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FERMIONIC DETERMINANT FOR TWO-DIMENSIONAL MASSIVE QED

by

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Abstract

We evaluate the fermionic determinant for two-dimensional massive QED for the case of zero topological-charge sector by means of Seeley's technique

Key-words: Field theory; Functional integral

1. Introduction

In the last years, a considerable effort has been done to understand some two-dimensional theories [1-10]. The hope is that some of the properties in two-dimensional models will be either independent of the dimension of space-time or will at least have generalizations to higher dimensions.

The problem of integrating over the fermionic field has received also some attention [2-9]. An advance along this line [5-9], which can be understood as a sort of path-integral version of the bosonization technique [2,10], has been achieved in some two-dimension models with massless fermions but in the case that the fermions are massive [2,10] this approach was not successfull.

We show in this paper for the case of zero topological-charge sector how we can evaluate the fermionic determinant for two-dimensional QED(QED_2) with massive fermions. The method used is the following: at first we implement a chiral change of variables, the Jacobian associated to the transformation is calculated by the method developed recently [6] and the remaining fermionic determinant is obtained by means of Seeley's asymptotic expansion [11].

In the next section we implement the chiral change of variables and calculate the associated Jacobian, in section III we evaluate the remaining fermionic determinant and briefly comment the result.

2. The change of fermionic variables

Let us now consider generating functional for Euclidean QED_2 with massive fermions; the behavior of all fields at infinite is assumed so that it is possible to compactify the space:

$$Z = \int D\bar{\psi} D\psi DA \; \exp\{\int [\, \bar{\psi} D\psi + \frac{1}{4} F^2_{\mu\nu} \,] d^2x \tag{1}$$

where $D = -\not{D} - m = -i\not{\partial} - e\not{A} - m.$

We want to perform the finite chiral transformation:

$$\begin{aligned} \psi(x) &= e^{\gamma_5 \alpha(x)} \eta(x) \\ \bar{\psi}(x) &= \bar{\eta}(x) e^{\gamma_5 \alpha(x)} \end{aligned} \tag{2}$$

This finite transformation will be achieved by successive infinitesimal changes. We introduce then a real parameter r $(0 \leq r \leq 1)$ so that

$$\begin{aligned} \eta_r(x) &= \bar{e}^{\gamma_5 r \alpha(x)} \psi(x) \\ \bar{\eta}_r(x) &= \bar{\psi}(x) e^{-\gamma_5 r \alpha(x)} \end{aligned} \tag{3}$$

and for $r = 1$ we reobtain the finite transformation (2). In order to calculate the Jacobian associated to (2) we perform the transformation (3) in (1); considering for simplicity only the fermionic part we get:

$$G = \int J(r)\mathcal{D}\bar{\eta}_r \mathcal{D}\eta_r \ \exp\{\int \bar{\eta}_r D_r \eta_r d^2x\} \tag{4}$$

where

$$D_r = e^{\gamma_5 r\alpha(x)} D e^{\gamma_5 r\alpha(x)} \tag{5}$$

and

$$\mathcal{D}\bar{\psi}\mathcal{D}\psi = J(r)\mathcal{D}\bar{\eta}_r \mathcal{D}\eta_r \tag{6}$$

we note that the Jacobian for the finite transformation is given by $J = J(r=1)$. Integrating over the fermionic variables in (4) and since G cannot depend on r:

$$\frac{dG}{dr} = 0 = \frac{dJ(r)}{dr} \det D_r + J(r)\frac{d}{dr}(\det D_r) \tag{7}$$

After integration of (7) over r we obtain:

$$J = J(1) = \exp\{-\int_0^1 \omega'(r)dr\} \tag{8}$$

where

$$\omega(r) = \ell n \det D_r \tag{9}$$

Regularizing the determinant by the zeta function method [12], we can write $\omega(r)$ as:

$$\omega(r) = -\frac{d}{ds}\zeta(s,D_r)\Big|_{s=0} \tag{10}$$

where

$$\zeta(s,D_r) = \sum_j \lambda_j^{-s} \tag{11}$$

with λ_j the eigenvalues of D_r. Now, in order to compute $\omega'(r)$, we obtain from (5):

