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Test of new Nonperturbative Concepts in d=1+1 Factorizing Models of QFT

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Abstract

This paper deals with operators (PFG), which are localized in Minkowski space wedges (Rindler or Bisognano-Wichmann) regions, and yet are free of those polarization clouds which are typical of vectors obtained by applying interacting operators with smaller localization regions or local fields to the vacuum. They relate the (off-shell) KMS properties for the thermal Unruh wedge localization to the on-shell concept of crossing symmtry in a surprizingly deep way, and create new understanding in an area of QFT which hitherto remained somewhat obscure from the point of view of particle physics. The first test of these concepts, carried out on the Karowski-Weisz-Smirnov axiomatic approach to factorizable d=1+1 models, yields not only a deep understanding, but also opens new ideas in the bootstrap-formfactor approach. The findings suggest the existence of a new nonperturbative framework of QFT in which interactions are not defined and implemented by (interaction parts) of Lagrangians or closely related functional integrals, but rather by certain properties of the wedge algebras.

The sharpening of localization starting from wedges is done by the process of "quantum localization" via the intersection of algebras instead of the classical localization in terms of the support of testing functions. This new paradigm in QFT also leads to change of emphasis away from short distance properties of individual fields deciding over the existence and renormalizability of models in favor of nontriviality of intersections of wedge algebras.

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I. OBSERVATIONS ON NONPERTURBATIVE PFG'S IN D=1+1

Let us start with the following surprising observation on a nonlocal modification of a d=1+1 massive free scalar fields $A(x)$. For the latter we use the notation:

$$
\begin{aligned}
A(x) &= \frac{1}{\sqrt{2\pi}} \int (e^{-ipx} a(p) + h.a.) \frac{dp}{2\omega} \\
&= \frac{1}{\sqrt{2\pi}} \int (e^{-im\rho sh(\chi-\theta)} a(\theta) + h.a.) d\theta, \;\; x^2 < 0 \\
&= \frac{1}{\sqrt{2\pi}} \int_{\mathsf{C}} e^{-im\rho sh(\chi-\theta)} a(\theta) d\theta, \;\; \mathsf{C} = \mathbb{R} \cup \{-i\pi + \mathbb{R}\}
\end{aligned}
\tag{1}
$$

where in the second line we have introduced the x- and momentum- space rapidities and specialized to the case of spacelike x, and in the third line we used the analytic properties of the exponential factors in order to arrive at a compact and (as it will turn out) useful contour representation. Note that the analytic continuation refers to the c-number function, whereas the formula $a(\theta - i\pi) \equiv a^*(\theta)$ is a definition and has nothing to do with analytic continuations of operators*. With this notational matter out of the way, we now write down our Ansatz

$$
\begin{aligned}
F(x) &= \frac{1}{\sqrt{2\pi}} \int_{\mathsf{C}} e^{-im\rho sh(\chi-\theta)} Z(\theta) d\theta, \quad Z(\theta)\Omega = 0 \\
& Z(\theta - i\pi) = Z^*(\theta)
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
Z(\theta_1)Z(\theta_2) &= S_{Z,Z}(\theta_1 - \theta_2) Z(\theta_2) Z(\theta_1) \\
Z(\theta_1)Z^*(\theta_2) &= \delta(\theta_1 - \theta_2) + S_{Z,Z^*}(\theta_1 - \theta_2) Z^*(\theta_2) Z(\theta_1)
\end{aligned}
\tag{3}
$$

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*Operators in QFT never possess analytic properties in x- or p-space. The notation and terminology in conformal field theory is a bit confusing, because although it is used for operators it really should refer to vector states and expectation values in certain representations of the abstract operators. The use of modular methods require more conceptual clarity than standard methods.

For the moment the $S's$ are simply Lorentz-covariant (only rapidity differences appear) functions which for algebraic consistency fulfil unitarity $\overline{S(\theta)} = S(-\theta)$. We assume (for simplicity) that the state space contains only one type of particle. Before continuing with the special situation we introduce two useful general definitions.

