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Superconducting Cosmic String in Minimal Einstein-Cartan Gravity

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Abstract

We show that it is possible to construct a consistent model describing a current-carrying cosmic string endowed with torsion and curvature in four dimensions. The string torsion is interpreted in analogy with *screw dislocations* in a three-dimensional crystalline solid. In the simplest case the torsion can be written as a real scalar field gradient that preserves the skew-symmetry. We consider a superconducting cosmic string (SCCS) in Einstein-Cartan theory of gravity. A solution representing a superconducting cosmic string was found by considering the weak-field approximation while the exterior solution was derived in an exact manner. The torsion contribution to the gravitational force and geodesics of a test-particle moving around the SCCS are analyzed. In particular, we point out two interesting astrophysical phenomena in which the higher magnitude force we derived may play a critical role: the dynamics of compact objects orbiting the screwed SCCS and accretion of matter onto it. The deficit angle associated to the SCCS can be obtained and compared with data from the Cosmic Background Explorer (COBE) satellite. We also derived a value for the torsion contribution to matter density fluctuations in the early Universe.

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1. INTRODUCTION

Cosmic strings have exact solutions [1] which represent topological defects that may have been formed during phase transitions in the realm of the Early Universe [2,3]. Such defects arise in some grand Unified gauge theories. The GUT defects carry a large energy density and hence are of interest in Cosmology as potential sources for primordial density perturbations. These fluctuations would leave their imprint in the cosmic microwave background radiation (CMBR); a prediction not ruled out by COBE satellite observations yet [4], and hence would act as seeds for structure formation and thus as builders of the largest-scale structures in the Universe [5,6], such as the very high redshift superclusters of galaxies as for instance the *great wall*. They may also help to explain the most energetic events in the Universe such as the cosmological gamma-ray bursts (GRBs) [7–9], ultra high energy cosmic rays (UHECRs) and very high energy neutrinos [8] and gravitational-wave bursts [9] and backgrounds [10]. All these are issues deserving continuous investigation by many physicists nowadays [11–13].

Witten [14] has shown that the cosmic strings may possess superconducting properties and may behave like bosonic [15–20] or fermionic strings [21]. In other works it was supposed that the relevant superconductivity is generated during or very soon after the primary phase transition in which the string is formed. Physicists know that such strings may be superconducting defects and may carry substantial currents which have important astrophysical and cosmological effects, as the possibility of generation of a primordial magnetic field by a network of charged-current-carrying cosmic strings [22,23]. Existence of loops of SCCSs (*vortons*) were also investigated [21,24]. Torsion gained popularity in the 1970s when it was used to develope a local Poincaré gauged theory of gravity [25]. The interest in torsion gained more importance with the advent of string theory [26] and it appears also in supergravity theories [27]. Theories of gravitation in a space-time with torsion modify the spacetime geometry through non-symmetric connections [28] if the matter fields giving rise to the space-time curvature are endowed with spin. Cartan torsion has been connected previously with ordinary cosmic strings [29] and also spinning cosmic strings [30,31] from quite distinct point of views.

In this work we consider the study case of bosonic SCCSs in the minimal extension to Riemann-Cartan space-time. One should regard such an extension as a first step of a comprehensive study of cosmic string models in the context of theories including torsion. We aim at dealing with most realistic models which demand supersymmetry (an essential ingredient of grand-unification theories, string theory, etc), so we ought to combine both gravitational and spin degrees of freedom in the same formalism, and thus torsion is required. Our results concerning proper supersymmetric cosmic string models are the main subject of forthcoming communications [32,34].

The main-stream of this paper is as follows: we explored the physics of screwed cosmic strings in section II. In section III we derive the set of Einstein-Cartan equations describing the dynamics of the SCCSs pervaded by torsion stresses. An external solution for the SCCS metric in this scenario in presented in section IV, while in section V we derive the corresponding one for the internal structure of the SCCS by using the weak-field approximation. The junction conditions are obtained in section VI, where we find the specific form for the metric components, and discuss briefly the most immediate implications of such a solution. Two applications are provided. One focusing on the deviation of a particle moving near a screwed string. It is shown that such a high intensity of the gravitational force from the screwed SCCS (when compared with the one generated by a current-carrying string) may have important effects on the dynamics of compact objects orbiting around it, and also on matter being accreted by the string itself. The second one exploits the possibility that the temperature fluctuations in the cosmic microwave background radiation could have been, at least partially, generated after the SCCSs having interacted with it. We obtain a neat expression for the deficit angle in this context and a comparison is done with data from COBE satellite. We end this paper with a short summary of the picture here suggested.

2. SCREW DISLOCATIONS INSIDE COSMIC STRINGS

Next we shall try to understand the torsion effects on the space-time metric when the models under consideration have no fermionic fields. Here we construct a consistent framework for the torsion field pervading a cosmic string and define the vortex configuration for this problem. We choose here to analyse the simplest case where the torsion appears. In this line of reasoning, it is possible to describe torsion as a gradient-like field [35,36]

$$S_{\mu\nu}{}^{\lambda} = \frac{1}{2}[\delta^{\lambda}_{\mu}\partial_{\nu}\Lambda - \delta^{\lambda}_{\nu}\partial_{\mu}\Lambda], \tag{2.1}$$

being the Λ field the source of torsion in the string. Thus the SCCS generates a space-time curvature and torsion. The construction and dependence of the Λ potential on the physical variables of our system shall be specified in this work in the case where a screw dislocation is supposed to appear.

We shall proceed by considering an Abelian Higgs model with $U(1)' \times U(1)$ symmetry, which is the simplest model allowing for symmetry breaking. In this model we have a scalar field φ; a gauge field C_μ which gives us the vortex of the string, and a scalar field $\tilde{\sigma}$ along with a gauge field A_μ which encodes the superconducting properties.

A. Physics of a screwed string

In order to properly characterise the kind of cosmic string we will focus on, i. e., a current-carrying one endowed with curvature and torsion, next we shall specify its overall physical constituents and dynamical evolution from an action principle. The action representing the SCCS in a space-time with torsion can be written as:

$$S = \int d^4x\sqrt{g}\left[\frac{1}{16\pi G}R(\{\}) + \alpha_1\nabla_\mu S^\mu + \alpha_2 S_{\mu\nu k}S^{\mu\nu k} + \alpha_3 S_{\mu\nu k}S^{\mu k\nu} + \alpha_4 S_\mu S^\mu\right] + S_m, \tag{2.2}$$

where $R(\{\})$ is the curvature scalar of the Riemannian theory and S_m is the matter action that describes the superconducting cosmic string (to be specified below), and the constant α_1 is connected with the torsion gradient term $\nabla_\mu S^\mu$. Here ∇_μ is a Riemannian covariant derivative which drops out from the action because it is a term involving a total derivative. $S_{\mu\nu k}$ and S_μ are $SO(1,3)$ irreducible components of the torsion. In general, the torsion can be decomposed into three components

$$S_{\mu\nu}{}^{\lambda} = A_{\mu\nu}{}^{\lambda} + \delta^{\lambda}_{\mu} S_{\nu} - \delta^{\lambda}_{\nu} S_{\mu} + \Theta^{\lambda}{}_{\mu\nu}, \tag{2.3}$$

where $\Theta_{[\alpha\beta\gamma]} \equiv 0$, $\Theta^{\alpha}_{\alpha\gamma} \equiv 0$, $A_{[\alpha\beta\gamma]} \equiv \Theta_{[\alpha\beta\gamma]}$. The traceless part of torsion can be written as $C^{\alpha}_{\beta\gamma} \equiv \Theta^{\alpha}_{\beta\gamma} + A^{\alpha}_{\beta\gamma}$, with $\Theta^{\alpha}_{\beta\gamma}$ being given by

$$\Theta^{\alpha}_{\beta\gamma} = \frac{i}{6} \varepsilon_{\mu\nu}{}^{k\lambda} \Theta_{\lambda}. \tag{2.4}$$

Because the minimal extension to Riemann-Cartan space is sufficient for our purpose we set $C^{\alpha}_{\beta\gamma} = 0$. The quadratic torsion terms in the action do not give any dynamics to the torsion and the motivation for introducing them may be found in reference [37].

