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On the Temperature Dependent Coupling Constant In the Vector N-Component $(\lambda\varphi^4)_D$ Model

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Abstract

For the massive $(\lambda\varphi^4)_D$ vector N-component model, in the large N limit we check that there is no first order phase transition induced by the thermal renormalized coupling constant whatever is the dimension D. Moreover for $D = 3$ we are able to give an exact formula for the temperature behavior of the coupling constant

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The effective potential at finite temperature for the $(\lambda\varphi^4)$ model has been investigated by several authors at the one-loop approximation and also taking into account the contribution from multiloop diagrams [1]. In many of these papers [2] is presented the question of whether a first or second order phase transition should be present in the model. In particular Arnold and Spinosa [3] investigated the possibility of a phase transition in the $(\lambda\varphi^4)_4$ model using the ring improved effective potential. The authors claim that a second order phase transition could exist, but that the method employed cannot be trusted to distinghish between a first or a second order phase transition. Tetradis and Wetterich [4] have tried to better understand the problem by investigating the N-component $(\lambda\varphi^4)_4$ model using renormalization group techniques, resulting in a claim for a second order phase transition. A different answer has been got for instance by Carrington and Takahashi [5] who found a first order transition in a pure scalar model at $D = 4$. In a slightly different context using composite operator techniques, Carmelia and Pi [6] are in disagreement with the conclusion for a first order phase transition.

In recent papers [7][8] and [9] the behavior of the coupling constants in temperature in connection to stability and phase transitions was investigated. In particular the possibility of a first order phase transition in the $(\lambda\varphi^4)$ model at $D = 3$ was still raised in ref.[8]. In this paper Malbouisson and Svaiter investigated the finite temperature behavior of the model in arbitrary dimension D at the one-loop approximation. The result is that the thermal correction to the renormalized squared mass is positive and increases with the temperature, while the thermal correction to the renormalized coupling constant, is negative and increasing in modulus with the temperature. This raises again the possibility of the vanishing of the thermal coupling constant at some temperature and its change of sign afterwards. Of course we can not be sure that this peculiar behavior will be preserved when higher order loops contributions are taken into account. For instance, two-loop corrections have been added by Ananos and Svaiter [9] in the context of the tricritical phenomenon with the result that in the high temperature regime two-loops corrections are positive and tend to compensate the lowering of the value of the thermal renormalized coupling constant. In any case the question concerning the thermal behavior of the coupling constant of scalar models in field theory seems to still be a controversial one.

In order to throw some light on the problem we examinate a model which allows us a non-perturbative approach. We look in this note as a simple example, the vector N-component $(\lambda\varphi^4)_D$ model in the limit of the leading order in $\frac{1}{N}$. This is not a very hard task since in this limite the model is soluble. In this case we have been able to obtain a definite answer to the question raised in ref.[8] and the above quoted ones.

We consider the model described by the Lagrange density,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi_a\partial_\mu\varphi_a + \frac{1}{2}m^2 : \varphi_a\varphi_a : + \frac{\lambda_0}{N}(: \varphi_a\varphi_a :)^2 \tag{1}$$

(summation over repeated indices are understood) where λ_0 is the bare coupling constant and m is the physical mass (see below). At leading order the two-point function is of order 0 in $\frac{1}{N}$ and the four-point function is of order 1 in $\frac{1}{N}$, in the limit of a very large numbers of colours, N. We note that Wick ordering of the product of fields makes unnecessary an explicit mass renormalization at the order in $\frac{1}{N}$ considered. The tadpoles are completely suppressed by Wick ordering. We consider thus in the following only the thermal behaviour of the temperature dependent renormalized zero-external momenta four-point function, which we take as our definition of the thermal renormalized coupling constant.

