

**Particle Physics and QFT at the Turn of the Century:  
Old principles with new concepts  
(an essay on local quantum physics)  
revised and updated version.**

**Invited contribution to the Issue 2000 of JMP**

Dedicated to John Roberts on the occasion of his 60<sup>th</sup> birthday

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**Abstract**

The present state of QFT is analysed from a new viewpoint whose mathematical basis is the modular theory of von Neumann algebras. Its physical consequences suggest new ways of dealing with interactions, symmetries, Hawking-Unruh thermal properties and possibly also extensions of the scheme of renormalized perturbation theory. Interactions are incorporated by using the fact that the S-matrix is a relative modular invariant of the interacting- relative to the incoming- net of wedge algebras. This new point of view allows many interesting comparisons with the standard quantization approach to QFT and is shown to be firmly rooted in the history of QFT. Its radical “change of paradigm” aspect becomes particularly visible in the quantum measurement problem.

**Key words:** Quantum Field Theory, S-matrix Theory, Tomita-Takesaki Modular Theory.

# 1 Looking at the Past with Hindsight

To a contemporary observer the area which half a century ago was very appropriately called particle- or high-energy- physics with QFT being its main theoretical tool, has gradually lost its homogeneous presentation and appears presently somewhat fractured into several highly specialized regions whose mutual relations are often lost. Beyond vage analogies one would be very hard-pressed to interpret e.g. the standard perturbative formulation (especially of gauge theory), conformal field theory and massive factorizing  $d=1+1$  models as manifestations of the same physical principles. For this reason the value of controllable low-dimensional models of QFT as a theoretical laboratory to understand and explore the general principles of Local Quantum Physics [1] has remained opaque, despite the considerable sophistication of their formalism which went into their presentation. As no other previous theory in its long history, including Relativity and Quantum Mechanics, QFT has resisted construction (apart from some low-dimensional superrenormalizable models) to the degree that we do not know up to this day whether those operators and their correlation functions which one postulates and perturbatively “approximates” really exists in the presence of 4-dim. nontrivial interactions<sup>1</sup>. The coexistence of such a curious state of affairs for almost 70 years with a set of perturbatively consistent rules and recipes of stunning predictive power is the most remarkable enigmatic heritage and a gift of the 20<sup>th</sup> century particle physics to the 21<sup>st</sup>.

In thid context we will have little to say about string theory which has separated from the S-matrix aspects of QFT more than 3 decades ago and still undergoes rapid changes. The reason is that in addition to the absence of any tangible contact with the nature of particle physics, string theory has failed to compare its underlying principles with those of QFT or even to formulate its own principles. A theory which claims to transcend QFT without offering at the same time new physical principles by which its underlying philosophy can be secured against physical equivalence [2] with field theoretic principles is difficult to position. and here we will not even try.

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<sup>1</sup>Despite numerous attempts to convert this problem into a small nuisance which will be repaired at the future Plank length physics, the problem did not go away. The problem of mathematical consistency of physical principles cannot be solved by referring it to the next still unknown layer of physical reality.

A different and potentially more productive kind of dissatisfaction with the present state of particle theory results from theories with impressive predictive power but whose conceptual basis leaves too much to be desired in order to be considered in the long run as completed and mature theories. Here the very successful Bohr-Sommerfeld theory could serve as an example if its incorporation into QM which showed its transitory character would not have happened so swiftly. Potential contemporary candidates are electro-weak theory and quantum chromodynamics. Most of their theoretical discoveries and crucial theoretical developments occurred in the first 5 years starting in the late 60<sup>ies</sup>, although some of the important experimental verifications came much later. Compared with the speed of theoretical progress during a good part of the 20<sup>th</sup> century, the time from the middle 70<sup>ies</sup> up to now begins to appear more and more as a time of stagnation. The fact that an increasing number of renown theoretical particle physicist have uneasy feelings to accept the present gauge theoretic models extended by the Higgs mechanism as a mature description which constitute a closed chapter in particle physics, shows that this is more than a overcritical interpretation on my part.

Experience with past crisis in particle theory (vis. the ultraviolet divergency crisis of the 40<sup>ies</sup> solved by the renormalization theory of the 50<sup>ies</sup>) suggests to use a combination of conservative adherence to physical principles and leave the revolutionary changes on the side of concepts and mathematical formalism.

Most of the remedies which for the last two decades enjoy popularity (as e.g. string theory and physics based on noncommutative geometry) were revolutionary on the side of physical principles as well<sup>2</sup>. As the history which led up to renormalization theory has shown, it is easier to be revolutionary if one allows modifications of principles (e.g. postulating an elementary length or fundamental cutoff, abandoning QFT in favor of a pure S-matrix approach) than to maintain principles and limit the changes to new concepts (physical reparametrization, changing the canonical formalism for causal perturbations). It is indicative that even when a change of principles became unavoidable, as in the case

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<sup>2</sup>A closer look reveals that they are in fact amazingly conservative the side of formalism (e.g. the use of functional integral representations of the Lagrangian quantization approach or ad hoc noncommutative modifications thereof).

of relativity and quantum theory, there was an intense conceptual struggle with the old principles including the use of sophisticated Gedankenexperiments. It seems that this intellectually demanding art has been lost in the last quarter of the 20<sup>th</sup> century.

In the following I would like to expose some recent ideas which maintain a strictly conservative attitude on the side of physical principles. So our wanderlust to step into the “blue yonder” (to borrow a phrase used by Feynman) will be controlled by the valuable compass of physical principles underlying local quantum physics and not primarily by the extension of existing formalisms. The scheme which allows the most natural and clear formulation for our purposes is nowadays referred to as algebraic QFT (AQFT) or local quantum physics (LQP) [1]. Its impractical and non-constructive aspects of which it often stood accused fortunately are increasingly a matter of the past, and in the following we will go a long way to demonstrate this. LQP as enriched by modular theory contains both of the two most successful aspects of past particle physics: the formalism of local quantum physics but blended and controlled with the Wigner particle concept and a new modular role of the S-matrix.

Since I do not want to pose too many technical/mathematical barriers around these new ideas, I use the more flexible essay style (“statements” instead of theorems). I also assume that the reader is familiar with the standard framework of QFT ([3]).

For motivation I will first present some weak spots in the standard textbook approach to QFT. Most of the presentations start with the canonical formalism (Heisenberg-Pauli) or with the (Euclidean) path-integral formalism (Feynman). Both of them are closely related quantization procedures. This means that they are based on a classical parallelism starting from a classical Lagrangian or Hamiltonian<sup>3</sup> in analogy to the way quantum mechanical systems are defined (and named after their classical Hamiltonian). But there is one significant difference to the quantum mechanical case. Whereas in the latter the canonical formalism and the Feynman-Kac path-integral representation have a firm mathematical status even in the presence of interactions, the use of these quantization methods in QFT is (with the already mentioned exception of free fields and superrenormalizable in-

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<sup>3</sup>In the case of Fermions it has been standard praxis (Berezin) to invent a classical reality in form of Grassmann dynamical variables in order to extend the quantization parallelism.

teractions) what one may call more of an “artistic” nature. This means that although the quantization requirements offer enough guiding power to start perturbative calculations, the final renormalized answers do not fulfil the original requirements: *the renormalized physical correlation functions simply do not obey the canonical commutation relations nor are they Feynman-Kac representable!* The only generically remaining structure unaffected by renormalization is Einstein causality/locality i.e. the statement of mutual (anti)commutation of fields separated by spacelike distances. In view of this delicate fact and despite the resulting lack of a logical conceptual balance between the quantization requirement and the physical renormalized answers, quantization in this sense became an accepted fait accompli. The remarkable success swept aside worries for what appeared just small mathematical imperfections.

What enhanced the willingness of many physicists to live comfortably with this conceptual flaw of the formulation of QFT was the fact that their mathematical friends also became attracted to the differential-geometric appeal of path integral quantization and often succumbed to its delicate artistic fascination to such a degree that its conceptual and mathematical flaws were ignored and the artistic computational tools became accepted as a kind of experimental mathematics (and in several cases even received the admirations and blessings of mathematicians). There is a lot of irony in the present state of affairs where QM (for which the Feynman-Kac setting is rigorous but in praxis too difficult and time-consuming) is presented with operator methods, and on the other hand QFT (for which the method is a nice but artistic device to get calculations started) is done almost exclusively in path-integral formulation. Anybody who tried to give a physically balanced course on QM using path integrals knows at least one side of these problems.

There exists an alternative method of deforming free fields with interaction Wick-polynomials within the setting of causal perturbation [4] which uses the above mentioned fact that causality (and not the Feynman-Kac representability) survives renormalization. The interaction polynomials in terms of free fields enter the causality- and unitarity- based equation for time-ordered or retarded function as a perturbative input. All iterative steps are then shown to be uniquely fixed by the mentioned principle and minimality requirements for an order-independent minimal scale dimension. The mathematical problem is

the extension of time-ordered distributions from a certain subspace of test functions with nondiagonal support to such containing supports with coalescing points. There is no actual infinity and the difference renormalizable/nonrenormalizable is the implementability of such a minimality requirement (which is tantamount to a unique theory with a finite number of physical parameters). This method explains the infinities of many of the textbook quantization method as a result of their unwarranted relation to classical structures. In other words the infinities of the old classical particle models of Poincaré and Lorentz enter *nolens volens* via quantization (which remains too close to classical ideas) and represent a technical nuisance <sup>4</sup> which needs repair analogous to the mentioned selfenergy problems of the classical particle models (as was first pointed out by Kramers). Despite differences in the conceptual setting the renormalized results of all approaches (with or without intermediate infinities) are identical and the apparent restrictive relation between the possible existence and renormalizability of a theory and the “good short distance behavior” of those particular “field coordinatizations” in terms of which the interaction density was defined is common to all approaches which use pointlike fields.

With this remark we have come to the point of departure of the new framework from the old setting: *the substitution of individual fields by nets of algebras corresponding to spacetime regions*. This step is to be seen in analogy to the spirit underlying the transition of old fashioned geometry in terms of coordinates to modern coordinate-free intrinsic differential geometry.

There were strong historical indications pointing towards a field-coordinate-free formulation of local quantum physics<sup>5</sup>; one of the earliest was the observation about the insensitivity of the (on-shell) scattering matrix with respect to changes of the interpolating local fields. In the traditional setting of Lagrangian this was done by carrying out extremely formal field transformations. As in geometry one meets of course also preferred

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<sup>4</sup>According to Wigner’s analysis particles in QFT enter (to the degree that the QFT possesses them) automatically via the representation theory of the Poicaré group; there is no room for separate particle models ala Poincaré/Lorentz.

<sup>5</sup>Since it is quite awkward to use the terminology “QFT without pointlike fields”, we follow Haag [1] and use Local Quantum Physics or algebraic QFT, in particular whenever we want the reader not to think in terms of the standard textbook methods.

field-coordinates which have characteristic intrinsic properties; notably conserved Noether currents and other natural local objects which result from the localization of (global) symmetries or have a direct relation to superselected charges. I would even go as far as saying that it was basically the arbitrary ad hoc nature of selection of particular fields in particle physics which led to the sometimes fanatical “cleansing” attitude against QFT which even entered the publications of some S-matrix purists and is hard to understand from a contemporary point of view. Our modular localization approach will demonstrate, that also the opposite ideology against S-matrix theory (for quotations of famous sayings see [22]) is unwarranted. Since  $S$  is an important relative modular invariant, a constructive method based on modular theory should use it together with the local algebras already in the construction and not only after the purely field theoretic calculations have been finished. Our approach combines on- and off- shell aspects in one formalism and in particular presents the construction of the observable algebras from an S-matrix point of view without introducing individual fields; hence it accomplishes those steps which in the old S-matrix theory were missing or even thought to be impossible.

In fact this coordinate-free formulation already exists for quite some time [1]. Up to very recently it was limited to structural questions and contributed little in the direction of classifications and investigations of concrete models (a fact which perhaps also explains the widespread ignorance about it). The main motivation for this essay is to inform the reader about two new constructive ideas, both related to the Tomita-Takesaki modular theory for wedge-localized algebras. The first idea uses “polarization- free generators” of the wedge algebra whose structure is closely related to the scattering matrix. This structure is e.g. behind the bootstrap-formfactor program for  $d=1+1$  factorizing theories. The second idea is to relate a higher dimensional massive QFT to a finite number of isomorphic copies of one chiral conformal field theory whose relative positions in one Hilbert space are defined in terms of “modular inclusions and intersections”. In picturesque terms this should be thought of as some sort of “chiral scanning” or AQFT-holography. One encodes the rather complex structure of higher-dimensional massive QFT into a family of very simple chiral conformal QFT and their relative modular position. Such modular reformulations may also shed new light on the existence problem of higher dimensional QFT since there is

good control of existence of their chiral conformal building blocks.

The organization of the content is as follows. As a “warm up” we explain in the next section a presentation of interaction-free systems without the use of field coordinates. We then use this formalism for a presentation of the Hawking-Unruh thermal aspects of modular localization. The section continues with a totally intrinsic characterization of what one means by interactions. These results suggest to look at wedge algebras as the smallest spacetime regions which offer the best compromise between particles and fields; in fact if the often cited “particle-field dualism” makes any sense at all, it is in this context of wedge localization. In the third section we explain the relative modular invariance of the S-matrix. The crucial concept here are certain wedge-localized operators which if applied once to the vacuum even in the presence of interactions do not generate particle/antiparticle vacuum polarization clouds but just pure one-particle vectors. By specialization to 2-dim. models without real particle creation, they are identified with Zamolodchikov-Faddeev operators which in this way acquire for the first time a profound spacetime interpretation. We also comment on wedge-localized states and operators in the presence of real pair creation away from factorizing models. The section ends with a modular extension of standard symmetries to “hidden” symmetries.

Section 3 presents the “re-conquest” of notions known from basic quantum mechanics within LQP with the help of the “split property”. In this section the conceptual change of paradigm of the new approach becomes most evident. A closely related aspect for which the split property turns out to be essential is the “localization-entropy”, the other thermal manifestation of localization in addition to localization-temperature which in the special case of wedge localization is the same up to an acceleration factor as the temperature in the Unruh Gedankenexperiment involving uniformly accelerated observers..

