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# SKIN EFFECT IN CONDUCTING FRACTALS

by

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ABSTRACT

The skin effect should be a neat tool for measuring fractal dimensionalities of rough conductors. By assuming the well known expression for the skin depth $\delta \propto 1/\sqrt{\omega}$ (where $\omega$ is the frequency of the electromagnetic field), we obtain, for the *electrical resistance* and for the *electromagnetic power dissipation*, power-law dependences (on the applied frequency) with *anomalous exponents* directly related to relevant fractal dimensionalities.

An increasing number of physical properties is nowadays shown to be interpretable in terms of fractal concepts [1]. Indeed many real systems, proteins [2,3] and porous coals [4,5] among several others, present, within appropriate ranges of the control parameters, a fractal nature and are characterized by fractal dimensionalities. As a matter of fact, since long, non integer dimensionalities have been fruitful in Physics (e.g., Field Theory [6], Critical Phenomena [7], Percolation Theory [8], Dynamical Systems [9], etc).

There are physical properties (e.g., spin relaxation [2,3], specific heat [10]) which in principle depend on *both* geometrical and non geometrical quantities. For example, within certain range of temperatures (T), the spin-lattice relaxation time $\tau_1$ of some proteins is given [2,3] by $1/\tau_1 \propto T^n$, where the non-integer exponent n contains purely geometrical information (the fractal dimensionality of the protein) as well as non geometrical one (related to phonon dynamics, and also to diffusion properties). There are other physical phenomena (e.g., small-angle X-ray scattering [4,5]), from which the exclusive extraction of purely geometrical quantities is possible. For example, the just mentioned scattered intensity is proportional [5] to a non-integer power of the (conveniently redefined) wave-vector, the exponent being simply related to the relevant fractal dimensionality of the system. This second type of phenomena can be used as probes to measure geometrical quantities, whose direct determination (by structural inspection involving, for instance, the mathematical definition of fractal dimensionalities) might be extremely hard, partially unreliable or even untractable. We argue in the present paper that the use of the skin effect should constitute a fine (and unexpensive) tool for simple and quite direct determination of relevant (but non trivial) dimensionalities of arbitrarily rough and irregular substances provided they present a non neglectable electrical conductivity ($\sigma$) at the appropriate range of frequencies. The basic idea is clear and elementary: the skin depth $\delta$ being [11] proportional to $1/\sqrt{\omega}$ ($\omega \equiv$ frequency of the electromagnetic field), increasing frequencies will probe the "fractality" of the system at finer and finer levels, thus providing relevant geometrical information.

We consider a (rough) conductor which, for simplicity, can be thought to be roughly cylinder-like (see Fig. 1). Its (fractal) longitudinal and transverse (perimeter) lengths will be respectively denoted by $\ell_L$ and $\ell_T$; the (fractal) area they support will be denoted by S. We assume an alternate electric potential (with voltage V and frequency $\omega$) to be applied between the two (equipotential) bases of the cylinder. The corresponding skin depth is given by [11].

$$\delta \propto 1/\sqrt{\omega} \tag{1}$$

where the proportionality factor depends upon electromagnetic properties of the (homogeneous) substance. We denote by $S_T$ ($S_T \simeq \ell_T \delta$) the (fractal) transverse area through which the current mainly flows (see Fig. 1), and by $v$ ($v \simeq S\delta$) the (fractal) volume it supports. The lengths $\ell_L$ and $\ell_T$ are related to the corresponding fractal dimensionalities [1] through the following formulae which we take as definitions of the characteristic parameters $d_L$, $d_T$ and $d_S$:

$$d_L - 1 = \ln \ell_L / \ln \delta^{-1} \tag{2}$$

hence

$$\ell_L = 1/\delta^{d_L - 1} , \tag{2'}$$

$$d_T - 1 = \ln \ell_T / \ln \delta^{-1} \tag{3}$$

hence

$$\ell_T = 1/\delta^{d_T - 1} \tag{3'}$$

and

$$d_S - 2 = \ln S / \ln \delta^{-1} \tag{4}$$

hence

$$S = 1/\delta^{d_S - 2} . \tag{4'}$$

In general it will be $1 \leq d_L, d_T < 2$ (the equality corresponds to smooth differentiable curves), and $2 \leq d_S < 3$ (the equality corresponds to a smooth differentiable surface).

