ISSN 0029 - 3865

CBPF-NF-021/84

PATH-INTEGRAL BOSONIZATION OF TWO-DIMENSIONAL MASSIVE Q.C.D.

by

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Abstract

We evaluate the fermionic determinant for two-dimensional QCD with massive fermions by means of Seeley's technique. Apart from a gluon-mass term this determinant contains a Wess-Zumino anomaly term and a non-abelian extension of the Sine-Gordon.

Key-words: Field theory; Two-dimensional models; Path Integral.

Quantum field models in two space-time dimensions have proven to be useful in order to investigate various phenomena which we believe may occur in more realistic theories.

Recently the interest in two-dimensional field theories has been renewed, in particular for two-dimensional Q.C.D. (QCD_2) [1-8]. The evaluation of the fermionic determinant, which comes from path-integration over the fermionic variables, can be understood as a sort of path-integral version of the bosonization techniques [9]. This determinant has been solved by several methods for the case of massless QCD_2 [1,4,7]. However, with massive fermions it was evaluated only for two-dimensional QED for the case of zero topological-charge sector [8].

It is the purpose if this letter to show how we can evaluate the fermionic determinant for QCD_2 with massive fermions. The method used is as follows. Firstly we implement a chiral change of variables. The corresponding jacobian is computed by a method recently developed in ref. [3] and the remaining fermionic determinant is evaluated by means of Seeley's asymptotic expansion [10].

Let us start by assuming the behavior of all fileds to be such that at infinite it is possible to compactify the space. The generating functional for Euclidean QCD_2 with massive fermions is:

$$Z = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A \, \exp\left\{-\int\left[\bar{\psi}\mathcal{D}\psi - \frac{1}{4}F^2_{\mu\nu} + \text{gauge fixing term}\right]d^2x\right\} \quad (1)$$

where

$$D = i\not{\partial} + g\not{A}^a\lambda_a + m \quad (2)$$

and λ_a are the generators of the colour group SU(2).

We choose the decoupling gauge [11]:

$$A\!\!\!/ = \frac{i}{g} e^{-\gamma_5 \phi} \partial\!\!\!/ e^{-\gamma_5 \phi} \tag{3}$$

where

$$\phi = \phi^a \lambda_a \tag{4}$$

Changing the fermionic variables to:

$$\begin{aligned} \psi(x) &= e^{\gamma_5 \phi} \eta(x) \\ \bar{\psi}(x) &= \bar{\eta}(x) e^{\gamma_5 \phi} \end{aligned} \tag{5}$$

we obtain the following expression for the generating functional in terms of these new variables η and $\bar{\eta}$:

$$Z = \int J \mathcal{D}A \mathcal{D}\bar{\eta} \mathcal{D}\eta \; \exp \left\{ - \int \left[\bar{\eta} (i \partial\!\!\!/ + m e^{2\gamma_5 \phi}) \eta - \frac{1}{4} F^2 + g.f.t. \right] d^2x \right\} \tag{6}$$

where J is the jacobian associated with the transformation (5) which gives the quantum content of this transformation for the generating functional.

Now, in order to evaluate J, one considers an extended transformation dependent on a parameter $r (0 \leq r \leq 1)$:

$$\begin{aligned} \psi(x) &= e^{r\gamma_5 \phi} \eta_r(x) \\ \bar{\psi}(x) &= \bar{\eta}_r(x) e^{r\gamma_5 \phi} \end{aligned} \tag{7}$$

The finite transformation (5) will be achieved by successive infinitesimal changes, as we vary r from 0 to 1. Following the method described in ref. [3], we obtain:

$$\ell n J = -\frac{g^2}{4\pi}\int d^2x\, tr(\not A \not A) - \frac{g^2}{2\pi}\int d^2x \int_0^1 dr\, Tr(\not A_r \not A_r \phi\gamma_5) -$$

$$-\frac{m^2}{4\pi}\int d^2x \; \underset{c}{Tr}\left[1 - \cosh(4\phi) \right] \tag{8}$$

where the first two traces are calculated for the colour and γ-matrices indices, while in the last one this is done only for colour, and

$$\not A_r = \frac{i}{g} e^{(r-1)\gamma_5\phi} \not\partial e^{(r-1)\gamma_5\phi} \tag{9}$$