$$D_{r+\Delta r} = D_r + A_1\Delta r + O(\Delta r^2) \qquad (12)$$

with

$$A_1 = \gamma_5\alpha(x)D_r + D_r\gamma_5\alpha(x) \qquad (13)$$

Then we have for $\omega'(r)$:

$$\omega'(r) = \lim_{\Delta r\to 0} -\frac{1}{\Delta r}\left[\zeta(0,D_r + A_1\Delta r) - \zeta(0,D_r)\right]$$

$$= 2\mathrm{Tr}(D_r^{-s}\gamma_5\alpha(x))\Big|_{s=0} \qquad (14)$$

In the last step we have used a property of the zeta function [6,13]. Then substituting (14) in (8) we have:

$$J = \exp\{-2\int_0^1 \mathrm{Tr}(D_r^{-s}\gamma_5\alpha(x)\Big|_{s=0} dr\} \qquad (15)$$

The trace in (15) can be rewritten according to Seeley [11] and we get

$$J = \exp\{-2\int d^2x\int_0^1 dr\mathrm{Tr}(k_0(D_r;x,x)\gamma_5)\alpha(x) \qquad (16)$$

and the Kernel k_0 can be explicitly evaluated by means of Seeley's coefficients [6,11].

Choosing the Lorentz gauge

$$A_\mu = -\frac{1}{e}\varepsilon_{\mu\nu}\partial_\nu\alpha \tag{17}$$

we obtain for D_r defined in (5)

$$D_r = -i\not\partial - e(1-r)\not A - me^{2r\gamma_5\alpha} \tag{18}$$

Now in order to compute $k_o(D_r;x,x)$ for this operator we follow Seeley's technique [6,11]. The symbol [11] of the differential operator D_r is

$$\sigma(D_r) = a_1 + a_o \tag{19}$$

where a_1 is the principal symbol of D_r,

$$a_1 = \not\xi \tag{20}$$

and a_o is given by:

$$a_o = -(1-r)\not A - me^{2r\gamma_5\alpha} \tag{21}$$

We have to construct the coefficients $b_{-1-j}(x,\xi,\lambda)$, j = 0, ...,d = 4, in order to use Seeley's result for $k_o(D_r;x,x)$:

$$K_o(D_r;x,x) = \frac{-i}{(2\pi)^2}\int_{|\xi|=1} d\xi \int_o^\infty b_{-3}(x,\xi,i\lambda)d\lambda \tag{22}$$

The b_i's satisfy the following relations:

$$b_{-1}(a_1 - \lambda) = I$$

$$b_{-1-\ell}(a_1 - \lambda) + \sum_{\alpha} \left(\frac{\partial}{\partial\xi}\right)^{\alpha} b_{-1-j}\left(-i\frac{\partial}{\partial x}\right)^{\alpha} a_{1-k}\frac{1}{\alpha!} = 0 \qquad (23)$$

with $\ell > 0$, the sum taken for $j < \ell$ and $j + k + |\alpha| = \ell$.

Evaluating these coefficients for the operator D_r we obtain

$$\mathrm{Tr}\left[K_o(D_r;x,x)\gamma_5\right] = -\frac{e}{2\pi}(1-r)F_{01} - \frac{m^2}{2\pi}\sinh(4r\alpha) \qquad (24)$$

Substituting (24) in (16) we get for the Jacobian:

$$J = \exp\{-\frac{e^2}{2\pi}\int A_\mu A_\mu d^2x - \frac{m^2}{4\pi}\int(1 - \cosh 4\alpha)d^2x\} \qquad (25)$$

3. Evaluation of the fermionic determinant

The generating functional given in (1) in terms of the new fermionic variables η and $\bar{\eta}$ is

$$Z = \int \mathcal{D}A\mathcal{D}\bar{\eta}\mathcal{D}\eta \; \exp\{-\int\left[\bar{\eta}(i\partial\!\!\!/ + me^{2\gamma_5\alpha})\eta + \frac{e^2}{2\pi}A^2 + \frac{m^2}{4\pi}(1 - \cosh 4\alpha) - \frac{1}{4}F^2\right]d^2x\} \quad (26)$$

We may note that the fermionic part of the Lagrangian is the model studied by H. Lehmann and K. Pohlmeyer [14]. Now, integrating over the new fermionic variable in (26) we get:

$$Z = \int \mathcal{D}A \; \det(i\partial\!\!\!/ + me^{2\gamma_5\alpha}) \; \exp\{-\int d^2x\left[\frac{e^2}{2\pi}A^2 + \frac{m^2}{4\pi}(1 - \cosh 4\alpha) - \frac{1}{4}F^2\right]\} \qquad (27)$$

