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TCP (TRUNCATED COMPOUND POISSON) DISTRIBUTION FOR PARTICLE MULTIPLICITY IN HIGH ENERGY COLLISIONS*

by

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ABSTRACT

On using the Poisson distribution truncated at zero for intermediate cluster decay in a compound Poisson process we obtain TCP distribution which describes quite well the multiplicity distributions in hadronic collisions. A comparison is made between TCP and NB (negative binomial) for UA5 data. The marked features of TCP curves at higher energies are that they are narrower than NB at high multiplicity tail, look narrower at very high energy and develop shoulders and oscillations, not present in NB or other currently proposed distributions, which become increasingly pronounced.

Key-words: Multiplicity distribution; High energy collisions; Compound Poisson process.

INTRODUCTION:

Recently a fair amount of experimental data has been accumulated on multiplicity distribution (md.) of charged particles in high energy collisions in the energy range of 10 GeV upto around 900 GeV. The main experiments are ,the UA5 Collaboration[1] at SPS $p\bar{p}$ collider ($E_{c.m}$=200,546,900 GeV), NA22 Collaboration[2] (pp and πp collisions at $E_{c.m}$=22 GeV),HRS collaboration[3] at PEP (e^+e^- annihilation at $E_{c.m}$=29 GeV) among many others.

The violation of KNO scaling[4] in the mds. at energies above 200 GeV was detected by UA5 group who found that the distributions grew broader with energy. They used the negative binomial (NB) distribution to fit the data in full phase space with remarkable success over a wide range of c.m. energies upto 900 GeV. As for the data in symmetric rapidity windows, the reduced moments calculated assuming NB distribution also give rise to good agreement. In a recent publication it was pointed out that the above mentioned mds. may as well be described very well by a truncated compound Poisson distribution (TCP)[5] obtained by compounding two Poisson distributions in contrast to the case of NB where a logarithmic distribution appears compounded with a Poisson one. The agreement of the calculated reduced moments using TCP with the experimental values is found to be as good as in the case of NB and the distribution curves of TCP and NB almost coincide at lower energies. At higher energies ,however, some significant differences begin to apper in the form of shoulders and oscillations in the case of TCP while they are absent in NB (and several other currently proposed hadroproduction distributions)[6]. For the same values of the two parameters (Sec.II) which characterize TCP or NB the former is always narrower than the latter at higher multiplicity points and it may be possible to test these distributions more closely by a careful analysis of the tails of the mds.,say,at 900 GeV and at Tevatron energies. An evidence of marked oscillations in the mds. at SSC, SLC and LEP would be in favour of TCP or a situation in between the two distributions. We remark that in the interpretation of a compound Poisson process as a two step[7] process in which intermediate clusters or clans[8] are formed the average number of clusters is found to decrease with energy from 6 to 3 for the UA5 data if we use TCP while it is found to increase and saturate around 8 for NB. We discuss the properties of TCP distribution in Sec.II and the expressions for the various moments are given. In Sec.III we compare the TCP predictions with the UA5 data and with some of the NB predictions.

II. TCP AND NB DISTRIBUTIONS:

Both TCP and NB belong to the general class of the so called (discrete) compound Poisson distributions which are infinitely divisible. A compound Poisson process may conveniently be described by the following generating function

$$G(t) = G(g(t);\langle N\rangle) = \exp(\langle N\rangle[g(t)-1]) = \sum t^n P_n \qquad (1)$$

which corresponds to the multiplicity distribution

$$P_n = \sum e^{-\langle N\rangle}(\langle N\rangle^N/N!)\,(1/n!)\,(d^n/dt^n)\,(g(t))^N\Big|_{t=0} \qquad (2)$$