Let us also note that Eq. (4') leads to

$$V \simeq S\delta \simeq \delta^{3-d_S} \tag{5}$$

which recovers, through appropriate change of the characteristic length, Eq. (4) of Ref. [5].

From Ohm's law it comes the following expression for the electrical resistance R

$$R \propto \frac{\ell_L^2}{S\delta} \tag{6}$$

Eq. (6) immediately yields[12]

$$R \propto 1/\delta^{1+2d_L-d_S} \tag{7}$$

where we have used Eqs. (2') and (4'). By taking into account Eq. (1), this relation can be rewritten as follows:

$$R \propto \omega^{(1+2d_L-d_S)/2} \tag{8}$$

This relation reproduces the standard one[11] ($R \propto \sqrt{\omega}$), corresponding to non fractal conductors, when $2d_L = d_S$ (which is a condition softer than the standard one $2d_L = d_S = 2$.

If, instead of assuming a current flow experiment (measure of R), we are rather interested in the power dissipation P associated with the substance located in the interior of a cavity *where a constant electromagnetic field density is maintained*, we have, that

$$P \propto V \propto \delta^{3-d_S} \tag{9}$$

where we used Eq. (5); finally, by using Eq. (1),

$$P \propto 1/\omega^{(3-d_S)/2} \quad . \tag{10}$$

This relation reproduces the standard one[11] ($P \propto 1/\sqrt{\omega}$), corresponding to non fractal conductors, when $d_S = 2$.

To summarize, both the graphs $\ell nR$ vs. $\ell n\omega$ and $\ell nP$ vs. $\ell n\omega$ are expected to exhibit straight lines, with non-integer slopes respectively given by $(1+2d_L-d_S)/2$ and $(d_S-3)/2$. Consequently these experiments should constitute a simple way for unambigously determining $d_L$ and $d_S$. A situation which frequently appears in nature is that of an isotropic fractal, in which case it is expected $d_T=d_L=d_S-1$ (see Refs. [1,13]), and consequently

$$R \propto \omega^{d_L/2} \tag{11}$$

and

$$P \propto \omega^{\frac{d_L-2}{2}} \tag{12}$$

In such situation a *single* experiment is already sufficient to characterize the fractal.

Real substances are obviously not expected to present a fractal behavior "all the way long" (i.e., for all frequencies, in our case). Consequently an actual experiment, say a resistance measurement, should in principle provide several regimes, making crossovers from one into the other at appropriate frequencies. There will always be a *low frequence regime*, corresponding to $\delta$ comparable to (or larger than) the linear sizes of the sample (hence R almost independent from $\omega$), and also a *high frequency regime*, corresponding to the break-down of condition $\omega \ll \sigma/\varepsilon$ ($\varepsilon \equiv$ dielectric constant), necessary [11] for the validity of Eq. (1) (if $\omega > \sigma/\varepsilon$, the frequency dependence of the polarizabi lity of the substance starts to play an important and non trivial role). In between those two regimes, the conductor may exhibit one or more *fractal and/or standard regimes*, correponding to scales of $\delta$ within which the surface of the conductor is seen as "rough" or say "smooth". In Fig. 2 we have illustrated these concepts by assuming in the intermediate frequency region, one fractal regime (slope $(1+2d_L - d_S)/2$ different from 1/2), followed by a standard one (slope 1/2).

This skin effect technique can for example be applied to fracture surfaces of metals. In particular it could be tested in those whose fractal dimensions were already studied by other means[13].

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[12] It must be pointed out that not only the *resolved* volume v contributes to the dissipated power, but the *whole* skin volume up to a depth $\delta$. However, as long as electric field inhomogeneities at the surface irregularities do not mask the purely geometric effect considered here (as it has been assumed throughout), the contribution to dissipation of the difference between the whole and the resolved volumes is small and has the same frequency dependence than the latter. As an example we considered the Koch snowflake of fractal dimension $d_f = \ell n4/\ell n3$ with main triangle of side $\ell$. For a resolution length $\delta$, identified with the skin depth, the resolved cross section is $3\ell^{d_f}\delta^{2-d_f}$ while the whole skin cross section up to a depth $\delta$ is $3(1+\sqrt{3}/20)\ell^{d_f}\delta^{2-d_f}$.