Definition 1 *A field operator $F(x)$ is called "one-particle* **p***olarization* **f***ree" (PF) if $F(x)\Omega$ and $F^*(x)\Omega$ have only one-particle components (for any one of the irreducible particle spaces in the theory)*

For Pf's the vector $F^{\#}(x)\Omega$ is on mass-shell i.e. has a Fourier transform in terms of $Z^*(\theta)\Omega$, with $Z(\theta)\Omega = 0$. Note that the definition does not yet require that $F(x)$ itself to be on-shell. We are however interested in $F(x)'s$ which upon smearing with test functions restricted to a subspace $\mathcal{L}$ generate algebras

$$\mathcal{A} = alg\left\{F(\hat{f}) = \int F(x)\hat{f}(x)d^dx \mid \hat{f} \in \mathcal{L}\right\}$$

which on the one hand are big enough in order to create a dense set of states if applied to Ω, but on the other hand allow for an equally big commutant algebra $\mathcal{A}'$, in short the PF's should generate an $\mathcal{A}$ which is cyclic and separating with respect to the vacuum. As a result of $F(\hat{f})A'\Omega = A'F(\hat{f})\Omega$ for $A' \in \mathcal{A}'$, the on-shell aspect of the vectors is transferred to the operators, i.e. formula (2) for $F(x)$ is valid. The $\mathcal{L}'s$ we have in mind are subspaces of localized test functions $\mathcal{L} = \left\{\hat{f} \mid supp\hat{f} \subset \mathcal{O}\right\}$. But as a result of an old theorem by Jost and the present author [1], this immediately limits the admissable localization properties. If the field is pointlike local, this theorem forces the F to be a free field, and by a slight adaptation of the proof this would continue to hold for $F's$ which have a compact Minkowski space localization. Even for noncompact localizations which are properly contained in a wedge (i.e. a Lorentz transformed of the standard wedge $x_1 > |x_0|$) this clash with interactions continues† and the only consistent value of the S-

†I am indebted to D. Buchholz for a discussion of this point.

functions in the above Ansatz are $S = \pm 1$ i.e. free Bosons/Fermions. The smallest region for which these arguments break down are wedges themselves. The following theorem shows that indeed wedge localization in d=1+1 is consistent with nontrivial interactions and the result emerging from the above Ansatz in formula (3) is quite surprising. One finds that the coefficients are related to each other and fulfil the complete Zamolodchikov-Faddeev algebra if and only if the $F(\hat{f})'s$ with $supp\hat{f} \in W$ generate wedge localized algebra, thus unraveling the physical significance of this formally introduced algebraic structure in terms of wedge localization. This is not the first time in physics that wedges play a prominent role. In Unruh's Minkowski space illustration of the origin of thermal aspects of quantum matter encapsulated by a horizon, in the first application of Tomita's modular theory by Bisognano and Wichmann and in the inverse use of the B-W theorem for the direct construction of local algebras [2], in all cases one encounters the fundamental role of wedge localization and wedge algebras. In the present case we find [2]:

Proposition 2 *The requirement of wedge localization of a PF operator* $F(f) = \int F(x)\hat{f}(x)d^2x$, $suppf \in W$ *with* F *fulfilling formula (2,3) is equivalent to the Zamolodchikov-Faddeev structure of the Z-algebra. The corresponding F's cannot be localized in smaller regions i.e. the localization of* $F(\hat{f})$ *with* $supp\hat{f} \in \mathcal{O} \subset W$ *is not in* $\mathcal{O}$ *but still uses all of* W.

Here the requirement of wedge localization is expressed by

$$[A(W), A(W')] = 0 \tag{4}$$

with the prime on a spacetime region denoting its causal disjoint which for a wedge W consists of the opposite wedge $W' = W^{opp}$.

Before doing the necessary calculation, let us put on record two more definitions of a general kind which are suggested by the proposition.

Definition 3 *We call PF's which generate the wedge algebra*[‡]

$$\mathcal{A}(W) = alg\left\{F(\hat{f}), \forall f \; supp\hat{f} \in W\right\}$$

PFG or one-particle **p***olarization* **f***ree wedge* **g***enerators [2].*

We omitted the w for wedge in our short hand notation because wedges are the "smallest" regions in Minkowski space which do not have the full space as the causal completion and support nontrivial PF's. So all PF's which appear in our approach are always PFG's in the sense of wedges. In view of the mass-shell aspect of our approach and the fact that we work more frequently in momentum space and its rapidity-parametrized mass-shell, we reserve the simpler notation (without hat) to the Fourier transforms.