In our context it corresponds to a two step process. At the first stage N independent Poisson distributed clusters or clans are produced and each of which then subsequently decays according to the probability distribution corresponding to the generating function $g(t)$ giving rise to a total number of n particles as observed experimentally. The TCP distribution[5] is obtained by choosing $g(t)=(e^{Bt}-1)/(e^B-1)$,where B is a constant ,corresponding to a Poisson distribution truncated at zero in order to allow for at least one particle inside the decaying cluster (see also Sec.III). The NB is obtained from $g(t)=\ln(1-qt)/\ln(1-q)$. We have $P(0)=e^{-\langle N\rangle}$ where $\langle N\rangle$ is the average number of clusters formed. For the average number of particles produced we obtain $\langle n\rangle=\langle N\rangle \langle n_c\rangle$ where $\langle n_c\rangle= g'(1)$ gives the average multiplicity of the cluster decay. Defining the shape parameter k from $(\langle n^2\rangle-\langle n\rangle^2)/\langle n\rangle^2 = 1/\langle n\rangle+1/k$ we find $\langle N\rangle=kg''(1)/(g'(1)^2)$ and $\langle n\rangle/k=g''(1)/g'(1)=B$ for TCP while it equals $q/(1-q)$ for NB while $\langle N\rangle/k= 1-e^{-\langle n\rangle/k}$ for TCP and $\ln(1+\langle n\rangle/k)$ for NB. The distributions under consideration are completely characterized in terms of two parameters k and $B\equiv\langle n\rangle/k$ which are determined from the fit to the experimental data. An important property of compound Poisson distribution is infinite divisibility[7] in that it can be represented as n-fold convolution of a probability distribution with itself (see (1)). We verify easily

$$G[g_1(t);\langle N_1\rangle]\, G[g_2(t);\langle N_2\rangle] = G[(\langle N_1\rangle g_1+\langle N_2\rangle g_2)/(\langle N_1\rangle+\langle N_2\rangle);\langle N_1\rangle+\langle N_2\rangle] \qquad (3)$$

Another property worth mentioning is that we may rewrite the generating function $G(g(t);\langle N\rangle)$ above in infinite product form

$$e^{-a(1-t)}e^{-b(1-t^2)}e^{-c(1-t^3)} \qquad (4)$$

where the numbers of singlets, doublets, triplets etc. of particles involved have independent Poisson distributions with means a,b,c etc.

For TCP case we may derive the following probability distribution

$$P_n = e^{-\langle N\rangle} (B^n/n!)\, A_n(k\, e^{-B}),$$

$$\langle N\rangle = k\,(1- e^{-B}), \qquad \langle n_c\rangle = B/(1-e^{-B}) \qquad (5)$$

where $B = \langle n\rangle/k$ and $A_n(x)$ are polynomials defined by the recurrence relation $A_{n+1}(x) = x\,[A_n(x) + A'_n(x)]$ satisfying $A_0(x) = 1$, $A_1(x) = x$ etc. In deriving P_n we used the expansion $\exp[x\,(e^t-1)] = \sum A_n(x)\, t^n/n!$. The numerical coefficients in these polynomials are Stirling numbers of second kind and they grow very large with increasing n. Computer program, however , can be set up easily to handle the numerical computation required for obtaining the mds.

The generating function $G(t)$ allows us to calculate efficiently the various moments[9] characteristic of TCP and NB distributions. For example, the raw moments about origin $\langle n^r\rangle = (d^r/dt^r)\, G(e^t)|_{t=0}$, the factorial cumulants or inclusive correlation integrals $f_r = (d^r/dt^r)\ln G(t)|_{t=1}$, the cumulant moments $\varkappa_r = (d^r/dt^r)\ln G(e^t)|_{t=0}$. The reduced moments C_r are defined by $\langle n^r\rangle/\langle n\rangle^r$. We list some of these moments for comparison, $B = \langle n\rangle/k$,

TCP distribution:

$$C_2 = (Bk+B+1)/\langle n\rangle$$
$$C_3 = (B^2k^2+3B^2k+B^2+3Bk+3B+1)/\langle n\rangle^2$$
$$C_4 = (B^3k^3+6B^3k^2+7B^3k+B^3+6B^2k^2+18B^2k+6B^2+7Bk+7B+1)/\langle n\rangle^3 \qquad (6)$$
$$f_n = B^n k$$

NB distribution:

$$C_2 = (Bk+B+1)/\langle n\rangle$$
$$C_3 = (B^2k^2+3B^2k+2B^2+3Bk+3B+1)/\langle n\rangle^2$$
$$C_4 = (B^3k^3+6B^3k^2+11B^3k+6B^3+6B^2k^2+18B^2k+12B^2+7Bk+7B+1)/\langle n\rangle^3 \qquad (7)$$
$$f_n = (n-1)!\,B^n k$$