In order to compute the determinant given in (27) we introduce again a parameter $r(0 \leq r \leq 1)$ and the operator D_r given as [9]:

$$D_r = i\not\partial + me^{2rf} \tag{28}$$

where $f = \gamma_5\alpha(x)$, for $r=1$ we reobtain the operator under consideration. Differentiating D_r with respect to r we obtain:

$$\frac{d}{dr}D_r = 2(D_r - i\not\partial)f \tag{29}$$

The determinant of D_r is regulated by the proper time method:

$$\ell n \det D_r^2 = \mathrm{Tr}\,\ell n\, D_r^2 = -\int_\varepsilon^\infty \frac{ds}{s}\mathrm{Tr}[\exp(-sD_r^2)] \tag{30}$$

where ε is an ultraviolet cutoff on the proper time integration. Differentiating (30) with respect to r and using property (29) we obtain:

$$\frac{d}{dr}\mathrm{Tr}\,\ell n\, D_r^2 = \int_\varepsilon^\infty ds\ \mathrm{Tr}[2D_r\mathring{D}_r\ \exp(-sD_r^2)] =$$

$$= 4\int_\varepsilon^\infty ds\ \mathrm{Tr}[fD_r^2\ \exp(-sD_r^2)] - 4i\int_\varepsilon^\infty ds\ \mathrm{Tr}[\not\partial fD_r\ \exp(-sD_r^2)] =$$

$$= -4\int_\varepsilon^\infty ds\ \frac{d}{ds}\ \mathrm{Tr}[f\ \exp(-sD_r^2)] - 4i\int_\varepsilon^\infty ds\ \mathrm{Tr}[\not\partial fD_r\ \exp(-sD_r^2)] =$$

$$= 4\mathrm{Tr}[f\ \exp(-\varepsilon D_r^2)] - 4i\int_\varepsilon^\infty ds\ \mathrm{Tr}[\not\partial fD_r\ \exp(-sD_r^2)] \tag{31}$$

The second term on the last line above does not give contribution since we are considering only fields with trivial topology.

For the first term Seeley [11] has shown that there is an asymptotic small ε expansion for the diagonal part of the exponential. For operators of the form:

$$D = -D_\rho D^\rho + X \tag{32}$$

where D_ρ is a covariant derivative and X a matrix valued function we have [3,15]:

$$\langle x|\exp(-\varepsilon D)|x\rangle \xrightarrow[\varepsilon\to 0]{} \frac{1}{(4\pi\varepsilon)^{d/2}}[1 + \varepsilon X + O(\varepsilon^2)] \tag{33}$$

where d is the dimensionality of space-time.

Now, calculating D_r^2, with D_r given in (28), substituting the asymptotic small expansion (33) in the differential equation (31) and using the well known properties of traces of γ-matrices we obtain the differential equation:

$$\frac{d}{dr}\mathrm{Tr}\,\ell n\, D_r^2 = \frac{m^2}{4\pi}\int d^2x\, \frac{d}{dr}\mathrm{Tr}\,[e^{4r\alpha\gamma_5}] \tag{34}$$

Integrating this equation with respect to r we obtain:

$$\mathrm{Tr}\,\ell n(i\partial\!\!\!/ + me^{2\alpha\gamma_5}) - \mathrm{Tr}\,\ell n(i\partial\!\!\!/ + m) = -\frac{m^2}{4\pi}\int d^2x(1 - \cosh 4\alpha) \tag{35}$$

Substituting this result (35) in (27) we get for the generating functional of QED_2

$$Z = \int \mathcal{D}A\, \exp\{-\int d^2x\,[-\frac{e^2}{4\pi}A^2 + \frac{m^2}{2\pi}(1 - \cosh 4\alpha) - \frac{1}{4}F^2]\} \tag{36}$$

which is the path-integral version of the bosonized QED_2 with massive fermions. As we are working in Euclidean space in the continuation to Minkowski space we would have $\alpha \to i\alpha$ and the hiperbolical cosine would transform to a simple cosine in agre ement with the results obtained by bosonization techniques [2,10].

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