In the futuristic last section I mention some potential areas of applications where one expects the modular ideas to enlarge the conceptual realm of models in the direction of what would be called ordinarily “nonrenormalizable”. We also present various other poorly studied consequences of modular theory, including an LQP version of “holography” and “chiral conformal scanning”.

This essay is intended to fill some of the space left between two other major articles in



this issue of JMP on the present state of Local Quantum physics; one is a broadly-based paper with a strong emphasis on recalling the history and the spirit of times of particle physics during a good part of the 20<sup>th</sup> century [5], and the other [6] presents an exhaustive account of more recent developments about modular structures of LQP. In fact our findings add additional weight to the the word “revolutionary” used by Borchers in reference to the recent development of modular theory in local quantum physics.

## 2 Modular Structure of LQP

For pedagogical explanations of the new modular concepts, the interaction free theories are still the simplest. As in some of the textbooks (Haag, Weinberg), one starts from the Wigner approach which assigns a unique irreducible representation of the Poincaré group with every admissible value of the mass and spin/helicity ( $m, s$ ). The Wigner theory also preempts the statistics of particles and assigns in the case of  $d=3+1$ , where the particles can only be Fermions/Bosons (with the exception of the essentially unexplored case of continuous spin), unique momentum space creation and annihilation operators acting in a multiparticle Fock space. The uniqueness is lost at the moment one uses a manifestly local formalism in terms of pointlike fields. The covariant field construction is synonymous with the introduction of intertwiners between the unique Wigner  $(m, s)$  representation and the multitude of Lorentz-covariant momentum dependent spinorial (dotted and undotted) tensors which under the homogenous L-group transform with the irreducible  $D^{[A,B]}(\Lambda)$  matrices.

$$u(p)D^{(s)}(R(\Lambda, p)) = D^{[A,B]}(\Lambda)u(\Lambda^{-1}p) \quad (1)$$

The only restriction imposed by this intertwining is:

$$|A - B| \leq s \leq A + B \quad (2)$$

which leaves infinitely many  $A, B$  (half integer) choices for a given  $s$ . Here the  $u(p)$  intertwiner is a rectangular matrix consisting of  $2s + 1$  column vectors  $u(p, s_3), s_3 = -s, \dots, +s$  of length  $(2A + 1)(2B + 1)$ . Its explicit construction using Clebsch-Gordan methods can

be found in Weinberg's book [3]. Analogously there exist antiparticle (opposite charge)  $v(p)$  intertwiners:  $D^{(s)*}(R(\Lambda, p) \longrightarrow D^{[A,B]}(\Lambda)$ . The covariant field is then of the form:

$$\begin{aligned} \psi^{[A,B]}(x) = & \frac{1}{(2\pi)^{3/2}} \int (e^{-ipx} \sum_{s_3} u(p_1, s_3) a(p_1, s_3) + \\ & + e^{ipx} \sum_{s_s} v(p_1, s_3) b^*(p_1, s_3)) \frac{d^3p}{2\omega} \end{aligned} \quad (3)$$

Since the range of the  $A$  and  $B$  (undotted/dotted) spinors is arbitrary apart from the fact that they must fulfil the inequality with respect to the given physical spin  $s^6$ , the number of covariant fields is countably infinite. Fortunately it turns out that this loss of uniqueness does not cause any harm in particle physics. If one defines the algebras  $\mathcal{P}(\mathcal{O})$  as the operator algebras generated from the smeared field with  $\text{supp } f \in \mathcal{O}$  [18], one realises that these localized algebras do not depend on the representative covariant field chosen from the  $(m,s)$  class. In fact all the different covariant fields which originate from the  $(m, s)$  representation share the same creation/annihilation operators. This gave rise to the linear part of the Borchers equivalence classes of relatively local fields. The full Borchers class [18] generalized the family of Wick polynomials to the realm of interactions and gave a structural explanation of the insensitivity of the S-operator.

## 2.1 Modular Aspects of Wigner Particle Theory

The conceptually and mathematically natural way to implement the idea of independence of physics from different field coordinatizations is to use instead of smeared unbounded fields (with their technically difficult domain properties) the associated von Neumann algebras of bounded operators [1] which have lost their reference to particular field coordinates. In the case at hand of the Wigner particle theory of free particles this step recovers the Wigner uniqueness of  $(m,s)$  particle representations (which got lost as a result of the introduction of covariant fields as explained in the previous section). The obvious question is therefore: *is it possible to extract the spacetime indexed net of algebras directly from the Wigner theory without the intermediate appearance of fields?* A question like this

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<sup>6</sup>For the massless case the helicity inequalities with respect to the spinorial indices are more restrictive, but one Wigner representation has still a countably infinite number of covariant representations.

was probably on Wigner’s mind when he was looking (without success) for a relativistic localization concept within his representation-theoretical framework.

Recently this question of covariant localizaton received a positive answer as a result of the introduction of “modular localization” [7][8]. The idea can be traced back to a seminal paper of Bisognano and Wichmann in which it was shown that the modular Tomita-Takesaki theory [1] of von Neumann algebras has not only some deep use in quantum statistical mechanics (as was already known from the Haag-Hugenholtz-Winnink work which appeared at the same time as Tomita’s notes [1]), but is also an inexorable part of field theoretic wedge-localization<sup>7</sup>. What one needs here is in some sense the inverse of the Bisognano-Wichmann arguments namely the use of modular theory for the actual construction of a net of wedge algebras and their smaller descendents via intersections. Its adaptation to the case at hand would look for a kind of pre-Tomita theory which can be formulated within the Wigner theory and with the help of CCR/CAR functors preempts the net structure of the interaction-free LQP. This is indeed feasible and the resulting formalism is mathematically not more complicated than the formalism of free fields. Since it has appeared in different publications, a short description should suffice for the purpose of this essay.

The pre-modular theory alluded to is a generalization of the Tomita theory to real Hilbert spaces positioned within a complex Hilbert space. For its adaptation to the Wigner theory one starts with the boost transformation associated with a wedge and its reflection transformation along the rim of the wedge. For the standard x-t wedge these are the  $\Lambda_{x-t}(\chi)$  Lorentz boost and the x-t reflection  $r_{x-t} : (x, t) \rightarrow (-x, -t)$  which according to well-known theorems is represented antiunitarily in the Wigner theory<sup>8</sup>. One then starts from the unitary boost group  $u(\Lambda(\chi))$  and forms by the standard functional

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<sup>7</sup>It is important to emphasize that physicists have a significant share in the discovery of modular theory in particular with physicists whose only contact with this theory arose through “non-commutative geometry” without revealing the natural physical origin.

<sup>8</sup>In case of charged particles the Wigner theory should be suitably extended by a particle/antiparticle doubling.

calculus the unbounded “analytic continuation”. In terms of modular notation we define

$$\begin{aligned} \mathfrak{s} &= j\delta^{\frac{1}{2}} \\ j &= U(r) \\ \delta^{it} &= u(\Lambda(-2\pi t)) \end{aligned} \tag{4}$$

where  $u(\Lambda(\chi))$  and  $u(r)$  are the unitary/antiunitary representations of these geometric transformations in the (if necessary doubled) Wigner theory. Note that  $U(r)$  is apart from a  $\pi$ -rotation around the x-axis the one-particle version of the TCP operator. On the other hand  $\mathfrak{s}$  is a very unusual object namely an unbounded antilinear operator which on its domain is involutive  $\mathfrak{s}^2 = 1$ . The real subspace

$$\mathfrak{s}\psi = \psi \tag{5}$$

which consists of momentum space wave functions which are boundary values of analytic functions in the lower  $i\pi$ -strip of the rapidity variable  $\theta$ . The -1 eigenvalues of  $S$  do not give rise to a new problem since multiplication of the +1 eigenfunctions with  $i$  convert them into the -1 eigenfunctions. The real subspace  $H_R(W)$  is closed in the complex Hilbert space topology but the complexification  $H_C(W)$  gives a space which is only dense in the complex Wigner space. This surprising fact (which is the Wigner one-particle analogue of the Reeh-Schlieder denseness of local field states in full quantum field theory) has no parallel in any other area of quantum physics. It suggests that the above mentioned unusual property of the  $\mathfrak{s}$ -operator may be the vehicle by which geometric physical properties of space time localization are encoded into the abstract domain properties of unbounded operators. Some rather straightforward checks reveal that this interpretation is consistent namely in the present setting this localization interpretation gives consistency with the net properties of the spaces  $H_R(\mathcal{O})$ 's

$$H_R(\mathcal{O}) \equiv \cap_{W \supset \mathcal{O}} H_R(W) \tag{6}$$

as well as with the conventional field theoretic construction using pointlike fields where it agrees with localized covariant functions defined in terms of support properties of Cauchy initial data. The relation of Wigner subspaces and localized subalgebras is accomplished

with the help of the CCR or CAR functors which map real subspaces  $H_R(\mathcal{O})$  into von Neumann  $\mathcal{A}(H_R(\mathcal{O}))$  subalgebras and which define a limited but rigorous meaning of the word “quantization”

$$J, \Delta, S = \Gamma(j, \delta, \mathfrak{s}) \tag{7}$$

where the functorial map  $\Gamma$  carries the functions of the Wigner theory into the Weyl operators in Fock space (for the fermionic CAR-algebras there is an additional modification). Whereas the “pre-modular” operators denoted by small letters act on the Wigner space, the modular operators  $J, \Delta$  have an  $Ad$  action on the von Neumann algebras which are functorially related to the subspaces and which makes them objects of the Tomita-Takesaki modular theory

$$SA\Omega = A^*\Omega, S = J\Delta^{\frac{1}{2}} \tag{8}$$

$$Ad\Delta^{it}\mathcal{A} = \mathcal{A} \tag{9}$$

$$AdJ\mathcal{A} = \mathcal{A}'$$

This time the S-operator is that of Tomita i.e. the unbounded densely defined operator which relates the dense set  $A\Omega$  to the dense set  $A^*\Omega$  and gives  $J$  and  $\Delta^{\frac{1}{2}}$  by polar decomposition. The nontrivial miraculous properties of this decomposition are the existence of an automorphism  $\sigma_\omega(t) = Ad\Delta^{it}$  which propagates operators within  $\mathcal{A}$  and only depends on the state  $\omega$  (and not on the implementing vector  $\Omega$ ) and a that of an antiunitary involution  $J$  which maps  $\mathcal{A}$  onto its commutant  $\mathcal{A}'$ . The theory of Tomita assures that these objects exist in general if only  $\Omega$  is a cyclic and separating vector with respect to  $\mathcal{A}$ . Our special case at hand, in which the algebras and the modular objects are constructed functorially from the Wigner theory, suggest that the modular structure for wedge algebras may always have a geometrical significance with a fundamental physical interpretation in any QFT. This is indeed true, and within the Wightman framework this was established by Bisognano and Wichmann [1].

The existence of this coordinate-free formulation for interaction free theories has immediate consequences. Although in the present form it is not yet suited to incorporate interactions without the use of field coordinates, it does shed an additional helpful light on

the standard causal perturbation theory. Among other things it formally explains why an interaction which has been defined in terms of concrete free fields can be rewritten without change of content in terms of any other field coordinates and that moreover Euler-Lagrange coordinates which associate free fields with a bilinear zero order Lagrangians  $\mathcal{L}_0$  are not necessary in a real time operator formulation.. Of course since an Euler-Lagrangian field coordinatization exists for each Wigner  $(m, s)$  representation, the physical results remain the same if on properly transforms the interaction density into this Euler-Lagrange description. Hence the use of such field is not a restriction of generality.

## 2.2 Thermal Aspects of Modular Localization

Another valuable suggestion which can be abstracted from the pre-modular structure of the Wigner theory concerns thermal aspects which originate from localization. In modular theory the dense set of vectors which are obtained by applying (local) von Neumann algebras in standard position to the standard (vacuum) vector forms a core for the Tomita operator  $S$ . The domain of  $S$  can then be described in terms of the +1 (or -1) closed real subspace of  $S$ . In terms of the “pre-modular” objects  $\mathfrak{s}$  in Wigner space and the modular Tomita operators  $S$  in Fock space we introduce the following nets of wedge-localized dense subspaces:

$$H_R(W) + iH_R(W) = \text{dom}(\mathfrak{s}) \subset H_{Wigner} \quad (10)$$

$$\mathcal{H}_R(W) + i\mathcal{H}_R(W) = \text{dom}(S) \subset \mathcal{H}_{Fock} \quad (11)$$

These dense subspaces become Hilbert spaces in their own right if we use the graph norm (the thermal norm) of the Tomita operators. For the  $\mathfrak{s}$ -operators in Wigner space we have:

$$\begin{aligned} (f, g)_{Wigner} &\rightarrow (f, g)_G \equiv (f, g)_{Wig} + \overline{(sf, sg)}_{Wig} \\ &= (f, g)_{Wig} + (f, \delta g)_{Wig} \end{aligned} \quad (12)$$

This graph topology insures that the wave functions are strip-analytic in the wedge rapidity  $\theta$ :

$$p_0 = m(p_\perp) \cosh \theta, \quad p_1 = m(p_\perp) \sinh \theta, \quad m(p_\perp) = \sqrt{m^2 + p_\perp^2} \quad (13)$$

$$\text{strip} : 0 < \text{Im}z < \pi, \quad z = \theta_1 + i\theta_2$$

where the "G-finiteness" (12) is precisely the analyticity prerequisite for the validity of the KMS property for the two-point function. In this way one finally arrives at (for scalar Bosons):

$$(f, g)_{Wig}^W \equiv (f, g)_{K, T=2\pi} \quad (14)$$

where on the left hand side the Wigner inner product is restricted to  $H_R(W) + iH_R(W)$  and the right hand side is the thermal inner product which contains the characteristic thermal  $\frac{\delta}{1-\delta}$  factor where  $\delta = e^{2\pi K}$  with  $K^9$  the infinitesimal generator of the L-boost. The fact that the boost  $K$  with a two-sided spectrum appears instead of the one-sided bounded Hamiltonian  $H$  reveals one difference between the two situations. For the heat bath temperature of a Hamiltonian dynamics the modular operator  $\delta = e^{-2\beta\mathbf{H}}$  is bounded on one particle wave functions, whereas the unboundedness of  $\delta = e^{2\pi K}$  enforces the localization (strip analyticity) of the Wigner wave functions i.e. the two-sidedness of the spectrum does not permit a KMS state on the full algebra. In fact localization-temperatures are inexorably linked with unbounded modular symmetry operators. With the localization-temperature  $T = 2\pi$  in this way having been made manifest, the only difference between localization-temperatures and heat bath temperatures (for a system enclosed in a box described by a Gibbs formula) on the level of field algebras in Fock space corresponds to the difference between hyperfinite type  $III_1$  and type  $I$  von Neumann algebras. But even this distinction disappears if one passes from the Gibbs box situation to the infinite volume thermodynamic limit: the GNS reconstruction using the limiting correlation functions reveals that the algebra has become hyperfinite type  $III_1$ .

