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A Model for Baryogenesis at Reheating

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Abstract

By using transformation properties of the Hamiltonian of homogenous and isotropic cosmologies, we derive a model describing the transition of the universe from the inflationary phase into the radiation-dominated phase, during the process of reheating. The baryon excess produced in the transition is in a fixed ratio to the radiation content of the radiation-dominated phase. We derive an expression for the baryon asymmetry n_B/n_γ as a function of known thermodynamic constants, the gravitational constant, the radius of the present universe and the present temperature of the cosmic background radiation. The only parameter in the formula is the total number of (massless) degrees of freeedom of relativistic particles composing the radiation density. We obtain $n_B/n_\gamma \sim 5.49 \times 10^{-10}$. Further physical consequences are discussed.

Key-words: Baryogenesis; Inflationary models; Reheating; Homogeneous and isotropic cosmologies.

The observed baryon asymmetry of the universe [1, 2], $n_B/n_\gamma \sim 10^{-10}$, poses some fundamental questions for the models of formation and evolution of the universe. In the last thirty years the focus on this problem has shifted from specifying this number simply as an initial condition, to considering that the baryon asymmetry evolved from an initially symmetric configuration as consequence of processes which violate baryon number conservation in an expanding universe [3].

In the inflationary paradigm, baryogenesis takes place possibly during reheating, when the vacuum energy of the inflaton field (which dominates the energy density of the universe in the inflationary phase) is converted into particles which will constitute the matter-energy content of the subsequent radiation-dominated phase of the universe. This radiation dominated phase will then evolve into our present observable matter-dominated universe, and contains already the baryon excess which is the ordinary matter composing galaxies. In the present letter we introduce a model to describe the transition from the inflationary phase to the radiation-dominated phase, with production of the baryon excess in a fixed ratio to the radiation content of the radiation-dominated phase. This model is based on transformation properties of the gravitational Hamiltonian of homogeneous and isotropic models, which allow to construct a parametrized set of configurations of the gravitational field describing a continuous transition from the inflationary configuration to the radiation-dominated configuration.

Let us consider the one degree of freedom Hamiltonian

$$H = \frac{1}{24}\, P_X^2 - 2kC_R - 2kC_M X + \frac{3}{2}\,\varepsilon X^2 - 2kC_W X^3 - 2(\Lambda + \cdots)X^4 \tag{1}$$

where P_X is the canonical momentum conjugated to X. The parameters are given in the above form for future reference. Under the transformation

$$X \to X + \alpha \tag{2}$$

with α an arbitrary continuous parameter, (1) transforms into [4]

$$\bar{H} = \frac{1}{24}\, P_X^2 - 2k\bar{C}_K - 2k\bar{C}_M X + \frac{3}{2}\,\bar{\varepsilon} X^2 - 2k\bar{C}_W X^3 - 2(\bar{\Lambda} + \cdots)X^4 \tag{3}$$

where

$$\begin{aligned}
\bar{C}_R &= C_R + C_M\alpha - \frac{3}{4}\,\frac{\varepsilon}{k}\,\alpha^2 + C_W\alpha^3 + (\Lambda + \cdots)\frac{\alpha^4}{k} \\
\bar{C}_M &= C_M + 3C_W\alpha^2 - \frac{3}{2}\,\frac{\varepsilon}{k}\,\alpha + 4(\Lambda + \cdots)\frac{\alpha^3}{k} \\
\bar{\varepsilon} &= \varepsilon - 4kC_W\alpha - 8(\Lambda + \cdots)\alpha^2 \\
\bar{C}_W &= C_W + 4(\Lambda + \cdots)\frac{\alpha}{k} \\
\bar{\Lambda} &= \Lambda + \cdots
\end{aligned} \tag{4}$$

If we can consistently interpret H and $\bar{H}$ as describing the same system in distinct configurations, then the transformation (2) takes the configuration $X(c)$ into the configuration $X(\bar{c})$ of the same system. If H and $\bar{H}$ correspond to different phases of the same system then (2) describes a continuous transition from one phase of the system into the other phase of the same system [5]. As we will see, this is the case of the Hamiltonian of the universe [6] (for instance, in the inflationary phase $C_R = C_M = C_W = 0$, or in RWF-phase $\Lambda = 0$), if in (1) we interpret X as the scale factor, ε as the curvature of the spatial sections, and k as Einstein's constant; $(\cdots)$ may be interpreted as quantum corrections to the gravitational Hamiltonian. The constants of motion C_R, C_M and C_W are related respectively to the radiation energy density, the matter density and the energy density of domain walls by

$$\begin{aligned} \rho_R &= C_R/X^4 \\ \rho_M &= C_M/X^3 \\ \rho_W &= C_W/X \ , \end{aligned} \tag{5}$$

Λ being obviously the cosmological constant.