Substituting the value of the jacobian (8) in the generating functional (6) and integrating with respect to the new fermionic variables, we obtain:

$$Z = \int \mathcal{D}A \; \det(i\not\partial + me^{2\gamma_5\phi}) \; \exp\left\{ -\int d^2x \left[\frac{g^2}{4\pi} Tr(\not A \not A) + \frac{g^2}{2\pi}\int_0^1 dr\, Tr(\not A_r \not A_r \phi\gamma_5) + \right.\right.$$

$$\left.\left. - \frac{m^2}{4\pi} \underset{c}{Tr}(1 - \cosh 4\phi) - \frac{1}{4}F^2 + g.f.t. \right] \right\} \tag{10}$$

Now, in order to compute the above determinant, we introduce a parameter $r (0 \leq r \leq 1)$ and the operator D_r given as [2]:

$$D_r = i\not\partial + me^{2rf} \tag{11}$$

where $f = \gamma_5\phi$; for $r = 1$ we obtain the operator under consideration. Differentiating D_r with respect to r we obtain:

$$\frac{d}{dr}D_r = 2(D_r - i\not\partial)f \qquad (12)$$

The determinant of D_r is regulated by the proper time method:

$$\ln \det D_r^2 = \mathrm{Tr}\ln D_r^2 = -\int_\varepsilon^\infty \frac{ds}{s}\mathrm{Tr}\left[\exp(-sD_r^2)\right] \qquad (13)$$

where ε is an ultraviolet cutoff on the proper time integration. Differentiating (13) with respect to r and using the property (12) we obtain the following differential equation for the determinant:

$$\frac{d}{dr}\mathrm{Tr}\ln D_r^2 = 4\mathrm{Tr}\left[f\ \exp(-\varepsilon D_r^2)\right] - 4i\int_\varepsilon^\infty ds\mathrm{Tr}\left[\not\partial f D_r\ \exp(-sD_r^2)\right] \qquad (14)$$

The second term does not give contribution since the fields under consideration have trivial topology. For the first term Seeley [10] has shown that there is an asymptotic small ε expansion for the diagonal part of the exponential. If we consider operators of the form:

$$D = -D_\rho D^\rho + X \qquad (15)$$

where D_ρ is a covariant derivative and X is a matrix valued function we have [12]:

$$\langle x|\exp(-\varepsilon D)|x\rangle \xrightarrow[\varepsilon\to 0]{} \frac{1}{(4\pi\varepsilon)^{d/2}}\left[1 + \varepsilon X + O(\varepsilon^2)\right] \qquad (16)$$

where d is the dimensionality of space-time.

Now, calculating D_r^2, with D_r given in (11), substituting the asymptotic small expansion (16) in the differential equation (14), using the well known properties of traces of γ-matrices and integrating with respect to r we obtain:

$$Tr\ell n(i\not\partial + me^{2\gamma_5\phi}) - Tr\ell n(i\not\partial + m) = -\frac{m^2}{4\pi}\int d^2x Tr_c(1 - \cosh 4\phi) \quad (17)$$

Substituting this result (17) in (10) we get for the generating functional of QCD_2 with massive fermions:

$$Z = \int \mathcal{D}A \, \exp\left\{-\int d^2x \left[\frac{g^2}{4\pi}Tr(\not A \not A) + \frac{g^2}{2\pi}\int_0^1 dr Tr(\not A_r \not A_r \phi\gamma_5) + \frac{m^2}{2\pi}Tr_c(1 - \cosh 4\phi) - \right.\right.$$
$$\left.\left. - \frac{1}{4}F^2 + g.f.t.\right]\right\} \quad (18)$$

The first and second terms are known [2,4] to be the non-abelian extension of the Schwinger's mechanism and the two-dimensional analogue of the Wess-Zumino functional [13] respectively. The third term is the non-ableian extension of the sine-Gordon, obtained in two-dimensional QED by bosonization techniques [9], provided that we continue to Minkowski space once one have been working in Euclidean space.

Acknowledgements

The author is indebted to C.G. Bollini, L.C. Botelho, J. J. Giambiagi, J.A. Mignaco for discussions, to F.A. Schaposnik for calling his attention to the papers of ref. [2,4] and to E.C. Valadares for careful reading the manuscript.

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