The most general connection $\Gamma_{\lambda\nu}{}^{\alpha}$ written in terms of the contortion tensor $K_{\lambda\nu}{}^{\alpha}$ is

$$\Gamma_{\lambda\nu}{}^{\alpha} = \{{}^{\alpha}_{\lambda\nu}\} + K_{\lambda\nu}{}^{\alpha}, \tag{2.5}$$

where $K_{\lambda\nu}{}^{\alpha}$ can be written in terms of the torsion field as:

$$K_{\lambda\nu}{}^{\alpha} = -\frac{1}{2}(S_{\lambda}{}^{\alpha}{}_{\nu} + S_{\nu}{}^{\alpha}{}_{\lambda} - S_{\lambda\nu}{}^{\alpha}). \tag{2.6}$$

For this minimally extended Riemann-Cartan space the affine connection can be written in terms of $g_{\mu\nu}$ and $S_{\alpha} = \partial_{\alpha}\Lambda$ as

$$\Gamma_{\lambda\nu}{}^{\alpha} = \{{}^{\alpha}_{\lambda\nu}\} + S^{\alpha} g_{\lambda\nu} - S_{\lambda}\delta^{\alpha}_{\nu} \tag{2.7}$$

so that S_{μ} is the only piece that contributes to torsion, which here is the escalar derivative defined by (2.1).

Then we may consider a theory of *gravitation* possessing torsion by writing that part of the action S_G stemming from the curvature scalar R as:

$$S_G = \int d^4x \sqrt{g} \left[\frac{1}{16\pi G} R(\{\}) - \frac{\alpha}{2} \partial_{\mu}\Lambda \partial^{\mu}\Lambda \right], \tag{2.8}$$

where the coupling constant α will be specified with the help of COBE data.

In this work we use Eq.(2.1) where Λ generally depends on all coordinates. We postpone for a moment to specify the formal dependence of the vortex ansatz upon torsion (see below). Meanwhile, for the single defect the solutions correspond to flat space everywhere, except on the defect itself where there is curvature or torsion or both. Here we focus on the case of SCCSs having curvature and screw dislocation (a type of torsion), however, the space outside the defect has propagation of the effects of both the current and torsion. Our anzats for the function Λ assumes r, θ and z dependence of the form:

$$\Lambda(r, \theta, z) = \Sigma(r) + K(\theta, z), \tag{2.9}$$

where

$$K(\theta, z) = k_1 z + k_2 \theta \tag{2.10}$$

represents the screw in the SCCS, with k_1 and k_2 being scaling constants of the problem. Function $K(\theta, z)$ represents the *screw disclination*-like distorsion of the string, while the $\Sigma(r)$ function defines that part of torsion carrying off the string information about itself. The asymptotic conditions reads:

$$\begin{array}{ll} \Sigma(r) = 0 & r = 0 \\ \Sigma(r) \neq 0 & r \to \infty. \end{array} \tag{2.11}$$

We can study the SCCS considering the Abelian Higgs model with two scalar fields, ϕ and $\tilde{\sigma}$. In this case, the action for all matter fields turns out to be:

$$S_m = \int d^4x\sqrt{g}[-\frac{1}{2}D_\mu\phi(D^\mu\phi)^* - \frac{1}{2}D_\mu\tilde{\sigma}(D^\mu\tilde{\sigma})^* - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} - V(|\varphi|, \tilde{\sigma})], \tag{2.12}$$

where $D_\mu\tilde{\sigma} = (\partial_\mu + ieA_\mu)\tilde{\sigma}$ and $D_\mu\phi = (\partial_\mu + iqC_\mu)\phi$ are the covariant derivatives, and $V(|\phi|, |\tilde{\sigma}|)]$ is the potential for symmetry breaking (to be specified below). The field strenghts are defined as

$$\begin{array}{l} F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\ H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu \end{array} \tag{2.13}$$

with A_μ and C_ν being the gauge fields. In fact, the covariant derivative D_μ possesses the generalized component of the connection, $\Gamma_{\alpha\beta}{}^{\mu}$, through the definition

$$D_\alpha B_\mu = \partial_\alpha B_\mu - \Gamma_{\alpha\mu}{}^{\beta} B_\beta, \tag{2.14}$$

which has symmetric and non-symmetric contributions to the connection. We stress that the non-symmetric components of D_μ are associated with torsion. However, by knowing that the coupling of the gauge field to torsion breaks the gauge invariance we are assuming that there are no components of $\Gamma_{\alpha\beta}{}^{\mu}$ that couple with the gauge fields [38]. This is justified by virtue of the anti-symmetry of $F_{\mu\nu}$ and $H_{\mu\nu}$. The derivatives of the gauge potential do not need to be covariant: $F_{\mu\nu}$ and $H_{\mu\nu}$ are genuine tensors under General Coordinate Transformations even if they are built up with ordinary derivatives. So, no connection, and consequently, no torsion couples to A_μ and C_μ through their associated field strengths.

This action Eq.(2.1) has a $U(1)' \times U(1)$ symmetry, where the $U(1)'$ group associated with the ϕ-field, is broken and gives rise to vortices of the Nielsen-Olesen [39]

$$\begin{aligned} \phi &= \varphi(r)e^{i\theta} \\ C_\mu &= \tfrac{1}{q}[P(r)-1]\delta^\theta_\mu \end{aligned} \tag{2.15}$$

parametrized in cylindrical coordinates (t, r, θ, z), where $r \geq 0$ and $0 \leq \theta < 2\pi$, with C_μ the gauge field. The boundary conditions for the fields $\varphi(r)$ and $P(r)$ are the same as those of ordinary cosmic strings [39]:

$$\begin{array}{ll} \varphi(r) = \eta \;\; r \to \infty & \qquad P(r) = 0 \;\; r \to \infty \\ \varphi(r) = 0 \;\; r \to 0 & \qquad P(r) = 1 \;\; r \to 0. \end{array} \tag{2.16}$$

The other group $U(1)$ entails the $\tilde{\sigma}$-field which we identify with Electromagnetism, which remains unbroken in vacuum, but broken in the string interior. The $\tilde{\sigma}$-field in the string core, where it acquires an expectation value, is responsible for a bosonic current being carried by the gauge field A_μ. The only non-vanishing components of the gauge fields are $A_z(r)$ and $A_t(r)$ and the current-carrier phase may be expressed $\zeta(z,t) = \omega_1 t - \omega_2 z$. Notwithstanding, we focus only on the magnetic case [20]. Their configurations are:

$$\begin{aligned} \tilde{\sigma} &= \sigma(r)e^{i\zeta(z,t)} \\ A_\mu &= \tfrac{1}{e}[A(r) - \tfrac{\partial\zeta(z,t)}{\partial z}]\delta^z_\mu. \end{aligned} , \tag{2.17}$$

because of the rotational symmetry of the string itself. The fields responsible for the cosmic string superconductivity have the following boundary conditions:

$$\begin{array}{ll} \frac{d}{dr}\sigma(r) = 0 \;\; r \to 0 & \qquad A(r) \neq 0 \;\; r \to \infty \\ \sigma(r) = 0 \qquad r \to \infty & \qquad A(r) = 1 \;\; r \to 0 \\ & \qquad \frac{dA(r)}{dr} = 0 \;\; r \to 0. \end{array} \tag{2.18}$$

On the other hand, the potential $V(\varphi, \sigma)$ triggering the symmetry breaking can be defined by:

$$V(\varphi, \sigma) = \frac{\lambda_\varphi}{4}(|\varphi|^2 - \eta^2)^2 + f|\varphi|^2|\sigma|^2 + \frac{\lambda_\sigma}{4}|\sigma|^4 - \frac{m_\sigma^2}{2}|\sigma|^2, \tag{2.19}$$

where λ_φ, λ_σ and f are coupling constants, and the boson mass being defined by m_σ.