Poisson distribution:

$$C_2 = (\langle n\rangle+1)/\langle n\rangle$$
$$C_3 = (\langle n\rangle^2+3\langle n\rangle+1)/\langle n\rangle^2$$
$$C_4 = (\langle n\rangle^3+6\langle n\rangle^2+7\langle n\rangle+1)/\langle n\rangle^3 \qquad (8)$$
$$f_n = 0 \text{ for } n \geq 2$$

It is clear that both TCP and NB deviate from Poisson distribution and are broader. For large values of k and B small the TCP distribution tends towards a Poisson like curve (see plots for various B and k). We note also that $\langle n_c\rangle \geq 1$ tends to unity in the limit $\langle n\rangle/k \to 0$.

III. COMPARISON WITH UA5 DATA:

The charge conservation constraint must be taken into account for fitting the full phase space all charged distributions. From a distribution defined for n = 0,1,2,3..we may by ignoring odd multiplicities derive a modified probability distribution which allows for only the even multiplicities. The corresponding generating function may be written as

$$G_e(t) = \sum t^n P_{en} = \alpha \sum [1+(-1)^n]\, P_n t^n = \alpha\, [G(t)+G(-t)] \qquad (9)$$

On demanding that $G_e(1) = 1$ we find $\alpha = 1/[1+G(-1)]$. The corresponding moment generating function is thus given by $G_e(e^t) = \alpha\, [\, G(e^t) + G(e^{t+i\pi})\,]$. We may check by exact computation or use simple arguments to show that $\langle n^r \rangle_e = \alpha \langle n^r \rangle$ to very great accuracy both for TCP and NB. The inverse renormalization factor for TCP is $\alpha^{-1} = (1+ e^{-k[1-\exp(-2B)]})$ while for NB it is given by $(1 + e^{-k\ln(1+2B)})$. It can become quite appreciable if both B and k are small. We remark that the even multiplicity distribution thus constructed is valid also for B<1 in contrast to the one constructed from (2), for example, by choosing $g(t) = [\exp(Bt^2)-1]/[\exp B -1]$ which fails to be a probability distribution for $\langle n \rangle < k$ which holds for lower energies of UA5 data. In Tab. we compare the reduced moments for full phase space data of UA5 with the predictions of TCP distribution. The agreement over the energy range from 10 GeV to 900 GeV is quite good. The average number of clusters $\langle N \rangle$ is found to decrease with energy from about 6 to 3 contrary to the case of NB where it increases to around 8 at 900 GeV. From the theoretically unexplained fact that[1] k (>1) decreases with energy while B increases it is found from (5) that the $\langle N \rangle$ decreases like k does for UA5 data above ≅ 200 GeV. With increasing energy TCP predicts less numerous and wider clusters with more energy content reminding us of the cosmic ray energy data.

The reduced moments C_2, C_3, C_4, C_5 show a smooth variation with parameters. Increasing the value of k lowers the reduced moments and the corresponding TCP distributions for k » ⟨n⟩ tend to become smooth Poisson like. Drawn are some TCP curves at several energies comparing them with the best NB fits(1) as found in UA5 data. The predictions at 2 TeV and 40 Tev are also included by extrapolating the energy dependence(1) of ⟨n⟩ and k. The marked feature of TCP curves at higher energies is that they become narrower than NB ones at higher multiplicity points even though for lower multiplicity values they are sligtly broader with shoulders and oscillations which become increasingly pronounced with energies as indicated by the prediction at 40 TeV. A carefull analysis of the tails of the experimental distributions and on the presence of shoulders and oscillations, say, in 900 GeV and the forthcoming FERMILAB data may be warranted.

We remark that for a compound Poisson distribution the cumulant moments and factorial cumulants f_r measure essentially the moments of the cluster decay multiplicity distribution. The errors in experimental data, however, do not allow us to favour one or thé other alternatives discussed here. For the data with fixed rapidity gaps(1) at 200,546 and 900 GeV we have checked that the moments computed from TCP, considering the experimental errors, are as good as those obtained from NB which is also evident from (6) and (7).