Having in mind the above transformation apparatus, let us consider the inflationary phase of the universe. The dynamics of this phase is dominated by the vacuum energy density of the inflaton field ϕ, $\rho_{VAC} \simeq V(\phi = const.)$, the Hamiltonian being given by

$$H = \frac{1}{24} P_X^2 + \frac{3}{2}\varepsilon X^2 - 2k\rho_{VAC} X^4 \tag{6}$$

where $k\rho_{VAC}$ plays the role of the cosmological constant.

Now in the process of reheating, whereby the universe undergoes a transition from the inflationary phase to the radiation-dominated phase, the vacuum energy of the inflaton field is converted into particles which will constitute the matter-energy content of our observable universe. We may assume that this transition occurs under two conditions: (i) the variation of the vacuum energy ρ_{VAC} with corresponding creation of particles is continuous as in a second order phase transition; (ii) the time-scale of this transition is much smaller than the time-scale of the Hubble expansion.

In the time-scale of the transition we may express the variation of the vacuum energy density as

$$\delta\rho_{VAC} = (\rho_{VAC} - \sigma) - \rho_{VAC} = -(\rho_M + \rho_R + \cdots) \tag{7}$$

where σ is a parameter varying continuously and monotonically from 0 to $\sigma_0 \leq \rho_{VAC}$. The decreasing of ρ_{VAC} corresponds to the increasing of $(\rho_R + \rho_M + \cdots)$, with the initial condition

$$\sigma = (\rho_R + \rho_M + \cdots) = 0 \tag{8}$$

at the beginning of reheating. Each intermediate configuration X in this transition depends continuously on σ, and must be a solution of the Hamiltonian resulting from $\rho_{VAC} \to \rho_{VAC} - \sigma$, with (7) and (8) holding. From (1) - (4) it then follows that in the transition $\rho_{VAC} \to \rho_{VAC} - \sigma$ (with (7) and (8) holding) the transformation

$$X \to X + \alpha$$

realizes a set of intermediate states $X(\alpha) = X(0) - \alpha$ through which the universe undergoes a continuous transition from the inflationary phase into the radiation-dominated phase, where α is a parameter increasing monotonically with σ and such that $\alpha(\sigma = 0) =$

0. We remark that during this transition the scale factor of the universe is contracted by a factor $1-\alpha/X(0)$. According to (4) the intermediate Hamiltonian is expressed [7]

$$\begin{aligned} H &= \frac{1}{24}\,P_X^2+\frac{3}{2}[\varepsilon-8k(\rho_V-\sigma)\alpha^2]X^2+ \\ &- 2k\left[-\frac{3}{2}\,\frac{\varepsilon\alpha}{k}\,+4(\rho_V-\sigma)\alpha^3\right]X+ \\ &- 2k\left[-\frac{3}{4}\,\frac{\varepsilon\alpha^2}{k}+(\rho_V-\sigma)\alpha^4\right]+ \\ &- 2k[4(\rho_V-\sigma)\alpha]X^3-2k(\rho_V-\sigma)X^4 \end{aligned} \tag{9}$$

If in the above process all the vacuum energy is converted into particles then the end of the transition will correspond to $\sigma_0=\rho_{VAC}$, and the Hamiltonian (6) is transformed into the RWF Hamiltonian(cf. (4) and (9))

$$\bar{H}=\frac{1}{24}\,P_X^2+\frac{3}{2}\,\varepsilon X^2-2kC_MX-2kC_R \tag{10}$$

where

$$\begin{aligned} C_M &= -\frac{3}{2}\,\varepsilon\frac{\alpha_0}{k} \\ C_R &= -\frac{3}{4}\,\varepsilon\frac{\alpha_0^2}{k} \end{aligned} \tag{11}$$

with $\alpha_0=\alpha(\sigma_0)$ given by [7], [8]

$$\alpha_0\simeq\sqrt{-\frac{4\varepsilon k}{3}\rho_{VAC}}X_{RH}^2$$

From (11) we can see that this description is physically consistent only for an open universe ($\varepsilon=-1$). The Hamiltonian (10) is the Hamiltonian for the subsequent radiation-dominated phase of the Universe, with ρ_M and ρ_R uncoupled. We can appropriately interpret $\rho_M=C_M/X^3$ as the density of the baryon excess generated in the transition $\rho_{VAC}\rightarrow$ particles which shall constitute the matter density of the present matter-dominated universe.