Definition 4 *We call the improvement of localization obtained by intersecting $\mathcal{A}(W)'s$ for different wedges an improvement of "quantum localization" [2], whereas the standard localization in $supp\hat{f}$ with the use of smeared out pointlike local fields $A(\hat{f})$ is referred to as classical (albeit in a* **quantum** *field theory).*

We now prove the above proposition by appealing to the so called KMS condition for localized algebras. This property originally arose in thermal systems in cases where the thermodynamical limit for the infinitely extended system cannot be described in terms of a Gibbs formula (volume divergencies), but it later turned out to be a characteristic property for all von Neumann algebras in a cyclic and separating state vector [3]. Local algebras in QFT are known to have this property with respect to the vacuum state at least as long as the localization region has a nontrivial causal complement, but they generally do not admit a natural thermodynamic limit description in terms of a sequence of increasing

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[‡]In this letter we do not discuss the necessity to distinguish between localized von Neumann algebras $\mathcal{A}(\mathcal{O})$ of bounded operators and polynomial algebras $\mathcal{P}(\mathcal{O})$ of affiliated unbounded operators as those formed from products of $F(f)$'s and their precise relation.

quantization boxes. In the case at hand, the wedge localized field algebras are known to have Lorentz boost as their KMS automorphism group with the KMS temperature equal to the Hawking-Unruh temperature $T_{loc} = 2\pi$.

Proof: Consider first the KMS property of the two-point function

$$\left\langle F(\hat{f}_1)F(\hat{f}_2)\right\rangle = \left\langle F(\hat{f}_2^{2\pi i})F(\hat{f}_1)\right\rangle = \left\langle F(\hat{f}_2^{\pi i})F(\hat{f}_1^{-i\pi})\right\rangle \tag{5}$$

Rewritten in terms of the $\hat{\mathrm{f}}$'s we have

$$\int f_1(\theta)\bar{f}_2(\theta)d\theta = \int f_2(\theta - i\pi)\bar{f}_1(\theta + i\pi)d\theta \tag{6}$$

which is an identity in view of the fact that the wedge support properties for the test functions f together with their reality condition imply $f(\theta - i\pi) = \bar{f}(\theta)$.

The 4-point function $\langle 1,2,3,4\rangle$ consists of 3 contributions, one from an intermediate vacuum state vector associated with the contraction scheme $\langle 12\rangle\langle 34\rangle$, another one from the direct intermediate two-particle contribution $\langle 14\rangle$ $\langle 23\rangle$and the third one from its exchanged (crossed) version $\langle 13\rangle\langle 24\rangle$. The latter is the only one which carries the interaction in form of the S-coefficients. In the would be KMS relation

$$\left\langle F(\hat{f}_1)F(\hat{f}_2)F(\hat{f}_3)F(\hat{f}_4)\right\rangle = \left\langle F(\hat{f}_4^{-2\pi i})F(\hat{f}_1)F(\hat{f}_2)F(\hat{f}_3)\right\rangle \tag{7}$$
$$f^z(\theta) \ : \ = a.c.f \mid_{\theta\rightarrow\theta+z}$$

the vacuum terms and the direct terms interchange their role on both sides of the equation and cancel out, whereas the crossed terms are related by analytic continuation. The required equality for the crossed term brings in the S-matrix via the relations ([3]) and yields

$$\begin{aligned} &\int\int d\theta d\theta' S(\theta-\theta')f_2(\theta)\bar{f}_4(\theta)f_1(\theta')\bar{f}_3(\theta') \\ &= \int\int d\theta d\theta' S(\theta-\theta')f_1(\theta)\bar{f}_3(\theta)f_4(\theta'-2\pi i)\bar{f}_2(\theta') \end{aligned} \tag{8}$$

Again using the above boundary relation for the wave functions we rewrite the last product in the second line as $\bar{f}_4(\theta' - i\pi)f_2(\theta' - i\pi)$ and performing a contour shift $\theta' \rightarrow \theta' + i\pi$, renaming $\theta \leftrightarrow \theta'$ and finally using the denseness of the wave functions in the Hilbert space, we obtain the crossing relation for S

$$S(\theta) = S(-\theta + i\pi) \tag{9}$$

Note that we already omitted the subscripts on S, since the identity $S_{Z,Z^*} = S_{Z,Z} \equiv S$ follows from the two different ways of calculating the crossed term, once by interchanging the two creation operators in $Z^*(\theta_3)Z^*(\theta_4)$ and then performing the direct contraction and another way by interchanging $Z(\theta_2)Z^*(\theta_3)$ and then being left with the vacuum contraction. Let us look at one more KMS relation for the six-point functions of the would be PFG's.

$$\left\langle F(\hat{f}_1)....F(\hat{f}_6)\right\rangle = \left\langle F(\hat{f}_6^{2\pi i})F(\hat{f}_1)...F(\hat{f}_5)\right\rangle \tag{10}$$