The existence of torsion does not alter the vortex configuration, i. e., torsion is only responsible for the appearance of further properties in the string but without destroying the cylindrical nature of the vortex nor the current configurations in the string. Thus, in the limit where the torsion gradient does vanish the string recovers its basic superconducting character.

3. EINSTEIN-CARTAN EQUATIONS

Let us consider a SCCS in a cylindrical coordinate system (t, r, θ, z), so that $r \geq 0$ and $0 \leq \theta < 2\pi$ with the metric defined in these coordinates as:

$$ds^2 = e^{2(\gamma-\psi)}(-dt^2 + dr^2) + \beta^2 e^{-2\psi} d\theta^2 + e^{2\psi} dz^2 \tag{3.20}$$

where γ, ψ and β depend only on r. We can write Einstein-Cartan equations in the quasi-Einsteinian form:

$$G^{\mu}_{\nu}(\{\}) = 8\pi G(2\alpha g^{\mu\alpha}\partial_\alpha \Sigma \partial_\nu \Sigma - \alpha \delta^{\mu}_{\nu} g^{\alpha\beta}\partial_\alpha \Sigma \partial_\beta \Sigma + T^{\mu}_{\nu}) \tag{3.21}$$

where $(\{\})$ stands for Riemannian geometric objects, and δ^{μ}_{ν} and T^{μ}_{ν} correspond to the identity and energy-momentum tensors, respectively. We can find the Einstein-Cartan equations as

$$G^{t}_{t}(\{\}) = 8\pi G\left(T^{t}_{scs\ t} - g^{rr}\alpha\Sigma'^2\right) = 8\pi G\tilde{T}^{t}_{t} \tag{3.22}$$

$$G^{r}_{r}(\{\}) = 8\pi G\left(T^{r}_{scs\ r} + g^{rr}\alpha\Sigma'^2\right) = 8\pi G\tilde{T}^{r}_{r} \tag{3.23}$$

$$G^{\theta}_{\theta}(\{\}) = 8\pi G\left(T^{\theta}_{scs\ \theta} - g^{rr}\alpha\Sigma'^2\right) = 8\pi G\tilde{T}^{\theta}_{\theta} \tag{3.24}$$

$$G^{z}_{z}(\{\}) = 8\pi G\left(T^{z}_{scs\ z} - g^{rr}\alpha\Sigma'^2\right) = 8\pi G\tilde{T}^{z}_{z}. \tag{3.25}$$

where $\tilde{T}^{\mu}_{\nu}$ tensor corresponds to an eneregy-momentum tensor contaning the torsion field.

We have seen that the dependence upon torsion is represented, in the quasi-Einstenian form, by the Σ-field that has an equation of motion given by Eq.(3.35) below, whose solution shall be addressed subsequently.

The SCCS energy-momentum tensor is defined by:

$$T^{\mu}_{(scs)\nu} = \frac{2}{\sqrt{g}}\frac{\delta S_m}{\delta g_{\mu\nu}}, \tag{3.26}$$

which yields:

$$T^{t}_{scs\ t} = -\frac{1}{2}\left\{e^{2(\psi-\gamma)}[\varphi'^2 + \sigma'^2] + \frac{e^{2\psi}}{\beta^2}\varphi^2 P^2 + e^{-2\psi}\sigma^2 A^2 + \right.$$
$$\left. + \frac{e^{2(2\psi-\gamma)}}{\beta^2}(\frac{P'}{q})^2 + e^{-2\gamma}(\frac{A'}{e})^2 + 2V(\varphi,\sigma)\right\} \tag{3.27}$$

$$T^r_{scs\ r} = \frac{1}{2}\left\{ e^{2(\psi-\gamma)}[\varphi'^2 + \sigma'^2] - \frac{e^{2\psi}}{\beta^2}\varphi^2 P^2 - e^{-2\psi}\sigma^2 A^2 + \right.$$
$$\left. + \frac{e^{2(\psi-\gamma)}}{\beta^2}(\frac{P'}{q})^2 + e^{-2\gamma}(\frac{A'}{e})^2 - 2V(\varphi,\sigma)\} \right\} \tag{3.28}$$

$$T^\theta_{scs\ \theta} = -\frac{1}{2}\left\{ e^{2(\psi-\gamma)}[\varphi'^2 + \sigma'^2] - \frac{e^{2\psi}}{\beta^2}\varphi^2 P^2 + e^{-2\psi}\sigma^2 A^2 + \right.$$
$$\left. - \frac{e^{2(\psi-\gamma)}}{\beta^2}(\frac{P'}{q})^2 + e^{-2\gamma}(\frac{A'}{e})^2 + 2V(\varphi,\sigma) \right\} \tag{3.29}$$

$$T^z_{scs\ z} = -\frac{1}{2}\left\{ e^{2(\psi-\gamma)}[\varphi'^2 + \sigma'^2] + \frac{e^{2\psi}}{\beta^2}\varphi^2 P^2 - e^{-2\psi}\sigma^2 A^2 + \right.$$
$$\left. + \frac{e^{2(\psi-\gamma)}}{\beta^2}(\frac{P'}{q})^2 - e^{-2\gamma}(\frac{A'}{e})^2 + 2V(\varphi,\sigma) \right\}. \tag{3.30}$$

In these expressions Eqs.(3.26-3.30) only the usual fields of the string are present. When addressing the solution of the Einstein-Cartan equations we shall see that the torsion will naturally appear in the metric components. The Euler-Lagrange equations result from variation of the Eq.(2.2) together with the conditions for the Nielsen-Olesen [39] vortex Eqs.(2.15-2.17), and yield:

$$\varphi'' + \frac{1}{r}\varphi' + \frac{\varphi P^2}{r^2} - \varphi[\lambda_\varphi(\varphi^2 - \eta^2) + 2f\sigma^2] = 0 \tag{3.31}$$

$$P'' + \frac{1}{r}P' - q^2\frac{\varphi^2 P}{r^2} = 0 \tag{3.32}$$

$$\sigma'' + \frac{1}{r}\sigma' + \sigma[A^2 + \lambda_\varphi(\varphi^2 + \lambda_\sigma\sigma^2 - m_\sigma^2)] = 0 \tag{3.33}$$

$$A'' + \frac{1}{r}A' + e^2\sigma^2 A = 0, \tag{3.34}$$

while the torsion wave equation for these conditions Eq.(2.11) is given by:

$$\Box_g \Sigma = 0. \tag{3.35}$$

Above a prime denotes differentiation with respect to the radial coordinate r. As they look, these equations cannot be solved exactly for the whole spacetime. However, we can find a solution by splitting space-time into two regions. One corresponding to the internal $r < r_0$ and another defining the external $r > r_0$. The external solution is derived in an exact treatment of the set of equations. The general solution for the SCCS will be found in the weak-field approximation together with junction conditions for the external metric.