Passing from Wigner one-particle theory to free field theory we may now consider matrix elements of wedge-localized operators between wedge-localized multiparticle states.

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<sup>9</sup>The Unruh Hamiltonian is different from the boost  $K$  by a factor  $\frac{1}{a}$  where  $a$  is the Unruh acceleration.

Then the KMS property allows to move the wedge-localized particle state as an antiparticle with the analytically continued rapidity  $\theta + i\pi$  from the ket to the bra. The simplest illustration is the two-particle matrix element of a free current of a charged scalar field  $j_\mu(x) =: \phi^* \overleftrightarrow{\partial}_\mu \phi : .$  The analytic relation

$$\langle p' | j_\mu(0) | p \rangle = \underset{z \rightarrow \theta + i\pi}{anal. cont.} \langle 0 | j_\mu(0) | p, \vec{p}'(z) \rangle \quad (15)$$

where  $\vec{p}'(z)$  represents the analytic rapidity parametrization of the antiparticle is the simplest form of a crossing relation. It is an identity which is known to hold also in each perturbative order of renormalizable interacting theories and which together with TCP-symmetry constitutes the most profound property of QFT. But it has never been derived in sufficient generality within a nonperturbative framework of QFT nor (different from TCP) has its relation to the causality and positive energy property of QFT been adequately understood. It is often thought of as a kind of on shell momentum space substitute for Einstein causality and its strengthened form, called Haag duality.

If crossing symmetry is really a general property of local QFT, then it should be the on shell manifestation of the off shell KMS property for modular wedge localization not only in the previous free case but also in the presence of interactions. In fact we will show in the next section that the main step towards a deeper understanding of crossing symmetry is the existence of certain on-shell operators  $\int F(x)f(x)dx$  ( $supp f \in W$ ) which generate the wedge algebra and upon application to the vacuum create a one-particle state vector without the vacuum polarization clouds which are characteristic for interacting operators in smaller than wedge localization regions. We will call them polarization-free generators or PFG's. In the case of  $d=1+1$  factorizing models their mass shell Fouriertransforms satisfy the Zamolodchikov-Faddeev algebraic relations<sup>10</sup> in the momentum space rapidity [10], and the derivation of crossing symmetry is similar (albeit more involved) to the previously mentioned case of formfactors in free theories.

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<sup>10</sup>As will become clear in the next section, although these operators are nonlocal, they generate the wedge localized algebra, and as a consequence the modular KMS formalism is applicable to them.



## 2.3 Wedge Localization for Special Interactions

A major challenge to ones conceptual abilities is the generalization of these modular attempts to the realm of interactions. Here the first step should be a clear intrinsic definition of what one means by interactions without the use of e.g. Lagrangians, Feynman rules or other ways of computing but solely based on intrinsic properties of correlation functions or nets of local algebras . The example of Wick polynomials in the free Borchers class, which despite their complicated looking vacuum correlation functions still represent only free theories in the veil of different field coordinates, gives a first taste of the magnitude of the problem. This will be addressed in the next subsection.

We start with the Fock space of free massive Bosons or Fermions. In order to save notation we will explain the main ideas first in the context of selfconjugate (neutral) scalar Bosons. Using the Bose statistics we will use for our definitions the “natural” rapidity-ordered notation for n-particle state vectors

$$a^*(\theta_1)a^*(\theta_2)\dots a^*(\theta_n)\Omega, \quad \theta_1 > \theta_2 > \dots > \theta_n \quad (16)$$

and define new creation operators  $Z^*(\theta)$  in case of  $\theta_i > \theta > \theta_{i+1}$  and with the previous convention

$$\begin{aligned} Z^*(\theta)a^*(\theta_1)\dots a^*(\theta_i)\dots a^*(\theta_n)\Omega = \\ S(\theta - \theta_1)\dots S(\theta - \theta_i)a^*(\theta_1)\dots a^*(\theta_i)a^*(\theta)\dots a^*(\theta_n)\Omega \end{aligned} \quad (17)$$

With  $Z(\theta)$  as the formal adjoint one finds the following two-particle commutation relations

$$\begin{aligned} Z^*(\theta)Z^*(\theta') &= S(\theta - \theta')Z^*(\theta')Z^*(\theta) \\ Z(\theta)Z^*(\theta') &= S(\theta' - \theta)Z^*(\theta')Z(\theta) + \delta(\theta - \theta') \end{aligned} \quad (18)$$

where the formal  $Z$  adjoint of  $Z^*$  is defined in the standard way. The \*-algebra property requires  $S(\theta) = S(\theta)^* = S(\theta)^{-1} = S(-\theta)$ . Although our notation already preempted the relation with the Zamolodchikov-Faddeev algebra, the conceptual setting here is quite different. We do not demand that the structure function  $S$  is the crossing symmetric  $S$ -matrix where certain poles represent bound states of particles. Rather we will show that

all these properties including their physical interpretation are consequences of modular wedge localization of PFG's formed from the  $Z$ 's. This structure leads in particular to

$$\begin{aligned} Z^*(\theta_1)\dots Z^*(\theta_n)\Omega &= a^*(\theta_1)\dots a^*(\theta_n)\Omega \\ Z^*(\theta_n)\dots Z^*(\theta_1)\Omega &= \prod_{i>j} S(\theta_i - \theta_j) a^*(\theta_1)\dots a^*(\theta_n)\Omega \end{aligned} \tag{19}$$

for the natural/opposite order with all other cases between these extreme orders. Note that for momentum space rapidities it is not necessary to say something about coinciding rapidities since only the  $L^2$  measure-theoretical sense and no continuity is relevant here. In fact the mathematical control of these operators i.e. the norm inequalities involving the number operator hold as for the standard creation/annihilation operators. Let us now imitate the free field construction and ask about the localization properties of these F-fields

$$F(x) = \frac{1}{\sqrt{2\pi}} \int (e^{-ipx} Z(\theta) + h.c.) \tag{20}$$

This field has all the standard properties of operator-valued tempered distributions, but it cannot be local if  $S$  depends on  $\theta$  since the on-shell property together with locality leads to the free field formula. In fact it will turn out (see next section) that the smeared operators  $F(f) = \int F(x)f(x)d^2x$  with

$$supp f \in W_0 = \{x; x^1 > |x^0|\} \tag{21}$$

have their localization in the standard wedge  $W$  and that, contrary to smeared pointlike localized fields, the wedge localization cannot be improved by improvements of the test function support inside  $W$ . Instead the only way to come to a local net of algebras (and, if needed, to their pointlike field generators) is by intersecting oppositely localized wedge algebras (see below). Anticipating their wedge localization properties these operators are our first examples of **polarization free generators** (PFG). Like free fields their one-time application to the vacuum creates a one-particle state without a (vacuum) polarization cloud admixture.

We want to show that the operators  $F(f)$  are generators of a wedge localized algebra

$$\mathcal{A}(W) = alg \{F(f); supp f \in W\} \tag{22}$$

As in the case of free fields the algebra may be defined as the weak closure of the  $C^*$ -algebra generated by the spectral projection operators in the spectral resolution

$$F(f) = \int \lambda dE_f(\lambda) \quad (23)$$

We first show that n-point functions of the  $F(f)$ 's obey a KMS condition with respect to the Lorentz-boost subgroup which leaves the wedge  $W_0$  invariant if and only if the commutation functions (in addition to their holomorphy properties in the  $\theta$ -strip) are crossing symmetric which is the symmetry of reflections through the point  $\theta = i\frac{\pi}{2}$  (with the additional appearance of the charge conjugation for non-neutral particles). One can show the following statement

*Statement:*([7]) The KMS-thermal property of the wedge algebra generated by the PFG's is equivalent to the crossing symmetry of the S-matrix

$$(\Omega, F(f_{1'})F(f_{2'})F(f_2)F(f_1)\Omega) \equiv \langle F(f_{1'})F(f_{2'})F(f_2)F(f_1) \rangle_{therm} \quad (24)$$

$$\stackrel{KMS}{=} \langle F(f_{2'})F(f_2)F(f_1)F(f_{1'}^{-2\pi i}) \rangle_{therm}$$

$$\Leftrightarrow S(\theta) = S(i\pi - \theta) \quad (25)$$

Here we only used the cyclic KMS property (the second line containing the imaginary  $2\pi$ -shift) for the four-point function. The relation is established by Fouriertransformation and contour shift  $\theta \rightarrow \theta - i\pi$ . One computes

$$F(\hat{f}_2)F(\hat{f}_1)\Omega = \int \int f_2(\theta_2 - i\pi)f_1(\theta_1 - i\pi)Z^*(\theta_1)Z^*(\theta_2)\Omega + c.t. \quad (26)$$

$$= \int \int f_2(\theta_2 - i\pi)f_1(\theta_1 - i\pi)\{\chi_{12}a^*(\theta_1)a^*(\theta_2)\Omega + \chi_{21}S(\theta_2 - \theta_1)a^*(\theta_2)a^*(\theta_1)\Omega\} + c\Omega \quad (27)$$

where the  $\chi$  are the characteristic function for the differently permuted  $\theta$ -orders. The analogous formula for the bra-vector is used to define the four-point function as an inner product. If S has a crossing symmetric pole in the in the physical strip of S the contour shift will produce an unwanted terms which wrecks the KMS relation. The only way out

is to modify the previous relation

$$F(\hat{f}_2)F(\hat{f}_1)\Omega = (F(\hat{f}_2)F(\hat{f}_1)\Omega)_{scat} \tag{28}$$

$$+ \int d\theta f_1(\theta_1 + i\theta_b)f_2(\theta_2 - i\theta_b) |\theta, b\rangle \langle \theta, b | Z^*(\theta - i\theta_b)Z^*(\theta + i\theta_b) | \Omega \rangle \tag{29}$$

The second contribution is compensated by the pole contribution from the contour shift. In general the shift will produce an uncompensated term from a crossed pole whose position is obtained by reflecting in the imaginary axis around  $i\frac{\pi}{2}$ , which creates the analogous crossed bound state contribution. In our simplified selfconjugate model it is the same term as above. In the presence of one or several poles one has to look at higher point functions. Despite the different conceptual setting one obtains the same formulas as those for the S-matrix bootstrap of factorizing models and hence one is entitled to make use of the bootstrap technology in this modular program. What is behind is the so-called GNS construction which converts the numerical poles in S and its higher bound versions into new states i.e. the original Fock-space structure has to be enlarged if we initially forgot to include the b-particles. Even though the description of the wedge algebra appears like QM, there is one important difference which is worthwhile noticing. This is the principle of “nuclear democracy” between particles. In QM there is a hierarchy between fundamental and bound: elementary states do not reappear as boundstates of others and in particular not of composites of themselves. We will see in the following that this realization of nuclear democracy for double cone algebras is not any more with particles and their binding, but rather with charges and their fusion. The reason is of course the appearance of polarization clouds below wedge localization. This nuclear democracy idea was the basis of the S-matrix bootstrap approach and was first made to work in special two-dimensional situations in [12][13][14].

With the derivation of crossing symmetry and the bound state and fusion structure of  $S$  we achieved our aim to present an example of the constructive power of the modular localization method. In fact our fusion formulas for multi F-vectors correctly interpret the Z-formulas in [10] [11] which if taken literally are not true. As an unexpected gratification we also obtained the equivalence between the crossing symmetry of particle physics and the thermal KMS properties of the Hawking-Unruh effect.

Strictly speaking the check of the KMS property with the Lorentz-boost as the automorphism of the wedge algebra does not yet prove that the modular theory is completely geometric. If we could show that the Tomita involution is equal to the TCP operator, we would be done. For this to hold, we define

$$J = \mathbf{S}_s J_0 \tag{30}$$

This relation between the incoming Tomita involution  $J_0$  for the free wedge algebra and that of the interacting theory  $J$  is nothing else as a reformulation of the TCP transformation for the scattering matrix in a general QFT. We can now directly check

$$\begin{aligned} \hat{Z}^*(\theta) &:= JZ^*(\theta)J \\ [\hat{Z}^*(\theta), Z^*(\theta')] &= 0 \end{aligned} \tag{31}$$

$$[\hat{Z}(\theta), Z^*(\theta')] = \delta(\theta - \theta') \tag{32}$$

In other words the two operators  $\hat{Z}^\#(\theta)$  and  $Z^\#(\theta')$  have relative canonical commutation relations which in turn leads to the relative commutativity

$$[\hat{F}(\hat{f}), F(g)] = 0, \text{ } \text{supp}\hat{f} \in W^{opp}, \text{ } \text{supp}g \in W \tag{33}$$

The  $F$  and  $\hat{F}$  PFG's generate algebras  $\mathcal{A}(W)$  and  $\mathcal{A}(W)' = \text{alg}\{\hat{F}(\hat{f}); \text{supp}\hat{f} \in W^{opp}\} = J\mathcal{A}(W)J$  and one easily checks

$$\begin{aligned} J\Delta^{\frac{1}{2}}F(f_1)\dots F(f_n)\Omega &= (F(f_1)\dots F(f_n))^*\Omega \\ \Delta^{it} &= U(\Lambda(2\pi i)) \end{aligned} \tag{34}$$

which is the defining relation for the Tomita operator  $S = J\Delta^{\frac{1}{2}}$ .