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CAPTION FOR FIGURES

Fig. 1 - Cylinder-like conducting sample with fractal external surface; $\ell_T$ refers to the perimeter. The alternate voltage is applied between the two (equipotential) bases. The indicated skin depth $\delta$ is out if scale (too large) if assumed to be appropriate for probing the "fractality" of the illustrated "rough" surface.

Fig. 2 - Possible result for a resistance measurement of a conducting rough sample, exhibiting a fractal regime (slope $>1/2$) which crosses over to a standard one (slope 1/2). $\omega_o$ ($R_o$) is a reference frequency (resistance) adapted to the particular sample.

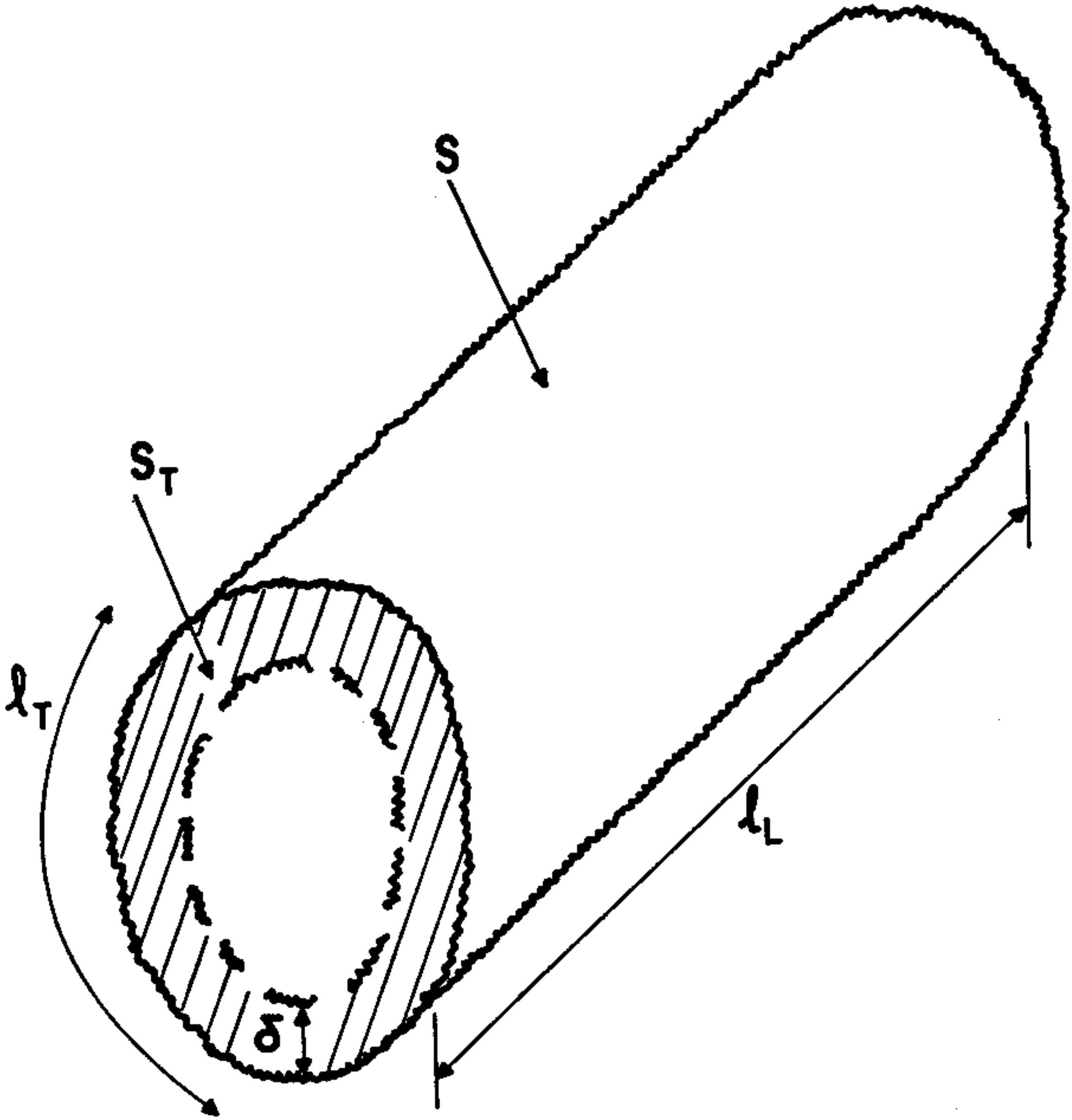


FIG.1

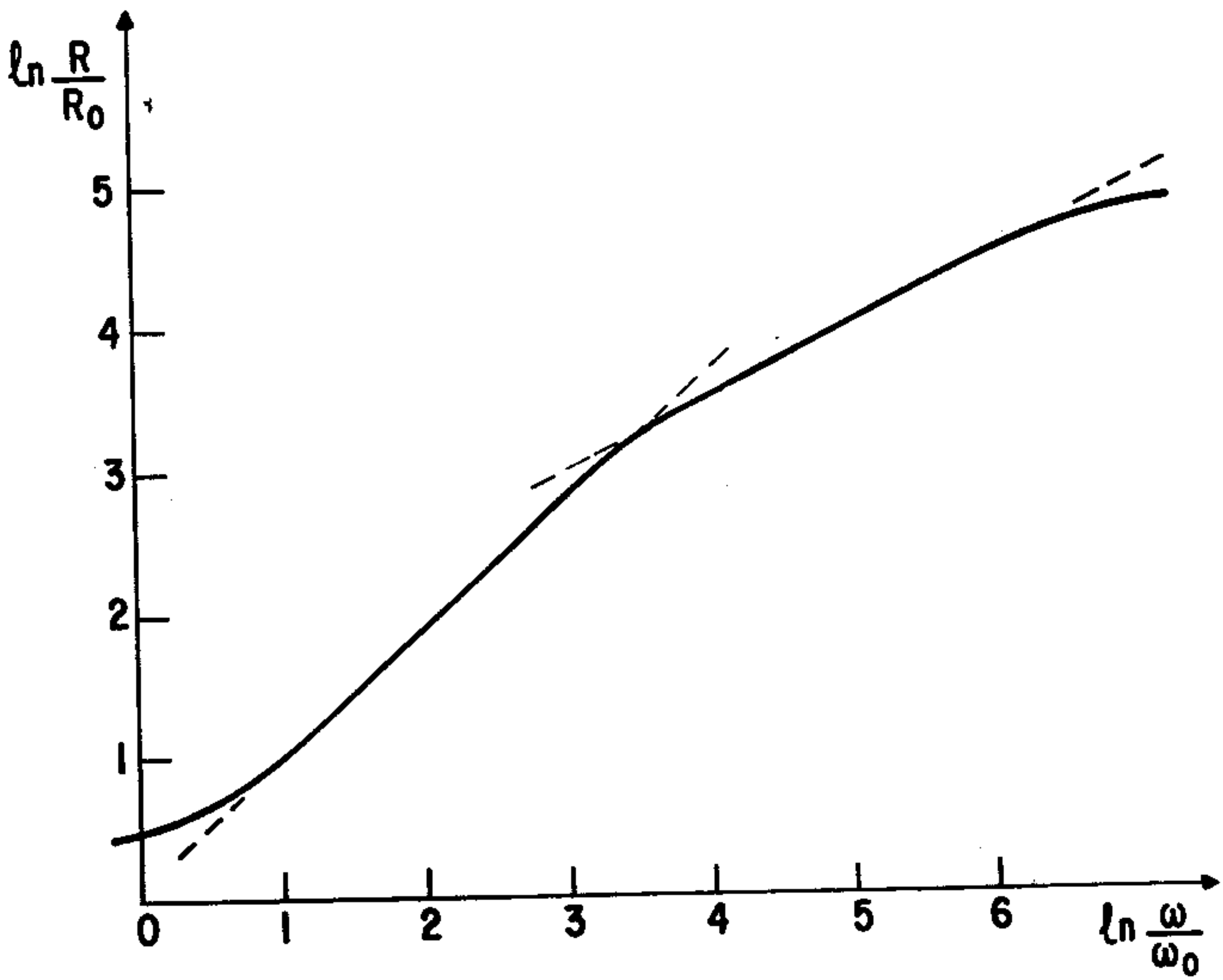


FIG.2