This time one has many more pairings In fact ordering with respect to pair contraction times 4-point functions one may again group the various terms in those for which the pairing contraction is between adjacent $Z's$ and those where this only can be achieved by exchanges. The first group satisfies the KMS condition because of the previous verification for the 2- and 4- point functions. For the crossed contributions the wave functions say f_i and $\bar{f}_k$. Those terms only compensate by shifting upper C-contours into lower ones and vice versa. If S would contain poles in the physical sheet, then there are additional contributions and the KMS property only holds if these poles occur in symmetric pairs i.e. in a crossing symmetric fashion.⌋

Here we will not pursue the fusion structure for the $Z's$ resulting from poles beyond noting that the particle spectrum already shows up in the fusion of the wedge localized $Z(f)'s$[§]. It should be stressed that the simple quantum mechanical picture of charge fusion in terms of particle bound states only holds for the above class of model with

[§]In fact it is only through the PFG's F(x) that the Z-F algebra and the fusion rules for the Z's receive a space-time interpretation. This close relation to a kind of "relativistic QM" only occurs pair interactions and not for more realistic models with real (on-shell) particle creation. All models except free fields, whether they are real particle conserving or not, have a rich virtual particle structure (see below), i.e. the particle content of operators A with compact localization e.g. $A \in \mathcal{A}(\mathcal{O})$ complies with the "folklore" that all particle matrix elements

$$^{out}\left\langle p_1, ..., p_k \left| A \right| q_1, ..., q_l \right\rangle^{in} \neq 0 \tag{11}$$

as long as they are not forced to vanish by superselection rules. Although we have explained the basic concepts in the case of an Ansatz with diagonal S-coefficients, one realizes immediately that one can generalize the formalism to *matrix-valued* "pair interactions". The operator formalism (the associativity) then leads to the Yang-Baxter conditions and the crossing relations are again equivalent to the KMS property for the wedge generators $F(f)$. In fact this family of theories with matrix-valued pair interactions constitutes a kind of long distance equivalence class in the sense that the long-distance limit of an arbitrary d=1+1 scattering matrix asymptotically falls into this pair interaction class better known under the name of "factorizing models".

The relation of the above observation with aspects of interaction in **l**ocal **q**uantum **p**hysics (LQP) becomes more manifest, if one reminds oneself that the Lorentz boost, which featured in the above KMS condition, also appears together with the TCP operator in the Tomita theory for the pair $(\mathcal{A}(W), \Omega)$. Fortunately, this physically extremely important result, first obtained by Bisognano and Wichmann, has meanwhile entered a textbook on LQP [3], so that we can effort to be very brief on its description. Consider the basic relation

$$S_T A\Omega = A^*\Omega, \quad A \in \mathcal{A}(W) \tag{12}$$

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on the level of wedge localization; the algebras resulting from intersections of wedge algebras loose this quantum mechanical aspect and show the full virtual particle creation/annihilation polarization structure.

which defines the antilinear, unbounded, closable, involutive (on its domain) Tomita operator S_T. Its polar decomposition

$$S_T = J\Delta^{\frac{1}{2}} \tag{13}$$

defines a positive unbounded $\Delta^{\frac{1}{2}}$ and an antiunitary involutive J and the nontrivial part of Tomita's theorem (with improvements by Takesaki) is that the unitaty Δ^{it} defines an automorphism of the algebra i.e. $\sigma_t(\mathcal{A}) \equiv \Delta^{it}\mathcal{A}\Delta^{-it} = \mathcal{A}$ and the J maps $\mathcal{A}$ antiunitarily into its commutant $j(\mathcal{A}) \equiv J\mathcal{A}J = \mathcal{A}'$. The wedge situation is a special illustration for the Tomita theory [3]. In that case both operators have well known physical aliases; the modular group is the one-parametric wedge affiliated Lorentz boost $\Delta^{it} = U(\Lambda(-2\pi t)$, and the J in d=1+1 LQP's is the fundamental TCP-operator; in higher dimensions it is only different from TCP by a π-rotation around the spatial wedge axis. The prerequisite for the general Tomita situation is that the vector in the pair (algebra, reference vector) is cyclic and separating i.e. there is no annihilation operators in the von Neumann algebra or equivalently: its commutant is cyclic relative to the reference vector. In LQP these properties are guarantied for localization regions $\mathcal{O}$ with nontrivial causal complement $\mathcal{O}'$ thanks to the Reeh-Schlieder theorem. Returning to our wedge situation we conclude from the Bisognano-Wichmann result that the commutant of $\mathcal{A}(W)$ is geometric i.e. fulfils Haag duality $\mathcal{A}(W)' = \mathcal{A}(W')$, a fact which can be shown to be modified by Klein factors in J in case of deviation from Bose statistics.