4. THE EXTERNAL SOLUTION

Now we proceed to solve the previous set of equations for an observer outside the SCCS stressed by torsion focusing on the external metric which satifies the constraint $r_0 \leq r \leq \infty$. In a torsionless space-time it is possible to find an exact external solution. The external contribution to the energy-momentum of the string reads

$$\mathcal{T}^{\mu}_{\nu} = \frac{1}{4}(g^{\mu\alpha}g^{\beta\rho}F_{\alpha\beta}F_{\nu\rho}) - \delta^{\mu}_{\nu}g^{\sigma\alpha}g^{\beta\rho}F_{\sigma\beta}F_{\alpha\rho}. \tag{4.1}$$

This tensor is the external energy-momentum tensor of a SCCS with no torsion. If we observe the asymptotic conditions Eq.(2.16) and Eq.(2.18) we see that the only field that does not vanish is the A_{μ}-field that is responsible for carrying off the string the effects of the current on it. The torsion contribution to the external energy-momentum tensor is given by

$$\mathcal{T}^{\mu}_{\nu_{tors}} = 2\alpha g^{\mu\alpha}\partial_{\alpha}\Sigma\partial_{\nu}\Sigma - \alpha\delta^{\mu}_{\nu}g^{\alpha\beta}\partial_{\alpha}\Sigma\partial_{\beta}\Sigma. \tag{4.2}$$

For this configuration the energy-momentum tensor displays the following symmetry properties:

$$\mathcal{T}^{t}_{t} = -\mathcal{T}^{r}_{r} = \mathcal{T}^{\theta}_{\theta} = -\mathcal{T}^{z}_{z}. \tag{4.3}$$

Then, the only one component of Λ in Eq.(3.35) to be solved is the $r-$dependent function $\Sigma(r)$. The solution reads:

$$\Sigma(r) = \lambda \ln(r/r_0). \tag{4.4}$$

Solution of equations (3.22)-(3.25) are found from the symmetries of the set of equations

$$\beta'' = 8\pi G\beta(\mathcal{T}^{t}_{t} + \mathcal{T}^{r}_{r})e^{2(\gamma-\psi)} \tag{4.5}$$

$$(\beta\gamma')' = 8\pi G\beta(\mathcal{T}^{r}_{r} + \mathcal{T}^{\theta}_{\theta})e^{2(\gamma-\psi)} \tag{4.6}$$

$$(\beta\psi')' = 8\pi G\beta(\mathcal{T}^{t}_{t} + \mathcal{T}^{r}_{r} + \mathcal{T}^{\theta}_{\theta} - \mathcal{T}^{z}_{z})e^{2(\gamma-\psi)}. \tag{4.7}$$

One may check that the torsion contribution to the energy-momentum components in the RHS of Eqs. (4.5)-(4.7) is vanishing. Hence the solutions of Eq.(4.5) and Eq.(4.6) are given by

$$\beta = Br \tag{4.8}$$

$$\gamma = m^2 \ln r/r_0. \tag{4.9}$$

To solve Eq.(4.7), and to find the ψ-solution, we can use the condition:

$$\varphi = 2\Sigma'^2 e^{2(\psi-\gamma)}. \tag{4.10}$$

This condition is different from the usual one [20] because the scalar curvature φ does not vanish, and opposedly it is linked to the torsion-field Σ. Then, this condition has the same form as the one for a SCCS in a scalar-tensor theory [34]. By making use of solutions (4.4), (4.8) and (4.9) we find:

$$\psi = n \ln (r/r_0) - \ln \frac{(r/r_0)^{2n} + k}{(1+k)}. \tag{4.11}$$

Thus we see that from the solutions of the SCCS (4.8-4.11) there exists a relationship between the parameters n, λ and m given by:

$$n^2 = \lambda^2 + m^2. \tag{4.12}$$

With the above results, we find that the external metric for the SCCS takes the form:

$$g_{tt} = -(r/r_0)^{-2\lambda^2}(r/r_0)^{2(n^2-n)}W^2(r), \tag{4.13}$$

$$g_{rr} = (r/r_0)^{-2\lambda^2}(r/r_0)^{2(n^2-n)}W^2(r), \tag{4.14}$$

$$g_{\theta\theta} = (B)^2(r/r_0)^{2-2n}W^2(r), \tag{4.15}$$

$$g_{zz} = (r/r_0)^{2n}/W^2(r), \tag{4.16}$$

with

$$W(r) = \frac{[(r/r_0)^{2n} + k]}{[1+k]}. \tag{4.17}$$

The external solution alone does not provide a complete description of the physical situation. We proceed hereafter to find the junction conditions to the internal metric in order to obtain an appropriate accounting for the nature of the source and its effects on the surrounding space-time.

5. SCCS SOLUTION: THE WEAK-FIELD APPROXIMATION

We have seen that it is possible to find an analytical external solution to Einstein-Cartan equation but not a general solution. Now let us find the Einstein-Cartan solutions for a SCCS by considering the weak-field approximation, where both the exterior and interior metrics are considered as only weakly perturbed from the flat space metric. Thus, the space-time metric may be expanded in terms of a small parameter ε about the values $g_{(0)\mu\nu} = diag(-1,1,1,1)$, then:

$$g_{\mu\nu} = g_{(0)\mu\nu} + \varepsilon h_{\mu\nu}, \tag{5.1}$$

while

$$\tilde{T}_{\mu\nu} = \tilde{T}_{(0)\mu\nu} + \varepsilon \tilde{T}_{(1)\mu\nu}. \tag{5.2}$$

The $\tilde{T}_{(0)\mu\nu}$ tensor corresponds to the energy-momentum tensor in a space-time with no curvarture. However, torsion is embeeded. $\tilde{T}_{(1)\mu\nu}$ represents the part of the energy-momentum tensor containing curvature and torsion. As we can see both tensors have dependence upon the torsion. The torsion contribution in this appraoch comes from the non-Riemannian part of the Einstein-Cartan tensor which can be written in a quasi-Einstenian form Eq.(3.21) with components given by Eqs.(3.22)-(3.25). Next we proceed to define some important quantities useful for the analysis to come.

The energy-momentum density of the thin SCCS is given by:

$$U = -2\pi \int_0^{r_0} \tilde{T}^t_{(0)t} r dr \tag{5.3}$$

which corresponds to the energy per unit length. The string tension reads:

$$T = -2\pi \int_0^{r_0} \tilde{T}^z_{(0)z} r dr. \tag{5.4}$$

The remaining components follows as

$$X = -2\pi \int_0^{r_0} \tilde{T}^r_{(0)r} r dr \tag{5.5}$$

and

$$Y = -2\pi \int_0^{r_0} \tilde{T}^\theta_{(0)\theta} r dr, \tag{5.6}$$

while the torsion density that satisfies Eq.(4.2) is given by

$$S^2 = 2\pi r^2 \Sigma'^2 \tag{5.7}$$

Combining these relations we can show that the energy conservation follows as:

$$\partial_r \tilde{T}^r_r + (\tilde{T}^r_r - \tilde{T}^\theta_\theta)(\frac{\partial_r \beta}{\beta} - \partial_r \psi) + \partial_r \gamma(\tilde{T}^r_r - \tilde{T}^t_t) - \partial_r \psi(\tilde{T}^z_z - \tilde{T}^t_t) = 0. \qquad (5.8)$$

In the weak-field approximation the expression (5.8) reduces to

$$r\frac{dT^r_{(0)r}}{dr} = (T^\theta_{(0)\theta} - T^r_{(0)r}), \qquad (5.9)$$

where $T_{(0)\mu\nu}$ represents the tensor with no torsion.

Moreover, in the *weak-field* approximation those terms contributing to torsion drop out the expression (5.9) and the relation turns to be as usual, that is, it only happens because the screw configuration does not propagate due to the choice Eq.(2.1). The only field deriving from torsion is related to $\Sigma(r)$. In forthcoming contributions we will address situations where for instance torsion stems from vector fields with full dependence upon the vector components. [32], a study case not occurring in the present work.