The KMS computation can be extended to “formfactors” i.e. mixed correlation functions containing in addition to  $F$ 's one generic operator  $A \in \mathcal{A}(W)$  so that the previous calculation results from the specialization  $A = 1$ . This is so because the connected parts of the mixed correlation function is related to the various  $(n, m)$  formfactors obtained by the different ways of distributing  $n+m$  particles in and out states using the relation between  $Z$ 's and Fock space creation and annihilation operators. These different formfactors are described by different boundary values of one analytic master function which is in turn

related to the various forward/backward on shell values which appear in one mixed A-F correlation function. We may start from the correlation function with one  $A$  to the left and say  $n$   $F$ 's to the right and write the KMS condition as

$$\langle AF(\hat{f}_n)\dots F(\hat{f}_2)F(\hat{f}_1) \rangle = \langle F(\hat{f}_1^{2\pi i})AF(\hat{f}_n)\dots F(\hat{f}_2) \rangle \quad (35)$$

The  $n$ -fold application of the  $F$ 's to the vacuum on the left hand side creates besides an  $n$ -particle term involving  $n$  operators  $Z^*$  to the vacuum (or KMS reference state vector)  $\Omega$  also contributions from a lower number of  $Z^*$ 's together with  $Z$ - $Z^*$  contractions. As with free fields, the  $n$ -particle contribution can be isolated by Wick-ordering<sup>11</sup>

$$\langle A : F(\hat{f}_n)\dots F(\hat{f}_2)F(\hat{f}_1) : \rangle = \langle F(\hat{f}_1^{2\pi i})A : F(\hat{f}_n)\dots F(\hat{f}_2) : \rangle \quad (36)$$

Rewritten in terms of  $Z$ 's and using the denseness of the  $f$ 's this relation reads

$$\begin{aligned} & \langle \Omega, AZ^*(\theta_n)\dots Z^*(\theta_2)Z^*(\theta_1 - 2\pi i)\Omega \rangle \\ &= \langle \Omega, Z(\theta_1 + i\pi)AZ^*(\theta_n)\dots Z^*(\theta_2)\Omega \rangle \\ &= \langle Z^*(\theta_1 - i\pi)\Omega, AZ^*(\theta_n)\dots Z^*(\theta_2)Z^*(\theta)\Omega \rangle \end{aligned} \quad (37)$$

The analytic continuation by  $2\pi i$  refers to the correlation function and not to the operators. For the natural order of rapidities  $\theta_n > \dots > \theta_1$  this yields the following crossing relation (assuming absence of boundstates)

$$\begin{aligned} & \langle \Omega, Aa_{in}^*(\theta_n)\dots a_{in}^*(\theta_2)a_{in}^*(\theta_1 - \pi i)\Omega \rangle \\ &= \langle a_{out}^*(\theta_1)\Omega, Aa_{in}^*(\theta_n)\dots a_{in}^*(\theta_2)\Omega \rangle \end{aligned} \quad (38)$$

The out scattering notation on the bra-vectors becomes only relevant upon iteration of the KMS condition since the bra  $Z$ 's have the opposite natural order. By iteration one finally obtains the general mixed matrix elements

$$\langle a_{out}^*(\theta_k)\dots a_{out}^*(\theta_1)\Omega, Aa_{in}^*(\theta_n)\dots a_{in}^*(\theta_{k-1})\Omega \rangle \quad (39)$$

as analytic continuations from  $\langle \Omega, AZ^*(\theta_n)\dots Z^*(\theta_2)Z^*(\theta_1)\Omega \rangle$  which a posteriori justifies the use of the name formfactors in connection with the mixed A-F correlation functions.

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<sup>11</sup>Note that as a result of the Z-F commutation relation the change of order within the Wick-ordered products will produce rapidity dependent factors

The upshot of this is that such an  $A$  must be of the form

$$A = \sum \frac{1}{n!} \int_C \dots \int_C a_n(\theta_1, \dots, \theta_n) : Z(\theta_1) \dots Z(\theta_n) : \quad (40)$$

where the  $a_n$  have a simple relation to the various formfactors of  $A$  (including bound states) whose different in-out distributions of momenta correspond to the different contributions to the integral from the upper/lower rim of the strip bounded by  $C$  consisting of two contributions, which are related by crossing. The transcription of the  $a_n$  coefficient functions into physical formfactors (39) complicates the notation, since in the presence of bound states there is a larger number of Fock space particle creation operators than the initial PFG wedge generators  $F$ . It is comforting to know that the wedge generators, despite their lack of vacuum polarization clouds, nevertheless contain the full (bound state) particle content. The wedge algebra structure for factorizing models is like a relativistic QM, but as soon as one sharpens the localization beyond wedge localization, the field theoretic vacuum structure will destroy this simple picture and replace it with the appearance of the characteristic virtual particle structure which separates local quantum physics from quantum mechanics.

In order to see by what mechanism the quantum mechanical picture is lost in the next step of localization, let us consider the construction of the double cone algebras as a relative commutants of shifted wedge (shifts by  $a$  inside the standard wedge)

$$\mathcal{A}(C_a) := \mathcal{A}(W_a)' \cap \mathcal{A}(W) \quad (41)$$

$$C_a = W_a^{opp} \cap W$$

For  $A \in \mathcal{A}(C_a) \subset \mathcal{A}(W)$  and  $F_a(\hat{f}_i) \in \mathcal{A}(W_a) \subset \mathcal{A}(W)$  the KMS condition for the  $W$ -localization reads as before, except that whenever a  $F_a(\hat{f}_i)$  is crossed to the left side of  $A$ , we may commute it back to the right side since  $[\mathcal{A}(C_a), F_a(\hat{f}_i)] = 0$ . The new relation resulting from the compact localization of  $A$  is

$$\begin{aligned} & \langle AF_a(\hat{f}_1) : F_a(\hat{f}_n) \dots F_a(\hat{f}_2) : \rangle \\ & = \langle A : F_a(\hat{f}_n) \dots F_a(\hat{f}_2) F_a(\hat{f}_1^{2\pi i}) : \rangle \end{aligned} \quad (42)$$

Note that the  $F_a(\hat{f}_1)$  in the first line is outside the Wick-ordering. Since it does neither act on the bra nor the ket vacuum, it contains both frequency parts. The creation part can be

combined with the other  $F$ 's under one common Wick-ordering whereas the annihilation part via contraction with one of the Wick-ordered  $F$ 's will give an expectation value of one  $A$  with  $(n-2)$   $F$ 's. Using the density of the  $f$ 's and going to rapidity space we obtain ([15]) the so-called kinematical pole relation [16]

$$Res_{\theta_{12}=i\pi} \langle AZ^*(\theta_n)\dots Z^*(\theta_2)Z^*(\theta_1) \rangle = 2i\mathbf{C}_{12} \langle AZ^*(\theta_n)\dots Z^*(\theta_3) \rangle (1 - S_{1n}\dots S_{13}) \quad (43)$$

Here the product of two-particle S-matrices results from commuting the  $Z(\theta_1)$  to the right so that it stands to the left of  $Z^*(\theta_2)$ , whereas the charge conjugation matrix  $\mathbf{C}$  only appears if we relax our assumption of selfconjugacy.

It is remarkable that this kinematical pole relation does not contain the size of the localization region for  $A$ . It is a relation which characterizes all operator spaces  $\mathcal{A}(\mathcal{O})$ ,  $\mathcal{O} \in W$  down to the pointlike limits. The individual localization sizes only influence the Paley-Wiener exponents in asymptotic imaginary rapidity directions.

The existence problem for the QFT associated with an admissible S-matrix (unitary, crossing symmetric, correct physical residua at one-particle poles) of a factorizing theory is the nontriviality of the relative commutant algebra i.e.  $\mathcal{A}(C_a) \neq \mathbf{C} \cdot 1$ . Intuitively the operators in double cone algebras are expected to behave similar to pointlike fields applied to the vacuum; namely one expects the full interacting polarization cloud structure. For the case at hand this is in fact a consequence of the above kinematical pole formula since this formula leads to a recursion which for nontrivial two-particle S-matrices is inconsistent with a finite number of terms in (40). Only if the bracket containing the S-products vanishes, the operator  $A$  is a composite of a free field.

The modular method has therefore converted the existence problem, which hitherto was dominated by the well-known ultraviolet behavior of special (Lagrangian) field-coordinates, into the problem of nontriviality of algebraic intersections or in more applied terms to the nontriviality of formfactor spaces. For special fields which have an intrinsic meaning as conserved currents and their related order/disorder structure (example: the conserved current and its Sine-Gordon potential in the massive Thirring model) one expects to be able to identify them individually and to compute their formfactors as well as their correlation function. The considerations in the next section will propose arguments that this modular construction method is not limited to factorizing models.



The determination of a relative commutant or an intersection of wedge algebras is even in the context of factorizing models not an easy matter. We expect that the use of the following “holographic” structure significantly simplifies this problem. We first perform a *lightlike translation* of the wedge into itself by letting it slide along the upper light ray by the amount given by the lightlike vector  $a_+$ . We obtain an inclusion of algebras and an associated relative commutant

$$\begin{aligned} \mathcal{A}(W_{a_{\pm}}) &\subset \mathcal{A}(W) \\ \mathcal{A}(W_{a_{\pm}})' &\cap \mathcal{A}(W) \end{aligned} \tag{44}$$

The intuitive picture is that the relative commutant lives on the  $a_{\pm}$  interval of the upper/lower light ray, since this is the only region inside  $W$  which is spacelike to the interior of the respective shifted wedges. This relative commutant subalgebra is a light ray part of the above double cone algebra, and it has an easier mathematical structure. One only has to take a generic operator in the wedge algebra which formally can be written as a power series in the generators  $Z$  and find those operators [7][9] which commute with the shifted  $F$ 's

$$[A, U(\epsilon_+)F(f)U^*(\epsilon_+)] = 0 \tag{45}$$

Since the shifted  $F$ 's are linear expressions in the  $Z$ 's, the  $n^{\text{th}}$  order polynomial contribution to the commutator comes from only two adjacent terms in  $A$  namely from  $a_{n+1}$  and  $a_{n-1}$  which correspond to the annihilation/creation term in  $F$ . The result is precisely the same as the one from the KMS property: the above kinematical pole formula (43), so we do not learn anything new beyond what was already observed with the KMS technique. However as will be explained below, the net obtained from the algebra

$$\mathcal{A}(\mathbf{R}_{\pm}) := \vee_{b_{\pm}} AdU(b_{\pm}) \left\{ \vee_{a_{\pm}} \mathcal{A}(W_{a_{\pm}})' \cap \mathcal{A}(W) \right\} \tag{46}$$

(in words the net of von Neumann algebras created by translating the relative commutants of size  $a_{\pm}$  with  $b_{\pm}$  along the upper/lower light rays) is a chiral conformal net on the respective subspace  $H_{\pm} = \overline{\mathcal{A}_{\pm}\Omega}$  which is indexed by intervals on the light ray. If our initial algebra were d=1+1 conformal theories, the total space would factorize  $H = H_+ \bar{\otimes} H_- =$

$\overline{(\mathcal{A}_+ \otimes \mathcal{A}_-)} \Omega$ , and we would recover the well-known fact that two-dimensional local theories factorize into the two light ray theories. For massive theories we expect  $H = \overline{\mathcal{A}_+ \Omega} = \overline{\mathcal{A}_- \Omega}$ , i.e. the Hilbert space obtained from one light ray horizon already contains all state vectors. This would correspond to the difference in classical propagation of characteristic massless versus massive data in  $d=1+1$ . There it is known that although for the massless case one needs the characteristic data on the two light rays, the massive case requires only one light ray. In fact there exists a rigorous proof that this classical behavior carries over to free quantum fields: with the exception of  $m=0$  massless theories, in all other cases (including light-front data for higher dimensional  $m=0$  situations) the vacuum is cyclic with respect to one light front  $H = \overline{\mathcal{A}_\pm \Omega}$  [17]. The proof is representation-theoretical and holds for all cases except the  $d=1+1$  massless case. The result may be written as an identity of global algebras

$$\mathcal{A}(W) = \mathcal{A}(\mathbf{R}_+^>) \tag{47}$$

where the superscript refers to the fact that we are considering the right half of the upper light ray (with the same relation for the lower light ray). This identity of global algebras, which we consider as an AQFT version of holography, does not extend to the natural net structure which consists of double cones in  $W$  resp. intervals on  $\mathbf{R}_+^>$ . This means that certain geometric actions as the lower light cone translation  $U(a_-)$  on the  $W$ -net will be extremely nonlocal in their action on  $\mathcal{A}(\mathbf{R}_+^>)$ . The appearance of these “hidden symmetries” is the prize one has to pay for the simplifications of lower dimensional holographic images. More remarks on holography for higher dimensional QFT can be found in a later part.

It almost goes without saying that the various restrictions we have imposed for pedagogical reasons on the  $Z$ -algebra structure (as diagonal structure of  $S$  and absence of poles) can easily be lifted.

## 2.4 Case with Real Particle Creation

For models with real particle creation it is not immediately clear how to construct PFG's, in fact it is not obvious whether they exist. On the other hand it is quite easy to see that for any smaller localization region (whose causal completion will not be as big as a wedge) there can be no PFG-like operators unless the theory is trivial (i.e. free in the sense of no interaction). This means that PFG's are ideal indicators for interactions because only polarization caused by interactions will appear<sup>12</sup>. With other words any operator with compact or even spacelike cone localization which couples the one-particle state with the vacuum if applied once to the vacuum will generate a polarization cloud on top of the one-particle state unless the particles are noninteracting. The proof of this theorem uses similar analytic techniques as that of the Jost-Schroer theorem[18]. In fact the proof follows almost literally the arguments of Mund [19] where these analytic techniques were recently used to show that the d=2+1 braid-group particles even in their “freest” form cannot be quantum mechanical objects i.e. they cannot be described by localized operators which carry a defined (incoming) particle number like free Bosons/Fermions and hence a nonrelativistic limit which maintains the plektonic spin-statistic connection will also maintain the vacuum polarization structure and hence be outside of quantum mechanics. In terms of a representation-theoretical setting of multi-particle states one loses the tensor product structure of n-particle scattering states in terms of Wigner one-particle states. For a more remarks on the “No-Go theorem for interacting PFG's with smaller than wedge localization” I refer to a forthcoming paper [20].