There is one more structural element following from "quantum localization" beyond wedge localization.

Proposition 5 *Operators localized in double cones $A \in A(\mathcal{O})$ obey a recursion relation in their expansion coefficients in terms of PFG operators*

$$A = \sum \frac{1}{n!} \int_C ... \int_C a_n(\theta_1, ...\theta_n) : Z(\theta_1)...Z(\theta_n) : d\theta_1...d\theta_n$$
$$= \sum \frac{1}{n!} \int ... \int \hat{a}_n(x_1, ...x_n) : F(x_1)...F(x_n) : d^2x_1...d^2x_n, \; supp\hat{a} \in W^{\otimes n}$$

$$ilim_{\theta\to\theta_1}(\theta-\theta_1)a_{n+1}(\theta,\theta_1,...,\theta_n)=(1-\prod_{i=2}^{n}S(\theta_1-\theta_i))a_{n-1}(\theta_2,..,\theta_n)$$

Remark 6 • *In order to compare (see below) with Smirnov's [4] axioms we wrote the recursion in rapidity space instead of in x-space light-ray restriction which would be more physical and natural to our modular approach. The series extends typically to infinity. Only for special operators (e.g. bilinears as the energy momentum tensor) in special models with rapidity independent S-matrices (e.g. Ising, Federbush) for which the bracket involving the product of two-particle S-matrices vanishes, the series restricts to a polynomial expression in Z. Therefore apart from these special cases, an operator $A\in\mathcal{A}(\mathcal{O})$ with $a_1\neq 0$ applied to the vacuum creates a one-particle component which an admixture of an infinite cloud of additional particles (particle-antiparticle polarization cloud). The above recursion together with Payley-Wiener type bounds for the increase of the $a_n's$ in imaginary θ-directions (depending on the shape and size of $\mathcal{O}$) characterize formfactors of operators from $\mathcal{A}(\mathcal{O})$.*

The proof follows rather straightforwardly from the quantum localization idea

$$\mathcal{A}(\mathcal{O})=\left[U(a)\mathcal{A}(W)U^{-1}(a)\right]'\cap\mathcal{A}(W) \tag{14}$$

i.e. we are considering the relative commutant inside the wedge algebra. Using the PFG's $F(f)$, the $A\in\mathcal{A}(\mathcal{O})$ are characterized by [2]

$$\left[A,F(\hat{f}_a)\right]=0,\ \forall\hat{f}\in W \tag{15}$$

where $\hat{f}_a(x)=\hat{f}(x-a)$, $a\in W$. One immediately realizes that the contribution of the commutator to the n^{th} power in F yields a relation between the a_{n-1} and a_{n+1} (from the creation/annihilation part of $F(\hat{f}_a)$). The details of this relation are easier, if one passes to the light-ray restriction which in the present approach turns out to be a quite nontrivial result of modular theory [2] [5] [6].

Proposition 7 *The relative commutant for light-like translations with $a_+ = (1,1)$ defines a "satellite" chiral conformal field theory via the (half) net on the (upper) +light ray*

$$\mathcal{A}(I_{a,e^{2\pi t}+a}) = U(a,a)\Delta^{-it}\left(\mathcal{A}(W_{a_+})' \cap \mathcal{A}(W)\right)\Delta^{it}U^{-1}(a,a) \tag{16}$$

where $I_{a,b}$ with $b > a \geq 0$ denotes an interval on the right upper light ray. This net is cyclic and separating with respect to the vacuum in the reduced Hilbert space

$$H_+ = \overline{\mathcal{M}_+\Omega} = P_+H \subset H = \overline{\mathcal{A}(W)\Omega} \tag{17}$$
$$\mathcal{M}_+ \equiv \cup_t A(I_{0,e^{2\pi t}}),\ E_+(\mathcal{A}(W)) = \mathcal{M}_+ = P_+\mathcal{A}(W)P_+$$

where the last relation defines a conditional expectation. The application of J to $\mathcal{M}_+$ gives the left lower part of this light ray, which is needed for the full net.