IV. CONCLUSIONS:

TCP distribution seems to be a good alternative candidate for representing the mds. observed over the energy range from 10 to 900 GeV in hadroproduction data. From the discussion above it is suggested that it should be equally suitable for other processes like e^+e^- annihilation and lepton-hadron collisions. It employs only Poisson distributions in its construction. It allows for shoulders and oscillations which become more and more pronounced with increasing energy making the distributions look 'narrower' compared to those of NB. For large values of k and B small TCP tends to Poisson which seems to be suitable for e^+e^- data at the now available energies[3]. A careful study of the high multiplicity tails, the lower multiplicity region and of oscillations in experimental data may be useful to clarify if one or the other or something in between is favoured. Finally it is highly desireable to develop a theoretical basis for the observed mds. starting from the microscopic theory (QCD) of quarks and gluons.

ACKNOWLEDGEMENT:

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FIGURE CAPTIONS:

Figs.1.1,1.2,1.3 :Comparison of the mds.arising from TCP and NB distributions at CM energies 19.7, 44.5, 200.0, 546, 900, 2000 and 40000 GeV for UA5 data. Here $z = n/\langle n\rangle$ and $B=\langle n\rangle/k$

Figs.2.1,2.2 :Comparison of TCP distributions for k =1.5,3.5 and 5.5 with a fixed value for B =2.0,5.0,8.0,12.0 and 32.0.

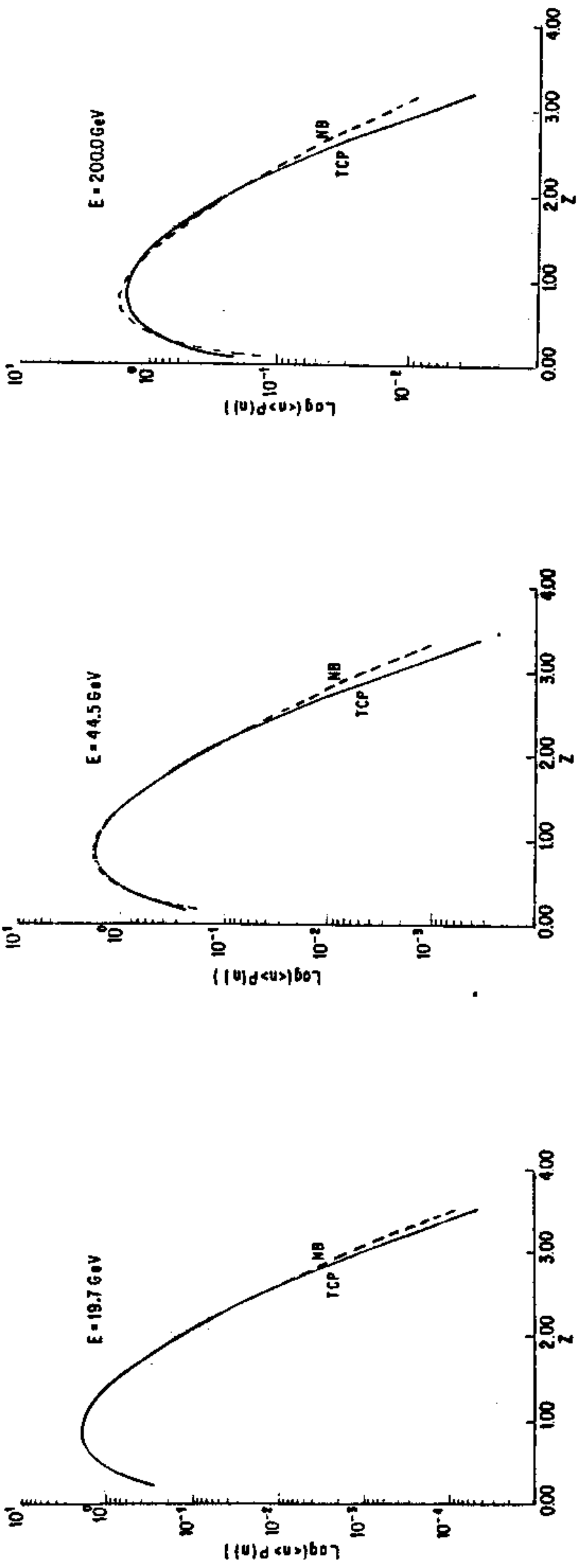
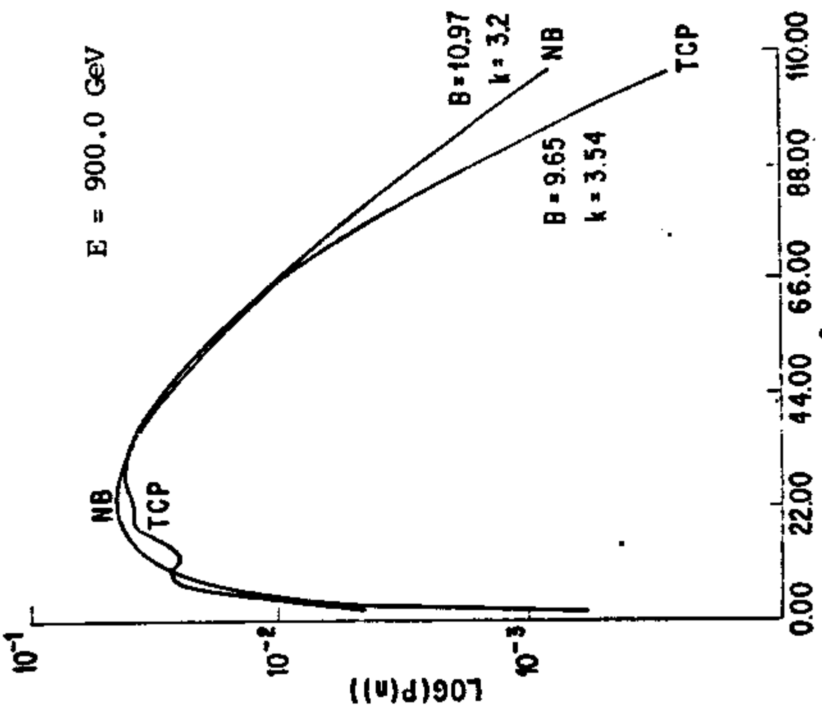