The four-point function for the N-component vector model is given pictorially by the sum of the diagrams in (fig.1). In the following we drop out the colour indices, and we perform as usual the Matsubara replacements, $\int \frac{d\omega}{2\pi} \rightarrow \frac{1}{\beta}\sum_{n=-\infty}^{\infty}, \omega \rightarrow \omega_n = \frac{2n\pi}{\beta}$, where β is the inverse temperature (we take $k = \hbar = 1$). Performing the sum over all diagrams indicated in (fig.1) we get for the temperature dependent four-point function with zero-external momenta (the renormalized coupling constant),

$$\Gamma_D^{(4)}(0,\beta) = \frac{1}{N}\frac{\lambda_0}{1-\lambda_0\Sigma(D,\beta)} \tag{2}$$

where $\Sigma(D,\beta)$ corresponds to single-bubble diagram present in fig.(1). To write down an expression for $\Sigma(D,\beta)$ for arbitrary dimension D, we briefly sketch some one-loop results described in ref.[8], which have been got by the concurrent use of dimensional and zeta-function analytic regularizations: taking the dimensionless parameters, $c^2 = \frac{m^2}{4\pi^2\mu^2}$, $(\beta\mu)^2 = a^{-1}$, $g = \frac{\lambda}{8\pi^2}$, $\frac{\varphi_0}{\mu} = \phi$, where φ_0 is the normalized vacuum expectation value of the field and μ is an arbitrary mass parameter introduced in order to deal with dimensionless quantities in the regularization procedures, the one-loop contribution to the finite temperature effective potential may be written in the form,

$$V(\beta,\phi) = \mu^D\sqrt{a}\sum_{s=1}^{\infty}\frac{(-1)}{2s}g^s\phi^{2s}\sum_{n=-\infty}^{\infty}\int\frac{d^{D-1}k}{(an^2+c^2+\mathbf{k}^2)^s}. \tag{3}$$

From the well-known formula,

$$\int\frac{d^dk}{(k^2+b^2)^s} = \frac{\pi^{\frac{1}{2}}}{\Gamma(s)}\Gamma(s-\frac{d}{2})\frac{1}{b^{2s-d}}, \tag{4}$$

we get

$$V(D,\beta) = \mu^D\sqrt{a}\sum_{s=1}^{\infty}f(D,s)g^s\phi^{2s}A_1^{c^2}(s-\frac{d}{2},a), \tag{5}$$

where $f(D,s)$ is a function proportional to $\Gamma(s-\frac{d}{2})$ and $A_1^{c^2}(s-\frac{d}{2})$ is one of the inhomogeneous Epstein zeta-functions defined by,

$$A_N^{c^2}(u;a_1,a_2,...a_n) = \sum_{n_1,...n_N=-\infty}^{\infty}(a_1n_1^2+...+a_Nn_N^2+c^2)^{-u}, \tag{6}$$

valid for $Re(u) > \frac{N}{2}$ (in our case $Re(s) > \frac{D}{2}$).

Then making use of some Mellin transform representations, the Epstein inhomogeneous zeta-functions may be extended to the whole complex u-plane (the s-plane in our case), and we obtain after some manipulations, the one-loop correction to the temperature dependent effective potential,

$$V(D,\beta) = \mu^D \sum_{s=1}^{\infty} g^s \phi^{2s} h(D,s)[2^{-(\frac{D}{2}-s+2)}\Gamma(s-\frac{D}{2})(\frac{m}{\mu})^{D-2s} + \sum_{m=1}^{\infty}(\frac{m}{\mu^2 n\beta})^{\frac{D}{2}-s} K_{\frac{D}{2}-s}(mn\beta)]. \tag{7}$$

The single-bubble function $\Sigma(D,\beta)$ is just the coefficient of the $4th$-power ($s=2$) of the field, in the above equation. Thus from the results of ref.[8] we may write $\Sigma(D,\beta)$ in the form

$$\Sigma(D,\beta) = A(D) - G(D) \tag{8}$$

where

$$A(D) = \frac{1}{8\pi^2} h(D,2) m^{D-4}\Gamma(2-\frac{D}{2}) \tag{9}$$

and

$$G(D,\beta) = \frac{3}{2}\frac{1}{(2\pi)^{\frac{D}{2}}} \sum_{n=1}^{\infty}(\frac{m}{n\beta})^{\frac{D}{2}-2} K_{\frac{D}{2}-2}(mn\beta) \tag{10}$$

