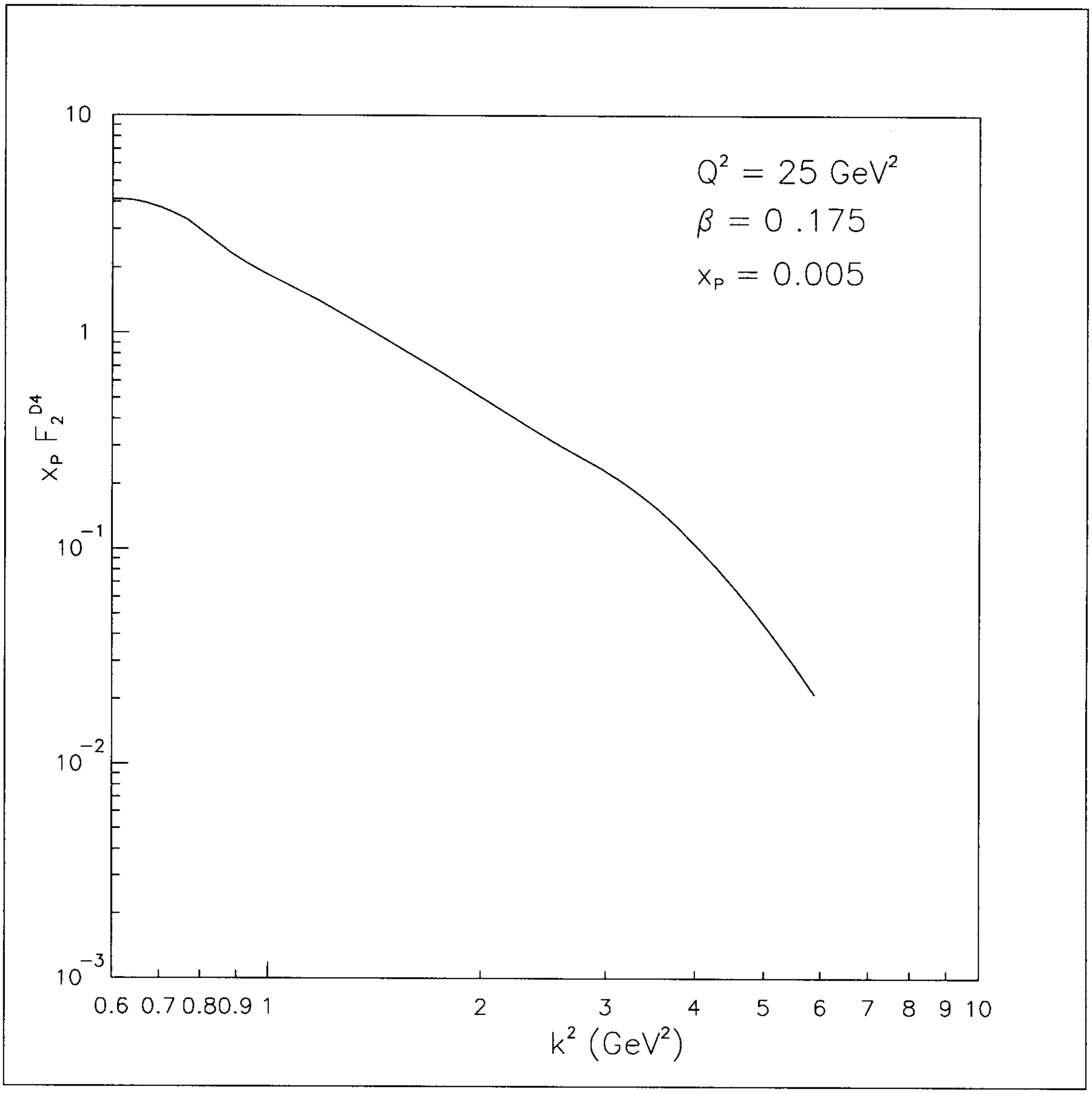


Figure 13: The transverse momentum distribution for the jet in the DD.