Fig. 1.1


E = 900.0 GeV
B = 10.97
k = 3.2
NB
TCP
B = 9.65
k = 3.54
NB
TCP
LOG(P(n))
n
0.00
22.00
44.00
66.00
88.00
110.00

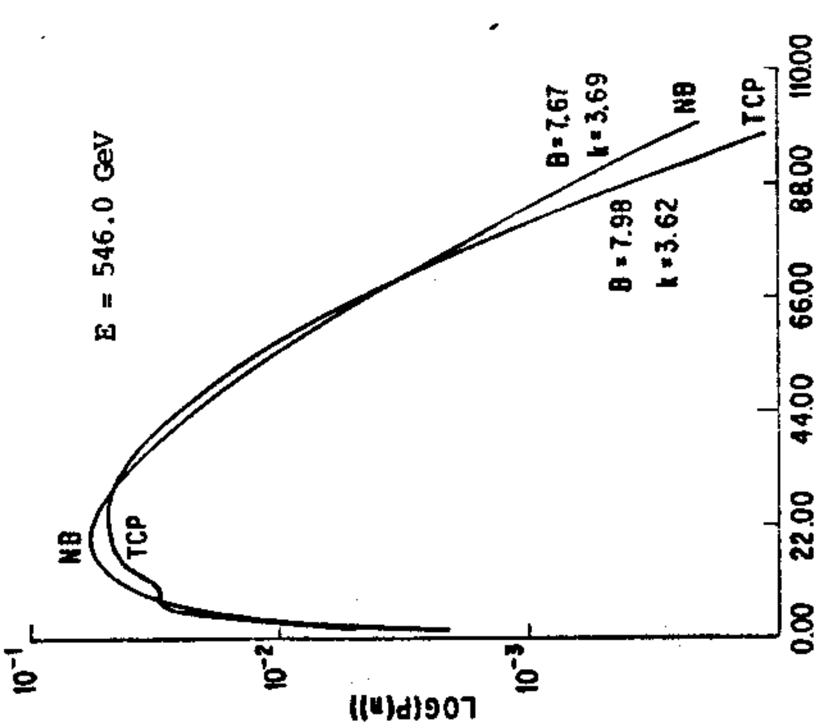

E = 546.0 GeV
B = 7.67
k = 3.69
NB
TCP
B = 7.98
k = 3.62
NB
TCP
LOG(P(n))
n
0.00
22.00
44.00
66.00
88.00
110.00