From (11) a relation between C_M and C_R results,

$$C_R=\frac{k}{3}C_M^2 \tag{12}$$

imposing a constraint between the radiation content and the matter-content (baryon matter) of the universe. This constraint is connected to the choice of (2) as a description of the intermediate states of the transition. With the use of thermodynamics relations in RWF cosmologies we derive from (12) and (5) that

$$\frac{n_\gamma}{n_B}=\sqrt{\frac{16\pi Ga}{3g_*}}\left(\frac{\mu_B}{2.7\beta}\right)X_0T_0 \tag{13}$$

Here n_γ/n_B is the ratio photon to baryon number density, G is the gravitational constant, a the black body constant, β the Boltzmann constant, μ_B the baryon mass (for which we take the value of the proton mass), X_0 the present radius of the universe and T_0 the present temperature of the cosmic background radiation. The numerical factor 2.7 comes from the expression of the average energy per photon in the radiation-dominated phase. The parameter g_* is the total number of (massless) degrees of freedom of relativistic particles compositing the radiation density; it appears in the relation $\rho_R = g_*/2\ \rho_\gamma$, where ρ_γ is the energy density of photons. For g_* we take the value $g_* \approx 106$ (cf. Ref. [2], chapter 3) which corresponds to the radiation component ρ_R at temperatures $T \gtrsim 300 GeV$. We assume that all particle species contributing to ρ_R are in equilibrium and have a common temperature, which is the photon temperature. Also since the product $X_0 T_0$ is a constant along the evolution of RWF universes, it follows that the ratio n_B/n_γ is constant in time, starting from the beginning of the radiation-dominated era. Expression (12) can be recast in the simpler form

$$\frac{n_\gamma}{n_B} = \sqrt{\frac{16\pi^3}{45 g_*}} \left(\frac{\mu_B}{\mu_P}\right) \left(\frac{\beta}{2.7\hbar c}\right) X_0 T_0 \tag{14}$$

where μ_P is Planck's mass. Using

$$\begin{aligned} X_0 &= 1.7 \times 10^{28} cm \\ T_0 &= 2.7^0 K \end{aligned}$$

we obtain from (14)

$$\frac{n_B}{n_\gamma} \sim 5.49 \times 10^{-10} \tag{15}$$

which is consistent with the observations.

If a perfect conversion of the vacuum energy into particles is not realized, a residual $\Delta\rho_{VAC}$ remains and the subsequent radiation-dominated period will contain in addition

spatial curvature correction :	$-8k\alpha^2(\Delta\rho_{VAC})$
matter density correction :	$4\alpha^3(\Delta\rho_{VAC})$
radiation density correction :	$\alpha^4(\Delta\rho_{VAC})$
domain-walls correction :	$4\alpha(\Delta\rho_{VAC})$
cosmological constant :	$k\Delta\rho_{VAC}$

with $\alpha = \alpha(\sigma_0)$, corresponding to the end of the transition with $\Delta\rho_{VAC} = \rho_{VAC} - \sigma_0$ (cf. Ref. [7]). By correction we mean that those quantities must be added to the corresponding constants in the Hamiltonian (10). We speculate whether the above matter and radiation corrections could be interpreted as cold and hot dark matter components, respectively.

Several questions arise related to the above model of the transition from the inflationary phase into the radiation-dominated phase during reheating. Does the consistency of (14) with observations imply that the set of intermediate configurations of the transition is actually generated by (2)? What is the implication of (2) for theories with baryon non-conservation in an expanding universe? Theories in which a net baryon asymmetry is produced should ultimately have their parameters constrained by (14). A final answer would obviously be a microscopic theory of the interaction of the inflaton field, the gravitational field described by the "scalar mode" X and other particle fields (with violation of C and CP) which would constrain the transition $\rho_{VAC} \rightarrow \rho_{VAC} - \sigma$ (cf. eq. (7)) to be realized through the intermediate states generated by (2).

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[4] We note that, under the transformation (2), the momentum P_X remains invariant. This results from the invariance of the action $\int P_X \left(\frac{dX}{d\eta}\right) d\eta$. Cf. also E.C.G. Sudarshan and N. Mukunda, Classical Mechanics, Wiley and Sons,New York (1974).

[5] We note that constants which are zero in one phase may be non-zero in the other, due to the transformations (4).

[6] In the conformal time parametrization.

[7] For $\sigma \leq \rho_{VAC}$, the relation between α and σ is given by (cf. (7))

$$AX^2(\alpha)\,\xi^4 + \frac{3}{4}\,\xi^2 - \frac{3}{4} = k\rho_{VAC}X^2(\alpha)$$

where $A \equiv k(\rho_{VAC} - \sigma)$ and $\xi = 1 + \alpha/X(\alpha)$.

[8] Using the first relation (11) we obtain

$$\rho_{VAC} \simeq \frac{k}{3}\left(\frac{\rho_0 X_0^3}{X_{RH}^2}\right)^2$$

where ρ_0 and X_0 are respectively the present matter density and radius of the universe. X_{RH} is the value of the scale factor at the end of reheating. These expressions are consistent only for an open universe.