For computing the overall metric we use the Einstein-Cartan in the quasi-Einsteinian Eq.(3.21), where it gets the form $G^{\mu\nu}(\{\}) = 8\pi G\tilde{T}^{\mu\nu}_{(0)}$ in the weak-field approximation, with the tensor $\tilde{T}_{(0)\mu\nu}$ (being first order one in G) containing torsion. After integration we have:

$$\int_0^{r_0} rdr(\tilde{T}^\theta_{(0)\theta} + \tilde{T}^r_{(0)r}) = \int_0^{r_0} rdr(T^\theta_{(0)\theta} + T^r_{(0)r}) = r_0^2 T^r_{(0)r}(r_0) = r_0^2\frac{A'^2(r_0)}{2e^2}. \qquad (5.10)$$

We can see that torsion does not appear in this combination of fields Eq.(5.10). We shall clarify the meaning of this issue later on.

To find the internal energy-momentum tensor it is more convenient to use Cartesian coordinates [20]. For this purpose we proceed to calculate the cross-section integrals of $\tilde{T}^x_{(0)x}$ and $\tilde{T}^y_{(0)y}$ so that the non-vanishing components of the energy-momentum tensor can now be written in cartesian coordinates as

$$\tilde{T}^t_{(0)t} = -\frac{1}{2}\left\{\varphi'^2 + \sigma'^2 + \frac{\varphi^2 P^2}{r^2} + \sigma^2 A^2 + \left(\frac{P'}{rq}\right)^2 + \left(\frac{A'}{e}\right)^2 + 2V + \alpha\Sigma'^2\right\} \qquad (5.11)$$

$$\tilde{T}^x_{(0)x} = (\cos\theta - \frac{1}{2})[\varphi'^2 + \sigma'^2 + \left(\frac{A'}{e}\right)^2 + \alpha\Sigma'^2] + (\sin\theta - \frac{1}{2})\frac{\varphi^2 P^2}{r^2} + \frac{1}{2}\left(\frac{P'}{q}\right)^2 - \frac{1}{2}\sigma^2 A^2 - 2V, \qquad (5.12)$$

$$\tilde{T}^y_{(0)y} = (\sin\theta - \frac{1}{2})[\varphi'^2 + \sigma'^2 + \left(\frac{A'}{e}\right)^2 + \alpha\Sigma'^2] + (\cos\theta - \frac{1}{2})\frac{\varphi^2 P^2}{r^2} + \frac{1}{2}\left(\frac{P'}{qr}\right)^2 - \frac{1}{2}\sigma^2 A^2 - 2V \qquad (5.13)$$

$$\tilde{T}^{z}_{(0)z} = -\frac{1}{2}\left\{\varphi'^2 + \sigma'^2 + \frac{\varphi^2 P^2}{r^2} - \sigma^2 A^2 + \left(\frac{P'}{rq}\right)^2 - \left(\frac{A'}{e}\right)^2 + 2V - \alpha\Sigma'^2\right\}, \tag{5.14}$$

This way we found:

$$\int r dr d\theta \tilde{T}^{x}_{(0)x} = \int r dr d\theta \tilde{T}^{y}_{(0)y} = \pi \int r dr [\left(\frac{P'}{qr}\right)^2 - \sigma^2 A^2 - V] = -\mathcal{W}. \tag{5.15}$$

Using the fact that

$$\tilde{T}^{r}_{(0)r} + \tilde{T}^{\theta}_{(0)\theta} = \tilde{T}^{x}_{(0)x} + \tilde{T}^{y}_{(0)y}, \tag{5.16}$$

then we have:

$$X + Y = 2\mathcal{W} = -r_0^2 \frac{A'^2(r_0)}{e^2}, \tag{5.17}$$

which can be computed by integration of Eq.(3.34)

$$\mathcal{J} = \int_0^{r_0} r dr \sigma^2 A, \qquad J = 2\pi \mathcal{J} \tag{5.18}$$

where J is the current density, to obtain

$$\mathcal{W} = -\frac{e^2}{2}\mathcal{J}^2. \tag{5.19}$$

However, electromagnetic $U(1)$ invariance requires the Noether current to be written as

$$J^\mu = \sigma^2(\nabla^\mu \psi + eA^\mu), \tag{5.20}$$

which is conserved in Einstein-Cartan theory. In this form $\mathcal{J}$ is the integrated norm of the conserved current. With these internal considerations we found the string structure. We can assume that the string is infinitely thin so that its stress-energy tensor is given by

$$\tilde{T}^{\mu\nu}_{string} = diag[U, -W, -W, -T]\delta(x)\delta(y). \tag{5.21}$$

It worths to note that definitions for both string energy U and tension T, as in equations (5.3)-(5.4), already incorporate information on the torsion. However, in cartesian coordinates the cross terms only exhibit dependence on the current for the string internal energy-momentum tensor. Notwithstanding, this tensor by itself is not conserved, we still need to take into consideration the string external fields, which will entail contributions from both current and torsion.

By virtue of the presence of the external current we use the form Eq.(5.21) for the string energy-momentum tensor as well as Eq.(4.1) and Eq.(4.2) for the external energy-momentum tensor in linearized solution to zeroth order in G. Thus the nonvanishing external components of Maxwell and torsion tensors are

$$\mathcal{T}_{(0)tt} = \frac{(J^2 + \alpha S^2)}{2\pi r^2} \tag{5.22}$$

$$\mathcal{T}_{(0)zz} = \frac{(J^2 - \alpha S^2)}{2\pi r^2} \tag{5.23}$$

$$\mathcal{T}_{(0)ij} = \frac{J^2}{2\pi r^4}(2x_i x_j - r^2\delta_{ij}) \tag{5.24}$$

where "i, j" denotes Cartesian components in the transverse plane (x,y).

As we can check the solution for the $\Sigma field$, in first order in G, is the same that is found in the external case given by Eq.(4.4) and have the same form of the external solution to $F_{\mu\nu}$, i. e., logarithmic divergent in the sense of distributions

$$\begin{aligned} &\nabla^2 ln(r/r_0) = 2\pi\delta(x)\delta(y) \\ &\nabla^2(\ln(r/r_0))^2 = 2/r^2 \\ &\nabla^2(r^2\partial_i\partial_j ln(r/r_0)) = 4\partial_i\partial_j ln(r/r_0). \end{aligned} \tag{5.25}$$

The energy-momentum tensor of the string source $\tilde{T}_{(0)\mu\nu}$, in cartesian coordinates, possesses no curvature, which is the well-known result [20,19,34], but does have torsion which produces the following energy-momentun tensor

$$\tilde{T}_{(0)tt} = U\delta(x)\delta(y) + \frac{(J^2 + \alpha S^2)}{4\pi}\nabla^2\left(ln\frac{r}{r_0}\right)^2, \tag{5.26}$$

$$\tilde{T}_{(0)zz} = -T\delta(x)\delta(y) + \frac{(J^2 - \alpha S^2)}{4\pi}\nabla^2\left(ln\frac{r}{r_0}\right)^2, \tag{5.27}$$

$$\tilde{T}_{(0)ij} = J^2\delta_{ij}\delta(x)\delta(y) - \frac{J^2}{2\pi}\partial_i\partial_j ln(r/r_0), \tag{5.28}$$

where the trace is given by

$$\tilde{T}_{(0)} = -(U + T - J^2)\delta(x)\delta(y) - \frac{\alpha S^2}{2\pi}\nabla^2\left(\ln\frac{r}{r_0}\right)^2. \tag{5.29}$$

Now let us find the matching conditions to the external solution. For this purpose we are using the linearized Einstein-Cartan equation in the form

$$\nabla^2 h_{\mu\nu} = -16\pi G(\tilde{T}_{(0)\mu\nu} - \frac{1}{2} g_{(0)\mu\nu} \tilde{T}_{(0)}). \tag{5.30}$$

The internal solution to equation (5.30) with source yields:

$$h_{tt} = -4G[J^2(\ln(r/r_0))^2 + (U - T + J^2)\ln(r/r_0)] \tag{5.31}$$

$$h_{zz} = -4G[J^2 \ln(r/r_0))^2 + (U - T - J^2)\ln(r/r_0)] \tag{5.32}$$

$$h_{ij} = -2GJ^2 r^2 \partial_i \partial_j \ln(r/r_0) - 4G\delta_{ij}\left[\Big((U + T + J^2)\ln(r/r_0)\Big) + S^2\left(\ln\frac{r}{r_0}\right)^2\right]. \tag{5.33}$$

This corresponds to the solution in cartesian coordinates. We note that torsion S appears explicitly in the transverse components of the metric. To analyse the solution for the junction condition to the external metric let us transform it back into cylindrical coordinates.