An existence proof of wedge-localized PFG which as unbounded operators associated with  $\mathcal{A}(W)$  (i.e. the proper PFG's for the purpose of this essay) is simple if one allows also unbounded PFG operators associated with the von Neumann algebras can be given. One first studies the wedge-localization spaces i.e. the vectors spanning the domain of  $\Delta^{\frac{1}{2}}$  which are the vectors in the thermal subspace  $H_R(W) + iH_R(W)$  where  $H_R(W)$  is the closed real subspace of solutions of the localization equation

$$S\psi = \psi, \quad S = J\Delta^{\frac{1}{2}}, \quad J = S_{scat}J_0 \tag{48}$$

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<sup>12</sup>The vacuum polarization clouds which are responsible for the localization entropy in the later section are also present in the free case.

This space has a nontrivial intersection with the one-particle subspace

$$H_R(W) \cap H_{Wigner}^{(1)} \neq 0 \quad (49)$$

which is a consequence of the fact that the modular operator  $\Delta^{it}$  is shared with that of the wedge algebra generated by the free asymptotic (incoming) fields. The possibility of representing each vector as an unbounded operator associated with  $\mathcal{A}(W)$  is guaranteed by modular theory and this applies in particular to a dense set of one-particle vectors.

In order to get a clue for the construction of the spaces we look at  $d=1+1$  theories which do not have any transversal extension to wedges. Furthermore we assume that there is only one kind of particle (absence of particle poles in the S-matrix) so that in terms of incoming particles one is in the situation of a Fock space with one kind of particle.

From the previous discussion we take the idea that we should look for a relation between the ordering of rapidities and the action of the scattering operator. Therefore we define a subspace indexed by two-particle wave functions as follows (omitting again the scat subscript):

$$\begin{aligned} \Psi_{f_2, f_1} &\equiv \int \int d\theta_1 d\theta_2 f_2(\theta_2) f_1(\theta_1) \Psi(\theta_2, \theta_1) \\ \Psi(\theta_2, \theta_1) &\sim \chi_{21} a^*(\theta_2) a^*(\theta_1) \Omega + \chi_{21} S a^*(\theta_1) a^*(\theta_2) \Omega \end{aligned} \quad (50)$$

It is easy to check that this vector fulfils (48) if the  $f$ 's have the properties of the previous section. The  $J_0$  sends the  $S$  into a  $S^*$  and the  $f$ 's into their complex conjugate whereas the  $S$  together with the unitarity reproduces the linear combination. Finally the  $\Delta^{\frac{1}{2}}$  makes a  $i\pi$  shift in the  $\theta$ 's which may be absorbed into the  $f^*$ 's with the result that the original  $f$ 's are reproduced.

The generalization to states indexed by 3  $f$ 's contain 6 contributions which correspond to the 6 permutations

$$\begin{aligned} \Psi_{f_3 f_2 f_1} &\equiv \int \int \int d\theta_1 d\theta_2 d\theta_3 f_3(\theta_3) f_2(\theta_2) f_1(\theta_1) \Psi(\theta_3, \theta_2, \theta_1) \\ \Psi(\theta_3, \theta_2, \theta_1) &\sim \chi_{321} a^*(\theta_3) a^*(\theta_2) a^*(\theta_1) \Omega + \chi_{312} S_{21} a^*(\theta_3) a^*(\theta_2) a^*(\theta_1) \Omega \\ &+ \chi_{231} S_{32} a^*(\theta_3) a^*(\theta_2) a^*(\theta_1) \Omega + \chi_{123} S_{321} a^*(\theta_3) a^*(\theta_2) a^*(\theta_1) \Omega \\ &+ \chi_{132} S_{321} \cdot S_{23}^* a^*(\theta_3) a^*(\theta_2) a^*(\theta_1) \Omega + \chi_{213} S_{321} \cdot S_{12}^* a^*(\theta_3) a^*(\theta_2) a^*(\theta_1) \Omega \end{aligned}$$

This expression results from writing each permutation as the nonoverlapping product of “mirror permutations”. The smallest mirror permutations are transpositions of adjacent factors as in the third and fourth term. For those one replaces the action of the permutation by the action of the S-matrix restricted to the adjacent transposed tensor factors (which is used as a subscript of S). An example for an overlapping product of transpositions is the product of two transpositions which have one element in common e.g.  $123 \rightarrow 132 \rightarrow 312$ ; this sequence of mirror permutations can not be associated with subsequent S-matrix actions on tensor products. However the composition  $123 \rightarrow 213 \rightarrow 312$  has a meaningful S-matrix counterpart: namely  $S \cdot S_{12} a^*(\theta_1) a^*(\theta_2) a^*(\theta_3) \Omega$  where  $S_{12}$  leaves the third tensor factor unchanged. The resulting vector under the  $S_{12}$  action has no well-defined incoming particle number and can also be written in tensor product notation as  $(S a^*(\theta_1) a^*(\theta_2) \Omega) \otimes a^*(\theta_3) \Omega$ . The third particle has remained a spectator and only enters the process when the final  $S$  is applied (which corresponds to the mirror permutation of all 3 objects). This action of nested mirror transformations is well-defined. In general if one mirror permutation is completely inside a larger one the scattering corresponding nested product of  $S$ 's is a well-defined physical meaning. The last two terms correspond are such nested mirror contributions. The inner products of such vectors with themselves will lead to matrix elements of the form

$$\langle \theta'_3, \theta'_2, \theta'_1 | S \cdot S_{12}^* | \theta_3, \theta_2, \theta_1 \rangle \quad (51)$$

In a graphical scattering representation particle 1 and 2 would scatter first and produce arbitrarily many particles (subject to the conservation laws for the total energy-momentum) which together with the third incoming particle (which hitherto was only a spectator) enter an additional scattering process of which only the 3-particle outgoing component is separated out by the matrix element in (51). The dot means summation over all admissible intermediate states and could be represented by e.g. a heavy line in the graphical representation in order to distinguish it from the one-particle lines. Whereas in the calculation of cross sections the summation over intermediate states lead to diagonal inclusive processes, the nested structure of the localized vectors correspond to non-diagonal inclusive processes. The proof that the space of vectors of the above form  $\Psi_{f_n \dots f_1}$  fulfil (48) is analogous to the previous case: the first and the fourth term change their role as well as

the second and third terms change role with the two nested terms.

For a 4-  $f$  labeled state vector  $\Psi_{f_4 f_3 f_2 f_1}$  there is the new possibility of having two -particle S's acting on two nonoverlapping pairs of in-particles before the action of either the identity or the full S-matrix is applied. For further details we refer to [21]. The full real wedge localization space is defined as the real closure of all the labeled spaces (labeled by wedge localized one-particle wave functions)

$$H_R(W) = \text{real closure} \left\{ \Psi_f, \Psi_{f_2 f_1}, \Psi_{f_3 f_2 f_1}, \Psi_{f_4 f_3 f_2 f_1}, \dots | \forall f_i \in H^{(1)}(W) \right\} \quad (52)$$

The remaining problem is whether one can generate the wedge localization spaces by the iterated application of PFG operators. The check of the equivalence between KMS and on-shell crossing symmetry would then proceed as before by forming inner products between these vectors. The understanding of the precise mathematical status of these PFG's was still an open problem at the time of writing. It is clear that in the case of real particle creation one loses the uniformization aspect in the rapidity in which the S-matrix and formfactors were meromorphic functions. In defining PFG's via inner products between localized vectors, we tacitly assume that the PFG's  $F(x)$  admit the usual interpretation of operator-valued tempered distributions since without being permitted to use the standard computational tools PFG's would be less useful. Here we confront a very curious situation in the relation mathematics/physics. Although mathematically PFG's always exist in LQP, the temperateness assumption and together with the translation invariance of their domain limit the PFG formalism to the  $d=1+1$  case without real particle creation. In some intuitive sense the presence of creation requires a strong weight at the infinitely remote regions which leads to the loss of temperateness. For more informations on this unexpected behaviour we refer to a forthcoming publication [20].

There are several reasons why constructions based on modular localization could be important for particle physics. Besides the improvement in the understanding the structure of interacting QFT one expects that they could lead to a perturbation theory of local nets which bypasses the use of the nonintrinsic field coordinatizations and also the appearance of short-distance ultraviolet divergencies. The perturbative construction of vacuum expectations of PFG's which generate wedge algebras is reminiscent to a the revival of the

perturbative version of the old dream to construct an S-matrix just using crossing symmetry (and the analyticity which is required for its formulation) in addition to unitarity. The old S-matrix bootstrap program failed, even on a perturbative level no formulation without the use of fields was found. But thanks to modular wedge localization we can now formulate a similar but structurally richer program which already showed its power in the case of factorizing models. It is clear now that the weak point of the old S-matrix bootstrap was not primarily in its concepts but rather in its almost ideological and unfounded stance against QFT and anything “off-shell”. For a recent review of S-matrix theory I refer to [22]. Finally the claim that it is a unique theory and that it constituted a “TOE” (a theory of everything, in this case everything minus quantum gravity) contributed to its downfall. The present modular localization approach is different on all counts. Even the avoidance of field coordinatizations in favor of nets has entirely pragmatic reasons. In sharpening the localization beyond wedges via algebraic intersections of wedge algebras instead of using the local coupling of fields with its short distance problems and rather ad hoc resulting separation into renormalizable/nonrenormalizable, one has the chance to shed an entirely new light on problems which are central to QFT.

## 2.5 Modular Origin of Quantum Symmetries

Modular theory reproduces all the standard spacetime and internal symmetries, but it also produces new symmetries which remained hidden to the Lagrangian approach.

Before we look at the hidden symmetries, it is interesting to note that even the standard symmetries (i.e. those having a classical Noetherian counterpart) reappear in a very unusual and interesting way. To illustrate this point let us ask how can we characterize a chiral conformal theory i.e. its algebraic description in terms of a net on the circle. The well-known answer is: *by two algebras which are in the relative position of “half-sided modular inclusion” (hsm)* [23]. The prototype are two half-circle algebras rotated by  $\frac{\pi}{2}$  relative to each other (the quarter-circle situation) [24]. The  $\frac{1}{4}$ -circle of their intersection is compressed towards one of its endpoints under the action of each of the dilations associated with the half-circle which are the modular groups of the associated algebras. In fact the compression only happens for one particular ( $\pm$ )sign of the dilation parameter

( $\pm hsm$ ). This together with the analytic results by Borchers coming from the energy positivity within the modular setting [25], inspired Wiesbrock to introduce a general theory of modular inclusions and modular intersection. With respect to chiral conformal theories Wiesbrock’s result was that the study of abstract “standard hsm-inclusions” is equivalent to the classification of chiral conformal nets.

Encouraged by this success, this modular inclusion concept was enriched by additional requirement of a more geometric nature whereupon it became possible to characterize also higher dimensional nonconformal nets in terms of the modular relations (inclusions, intersections) of a finite family of von Neumann algebras. The surprising aspects of these investigation was that both the spacetime symmetries (the Poincaré or conformal symmetries) as well as the physics-encoding net structure follow from abstract relations (modular inclusions, intersections) between a finite number of copies of one and the same unique von Neumann algebra (the hyperfinite  $\text{III}_1$ -factor). In view of the fact that the modular groups of most causally complete regions act as unknown non-pointlike transformations, it was interesting to get more information about their interpretation in terms of physical symmetries [15]. Again it appeared reasonable to study of this question in the simplest context of chiral conformal theories. In contrast to higher dimensions chiral theories do have infinitely many geometrically acting one-parametric diffeomorphisms which are unitarily implemented by unitaries which change the vacuum. It turns out that by taking the large parameter limit (see next section for an example) the transformed correlation functions stabilize and define a new state over the algebra which is invariant under the respective subgroup. A closer examination reveals that these states have a modular interpretation with respect to multi-interval algebras which are cyclic and separating with respect to this state (but loose this cyclicity upon restriction to one algebra). This explains the modular aspects of all spacetime regions on the circle, including disconnected ones. By contrast, in higher dimensions the modular groups of massive theories (with the exception of wedge regions) are for no choice of states pointlike<sup>13</sup>; they preserve the causal closure of the localization region but act nonlocally inside (they would act on localizing test functions in a support-preserving but otherwise nonlocal fuzzy way). By analogy one should then

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<sup>13</sup>The best one can hope for is that they act asymptotically pointlike near the causal horizon.



view a suitably defined universal infinite parametric modular group generated by all the individual modular groups of spacetime regions as the hidden symmetry analogue of the chiral diffeomorphism group. The Poincaré group is the maximal geometric subgroup and it is generated from a finite subset of  $(\mathcal{A}(W), \Omega)$   $W \in \mathcal{W}$ . One also meets “partially hidden” symmetries in the spacetime analysis of modular inclusions/intersections i.e. automorphisms which act geometrically on subnets.

The present method of analysis based on modular groups is not the only one; a very interesting alternative approach based on the modular involutions  $J$  has been proposed by [26].

Closely related to the issue of hidden symmetries is the inverse of the Unruh observation namely the question of existence of a geometrical interpretation “behind the horizon” of the von Neumann commutant of a thermal heat bath system. Conditions under which this is possible have been studied in [27][28]

The reduction of LQP to the study of inclusions and intersections has changed the underlying philosophical basis of particle physics. The different outlook had been occasionally described by Haag in terms of a change from the Newtonian picture of reality as a manifold filled with a material content (relativity and quantum mechanics included) to the world of monades of Leibnitz, which although lacking individuality, create a rich reality by their interrelations.

The reader is invited to try to translate Leibnitz’s monades into hyperfinite type  $\text{III}_1$  von Neumann factors. The latter are as structureless entities and like points in geometry without individuality with one important difference: one factor can be included in the other and both can have nontrivial intersection. One should mention that this mode of thinking is also quite visible in the mathematics discovered by Alain Connes and in Vaughn Jones subfactor theory.

### 3 Local Quantum Physics versus Quantum Mechanics: a Change of Paradigm

The consequences of modular localization as explained in the previous section are not the only source of radical conceptual change in QFT. Another equally drastic conceptual change change of paradigm (however with a strict adherence to the physical principles of LQP) is the “degree of freedom” or phase space property of QFT and the positioning of QM versus QFT.