In eqs.(7, 10) K_μ are the Bessel functions of the third kind and $h(D,s)$ is given by

$$h(D,s) = \frac{1}{2^{\frac{D}{2}-s-1}} \pi^{2s-\frac{D}{2}} \frac{(-1)^{s+1}}{\Gamma(s+1)} \tag{11}$$

From the properties of the Bessel functions it may be seen from eq.(10) that for any dimension D

$$G(D,\beta)_{\beta\to\infty} \to 0 \tag{12}$$

$$G(D,\beta)_{\beta\to 0} \to \infty \tag{13}$$

We conclude also from those properties that $G(D,\beta)$ is always positive for any values of D and β. From eqs.(2, 8) we have,

$$\Gamma_D^{(4)}(0,\beta) = \frac{1}{N}\frac{\lambda_0}{1-\lambda_0(A(D)-G(D))} \tag{14}$$

Then, let us define the zero temperature renormalized coupling constant λ_R as

$$\frac{1}{N}\lambda_R = lim_{\beta\to\infty}\Gamma_D^{(4)}(0,\beta). \tag{15}$$

From eq.(15) and eq.(12) we get,

$$\lambda_R = \frac{\lambda_0}{1-\lambda_0 A(D)}, \tag{16}$$

so, from eqs.(14) and (16), we obtain for the thermal renormalized coupling constant,

$$\Gamma_D^{(4)}(0,\beta) = \frac{1}{N}\frac{\lambda_R}{1+\lambda_R G(D,\beta)}. \tag{17}$$

It is easy to see that the above procedure leading to eq.(17) corresponds on more familiar grounds, to sum up all the chain of bubbles graphs of fig.(1) with all possible combinations of $\frac{\lambda_R}{N}$ and $\frac{\delta\lambda}{N}$ at the vertices, where the counterterm $\delta\lambda = \lambda_0 - \lambda_R$. It is nothing but the resummation of all perturbative contributions including counterterms from the chain of bubbles and the subtraction of the divergent (polar) parts written in a compact form. These subtractions are performed even in the case of odd dimension, D, in which case there are no poles of Γ-functions.

We see from eqs.(17), (12) and (13) and the positivity of $G(D,\beta)$ that for any dimension D there is no finite temperature such that the thermal renormalized coupling constant vanish. Therefore, at leading order in $\frac{1}{N}$, in the context of the N-component vector $\lambda\varphi_D^4$ model the answer to the question raised in [8] and the preceeding ones on the existence of a first order phase transition is clearly negative. Eq.(17) is for any dimension D, a decreasing positive function of the temperature. It generalises for any dimension the remarks of ref.[8] and those by Fujimoto et al. done for $D = 4$ [11]. Moreover, an exact result may be obtained in dimension $D = 3$. From the relationships [10],

$$K_{n+\frac{1}{2}}(z) = K_{-n-\frac{1}{2}}(z) \tag{18}$$

$$K_{\frac{1}{2}} = (\frac{\pi}{2z})^{\frac{1}{2}} e^{-z}, \tag{19}$$

we get from eq.(10), after summing the resulting geometric series,

$$G(D,\beta) = \frac{3}{8m\pi}\frac{1}{e^{m\beta}-1}, \tag{20}$$

which gives from eq.(17) the exact relationship,

$$\Gamma_3^{(4)}(0,\beta) = \frac{1}{N}\frac{8m\pi\lambda_R(e^{\frac{m}{T}}-1)}{(8m\pi)e^{\frac{m}{T}}+3\lambda_R-1} = \frac{1}{N}\lambda_R(T, D=3). \tag{21}$$

A plot of $\lambda_R(T, D = 3)$ is given in (fig.2).