6. MATCHING CONDITIONS

It is possible to find the matching conditions [41] to the external solution. In the case of a space-time with torsion we can find the junction conditions using the fact that $[\{{}^{\alpha}_{\mu\nu}\}]^{(+)}_{r=r_0} = [\{{}^{\alpha}_{\mu\nu}\}]^{(-)}_{r=r_0}$, and the metricity condition $[\nabla_\rho g_{\mu\nu}]^{+}_{r=r_0} = [\nabla_\rho g_{\mu\nu}]^{-}_{r=r_0} = 0$, to find the continuity conditions

$$\begin{aligned} [g_{\mu\nu}]^{(-)}_{r=r_0} &= [g_{\mu\nu}]^{(+)}_{r=r_0}, \\ [\tfrac{\partial g_{\mu\nu}}{\partial x^\alpha}]^{(+)}_{r=r_0} + 2[g_{\alpha\rho} K_{(\mu\nu)}^{\ \ \rho}]^{(+)}_{r=r_0} &= [\tfrac{\partial g_{\mu\nu}}{\partial x^\alpha}]^{(-)}_{r=r_0} + 2[g_{\alpha\rho} K_{(\mu\nu)}^{\ \ \rho}]^{(-)}_{r=r_0} \end{aligned} \tag{6.1}$$

Where $(-)$ represents the internal region and $(+)$ corresponds the external region around $r = r_0$. In analysing the junction conditions we notice that the contortion contributions do not appear neither in the internal nor in the external regions [41,42].

To match our solution with the external metric we used the metric in cylindrical coordinates, which is obtained from the coordinate transformations:

$$r^2 \partial_i \partial_j ln(r/r_0) dx^i dx^j = r^2 d\theta^2 - dr^2, \tag{6.2}$$

to get

$$g_{tt} = -\{1 + 4G[J^2(\ln(r/r_0))^2 + (U - T + J^2)\ln(r/r_0)]\} \tag{6.3}$$

$$g_{zz} = \{1 - 4G[J^2(\ln(r/r_0))^2 + (U - T - J^2)\ln(r/r_0)]\} \tag{6.4}$$

$$g_{rr} = \{1 + 2GJ^2 - 4G\left[(U + T + J^2)\ln(r/r_0) + S^2\left(\ln\frac{r}{r_0}\right)^2\right]\} \tag{6.5}$$

$$g_{\theta\theta} = r^2\{1 - 2GJ^2 - 4G\left[(U + T + J^2)\ln(r/r_0) + S^2\left(\ln\frac{r}{r_0}\right)^2\right]\}. \tag{6.6}$$

Unfortunately, for this goal we cannot use the metric Eqs.(6.3-6.6) as it stands. Therefore, we have to change the radial coordinate to ρ, using the constraint (symmetry) $g_{\rho\rho} = -g_{tt}$, to have, to first order in G,

$$\rho = r[1 + a_1 - a_2\ln(r/r_0) - a_3(\ln(r/r_0)^2]. \tag{6.7}$$

In this case we have $a_1 = G(4(U - \alpha S^2) + J^2)$, $a_2 = 4G(U - \alpha S^2)$ and $a_3 = 2G(J^2 + \alpha S^2)$, which corresponds to the magnetic configuration of the string fields [20]. The transformed metric yields:

$$g_{tt} = -\{1 + 4G[J^2(\ln(\rho/r_0))^2 + (U - T + J^2)\ln(\rho/r_0)]\} = -g_{\rho\rho} \tag{6.8}$$

$$g_{zz} = \{1 - 4G[(J^2 + \alpha S^2)(\ln(\rho/r_0))^2 + (U - T + J^2)\ln(\rho/r_0)]\} \tag{6.9}$$

$$g_{\theta\theta} = \rho^2\{1 - 8G(U - \alpha S^2 + \frac{J^2}{2}) + 4G(U - T - J^2 - \alpha S^2)\ln(\rho/r_0)] + 4G(J^2 + S^2)(\ln(\rho/r_0))^2\}. \tag{6.10}$$

Now we can find the external parameters B, n and m as functions of the source structure. If we consider the junction of the equation (6.1), after the linearization, and using the limit $|n\ln(\rho/r_0)| << 1$, we have:

$$\left[\frac{dg_{zz}}{d\rho}\right]^{(+)} \sim 2\left[-\frac{4n^2k}{\rho(1+k)^2}ln(\rho/r_0) - \frac{n}{\rho}\left(\frac{1-k}{1+k}\right)\right]^{(+)}, \tag{6.11}$$

and the internal solution Eq.(6.9) has the derivative:

$$\left[\frac{dg_{zz}}{d\rho}\right]^{(-)} \sim \left[-\frac{4G}{\rho}[2J^2\ln(\rho/r_0) + (U - T - J^2)]\right]^{(-)}. \tag{6.12}$$

At point $\rho = r_0$ we find the junction condition:

$$n\left(\frac{1-k}{1+k}\right) = 2G\left(U - T - J^2\right). \tag{6.13}$$

Using now an analogous procedure to $g_{\theta\theta}$ considering the external metric Eq.(4.15) and internal metric Eq.(6.10) we have $\beta^2 = (1 - b)\rho^2$, with

$$b = 8G\{U + J^2(\frac{1}{2} + \ln(\rho/r_0)) - \alpha S^2(1 - \ln(\rho/r_0))\}. \tag{6.14}$$

Thus, at $\rho = r_0$ and $\beta(0) = 1$ we find that the expression for B as a function of the source is given by

$$B^2 = 1 - 8G\left(U - \alpha S^2 + \frac{J^2}{2}\right). \tag{6.15}$$

If we now proceed to make the junction of component g_{tt}, from our previous results, we find the expression for m being given by

$$m^2 = 4GJ^2. \tag{6.16}$$

and by definition Eq.(5.7) we have $2\pi\lambda^2 = S^2$. This expression completes the derivation of the full metric components.

7. BRIEF DISCUSSION

In analysing the metric of the SCCS with torsion we note that the contribution of torsion appears in the $\theta\theta$-metric component, which is important in astrophysical applications such as gravitational lensing studies because this component is linked to the deficit angle that should be written as a function of the parameter b Eq.(6.14). However, as it is apparent from Eq.(6.14) that the torsion introduces a reductional factor on the overall magnitude of the deficit angle. This would imply that the apparent change in position of double images from a single lensed source, for instance, or else the CMBR temperature angular scale of variation, could be reduced in this context when compared to the standard estimates for ordinary and superconducting string endowing no torsion. This topic will be discuss better later on where we make some applications to the COBE data.

Next we present two preliminary applications of the formalism here introduced assuming that such a kind of torsioned SCCSs really exist. Firstly, we focus on the issue of the deviation of a particle moving near the string, and later on we attempt to perform a comparative analysis of the effects this sort of string may produce on the CMBR, supposed to interact with it as discussed in this paper, to the observations performed by the COBE satellite.