#### 3.1 The LQP Phase Space

Again this has a quite interesting history behind it, although some of its more dramatic consequences were only noticed in more recent times. It goes back to attempts by Haag and Swieca to make some of the consequences of the density of local states as expressed in the Reeh-Schlieder density theorem<sup>14</sup> more physically acceptable by introducing additional concepts [1]. Whereas in quantum mechanics the box localization separates the physical description via tensor-product factorization into an “inside and outside Hilbert space” (and a corresponding tensor-product of full operator algebras), the long range vacuum structure due to the omnipresence of vacuum fluctuations destroys such a picture and replaces it by an extreme opposite denseness (cyclicity) property of localized state vectors, the so-called Reeh-Schlieder property. This denseness property of localized states

$$\overline{\mathcal{A}(\mathcal{O})\Omega} = H$$

has been sometimes provocatively referred to by some of the protagonists of these investigations as the “particle creation behind the moon”-paradox: by applying appropriate observables localized in spacetime to the vacuum one may approximate any local change anywhere instantaneously. Even if one (as one learned from the analysis of ERP

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<sup>14</sup>The Reeh-Schlieder denseness theorem [18] is often presented together with the assertion of a one-to-one correspondence between localized operators and vectors in the dense subspace of localized states, the so called separability property (the “operator-state correspondence”) . Modular theory allows a profound understanding and relates denseness and separability as dual properties in the sense of von Neumann’s commutant notion.

Gedankenexperiment) is prepared to make a distinction between causal ties of events and long range correlations in states, this does not explain why there is such an impressive conceptual difference between the tensor factorization of quantum mechanical localization and the localization in LQP.

In an attempt to reconcile the strange-looking aspects with common sense in quantum theory, Haag and Swieca introduced the notion of phase space into LQP. They restricted the local vector states by the requirement that  $P_E \mathcal{A}^{(1)}(\mathcal{O})\Omega$  be a compact set of vectors in  $H$ . Here the superscript on  $\mathcal{A}(\mathcal{O})$  denotes the unit ball in the operator norm of the local algebra and  $P_E$  is the projector on vectors of energy smaller than  $E$  which feature in the spectral representation of the hamiltonian  $\mathbf{H} = \int E dP_E$ . They argue that the creation of “behind the moon states” in an earthly laboratory is not possible with a limited supply of energy i.e. the incredible small vacuum polarization correlations which exist as a matter of principle even over large distances can not be sufficiently amplified in the desired region with a limited energy supply. Using the same type of intuition but sharper estimates, Buchholz and Wichmann proposed a variant of this requirement which became known under the name nuclearity requirement and has the advantage that it is easier to use in calculations and closer to properties of thermal states. It reads

$$P_E \mathcal{A}(\mathcal{O})\Omega \text{ or } e^{-\beta \mathbf{H}} \mathcal{A}(\mathcal{O})\Omega \text{ is nuclear} \quad (53)$$

This amounts to the nuclearity of the map  $\Theta : \mathcal{A}(\mathcal{O}) \rightarrow e^{-\beta H} \mathcal{A}(\mathcal{O})\Omega$  i.e. the requirement that this map has a representation

$$\Theta A = \sum \phi_i(A) \psi_i \quad (54)$$

where the  $\phi_i$  are bounded linear forms on the algebra and the  $\psi_i$  are vectors in the Hilbert space such that

$$\sum \|\phi_i\| \|\psi_i\| < \infty \quad (55)$$

$$\|\Theta\|_1 := \inf \sum \phi_i(A) \psi_i \quad (56)$$

with the norms having the respective natural meaning and the last equation defines a new “nuclear norm” [1]. The requirement implies that the image set in the Hilbert space

is “nuclear” and a fortiori compact as demanded by Haag-Swieca. In physics terms such maps are only nuclear if the mass spectrum of LQP is not too accumulative in finite mass intervals; the excluded cases are those which in quantum statistical mechanics would cause the strange appearance of a maximal “Hagedorn” temperature or the complete loss of thermal concepts, so that one expects a close relation between nuclearity and the thermal aspects of QFT. Indeed the nuclearity assures that a QFT, which was given in terms of its vacuum representation, also exists in a thermal state. In fact the nuclearity index turns out to be the counterpart of the quantum mechanical Gibbs partition function [29][1] for open systems and behaves in an entirely analogous way to the Gibbs formula in a closed quantization box. The nuclearity property and the resulting phase space properties of LQP (localization in spacetime and limitation of energy) goes a long way to reconcile the local denseness of state property with common sense in that it associates with an approximating sequence of “particle behind the moon creation” an ever increasing expenditure in energy.

### 3.2 The Split Property

Before we link nuclearity with the pivotal “split property”, let us motivate the latter taking a helping hand from the history of QFT. The peculiarities of the above degrees-of-freedom-counting are very much related to one of the oldest “exotic” and at the same time characteristic aspects of QFT, namely vacuum polarization. As first noticed by Heisenberg (and later elaborated and used by Euler, Weisskopf and many others), the partial charge:

$$Q_V = \int_V j_0(x) d^3x = \infty \quad (57)$$

diverges as a result of uncontrolled vacuum particle/antiparticle fluctuations near the boundary. In order to quantify this divergence one acts with more carefully defined partial charges on the vacuum (s=dimension of space)

$$Q_R = \int j_0(x) f(x_0) g\left(\frac{\mathbf{x}}{R}\right) d^s x \quad (58)$$

The vectors  $Q_R\Omega$  only converge weakly for  $R \rightarrow \infty$  on a dense domain. Their norms diverge as [30]

$$(Q_R\Omega, Q_R\Omega) \leq \text{const} \cdot R^{s-1} \tag{59}$$

$$\sim \text{area}$$

The surface character of this vacuum polarization is reflected in the area behavior. Different from the vacuum polarization clouds in the previous sections this surface vacuum polarization exists even without interactions.

The algebraic counterpart of this age-old observation is the so called “split property”, namely the statement [1] that if one leaves between say the double cone (the inside of a “relativistic box”) observable algebra  $\mathcal{A}(\mathcal{O})$  and its causal disjoint (its relativistic outside)  $\mathcal{A}(\mathcal{O}')$  a “collar” (geometrical picture of the relative commutant)  $\mathcal{O}'_1 \cap \mathcal{O}$ , i.e.

$$\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{O}_1), \quad \mathcal{O} \ll \mathcal{O}_1, \quad \text{properly} \tag{60}$$

then it is possible to construct in a canonical way a type  $I$  tensor factor  $\mathcal{N}$  which extends in a “fuzzy” manner into the collar  $\mathcal{A}(\mathcal{O})' \cap \mathcal{A}(\mathcal{O}_1)$  i.e.  $\mathcal{A}(\mathcal{O}) \subset \mathcal{N} \subset \mathcal{A}(\mathcal{O}_1)$ . With respect to  $\mathcal{N}$  the Hilbert space factorizes i.e. as in QM there are states with no fluctuations (or no entanglement) for the “smoothened” operators in  $\mathcal{N}$ . Whereas the original vacuum will be entangled from the box point of view, there also exists a disentangled product vacuum on  $\mathcal{N}$ . The algebraic analogue of a smoothing of the boundary by a test function is the construction of a factorization of the vacuum with respect to a suitably constructed type  $I$  factor algebra which uses the above collar extension of  $\mathcal{A}(\mathcal{O})$ . It turns out that there is a canonical, i.e. mathematically distinguished factorization, which lends itself to define a natural “localizing map”  $\Phi$  and which has given valuable insight into an intrinsic LQP version of Noether’s theorem [1], i.e. one which does not rely on quantizing classical Noether currents. It is this “split inclusion” which allows to bring back the familiar structure of pure states, tensor product factorization, entanglement and all the other properties at the heart of standard quantum theory and the measurement process. However despite all the efforts to return to structures known from QM, the original vacuum retains its thermal (entanglement) properties with respect to all localized algebras, even with respect to the “fuzzy” localized  $\mathcal{N}$ .

Let us collect in the following some useful mathematical definitions and formulas for “standard split inclusions” [31]

*Def.:* An inclusion  $\Lambda = (\mathcal{A}, \mathcal{B}, \Omega)$  of factors is called standard split if the collar  $\mathcal{A}' \cap \mathcal{B}$  as well as  $\mathcal{A}, \mathcal{B}$  together with  $\Omega$  are standard in the previous sense, and if in addition it is possible to place a type I $_{\infty}$  factor  $\mathcal{N}$  between  $\mathcal{A}$  and  $\mathcal{B}$ .

In this situation there exists a canonical isomorphism of  $\mathcal{A} \vee \mathcal{B}'$  to the tensor product  $\mathcal{A} \bar{\otimes} \mathcal{B}'$  which is implemented by a unitary  $U(\Lambda) : H_{\Lambda} \rightarrow H_1 \bar{\otimes} H_2$  (the “localizing map”) with

$$U(\Lambda)(AB')U^*(\Lambda) = A \bar{\otimes} B' \quad (61)$$

$$A \in \mathcal{A}, B' \in \mathcal{B}'$$

$$U^*(\Lambda)(\Omega \bar{\otimes} \Omega) \equiv \eta_{\Lambda} \in H_{\Lambda}$$

$$\langle \eta_{\Lambda} | AB' | \eta_{\Lambda} \rangle = \omega(A)\omega(B') \neq \omega(AB')$$

This map permits to define a canonical intermediate type I factor  $\mathcal{N}_{\Lambda}$  (which may differ from the  $\mathcal{N}$  in the definition)

$$\mathcal{N}_{\Lambda} := U^*(\Lambda)B(H_1) \otimes \mathbf{1}U(\Lambda) \subset \mathcal{B} \subset B(H_{\Lambda}) \quad (62)$$

It is possible to give an explicit formula for this canonical intermediate algebra in terms of the modular conjugation  $J = U^*(\Lambda)J_{\mathcal{A}} \otimes J_{\mathcal{B}}U(\Lambda)$  of the collar algebra  $(\mathcal{A}' \cap \mathcal{B}, \Omega)$  [31]

$$\mathcal{N}_{\Lambda} = \mathcal{A} \vee J\mathcal{A}J = \mathcal{B} \wedge J\mathcal{B}J \quad (63)$$

The tensor product representation gives the following equivalent tensor product representation formulae for the various algebras

$$\mathcal{A} \sim \mathcal{A} \otimes \mathbf{1} \quad (64)$$

$$\mathcal{B}' \sim \mathbf{1} \otimes \mathcal{B}'$$

$$N_{\Lambda} \sim B(H_{\Lambda}) \otimes \mathbf{1}$$

As explained in [31], the uniqueness of  $U(\Lambda)$  and  $N_{\Lambda}$  is achieved with the help of the “natural cones”  $\mathcal{P}_{\Omega}(\mathcal{A} \vee \mathcal{B}')$  and  $\mathcal{P}_{\Omega \otimes \Omega}(\mathcal{A} \otimes \mathcal{B}')$ . These are cones in Hilbert space whose

position in  $H_\Lambda$  together with their facial subcone structures preempt the full algebra structure on a spatial level. The corresponding marvelous theorem of Connes [32] goes far beyond the previously mentioned state vector/field relation which follows from the Reeh-Schlieder density theorem.

Returning to our physical problem, we note that we have succeeded to find the right analogue of the QM box for open LQP subsystems. Contrary to the hyperfinite type  $III_1$  algebras for causally closed double cone regions with their sharp light cone boundaries (“quantum horizons”), the “fuzzy box” type I factor  $N_\Lambda$  constructed above (apart from its fuzzy geometrical aspects) permits all the properties we know from QM: pure states, inside/outside tensor-factorization, (dis)entanglement etc. Whereas  $\mathcal{A}$  as a type  $III$  algebra is “intrinsically entangled”<sup>15</sup>, the fuzzy box is a conventional quantum mechanical algebra whose only unusual aspect is that the restriction of the vacuum generates entanglement and a Hawking-Unruh temperature. Mathematically this means that the state  $\omega|_{\mathcal{A}\bar{\otimes}\mathcal{B}'}$  represented in the tensor product cone  $P_{\Omega\otimes\Omega}(\mathcal{A}\bar{\otimes}\mathcal{B}')$  is not the tensor-product of those of the separate restrictions of  $\omega$  to  $\mathcal{A}$  and  $\mathcal{B}'$  but rather a highly entangled KMS temperature state. This is obviously the result of vacuum fluctuations i.e. the fact that a physical vacuum in a LQP, different from the no-particle state of Schrödinger QM, correlates spatially separated regions. Note also that the restriction of the product state  $\omega \otimes \omega$  to  $\mathcal{B}$  or  $\mathcal{B}'$  is not faithful resp. cyclic on the corresponding vectors and therefore the application of those algebras to the representative vectors  $\eta_{\omega\otimes\omega}$  yields projectors (e.g.  $P_\Lambda = U^*(\Lambda)B(H_1)\bar{\otimes}1U(\Lambda)$ ).

### 3.3 Localization-Entropy

Since the fuzzy box algebra  $N_\Lambda$  is of quantum mechanical type  $I$ , we are allowed to use the usual trace formalism based on the density matrix description, i.e. the vacuum state is a highly entangled density matrix  $\rho_\Omega$  on  $\mathcal{N}_\Lambda$  which leads to a well-defined von Neumann

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<sup>15</sup>Such algebras have neither pure states nor can they appear as tensor-factors in the factorization of bigger algebras. Their properties from the quantum measurement point of view are nicely explained in [33].

entropy

$$(\Omega, A\Omega) = \text{tr} \rho_\Lambda A, \quad A \in \mathcal{A} \quad (65)$$

$$S(\rho_\Lambda) = -\text{tr} \rho_\Lambda \log \rho_\Lambda \quad (66)$$

It turns out to be quite difficult to actually compute  $\rho_\Lambda$  which describes the von Neumann entropy of the fuzzy box  $S(\rho_\Lambda)$ . Taking into account the above historical remarks on the early observations of vacuum-fluctuations near the boundary of a box softened by test functions (59), we expect that only degrees of freedom in the fuzzy surface around the horizon contribute to this localization-entropy.