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Figure captions

Fig.(1) – In this figure the four-field vertex is splitted to indicate colour circulation. To each vertex there is a factor $\frac{1}{N}$ and for each single bubble a colour circulation factor N

Fig(2) – A plot of the coupling constant(in units of $\frac{1}{N}$ as a function of the temperature in dimension 3.

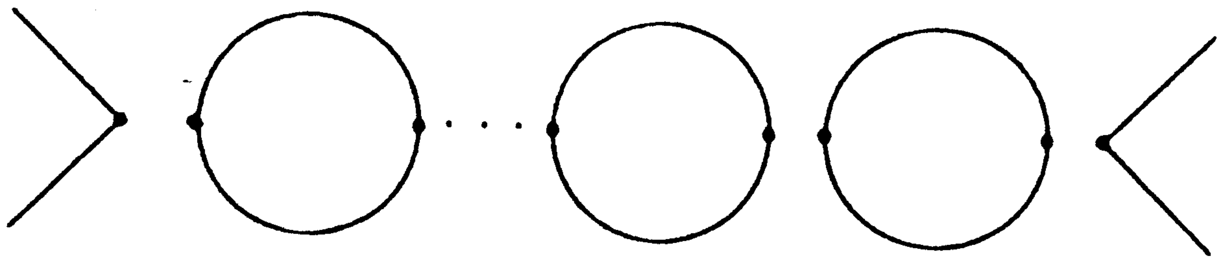

Fig. 1

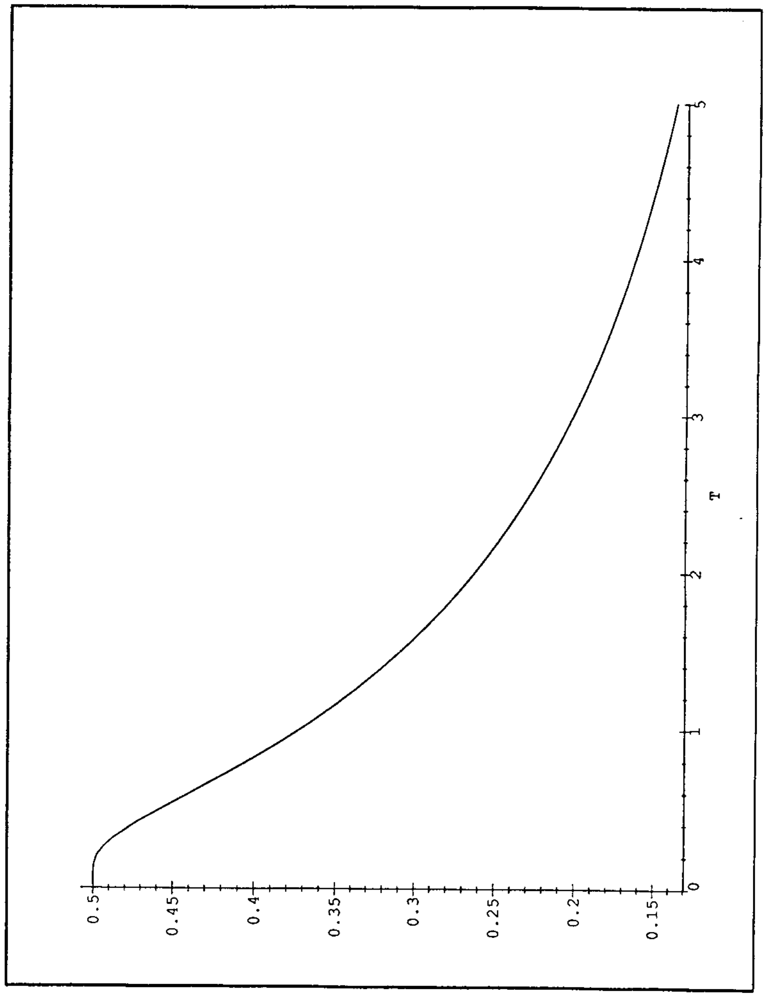

Fig. 2

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