Remark 8 *The most surprising aspect of this proposition is that this light-ray affiliated chiral conformal theory exhibits the "blow-up" property i.e. can be activated to reconstitute the two-dimensional net by association of the -light ray translation*

$$\mathcal{A}(W) = alg \cup_{a>0} \{adU_-(-a)(\mathcal{M}_+)\} \tag{18}$$
$$\mathcal{A} = \mathcal{A}(W) \vee \mathcal{A}(W)'$$

The Moebius groups SL(2,R)$_\pm$ account for 6 parameters in contradistinction to the 3 parameters of the two-dimensional Poincaré group of the massive theory. Most of the former are "hidden" and the original theory perceives these additional symmetries only in its $P_\pm$ projections (for the proofs see [2] [5] [8]) The light-ray reduction reduces the derivation of the recursion relation to a one-dimensional LQP problem and the reader may carry out the missing algebra without much effort. This reduction also helps significantly in the demonstration that the $\mathcal{A}(\mathcal{O})$ spaces are non-trivial i.e. contain more elements than multiples of the identity.

It is worth emphasizing that the existence problem** for nontrivial QFT's, which in the quantization (Lagrangian, functional integral) approach required a sufficiently mild behavior for short distances in the correlations of the Lagrangian field with the perturbative renormalizability requirement being $dim\mathcal{L}_{int} \leq dim$spacetime, the modular approach, which does not use individual "field-coordinatizations", relates the existence of nontrivial field theories associated with interacting PFG's to the nontriviality of intersections which represent double cone algebras. Of course in the present state of our knowledge we cannot exclude the possibility that there may be a hidden relation between nontriviality of such intersections and the short distance behavior of a special (Lagrangian) field coordinate in the equivalence class of all possible field coordinates. Note also that the above constructions is formal and only determine operators in the sense of bilinear forms and a more rigorous (bounded) operator approach which could handle the nontriviality problem for the intersections requires the conversion of the wedge generators into bounded operators similar to the transcription of smeared free fields into Weyl algebras. These (important) technical matter, which in more standard field theoretic terminology is related to the existence of correlation functions for a known space of formfactors, will be dealt with in a separate paper.

At this point it is appropriate to address the question of what we learned from this approach as compared to the Karowski-Weisz-Smirnov "axiomatics" [7] [4] for factorizing models. Actually this terminology is not quite fair since a considerable part of that axiomatics has been reduced to specializations of general field theoretic properties via the LSZ formalism, although this does not include the algebraic and analytic aspects of the fundamental crossing property. But since the LSZ formalism itself can be derived from the basic causality and spectral properties of say Wightman QFT, one may even want

**This problem has in no way been solved by the duality between large/small coupling constants; it only has been shifted to the problem of existence of a QFT with those two asymptotes.

to have a more direct physical spacetime understanding of the other properties. This is achieved by realizing that the a_n-coefficients have the interpretation of the connected part of formfactors of operators $A \in \mathcal{A}(\mathcal{O})$, for selfconjugate models

$$a_n(\theta_1, ..., \theta_n) = \langle \Omega \left| A \right| \theta_1, ..., \theta_n \rangle^{in} = \langle \Omega \left| A \right| \theta_1, ..., \theta_n \rangle^{in}_{conn} \qquad (19)$$
$$\theta_1 < \theta_2 < .. < \theta_n$$

$$a_n(\theta_1, ..\theta_\nu, \theta_{\nu+1} - i\pi, .., \theta_n - i\pi) \qquad (20)$$
$$= {}^{out}\langle \theta_1, ..\theta_\nu \left| A \right| \theta_{\nu+1}, ..; \theta_n \rangle^{in}_{conn}$$

The relations for different orderings of $\theta's$ follows from the algebraic structures of the $Z's$. Note that the modular approach determines spaces of formfactors of $\mathcal{A}(\mathcal{O})$ and therefore disentagles the problem of constructing a QFT from the choice of using individual fields i.e. the problem of constructing a convenient analogue of the Wick basis for the composite free fields. In the diagonal case this connection between $Z's$ and in- and out- creation/annihilation operators can be seen directly via representing the $Z's$ in a bosonic/fermionic Fock space of the incoming particles in the form

$$Z(\theta) = a_{in}(\theta) e^{i \int_{-\infty}^{\theta} a_{in}^*(\theta) a_{in}(\theta) d\theta} \qquad (21)$$

However such representations are not known for the nondiagonal case. But once one obtained the double cone localized operators the theory itself (scattering theory as a consequence of the locality+spectral structure) assures the existence of Z in terms of incoming particle creation/annihilation operators, albeit not in terms of simple exponential formulas. The modular theory for wedges in terms of PFG's really explains the KWS axiomatics by integrating it back into the fundamental principles of general QFT. In particular the notoriously difficult crossing symmetry for the first time finds its deeper explanation in Hawking-Unruh thermal KMS properties once one realizes that a curved space-time Killing vector (a classical concept) is not as important quantum localization of operator algebras. With these remarks we have achieved our goal of deriving and explaining all axioms of the KWS approach in terms of localization properties of PFG's with pair interactions.