In order to overcome the computational problems one could try to employ similar definitions of localization-entropy which have a similar intuitive content and avoid the direct construction of  $N_\Lambda$ . The definition which seems to be most suitable for computations is<sup>16</sup> that of the mathematician Kosaki who extended Araki's definition of relative entropy<sup>17</sup> by a variational formula. Araki's definition uses his relative modular theory with respect to a von Neumann algebra  $\mathcal{M}$

$$S(\omega_1|\omega_2)_M = -\langle \log \Delta_{\omega_1, \omega_2} \rangle \quad (67)$$

and Kosaki [35] converted this (in the most general setting) into a variational formula

$$S(\omega_1|\omega_2)_M = \sup \int_0^1 \left[ \frac{\omega(1)}{1+t} - \omega_1(y^*(t)y(t)) - \frac{1}{t} \omega_2(x^*(t)x(t)) \frac{dt}{t} \right] \quad (68)$$

$$x(t) = 1 - y(t), \quad x(t) \in \mathcal{M}$$

where in our case  $\omega_1 = \omega \times \omega$ ,  $\omega_2 = \omega$ ,  $\mathcal{M} = \mathcal{A} \vee \mathcal{B}'$ . An additional simplification should be gained by studying these localization entropies first in conformal QFT; the reason being that the modular aspects tend to be more geometrical. They offer the additional advantage

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<sup>16</sup>The suggestion to use this (or another closely related) definition I owe to Heide Narnhofer who was the first to study the issue of localization-entropy [34].

<sup>17</sup>This entropy concept was recently successfully used by R. Longo [36] in order to generalize some aspects of the Kac-Wakimoto formula from the special setting of rational conformal theories to the theory of superselection sectors.



of reducing the nuclearity (and hence the split-) property to the tracial condition

$$\text{tr} e^{-\beta n^\mu L_\mu} < \infty \quad (69)$$

$$L_\mu = P_\mu + IP_\mu I \quad (70)$$

where  $I$  denotes the geometric conformal inversion and  $L_\mu$  turns out to be an operator with discrete spectrum ( $L_\pm$  are the well-known rotation generators of the d=1+1 chiral decomposition) with  $L_0$  positive definite.

Let us first look at chiral conformal nets indexed by intervals on a light ray. The simplest split is obtained by choosing an interval of length  $2a$  symmetrically around the origin and a slightly bigger one of length  $2b$  enclosing the first such that the collar size of the split situation is  $d = b - a$  and  $\mathcal{A} = \mathcal{A}(I_a)$ ,  $\mathcal{B} = \mathcal{A}(I_b)$ . It is easy to see that the localization-entropy (with any of the possible definitions) for this situation can only depend on the harmonic ratio of these 4 points. The modular group  $\sigma_{\omega \times \omega}(t)$  is the tensor product of the  $\sigma'_\omega$ s and therefore known since the modular group for the vacuum restricted to  $\mathcal{A}$  or  $\mathcal{B}'$  is geometric.

The nongeometric culprit is the vacuum restricted to the 2-interval algebra  $\mathcal{A} \vee \mathcal{B}'$ . The “geometrically natural” state for  $\mathcal{A} \vee \mathcal{B}'$  is not the vacuum but rather that state which is left invariant under the diffeomorphism which leaves precisely the 4 -endpoints fixed. This is not a Moebius transformation, but it is closely related. It is well-known that by the successive application [15]

$$Möb_2 \equiv (z \rightarrow \sqrt{z}) \cdot Möb \cdot (z \rightarrow z^2) \quad (71)$$

where we have used the compact  $z = e^{i\varphi}$  coordinates instead of the light ray line, one obtains a well-defined diffeomorphism ( $2^{nd}$  quasisymmetric deformation of  $Möb$ ) on the circle (not in the complex plain!). These are precisely the diffeomorphisms mentioned before in connection with enlarging the realm of geometric modular groups beyond those which are visible through the vacuum properties. In fact one easily check that e.g.  $U(Dil_2(\tau))$  which fixes the 4 endpoints  $0, 1, -\infty, -1$  and acts geometrically on chiral fields  $A(x)$  (for simplicity take free fields) leads to a limit

$$\lim_{t \rightarrow \infty} \langle \Omega | A(x_1, \tau) \dots A(x_n, \tau) | \Omega \rangle \equiv \omega_2(A(x_1) \dots A(x_n)) \quad (72)$$

$$A(x, \tau) \equiv AdU(Dil_2(\tau))A(x)$$

which defines a state  $\omega_2$  such that  $(\mathcal{A} \vee \mathcal{B}', \Omega_2)$  turns out to have  $AdU(Dil_2(\tau))$  as its modular group. This state agrees precisely with the one constructed in [15]. The modular groups of higher dimensional double cone in conformal theories are known and their proximity to the two-dimensional case ( $a \rightarrow r, x_{\pm} \rightarrow r_{\pm} = t^0 \pm r$ ) suggest that all the modular constructions have a higher dimensional generalization.

The calculational idea is now to compute first

$$S(\omega \times \omega | \omega_2)_{\mathcal{A} \vee \mathcal{B}'} \tag{73}$$

and then to use the dominance of  $\omega$  by  $\omega_2$  to bound the original split entropy. Our conjecture is that for  $d > 2$  the split entropy behaves as

$$S(\omega \times \omega | \omega)_{\mathcal{A} \vee \mathcal{B}'} \simeq \left(\frac{a}{d}\right)^{d-2} \tag{74}$$

$$\frac{a}{d} \gg 1 \tag{75}$$

for small  $d$  or large  $a$  such that the ratio becomes large. This would entail the area law of the localization-entropy (associated with the causality horizon) in conformal field theories. Since massive theories according to common wisdom are short-distance dominated by conformal theories, the short distance behavior in the size of the fluctuation collar  $d \rightarrow 0$  has the same divergence, and barring the presence of a competing (pathological)  $\frac{m}{d}$  singularity, the short distance divergence remains coupled to the surface dependence.

The main reason for emphasizing this conjecture (analogies are not yet proofs) on the quantum version associated with the classical Bekenstein area law<sup>18</sup> in an essay like this is that there has been hardly any subject in the last decade which has received such an amazing amount of speculative attention going as far as postulating some new degrees of freedom. This is quite surprising in view of the fact that the localization-temperature has a rather mundane explanation in terms of the KMS properties of the restricted vacuum on conventional degrees of freedom. The situation resembles that of the speculative ideas of how to get rid of the ultraviolet divergencies before renormalization. Although I do not know the result in the present case, I would favor the LQP spirit of

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<sup>18</sup>The causal horizons in Minkowski QFT and the Unruh effect is analogous but not identical to black hole physics. Unruh states and Hartle-Hawking states are different but share the thermal aspects [37].

limiting all revolutionary ideas to physical and mathematical concepts and not to muddle with physical principles (as it was done without success with QFT in pre-renormalization times).

### 3.4 The LQP Paradigm: Quantum Measurement

Despite its conservative way of dealing with physical principles AQFT leads to radical change of paradigm. This is nowhere more visible than in its relation to quantum mechanics and the measurement process. As we have seen, the standard concepts about purity and entanglement of states loose their meaning i.e. LQP is quite remote from what is done in quantum information theory (note that the word “local” there has a very different meaning). Instead of tensor factorization associated with the inside/outside localization in quantum mechanics, the sharp relativistic boxes (double cones) do not have pure states and an attempt to use them together with their causally disjoint outside for the introduction of the entanglement concept along this inside/outside division will fail: *all states are intrinsically entangled vector states thus rendering the distinction meaningless* [33]. Even if we use the factorization along fuzzy boxes and their outside, we only recover these concepts at the expense of a thermally parametrized highly mixed vacuum including all its local excitations which constitute the natural set of states in particle physics. As a result, most of the famous Gedankenexperiments as e.g. the “Schrödinger cat” receive important qualitative modifications. But all effects are of the ridiculously small order of the Unruh temperature (at feasible acceleration values). Thus quite different from the recently measured decoherence times for “small Schrödinger cats” (a very small number of photons in a cavity probed with atoms), the additional effects of modular localization i.e. the difference between sharp and fuzzy boxes and the entangled nature of the vacuum state with respect to any of them will never be directly accessible. Rather one is limited to study the indirect manifestations of e.g. the Unruh (wedge) thermality within particle physics. In the previous section we learned that the crossing symmetry is equivalent to the KMS thermal properties of the Hawking-Unruh effect. As such it is a very large effect. Crossing symmetry is a property which was used in dispersion theory and the Kramer-Kronig dispersion relations for particles were experimentally tested a long time

ago.

## 4 A Peek at 21<sup>st</sup> Century Local Quantum Physics

A glance at the future consist mostly of personal expectations and, if one looks at the many attempts at predictions about the future and the many resulting unfulfilled promises during the last two decades, one gets a little bit discouraged. But just in order to prove that the modular framework is also capable to lead to interesting conjectures and expectations let me present some of them.

### 4.1 Extension of Renormalized Perturbation Theory?

There is certainly general agreement that gauge theories belong to the most important contributions to 20<sup>th</sup> century particle physics. But on the other hand they hardly constitute a closed mature chapter in particle physics. In fact it is very indicative that all the important observations about them have been made within the first 5 years after their (re-)discovery and adaptation to the purposes of particle physics at the end of the 60<sup>ies</sup> and that the rate of progress levelled off steeply afterwards. So it is natural to ask if one could expect the modular localization method to contribute to their future development. I believe that this question will have a positive answer.

In order to explain my reasons I find it convenient to place the problem behind gauge theory into the slightly physically more general context of search for renormalizable theories in which massive higher spin particles participate. It is well known that within the causal perturbative approach (as with any alternative approach based on Lagrangian quantization) massive theories with spin  $s \geq 1$  necessarily produce interaction densities  $W$  (i.e. scalar Wick-polynomials in free fields) of at least third degree whose operator short distance dimension  $dim W \geq 5$  surpasses the value 4 allowed by renormalizable power counting. The reason is of course that the operator dimension of physical quantum vector fields  $dim A_\mu = 2$  is too high as compared with its classical counterpart  $dim A_\mu^{class} = 1$ . In fact this is not a consequence of a bad selection of a covariant field associated with the (m,s) Wigner particle description; any other choice would have given at least a value

2. Can one think of an ad hoc covariantization which reduces this value to 1 and at the same time does not destroy the hope that the resulting violation of the quantum aspects of the covariant description the spin-1 Wigner particle has permanently wrecked the physical aspects? To be more specific, is it conceivable that the “ghost degrees of freedom” which achieve such a reduction of the covariantized propagation degree act like a mysterious kind of catalyzer which are not visible in the original problem and leave no traces in the final physical answer but nevertheless play a beneficial intermediate role?

Everybody knows that the answer is positive and that this is formally done with BRS ghosts in Fock space. The reason why this mathematical trick preempts the final return to physics is the fact that it amounts to a cohomological representation. In fact, and this is our new addition [38], in the massive case this can already be implemented on the level of the  $(m,s)$  one-particle Wigner space

$$H_{Wig} = \frac{\ker \mathfrak{s}}{im \mathfrak{s}} \quad (76)$$

where  $\mathfrak{s}$  is a cohomological operator  $\mathfrak{s}^2 = 0$  which acts on the ghost-extended Wigner space  $H_{Wig}$  (not to be confused with the pre-Tomita operator<sup>19</sup>). The Fock space operator version of this cohomological Wigner space representation for the operator algebras

$$\mathcal{A}_{phys} = \frac{\ker Q}{im Q} \quad (77)$$

(where the formal operator  $Q$  acts on the extended algebra by commutation) is nothing but a special version of the BRST formalism in which the position of the physical space with respect to the ghost-extended space does not change with the perturbative order. This simple formalism would not have been available with vanishing mass because in that case the free fields in zero order would not have been interpretable as the in-fields in the sense of time-dependent scattering theory (appearance of infrared-divergencies). Massive field theories, even if analytically more complicated, are conceptually simpler. The findings of this way of looking at spin=1 interactions can be described as follows [38]

- Physical consistency within the renormalizability requirement demands the existence of additional physical degrees of freedom which in their simplest (and probably

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<sup>19</sup>This should be viewed as an operator version of the Faddeev-Popov trick.

only) realizations are scalar particles as in the Higgs mechanism of gauge theories but without vacuum condensates which was characteristic of that mechanism. The intrinsic role of this field is the implementation of the Schwinger-Swieca charge screening.

- Some of the “elementary” physical fields (i.e. those which interpolate the perturbative particles) appear composite in the extended formalism. The rules for a direct characterization of physical fields remain presently complicated and their intrinsic nature is essentially not understood; they certainly do not follow simple invariance rules as the fixpoint algebras under a group symmetry, rather their representation in the extended formalism lead to ever changing linear combinations of composites.
- Apart from the renormalization induced self-interaction of the scalar Higgs analogues, the renormalization requirement is more restrictive<sup>20</sup> than expected and governed by just one coupling strength. In this sense the renormalization within the causal setting leads to gauge structure of the coupling: the gauge groups are not put in but result from the assumed particle multiplicities in conjunction with the cohomological trick which is part of renormalization and has nothing to do with group symmetry. In the standard presentation this appears the other way around and goes with the dictum: local gauge symmetry implies renormalizability.

Here we have tacitly assumed that there are several mutually interacting spin one objects in order to avoid the abelian case. In the case of abelian vectormesons there are two renormalizable models: the above one in which all physical matter fields (including the new scalar degree of freedom) have their expected short-distance dimension, and “massive QED” for which e.g. the physical spinor matter field has an ever increasing short distance behavior (i.e. it is an unrenormalizable field within a otherwise renormalizable theory) or a renormalizable representation in its unphysical (“gauge dependent”) extended realization.

This last remark suggests the following question: *is it conceivable that there are theories which are partially renormalizable i.e. in which suitably restricted observable sub-*

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<sup>20</sup>Classically the appearance of more Lorentz indices for increasing spin would enlarge the possibilities of invariant couplings

*algebras have a normal short-distance behavior?* Could it be that Lagrangian field coordinates (in particular if they belong to higher spin) are not minimal in the sense of short distances i.e. the same theory allows better behaved field coordinatizations which are not Lagrangian? What at all is the physical meaning of “short distance” in a field-coordinatization-free formulation in the LQP spirit; short distance behavior of what?