8. PARTICLE DEFLECTION NEAR A SCREWED SCCS

We know that when the string possesses current there appear gravitational forces. We shall consider the effect that torsion plays on the gravitational force generated by SCCS on a particle moving around the defect, initially with no charge, and endowed with quadri-velocity $u^\mu = \frac{dx^\mu}{d\tau} \sim (1, \mathbf{v})$. In the weak field approach, as we have seen (see Eq.(2.5)), the connection can be written as:

$$\Gamma_{\mu\nu}^{\ \lambda} \equiv \frac{1}{2} g_{(0)}^{\lambda\rho} (\partial_\nu h_{\mu\rho} + \partial_\mu h_{\nu\rho} - \partial_\rho h_{\mu\nu}) + g_{(0)}^{\lambda\rho} K_{\mu\nu\rho}. \tag{8.1}$$

The general expression for the geodesic equation including torsion reads:

$$\frac{du^\lambda}{d\tau} + \Gamma^\lambda{}_{(\mu\nu)} u^\mu u^\nu = 0, \tag{8.2}$$

where $\Gamma^\lambda{}_{(\mu\nu)}$ is the $\mu\nu$ symmetric part of the connection, which can be written in terms of the torsion as:

$$\Gamma^\lambda{}_{(\mu\nu)} = \left\{ {}^{\ \lambda}_{\mu\nu} \right\} + S^\lambda_{(\mu\nu)}. \tag{8.3}$$

Thus, we consider the particle speed $|\mathbf{v}| \leq 1$, condition under which the geodesic equation becomes:

$$\frac{d^2 x^i}{d\tau^2} + \Gamma^i_{tt} = 0, \tag{8.4}$$

where i is the spatial coordinate, leading to

$$\frac{d^2 u^r}{d\tau^2} - g_{(0)}^{rr} \frac{1}{2} (\partial_r h_{tt} + g_{(0)tt} \frac{S}{\rho}) = 0. \tag{8.5}$$

In this manner the gravitational force of the string (per unit length) gets the form

$$F_G = \frac{1}{2} (\nabla h_{tt} - \frac{S}{\rho}), \tag{8.6}$$

with $g_{tt} = -1 - h_{tt}$ in Eq.(6.8). Thus we identify

$$h_{tt} = -4G[J^2 (ln(\rho/r_0))^2 + (U - T + J^2) ln(\rho/r_0)] \tag{8.7}$$

We also note that the gravitational force is related to the h_{tt} component that has no explicit dependence on torsion. From which the expression for the force can be explicitly written as

$$F_G = -\frac{1}{\rho} \left[2GJ^2 \left(1 + \frac{(U-T)}{J^2} + 2\ln(\rho/r_0) \right) + S \right]. \tag{8.8}$$

A swift perusal of the last equation allows us to understand the essential role torsion plays in the context of the present formalism. As we show below this extra-term encloses a magnification of the total force a particle close to the SCCS will undergo.

In the limit case where $J^2 = 0$ and torsion S^2 vanish it is easy to see that the string turns to be an ordinary cosmic string if we consider (U=T), that is, we work with a string with no structure no gravitational field [22]. In this way, a test particle localized in the external region undergoes no gravitational force at all. However, if torsion is present, even in the case the string has no current, an attractive gravitational force appears. In the context of the SCCS torsion acts as a enhancer of the force a test particle feels outside the string. In our summary we discuss a bit further potential applications of this new result to astrophysics and cosmology.

9. ANGULAR DEFICIT AND COBE MAP

Recently Palle [43] and Garcia de Andrade [44] have shown that the COBE data are compatible with the Einstein-Cartan gravity. In this section, we analyse the effects of a screwed superconducting cosmic string on the primordial microwave background radiation using the COBE data. To this end, we need to compute the angular deficit introduced by torsion. The hidden idea here is that the cosmic large-scale density fluctuations could have had origin during the appearance of Cosmic String defects or due to interaction with them during the late stages of the Universe's evolution. The torsion would modify properties of light and radiation interacting with a cosmic string pervaded by screw dislocations in such a way that the density fluctuations induced might match those ones measured by COBE [4]. The DMR (Differential Microwave Radiometer) instrument of COBE has provided temperature sky maps leading to the rms sky variation where the beam separation in the COBE experiment is $\theta_1 - \theta_2 = 60^o$.

Each string that effects the photon beam induces a temperature variation [45,46], in largeness order, as:

$$\frac{\delta T}{T} \sim \delta \leq 10^{-6}(\mathrm{COBE}) \tag{9.1}$$

where δ is the angular deficit. If we consider the metric Eqs.(6.8)-(6.10), projected into the space-time perpendicular to the string, i. e., $dz = 0$, then we have:

$$ds_{\perp}^2 = (1 - h_{tt})[-dt^2 + dr^2 + (1 - b)r^2 d\theta^2], \tag{9.2}$$

with h_{tt} given by Eq.(8.7), and b calculated from junction conditions Eq.(6.14). Then, in first order in G, the deficit angle gets:

$$\delta = b\pi = 8\pi G\{U + J^2(\frac{1}{2} + \ln(\rho/r_0)) - \alpha S^2(1 - \ln(\rho/r_0))\}. \tag{9.3}$$

We can interpret this angular deficit δ as being due to three different contributions: δ_s to an ordinary cosmic string, δ_J to the current and δ_{tors} to the torsion field, respectively. In the case of the ordinary cosmic string the angular deficit is given by $\delta_s = 8\pi GU$, which in this work corresponds to the case where both current J and torsion S vanish. In this situation [47], it is demonstrated that cosmic string models are more consistent with the COBE data [48] for a wider range of cosmological parameters than the standard CDM models, and the numerical simulations have confirmed these preditions [49].

When the cosmic string carries current, we have used results of Ref. [50] for the current, that is a configuration with the maximum current $J \sim \eta$, for $\eta = 10^{16}GeV$ as well-known for grand unification theories. In such a case, we found $\delta_J = 8\pi GJ^2 \sim 10^{-6}$ or less what is compatible with COBE date. As it is easy to see we neglected the logarithmic term because we consider the experiment is being performed close to the string surroundings.

However, in the situation where the cosmic string is stressed by torsion the issue is more difficult because we have no idea about the energy density torsion put into the Universe via cosmic strings. Thereby, if a cosmic string actually formed and it is a good mechanism to generate density fluctuations that can be measured by COBE, then we can estimate the density of torsion the string induces in the cosmic background. To this purpose we choose the value $\alpha \sim 10^{38}GeV^{-2} \sim 1/G$ [36], in this case we have the torsion energy density $S \sim 10^{-3}$ with $\delta_{tors} = 8\pi G\alpha S^2$. As one can check by substituting in the previous section, the inferred value for S enlarges the intensity of the net force underwent by a test particle encircling the SCCS.

10. SUMMARY

Here we have examined the issue of a superconducting Abelian gauge cosmic string produced during these early transitions and endowed with torsion using the Einstein-Cartan theory. Although the relation of the torsion vector S_α with the scalar fields Λ allows one to compare our approach for SCCSs for instance to the one in scalar tensor theories of gravity [33,34], our result appears to differ considerably from scalar-tensor SCCS studies [40,34]. This is so, essentially, because in this approach we are considering the torsion in analogy with a field inside the string, an integral part of the SCCS structure.

It is possible for torsion to have had a physically relevant role during the early stages of the Universe's evolution. In this lines, torsion fields may be potential sources of dynamical stresses which, when coupled to other fundamental fields (i. e., the gravitational field), might have performed an important action during the phase transitions leading to formation of topological defects, such as the SCCSs here we focused on. It therefore seems a crucial issue to investigate basic models and scenarios involving cosmic defects within the torsion context. We showed that in this picture there exists the possibility for SCCSs to effect the spectrum of primordial density perturbations, whose imprints could be seen in the relic cosmic microwave background radiation as observed by COBE.