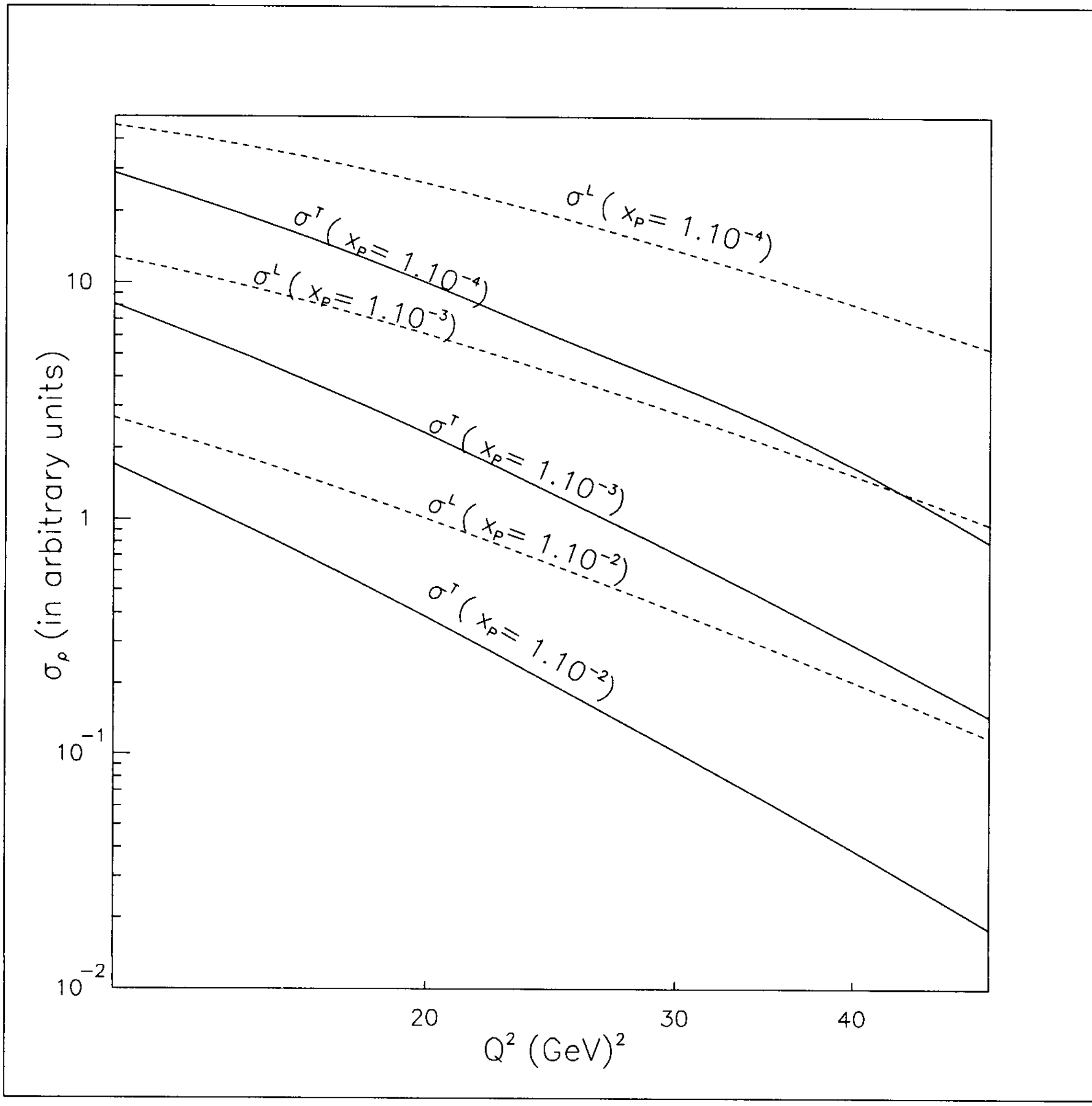


Figure 14: The cross section for ρ production (in arbitrary units) for Eqs.(77) and (78).