There is also an interesting extension of the KWS axiomatics in form of a pair of satellite chiral conformal theories. In contradistinction to the standard short distance association the light ray association via modular theory is not just a one way street; the blow-up property with the help of adjoining the opposite light cone translation allows to return, so that hidden conformal symmetries become relevant for the massive theory or more precisely for the massive theory projected into the $H_{\pm}$ subspaces. Similar ideas about chiral conformal "sattelites" have been recently used in the context of degrees of freedom counting in higher dimension [8] [5]. The resulting light ray projection ("holographic property") and its inverse (the "blow-up" property) obtained from modular inclusion/intersection methods are generic properties of Local Quantum Physics and do not rely on string theoretic ideas. The present construction principle can also be used for the direct systematic construction of chiral conformal theories. For the construction of W-like algebras one starts with PFG generators on a half line. Modular theory assures that in principle every system of S-coefficients fulfilling the Z-F algebra leads to a bosonic/fermionic conformal theory granted that the previous relative commutator algebra is non-trivial. This is a construction scheme which could not have been guessed within the framework of pointlike fields. Another apparently simple but untested idea suggested by the present concepts is the classification of wedge algebras with non-geometric commutator algebras via statistics Klein factors or constant S-matrices in J. Examples are the Ising field theory and the order/disorder fields. For the more interesting case of plektonic R-matrices which appear in the exchange algebras [9] of charge carrying fields, one knows that these algebras in contradistinction to bosonic/fermionic (e.g. W-algebras) are incomplete since the distributional character at coalescent points is left unspecified. This is not the case if one uses the R-data as an input into plektonic $Z^{\#}(\theta)$. The Hilbert space obtained by iterative application of Z-creation operators is not compatible with a Fock space structure. Rather the n-particle subspace has the structure of a path space as known from the representation theory of intertwiner algebras. The combinatorial complications should be offset by the simplicity of constant S-matrices. As the operator representation of the massive Ising model shows, the case of constant S should even have a simple coefficient series in the massive case.

II. REMARKS ABOUT THE GENERAL CASE

The amazing new spacetime insight from modular theory into the workings of the nonperturbative non-Lagrangian KWS bootstap formfactor approach [7] [4] has been obtained by specializing general principles and concepts to situations with no real particle creations (factorizing models) which can only occur in d=1+1[††]. But even the very special situation studied in this letter suggests fresh ideas on nonperturbative QFT which remained hidden in the quantization approach. Let us look at one related to the age old dream of having objects with a better short-distance behavior than the pointlike fields in the standard quantization approach. Modular theory achieves this by starting with PFG's which are (thanks to their spatial extension) naturally short-distance well-behaved, although for totally different reasons than the cut-off quantized fields: PFG's are not modified pointlike fields, but they are intrinsically nonlocal objects which are attached to the full wedge. However they are *nonlocal objects* which generate wedge algebras *in a* would be *local theory*. Only by studying quantum localization via intersections of these algebras does one get to the more local operators; in fact the intersections are either trivial (multiples of the identity), or they define a local theory of which the pointlike generators are the fields co-

[††]There exist also PFG's in situations with creation, if one restricts to d=1+1 scattering of waves instead of particles. This was pointed out to me by Buchholz [10]. Any nontrivial zero mass scattering model of the type envisaged by Buchholz will give rise to nontrivial PFG's.

ordinates of the standard approach. Note that the latter only appear at the very end and not ab inicio as in the quantization approach. The known examples of factorizing models show that the short distance behavior of even the best local interpolating fields can be as bad as an arbitrarily high inverse power, thus contradicting the standard perturbative renormalizability criterion in terms of power counting. Therefore the present situation of having nonlocal operators (the PFG's) which only make sense in a local theory is a far shot away from ad hoc cut-offs or regulators imposed on quantized local fields (which clash with the principles of a local theory and need to be removed at the end). The new objects which generate the intersection algebras (the PFG's) are completely reprocessed; the intersection method leads to new generators which are not modifications of individual old operators as in the standard approch where the interacting local composites are viewed as deformed Wick products of free composites. In our factorizing model illustrations the double cone generators turned out to be infinite power series in the PFG's. For a more elegant intrinsic description one probably needs a better knowledge of the modular theory of the intersection algebras.