We also showed that torsion has a non-negligible contribution to the geodesic equation obtained from the contortion term. From a physical point of view, this contribution is responsible for the appearance of a stronger attractive force acting on a test-particle. Using the COBE data we found that the torsion density contribution S is the order of 10^{-3}. If we compute the force strength, Eq.(8.8), in association with the above estimative and data coming from COBE observations, we can show that the torsion contribution to this force is 10^3 times bigger than the corresponding to a current-carrying string compared to the one induced by the gravitational interaction itself.

This peculiar fact may have meaningful astrophysical and cosmological effects. Let us imagine for while a compact object (CO): a black hole or an exotic cosmic relic such as a boson, strange o mirror star, for instance, orbiting around the SCCS. Because the acceleration induced on the radial component of its orbital motion is about one thousand stronger than in ordinary cases, then we can expect the changes it provokes in the quadrupole moment of the system (SCCS + CO) to be enhanced by a large factor so that the gravitational wave (GW) signal expected from the CO inspiraling onto the SCCS could be above the lower strain sensitivity threshold of planned LIGO, VIRGO, GEO-600, etc. interferometric GW observatories, for distances even as the Hubble radius. Moreover, this very strong force may also turn the SCCS a potential source of hard X-ray and γ-ray transient emissions. These radiations can be emitted by matter (primordial gas and/or dust clouds, or something else) accreting onto the SCCS as the material gets closer and becomes heated due to the powerful tidal stripping. All these issues we plan to address in a forthcoming work [51].

Finally, we advance that other interesting torsion effects relevant to cosmological problems such as the Sachs-Wolfe effect in space-time with torsion and production multiple images by intervening screwed SCCS lying at cosmological distances will be addressed in a work in preparation [32]. Superconducting cosmic strings with torsion may also serve to place an upper limit on the space-time torsion itself, that is, on the degree of spin of our Universe.

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REFERENCES

[1] A.Vilenkin, Phys. Rev. **D 23**,852 , (1981); W.A.Hiscock, Phys. Rev. **D 31**, 3288, (1985); J.R.GottIII, Astrophys. Journal, **288**, 422, (1985); D. Garfinkel, Phys. Rev. **D 32** 1323, (1985).

[2] T.W. Kibble, J. of Phys. **A9**, 1387 (1976).

[3] M.B. Hindmarsh and T.W.B. Kibble, hep-ph/9411342, (1994).

[4] G. F. Smoot, Lectures at D.Chalonge School on Large-Scale Structure of the Universe, Erice, Dic 12-17 (1999).

[5] T.W.B. Kibble, Phys. Rep **67**, 183, (1980).

[6] J. Magueijo and R.H. Brandenberger, astro-ph/0002030, (2000).

[7] R. H. Brandenberger, A. T. Sornborger and M. Trodden, Phys. Rev. D., 48, 940 (1993).

[8] V. Berezinshy, B. Hnatyk and A. Vilenkin, astro-ph/0001213 (2000).

[9] H.J.Mosquera Cuesta and C.N.Ferreira, "Bursts of Gravitational Radiation from Superconducting Cosmic Strings and the Neutrino Mass Spectrum", in preparation.

[10] B.Allen and P.Casper, Phys. Rev **D 51**, 1546 (1995), gr-qc/9407023; Phys. Rev **D 50**, 2496 (1994), gr-qc/9405005.

[11] S.W.Hawking and S.F.Ross, gr-qc/9506020 (1995).

[12] J.Audretsch and A.Economou, Phys Rev **D 44**, 3774 (1991).

[13] Y.A.Sitenko and A.V. Mishchenko, hep-th/9711121 (1999).

[14] E. Witten, Nucl.Phys. **B249**, 557 (1985).

[15] A.Vilenkin and T.Vachaspati, Phys.Rev. Lett. **58**, 1041, (1987).

[16] I.Moss and S.Poletti, Phys. Lett **B 199**, 34, (1987).

[17] A.Babul, T.Piran and D.N.Spergel, Phys. Lett **B 209**, 477, (1988).

[18] P.Amsterdamski and P.Laguna-Castillo, Phys. Rev D 37 , 877, (1988).

[19] B.Linet Class.Quant.Grav **6**, 435, (1989).

[20] P. Peter and D. Puy, Phys. Rev. D, 48, 5546 (1993).

[21] S.C.Davis, hep-ph/9901417 , (1999); S.C.Davis, W.B.Perkins and A.C.Davis hep-ph/9912356 , (1999).

[22] K. Dimopoulos , Phys.Rev **D57**, 4629 (1998).

[23] R.H.Brandenberger, A.C.Davis, A.M.Matheson and M.Trodden, Phys lett. **B 293**, 347 (1974).

[24] M.J.Thatcher and M.J.Morgan, Phys. Rev **D 59**, 065017 (1999).

[25] P.G.Bergmann and V. De Sabbata, "Cosmology and Gravitation " Ed. Plenum (1980).

[26] J.Scherk and J.H.Schwartz, Phys lett. **B 52**, 347 (1974).

[27] B.Julia, E. Remmer, and J. Scherk, Phys Lett. **76 B**, 409 (1978); E.S.Fradkin and M.A.Vasseliev, Phys. Lett **85 B**, 47 (1979).

[28] F.W.Hehl et al., Rev. Mod Phys. **48**,393 (1976).

[29] L.C.Garcia de Andrade, Mod.Phys.Lett. **A13**, 2227 (1998).

[30] P.S.Letelier, Class. and Quant. Grav.**12**, 471 (1995).

[31] P.S.Letelier, Class. and Quant. Grav. **12**, 2224, (1995).

[32] H.J.Mosquera Cuesta and C.N.Ferreira, " Cosmic Strings in Conformal Field Theory with Torsion" in preparation.

[33] C. N Ferreira, H. J. Mosquera Cuesta and M. E. X. Guimarães "Torsion effects on superconducting cosmic strings in Einstein gravity and scalar-tensor theories of gravity", in preparation.

[34] C.N.Ferreira, M.E.X. Guimarães and J.A.Helayel-Neto, Current-Carrying Cosmic Strings in Scalar-Tensor Gravities, submeted to Nuclear Physics.

[35] H.Kleinert, Phys.Lett. **B440** 283 (1998).

[36] M.Novello, Phys.Lett. **A59**, 105 (1976).

[37] I. Buchbinder, S.D. Odintsov and I.Shapiro " Effective Action in Quantum Gravity " , Institute of Physics Publishing Bristol and Philadelphia (1992).

[38] R.Hammond, gr-qc/9904033 (1999); Class. and Quant. Grav. **13**,L73 (1996).

[39] Nielsen H.B and Olesen P, Nucl.Phys. **61**, 45, (1973).

[40] C.N.Ferreira, M.E.X.Guimarães and J.A.Helayel-Neto, gr-qc/9910021 (1999). contributed talk at COSMOS-99, Septemb., Trieste, Italy (1999).

[41] W.Arkuszewski, W.Kopczynski and V.N.Ponomariev, Commun.Math.Phys. **45**, 183 (1975).

[42] R.A.Puntigam and H.H.Soleng, Class. Quantum Grav. **14**, 1129 (1997).

[43] D.Palle, astro-ph/9811408, (1998).

[44] L.C.Garcia de Andrade, Phys.Lett. **B 468**, 23, (1999).

[45] N.Kaiser and Stebbins, Nature **310**, 391 (1984).

[46] A.Stebbins, Astrophys. J. **327**, 584 (1988).

[47] L.Perivolaropoulos, hep-ph/9208247 (1992).

[48] D.Bennett, A.Stebbins and F.R.Bouchet, Fermilab preprint (1992).

[49] F.R.Bouchet, D.Bennett and A.Stebbins, Nature, **335**, 410 (1988).

[50] A.Babul, T.Piran and D.N.Spergel, Phys. Lett **B 202**, 307, (1988).

[51] H.J.Mosquera Cuesta and C.N.Ferreira, in preparation