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GHOST BASIS FOR NEUTRINO

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Recently some solutions of Dirac's equation in gravitational fields have been presented[1] with the curious property of having a null energy-momentum tensor but a non-null current density. Some authors have interpreted this as being a sort of ghost neutrinos in the sense that they can be acted upon by a gravitational field although they cannot create gravitation.

One's first reaction to these solutions may be a rejection of them on physical grounds. However, the fact that they seem to be present in any kind of geometry makes worthwhile a less superficial investigation. In order to be able to study some details of these ghost we have to find not only one but a class of these solutions in a given geometry.

Let us restrict our comments on the present paper to the neutrino cosmological model recently founded by myself and I.D. Soares[2].

This model contains arbitrary functions of the time-coordinate which permits to create such an infinity class of ghost solutions.

The line element has the form

(1) $$ds^2 = dt^2 - 2A(t)dzdt - C^2(t)(dx^2 + dy^2)$$

In this geometry, for a spinor ψ which depends only on t, Dirac's equation assumes the form:

(2) $$(\gamma^o-\gamma^1)\dot{\psi} - \frac{1}{2}\frac{\dot{A}}{A}(\gamma^1-\gamma^o)\psi + \frac{\dot{C}}{C}(\gamma^o-\gamma^1)\psi = 0$$

This equation is identically satisfied by choosing ψ to have the form(*)

(3) $$\psi = \begin{pmatrix} \phi \\ \sigma'\phi \end{pmatrix}$$

ϕ is a 2-component spinor with arbitrary dependence on t and $\sigma' = \begin{pmatrix} 0 1 \\ 1 0 \end{pmatrix}$.

The energy-momentum tensor of the neutrino is given by

(4) $$T_{MN} = \overline{\psi}\,\gamma_{(M}\,D_{N)}\,\psi - D_{(M}\,\overline{\psi}\,\gamma_{N)}\psi$$

For the local tetrads we choose:

$$e^o_{(0)} = 1, \quad e^o_{(1)} = e^1_{(1)} = -A(t) \quad \text{and} \quad e^2_{(2)} = e^3_{(3)} = C(t)$$

(coordinate indices are written under parenthesis).

In this system of tetrads the only non-null components of T_{MN} are

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(*) Our convention is the same as in ref [2].

$$\gamma^K = \begin{pmatrix} 0 & \sigma^K \\ -\sigma^K & 0 \end{pmatrix} \quad ; \quad \gamma^o = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$; the σ^K are Pauli matrices.

(5) $$T_{00} = T_{11} = - T_{01} = - 8\ \mathrm{Im}(\phi^{+}\dot{\phi})$$

(a dot means t-derivative).

Einstein's equation gives a unique relation between the metrical coefficients A, C and the spinor ψ:

(6) $$\frac{\ddot{C}}{C} - \frac{\dot{A}}{A}\frac{\dot{C}}{C} = - 2\ \mathrm{Im}(\phi^{+}\dot{\phi})$$

From expression (5) the condition of ghostness is

(7) $$\mathrm{Im}(\phi^{+}\dot{\phi}) = 0$$

We remark that given two ghost solution ψ_1 and ψ_2 the linear combination $\alpha\psi_1 + \beta\psi_2$ is still a solution of Dirac's equation but it is not necessarily a ghost for arbitrary complex numbers α and β. This is a direct consequence of linearity of Dirac's equation and non-linearity of the energy-momentum tensor.

There is two independent types of ghost spinor present in our geometry. They are

(8a) $$U_K = T_K(t)\ U_I$$

(8b) $$W_\ell = T_\ell(t)\ U_{II}$$

in which $U_I = \begin{pmatrix}1\\0\end{pmatrix}$, $U_{II} = \begin{pmatrix}0\\1\end{pmatrix}$, T_K is an arbitrary real function of time.

By choosing a set of functions $\{T_K\}$ to constitute a basis for functions of t of a given class we can develop any spinor of our geometry into an infinity sum of ghost spinors. Indeed, we can write

(9) $$\phi(t) = \sum_K \alpha_K U_K + \sum_K \beta_K W_K$$

in which α_K, β_K are complex numbers.

The energy of the field is given by

(10) $$E = \sum_K \sum_\ell \mathrm{Im}(\alpha_K^{*} \alpha_\ell + \beta_K^{*} \beta_\ell) T_K \dot{T}_\ell$$

It seems worthwhile to remark that these neutrinos do not have in general a definite helicity. However, if we insist that they must have helicity $\varepsilon = \pm 1$ then the β's and α's are related by $\beta_K = \varepsilon \alpha_K$.

Expression (10) tells us the following:

(i) In general, by adding ghost neutrinos we do not obtain a ghost state.

(ii) Any neutrino that generates the above geometry is a linear combination of infinity ghost states.

In a sense we could say that <u>the state of the neutrino that creates curvature is composed of ghost-states each of which do not create curvature</u>.

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