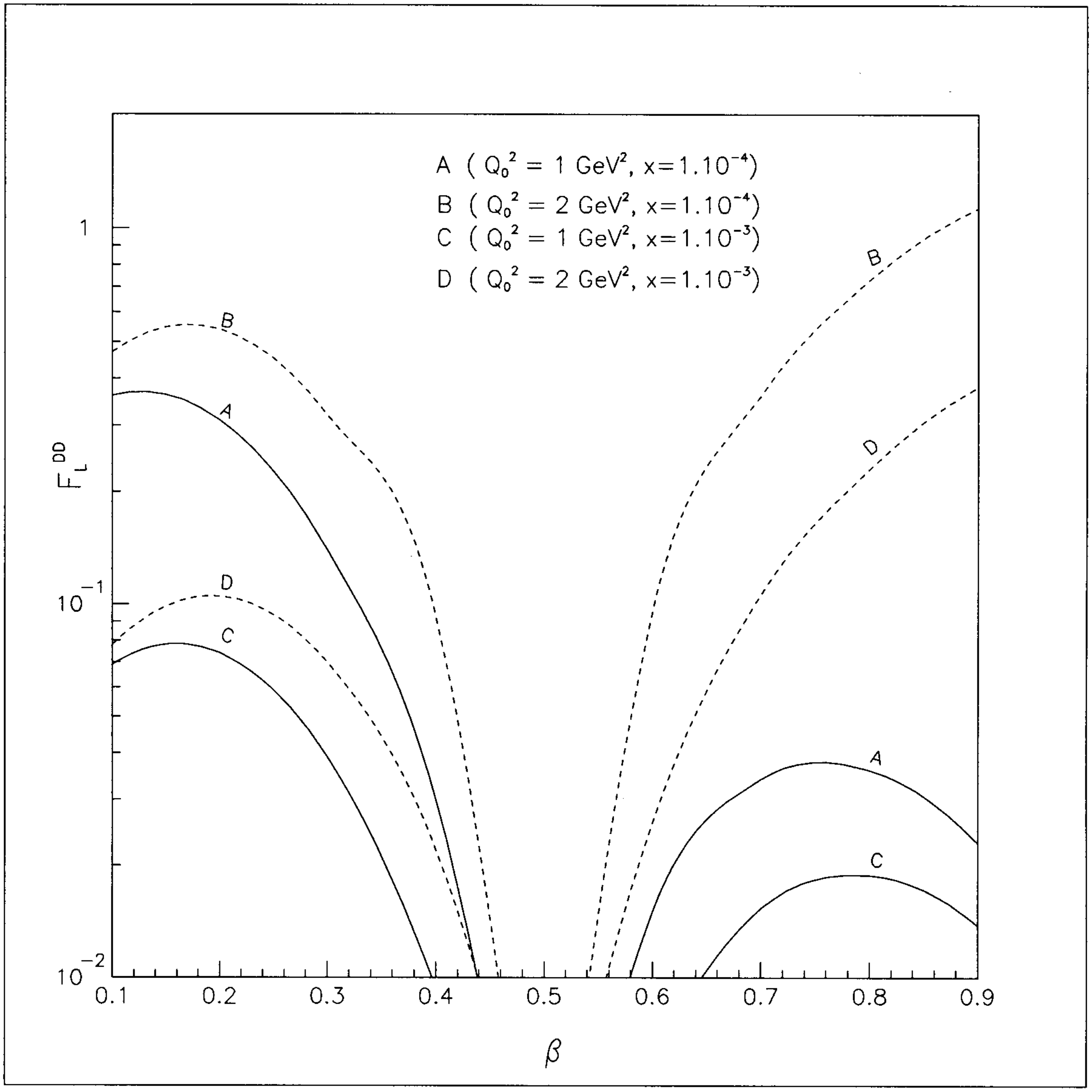
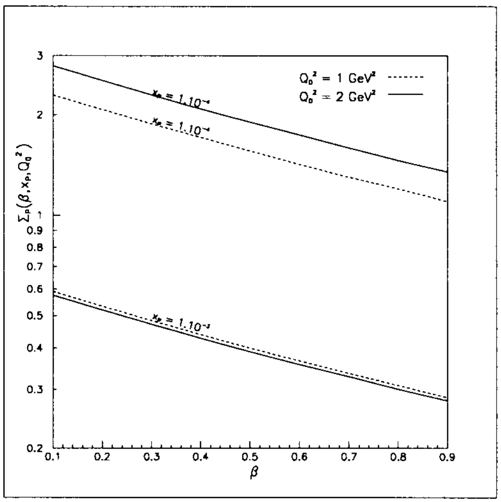


Figure 15: The initial quark distribution for the evolution equation for longitudinally polarised photon.



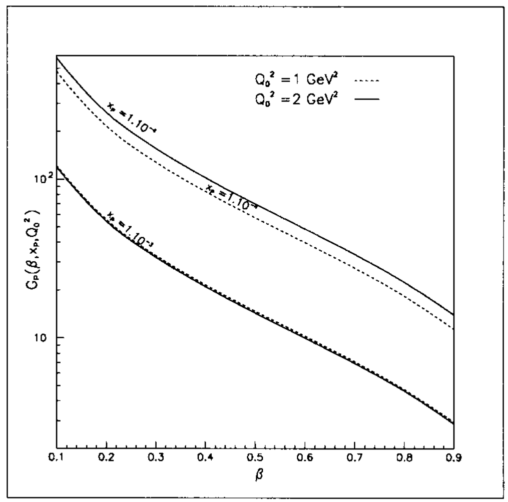


Figure 16: The initial quark distribution for the evolution equation for transversely polarised photon.