Fig. 1.2

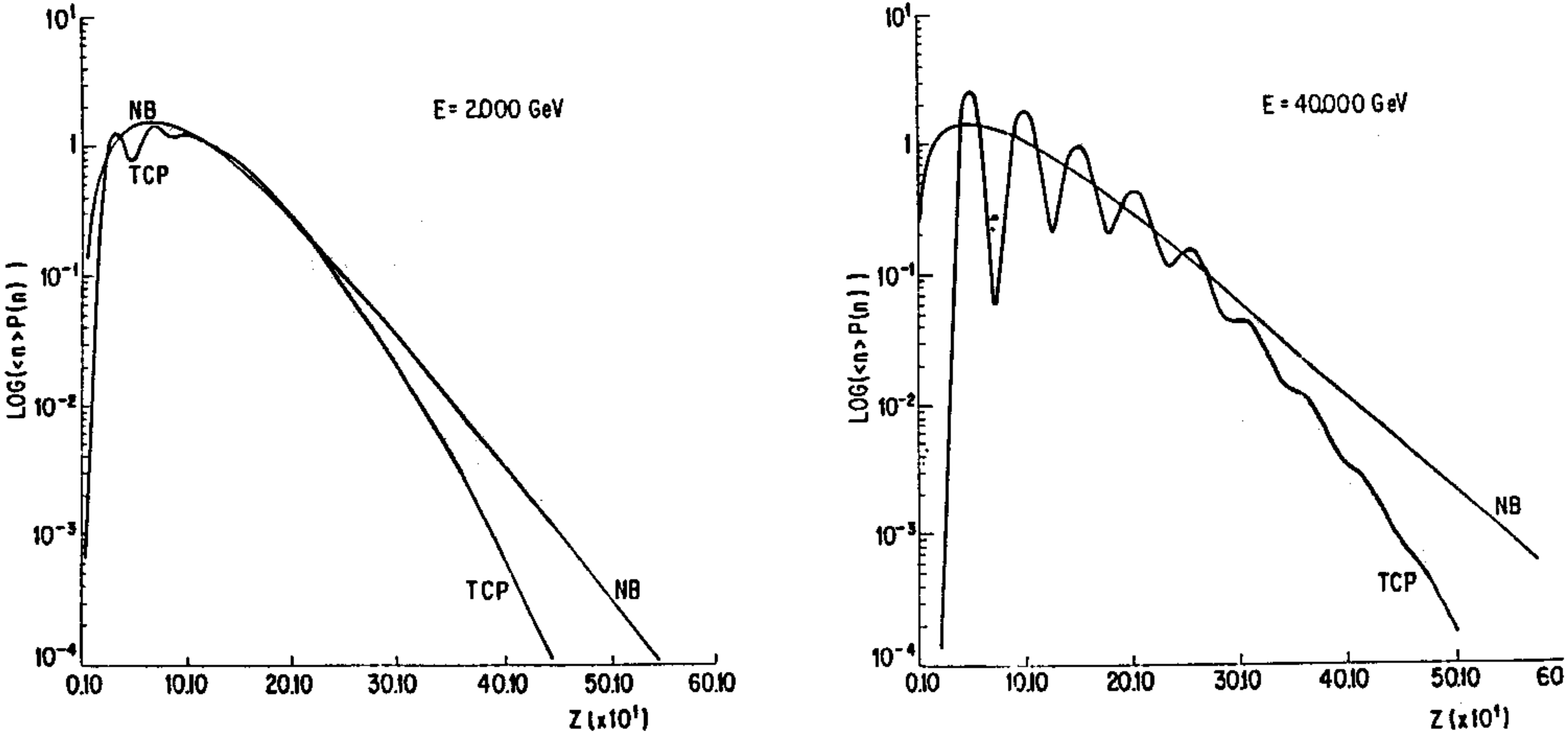


Fig. 1.3

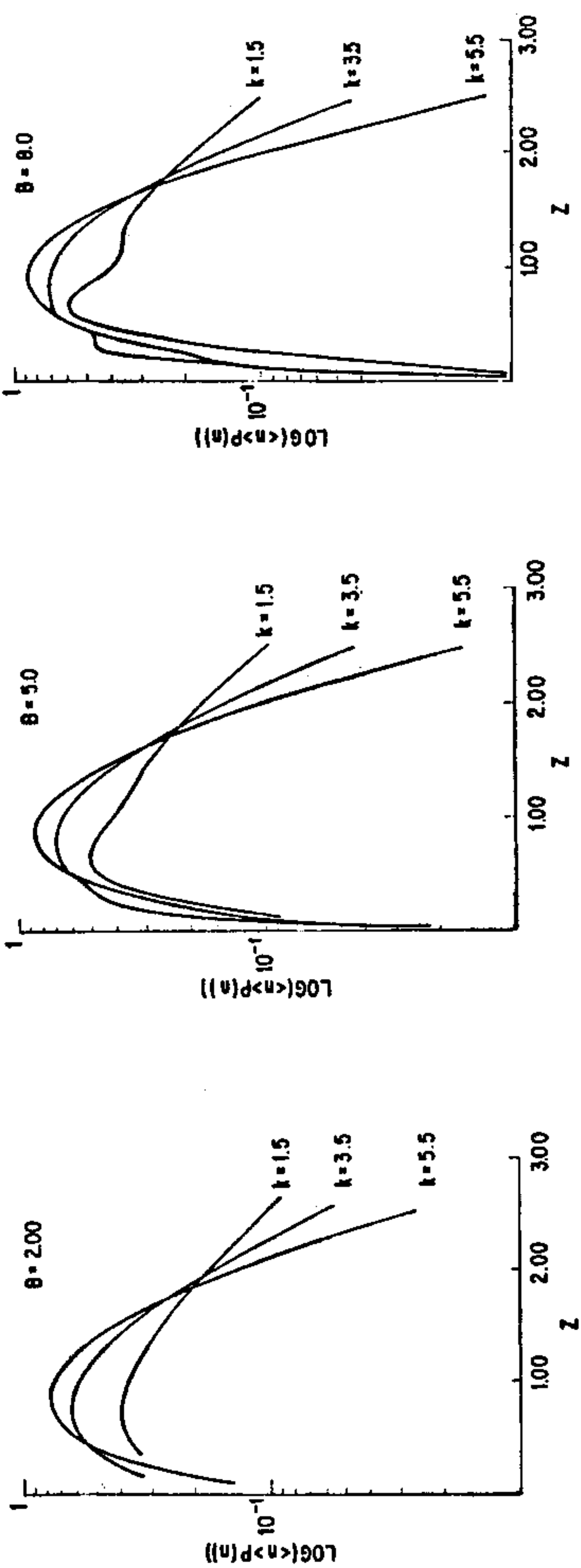


Fig. 2.1

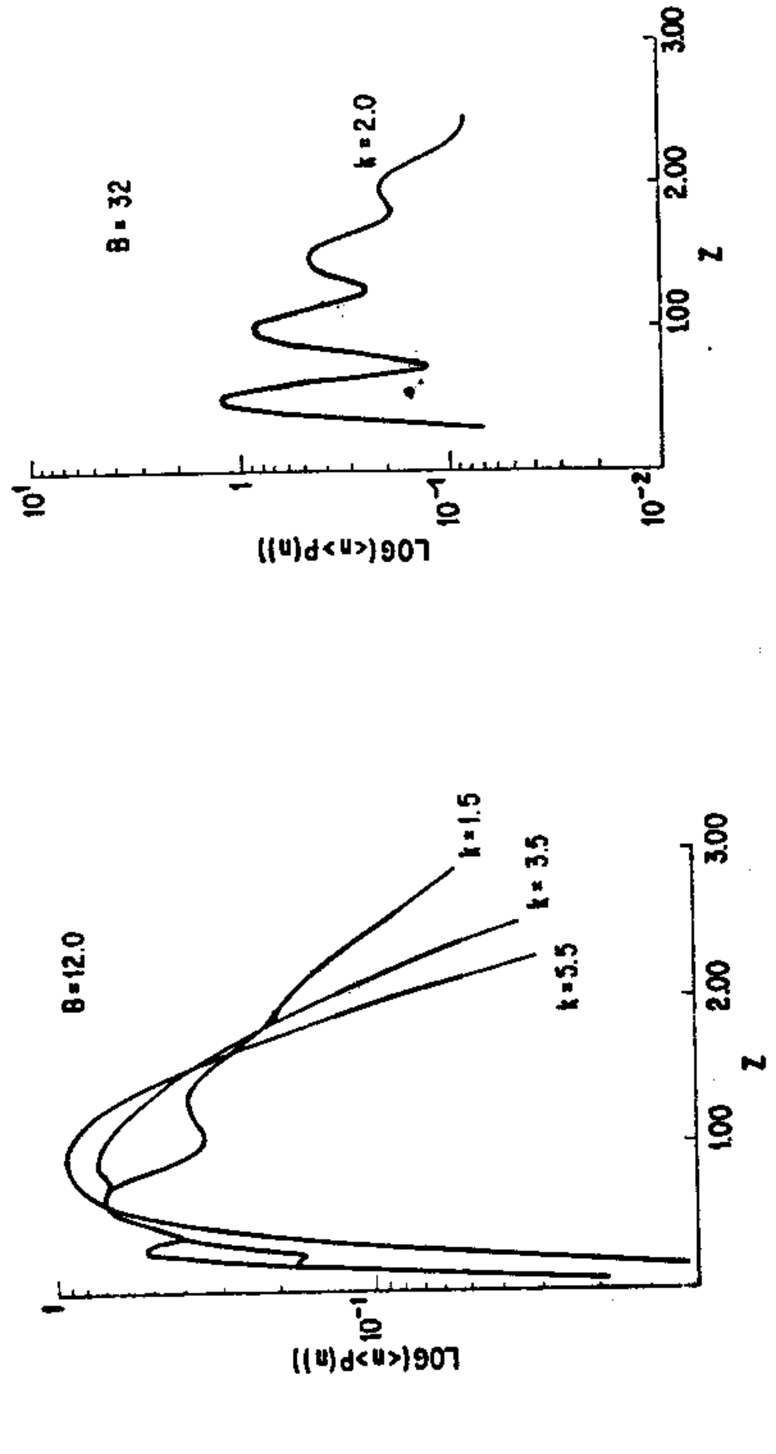

B = 32
k = 2.0
LOG(<n>P(n))
10^1
1
10^{-1}
10^{-2}
1.00
2.00
3.00
Z
B=12.0
k=1.5
k=3.5
k=5.5
LOG(<n>P(n))
1
10^{-1}
1.00
2.00
3.00
Z

Fig. 2.2

$E_{c.m.}$ GeV	$\bar{n}$ (input)	$B = \bar{n}/k$	$\bar{N}$	⟨n⟩	C_2	C_3	C_4	C_5
900	34.6	10.76	3.21	33.26 34.6±1.2	1.39 1.34±0.033	2.32 2.22±0.13	4.43 4.30±0.40	9.59 9.3±1.1