Especially this last question brings us back to the main theme of this essay: the modular localization approach. Since the wedge-localized algebra is a field-coordinate independent object and the local net of spaces and algebras is obtained by intersection of wedge-localized situations, such a procedure would directly confront these questions. There is no worry about ghosts reappearing in such a setting since the short distance behavior of pointlike objects has gone which was their reason d’etre.

*In fact the modular formalism can be interpreted as an extension of the Wigner theory to the realm of interactions.* Its starting point, the wedge algebra is on-shell<sup>21</sup> The improvement of localization i.e. the transition to off-shell double cone subspaces and algebras is done by intersections and in no way calls for ghosts or touches in any other way the standard short-distance issue. So the interesting remaining problem is: can these ideas be supplemented with some new perturbative technology which extends the realm of the standard renormalization theory. This implies in particular the reproduction of the correct old results.

Looking back to the particle physics of the 60<sup>ies</sup>, one even gets the impression that the ill-fated S-matrix bootstrap approach was an attempt in this direction [22]. For an outside observer as the present author it is very hard to find out why the program of perturbative constructions of crossing symmetric S-matrices by pure on-shell methods failed. Technically speaking it had to do with the generalization of the Mandelstam representation to more than 4 points. But physically these technical points remained obscure because there was a lot of rampant analyticity (guessed on the basis of extrapolation from low order perturbation theory) which conceptually was unaccounted for. Not even the conceptual basis of crossing symmetry was properly understood. On the other hand the analyticity which

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<sup>21</sup>The fact that on-shell quantities are free of ghosts has been used in the tree approximation unitarity S-matrix arguments in favour of a gauge theoretic description of vector mesons.

enters the present modular approach has a well-understood conceptual position in terms of the principles of LQP. The new modular framework is therefore expected to do much better. Indeed the transition from crossing symmetry to the thermal KMS properties for the correlations of PFG's as in section 2 is expected to give a physically richer and formally more systematic starting point than the old bootstrap approach since it uses field theoretic concepts and formalism already for the introduction of the on-shell wedge-localized algebras. Needless to say that the modular approach does not support the “cleansing ideology” of the S-matrix bootstrap approach against off-shell concepts from QFT. To the contrary, the modular structure, more than any other method of particle physics, places causality and spacetime localization back onto the centre of the stage. In doing this it sheds new and quite unexpected light on the old on-shell/off-shell dichotomy of particle physics which remained inaccessible to differential geometric methods. It promises to elevate the intrinsic spirit of Wigner's 1939 quantum theory of free relativistic particles to the level of interacting local quantum physics. On the perturbative level one expects the feasibility of a deformation theory of field-coordinatization-free wedge-localized nets with the off-shell steps following suit via intersections.

It is well-known that infrared problems indicate a change of the Wigner particle picture [39]. In the present proposal this shows up in the appearance of violent (off-shell) infrared divergencies due to the breakdown of the Fock-space structure and the loss of physically defined (by scattering theory) reference (free) fields. In terms of the above BRST-like cohomological extension in the setting of point-like fields this means that e.g. the physical  $\psi$ -fields (describing the spinor matter) which are equal to the original  $\psi$ -fields, do not have zero mass limits. This is a manifestation of charge liberation which is the inverse mechanism to the afore-mentioned Schwinger-Swieca charge screening. From general LQP structure results we expect that charge-carrying fields in QED-like theories do not admit compact localization since the accompanying photon clouds are necessarily semiinfinite noncompact objects<sup>22</sup>. Therefore one must modify the physical  $\psi$ -fields before taking the massless limit in such a way that the worsening of localization is preempted. It

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<sup>22</sup>The photon clouds require semiinfinite spacelike cone regions for their localization. This is preempted on a formal level by the spacelike Mandelstam strings of gauge theory.



is interesting to note that this must go together with the expected de-coupling of the Higgs-like degrees of freedom. Both phenomena should show up after projection to the physical perturbation theory. The infrared issue and the resulting modification of particle structure can also be dealt with in the standard gauge approach by separating the algebraic aspects from those due to states [40]. Finally one should also mention that there are other less conservative ideas which promise to adjust the (semi)classical gauge idea directly to the noncommutative setting. Their motivation is different from the above attempts of extending renormalized perturbation theory beyond its present borders (and keeping the existing renormalized results unchanged).

## 4.2 Conformal Scanning?

For the analysis of nonperturbative aspects modular theory offers a different method which was already alluded to before, namely the reduction of a complicated higher dimensional massive theory to a finite number of copies of a simpler chiral conformal theory which reside in a common Hilbert space and have a carefully tuned relative position to each other. This use of chiral “holography” or “scanning” for general QFT is possible because the LQP version of chiral conformal theory is more general than the standard framework which ties chiral theories to the representation theory of a two-dimensional energy-momentum tensor with zero physical mass. As we have seen in section 2 the wedge algebra of a higher dimensional theory with its light ray translations and the Lorentz-boost is naturally encoded into the half light ray algebra. By its construction via modular inclusion the light ray theory has automatically a conformal rotation i.e. is fully Möbius-covariant, i.e. the more general version leads to the same vacuum structure as the standard.. The spectrum of the light ray translation is gapless as it should be in a chiral conformal theory, since light cone momenta are always gapless. The abstract chiral light ray theory does however not possess an energy-momentum tensor with a  $L_n$  Virasoro algebra structure which is the hall-mark of an autonomous two-dimensional conformal field theory. The physical mass-gap spectrum can be recovered in the chiral light-ray holography of the wedge by a careful re-interpretation of the geometric transformations in the wedge. In this way the light ray translation on the lower wedge horizon becomes a “hidden symmetry”, namely a

totally nonlocal (“fuzzy”) transformation; whereas the transversal translations generated by  $\vec{P}_\perp$  are presenting themselves in the light ray world as a kind of noncompact inner symmetry. The local generator  $P_+$  of the light ray translation together with its hidden counterpart  $P_-$  and the fake internal symmetry generator  $\vec{P}_\perp$  define the massive physical spectrum of

$$P^\mu P_\mu \equiv P_+ P_- - \vec{P}_\perp^2 \tag{78}$$

In view of this additional partially hidden structure of chiral theories originating from holographic projections of higher dimensional massive ones as compared to the standard ones (based on the existence of a Virasoro type energy-momentum tensor), it is sometimes helpful to picture the chiral projections as associated with the d-1 dimensional (upper) horizon of the wedge. This does no harm as long as one remains aware of the fact that this picture does not include the net structure associated with the  $P_-$  and  $\vec{P}_\perp$  translations. The remaining L-transformations which are not symmetries of the standard wedge  $W$ , are transforming  $\mathcal{A}(W)$  into a differently positioned  $\mathcal{A}(W')$  i.e. are isomorphisms within the total algebra  $B(H)$ . For  $d=2+1$  one only needs one particular operator from the one-parametric family of “tilting” boosts which fix the upper light ray. Such transformations are well-known from the Wigner “little group” of light like vectors. In the present case of  $d=2+1$  the little group is generated by just one “translation” (within the L-group). Any special transformation from this 1-parametric family different from the identity will via a  $W'$  and its holographic light cone projection  $\mathcal{A}'(R_+)$  lead to an isomorphism within  $B(H)$  of  $\mathcal{A}(R_+)$  to  $\mathcal{A}'(R_+)$ . It is plausible that such isomorphism between two differently positioned light ray algebras can encode the missing covariances and net structure. This can be demonstrated by applying the theory of modular intersections to the two light ray (alias wedge) algebras. In dimension d one needs precisely d-2 specially positioned chiral theories in order to recover the full Poincaré symmetry and the d-dimensional net structure. As far as counting parameters is concerned, this corresponds precisely to a light front holography onto the horizon of the wedge, but a better picture is that of a scanning by d-1 (isomorphic) copies of a chiral theory. In order to apply these ideas for practical constructive purposes in higher dimensional field theories, one should look for an extension of the notion of modular intersection to more than two algebras. Using a

similar historical analogy as above (modular wedge localization method  $\simeq$  extension of Wigner representation theory), it is tempting to interpret the modular holography as a clarification and extension of light cone (or  $p \rightarrow \infty$  frame) physics.

In order to accomplish such a program, the understanding of chiral conformal field theories themselves should be improved. Its present sectarian role with respect to higher dimensional QFT and the general principles is clearly caused by the heavy reliance on special algebras (energy-momentum tensor, affine, current) which have no higher dimensional counterpart. On the other hand the theory of superselection rules and their consequences for particle/field statistics is common to all theories. In the particular case at hand [41] the admissible statistics belongs to the braid group and can be in fact classified by Markov traces on the braid group which via GNS construction lead to combinatorical type II von Neumann algebras (sometimes inappropriately called “topological field theories”). They contain the statistics information in such a way that the permutation group statistics emerges as a special case. The missing field theoretic part is the use of the quantized statistics (the statistical dimensions follow the famous trigonometric Jones formula) for the construction of the spacetime carriers of these superselected charges. The ultimate test should consist in the derivation of FQS-quantization of the central charge from the physically more universal statistics quantization. It is clear that the modular theory must play an important role [42].

### 4.3 A higher dimensional Theory of Anomalous Dimension?

In order to avoid the impression that the conservative attitude of LQP with respect to physical principles prevents addressing presently fashionable subjects, I would like to explain some speculative ideas on so-called SYM models. This is clearly part of the general question of nontrivial aspects of higher dimensional conformal QFT. As in the well-studied  $d=1+1$  conformal theories, interpolating local fields which create Wigner particles are necessarily canonical free fields. Hence nontrivial fields cannot be associated with Wigner particles and must have noncanonical anomalous dimensions (which at best can be associated with infraparticles). So the first step in unraveling the structure of  $d>1+1$  conformal theories should be the understanding of its spectrum of anomalous

dimensions. For  $d=1+1$  conformal models such a theory of anomalous dimension (critical indices of associated critical statistical mechanics) exists; these numbers are determined (modulo  $2\pi$ ) by the statistical phases of the braid group statistics of the fields (the R-matrices of the exchange algebra). The classification of physically admissible braid group statistics is a well-defined mathematical problem which can be separated from the space-time aspects of QFT and treated by the technique of Markov-traces. The construction of nets fulfilling exchange algebra relations can be converted into a well-defined problem of modular theory. Can one achieve a similar situation with respect to anomalous dimensions ( $\simeq$  critical indices) in higher dimensional conformal theories? The answer is positive for theories which admit observable algebras which fulfil timelike commutativity i.e. which propagate only in lightlike directions (Huygens principle) as zero mass free fields. There are arguments that by choosing the observable algebra sufficiently small, this can always be achieved. One would like to interpret anomalous dimension fields as carriers of superselection charges associated with the timelike local observable algebra and one glance at the two-point function reveals that one should expect timelike braidgroup commutation relations associated with the timelike ordering structure<sup>23</sup>. This is indeed what a systematic DHR analysis in terms of localized endomorphisms confirms. We obtain the whole superselection formalism with braidgroup (R-matrix) commutation relations except that the statistic interpretation is missing: from the viewpoint of spacelike commutation relations we are dealing with Bosons/Fermions. The two- and three-point functions of the observable fields suffer the usual conformal restrictions i.e. they are determined by their dimensions modulo a normalization constant which carries the memory about the interaction. If supersymmetry “protects” these parameters against changes due to interactions, then such a model is in dangerous proximity of a free field theory.

My conviction that the present modular framework and more generally the LQP approach will have a rich future stems primarily from the fact that the intrinsic logic of LQP is strong and convincing that it appears a safer guide than that obtained from the quantization approach. Whereas the canonical formalism, the interaction picture, the

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<sup>23</sup>In fact the time-like net in the forward light cone admits a projection onto the timelike line which is a chiral conformal theory without the Virasoro structure.

formalism of time-ordering etc. can (and has been) be used outside of relativistic QFT, the modular approach is *totally specific for real time LQP*. In fact it is the only truly noncommutative entrance into QFT which came really from physics (rather than physical illustrations of mathematical concepts as done e.g. with noncommutative geometry). Admittedly, it is an area, which because of its strong conceptual roots and demanding mathematical apparatus is not easy to enter; neither does the subject render itself to fast publications. But in compensation, even if progress at times is very slow, it carries a conceptual profoundness and mathematical solidity which, if coupled with the belief in the guiding power of physical principles (especially through times of crisis), is hard to match.

A superficial observer would conclude from the present account that particle physics is strong and healthy with a promising 21<sup>st</sup> century future. Such an observer has missed to notice the radical change of values which also profoundly altered the exact sciences. An outburst of stunning creativity as it happened at the beginning of last century (Plank, Einstein, Bohr, Heisenberg) is only possible under very special sociological conditions in which the search for scientific truth and universality has a high social ranking and were new emerging ideas in sciences were always confronted with historical and traditional aspects. These are not necessarily the good times as the explosion of sciences and the arts in the imperial as well as in the humiliated post-war Germany of the Weimar republic shows.

Present sociology and philosophy of life is totally different. The high social ranking of shareholder-values and globalization over productive values in modern capitalism has found its counterpart also in particle physics. It consists of using ones knowledge, including mathematical sophistication primarily for improving ones status within a scientific community and not for the benefit of furthering science. This works because it is tacitely accepted by a majority. In earlier times there still existed a perceived difference of “physics” and what at one or the other time “physicist were doing”, whereas nowadays this distinction disappeared. How can one otherwise explain that theories which already exist for 30 years and besides making their inventors famous never contributed anything to particle physics enjoy such popularity? And how can one explain that rewards are

starting to be given to inventors, thus setting examples for the young generation? The acquired profound knowledge about quantum field theory is now rapidly getting lost and it is a truly amazing experience to meet young people who do not have the slightest idea about scattering theory, dispersion theory and the Wigner particle theory although they know more than necessary about Calabi-Yao spaces, Riemann surfaces and all those theories which hide behind big Latin Letters. At most places it is already impossible to have a carrier in physics outside these trends; the academic freedom is rapidly losing its economic basis. Fast returns as with shareholder values are incompatible with the flourishing of particle physics. If this trend continues another 10 years, the profound knowledge about real problems of 20<sup>th</sup> century particle physics and QFT will have been lost with the young generation. Even if one believes that truths in exact sciences will always eventually find its way, one does not want to be proven correct on such a rather pessimistic outlook.

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