This raises the question if the PFG's $F(x)$ in their property as wedge algebra generators could exist also in higher dimensions. In that case, as a result of the always present real particle creation, their more than one time application to the vacuum would generate state vectors whose real particle content is already very complicated. As often in QFT with such structural questions, it is easier to see what does not work, i. e. to prove No-Go theorems. Indeed if the interacting PFG's exist at all, their causally closed living space $\mathcal{O}$ cannot be (even a tiny little bit) smaller than a wedge $\mathcal{O} \subset W$. As was already stated at the beginning, if there would be spacelike separated "get away" directions with an arbitrarily small conic surrounding which are contained in W but not in $\mathcal{O}$, it is fairly easy to generalize the proof of the Jost-Schroer theorem [1] and show that the commutators of such PFG's must be a c-number which is determined by their two-point function. However the method used in those No-Go theorems has no extension to the wedge region. If wedge algebras can indeed be generated by PFG's, one expects again that modular theory does not only relate them to the S-matrix so that their correlations can be expressed in terms of products of S-matrix elements and furthermore that the elusive crossing symmetries for the S-matrix and formfactors find their explanation in the thermal KMS properties. This surprising relation between particle physics and the thermal properties of (Hawking-Unruh) wedge horizons has attracted the attention of many physicist, the ideas most close to those of the present work and several older articles [11] of the present author are those in [12]. However it should be clear that as long as higher dimensional PFG's, which are the mediators between off- and on-shell, have yet to be constructed, there is no proof beyond the one in the present work, despite some claims to the contrary in the cited literature.

The main problem in the extension to theories with real particle creation is to replace the Z-F commutation relations by analytic formulas which relate the various rapidity orderings of $\mathcal{Z}^{\#\prime}s$ inside correlation functions with iterated applications of S-operators acting on subsets of in states. The important two-dimensional ordering in the rapidities can also be achieved in higher dimensions by replacing the mass by an "effective" mass $m(p_\perp) = \sqrt{p_\perp^2 + m^2}$ in terms of transversal wedge components. In the relation of the rapidity ordering to the S-matrix one expects that the decomposition into cycles will be important because the cycles belong naturally to sub-S-matrices. A future clarification of the possibility of PFG's and a wedge based approach in higher dimension would be very desirable since presently there exists no systematic nonperturbative nonquasiclassical method. The revolutionary aspect of the present message abstracted from the pioniering formfactor approach lies not so much in pushing an existing quantization formalism upword by many orders of magnitude, but rather in the wealth of new concepts based on well established physical principles.

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[1] [1] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics and all That*, Benjamin 1964

[2] [2] B. Schroer and H.-W. Wiesbrock, *Modular Constructions of Quantum Field Theories with Interactions*, hep-th/9812251, and previous papers quoted therin.

[3] [3] R. Haag, *Local Quantum Physics*, Springer 1992

[4] [4] F. A. Smirnov, *Form Factors in Completely Integrable Models, Adv Series in Math. Phys. 14*, World Scientific 1992. Our operator notation is more close to M. Yu Lashkevich, *Sectors of Mutually Local Fields in Integrable Models of Quantum Field Theory*, hep-th/9406118

[5] [5] B. Schroer and H.-W. Wiesbrock, *Looking beyond the Thermal Horizon: Hidden Symmetries in Chiral Models*, hep-th/9901031

[6] [6] D. Guido, R. Longo and H.-W. Wiesbrock, *Commun. Math. Phys.* **192**, 217 (1998)

[7] [7] M. Karowski and Weisz, *Nucl.Phys.* B139, 445 (1978)

[8] [8] D. Guido, R. Longo, J. E. Roberts and R. Verch, *A general framework for charged sectors in quantum field theory on curved spacetime*, in preparation

[9] [9] K. H. Rehren and B. Schroer, *Nucl. Phys.* **B312**, 715 (1989)

[10] [10] D. Buchholz, *Commun. Math. Phys.* **45**, 1 (1975)

[11] [11] B. Schroer, *Annals of Physics* Vol. **255**, 270 (1997)

[12] [12] M. Niedermaier, *Commun. Math. Phys.* **196**, 411 (1998)