546	29.1	8.02	3.63	28.35 29.1±0.9	1.34 1.31±0.03	2.15 2.12±0.11	3.91 4.05±0.32	7.92 8.8±1.0
200	21.4	4.56	4.64	21.20 21.4±0.8	1.27 1.26±0.03	1.89 1.91±0.12	3.19 3.30±0.30	5.94 6.6±0.9
62.6	13.63	1.73	6.49	13.62 13.63±0.16	1.20 1.20±0.01	1.65 1.67±0.03	2.53 2.60±0.08	4.28 4.4±0.2
52.6	12.76	1.68	6.18	12.75 12.76±0.14	1.21 1.21±0.01	1.69 1.70±0.03	2.63 2.70±0.09	4.52 4.8±0.3
44.5	12.08	1.42	6.46	12.08 12.08±0.13	1.20 1.20±0.01	1.65 1.67±0.03	2.53 2.63±0.10	4.27 4.6±0.3
27.6	9.77	1.05	6.04	9.77 9.77±0.16	1.21 1.21±0.01	1.69 1.72±0.05	2.63 2.76±0.13	4.52 5.0±0.4
19.7	8.56	0.489	6.77	8.56 8.56±0.11	1.17 1.174±0.010	1.56 1.57±0.03	2.29 2.34±0.08	3.68 3.8±0.2
11.5	6.35	0.219	5.70	6.35 6.35±0.08	1.19 1.192±0.009	1.62 1.63±0.03	2.44 2.49±0.08	4.04 4.2±0.2

<u>TABLE</u>

Comparison of moments C_q obtained from TCP distribution with experimental data[1] indicated below each computed value. $\bar{N}$ indicates the average number of clusters for each energy.

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