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Radiative processes for Rindler and accelerating observers and the stress-tensor detector

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Abstract

We consider a monopole detector interacting with a massive scalar field. Using the rotating wave approximation the radiative processes is discused from the accelerated frame point of view. After this we obtain the Minkowski vacuum stress tensor measured by the accelerated observer using a non-gravitational stress tensor detector as discussed by Ford and Roman (PRD 48, 776 (1993)). Finally, we analyse radiative processes of the monopole detector travelling in a world line that is inertial in the infinite past and has a constant proper acceleration in the infinite future.

Key-words: Spontaneous emission; Vacuum fluctuations.

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1 Introduction

It has been known that an uniformly accelerated detector interacting with a massless scalar field in the Minkowski vacuum behaves like an inertial detector in equilibrium with a thermal bath at temperature $\beta^{-1} = \frac{1}{2\pi\alpha}$, where α^{-1} is the proper acceleration of the detector[1].

In a recent paper, Svaiter and Svaiter [2], studied the spontaneous and induced emission problem, using a very simple model of an atom consisting of a pointlike object with an internal structure defining two energy levels introduced by DeWitt [3]. Assuming that the atom (detector) interacts with a real massless scalar field, and it is travelling in inertial or non inertial world lines, the authors obtained the probability of transition per unit proper time as $\frac{dF(E,\Delta\tau)}{d\Delta\tau}$, (normalized by the selectivity of the detector) between different eigenstates of the detector and also presented the rate of spontaneous excitation after a finite observation or switching time ΔT. The extension of these calculations for the detector in the presence of paralel plates at zero and finite temperature was given by Ford, Svaiter and Lira [4]. A more involved mathematically case of the monopole detector in the presence of cosmic strings was discussed more recently in ref.[5].

It is useful to review how the idea of spontaneous emission arises in quantum optics. First, the interaction between the atom and the field is ignored; then the atom has stationary states with well defined energy. After this step, we introduce the interaction between the atom and the field as a perturbation. It is easy to show that only the ground state of the atom stays with well defined energy. All the excited energy levels have a width, and will decay spontaneously. A different approach uses the role of the vacuum fluctuations in the spontaneous emission processes. Using perturbation theory it can be shown that in first order approximation the asymptotic probability per unit time of decay is given by the Fourier transform of the positive Wightman function in the world-line of the atom. In this approach, it is assumed well defined levels of the atom, even after turning on the interaction between the atom and the field, and time-dependent perturbation avoids the calculations of the finite energy width. If we prepare the atom in the excited state, it will decay spontaneously by the effects of the vacuum fluctuations. Spontaneous emission can be interpreted as stimulated emission induced by vacuum fluctuations. Note that the fact that the excited states are not eigenstates of the full Hamiltonian of the system is automatically taken into account in the later scheme.

The purpose of this paper is to discuss radiative processes from the accelerated point of view and also to discuss the Minkowski stress-tensor measured by this observer using a generalization of the monopole detector given by Ford and Roman [6]. This derivatively coupled detector was analysed a long time ago by Hinton [7]. It was shown that in two-dimensional space-time, both detectors (the monopole and the derivative detector) agree. Nevertheless in a four-dimensional space-time there are discrepancies in the response function of both detectors. This fact raises a question of which of them is the true particle detector. Part of this question we will treat latter.

The main difference among our approach and all the previous papers is that we use the rotating wave approximation. In a real quantum detector prepared in the ground state, the detector goes to an excited state by an absorption process. Of course we are assuming asymptoticaly measurements, i.e. the observation time is large when compared with times on the order E^{-1}, where E is the energy gap between the excited and the ground state of the detector. Consequently it is possible to assume the normally ordered field correlation function in the probability of transition i.e., the rotating wave approximation. Since the detector measures frequencies with respect to its proper time, we have to use the normally ordered field correlation functions with respect to tho Rindler's time.

With the formalism which we developed, it is possible to obtain the transition rates for the accelerated detector with different proper accelerations in both, the begining and in the end of the observation time. In order to simplify the calculations for finite time switching detectors we will introduce the switching in the transition rate. As we will see, although this procedure is not exact from the mathematical point of view, it will reproduce known results.

The paper is prepared as follows. In section II we discused radiative processes in a frame of reference comoving with the monopole detector. In section III we repeat the calculations using the derivativelly coupled detector. In section IV, the asymptotic accelerated detector is discussed. Conclusions are given in section V. In this paper we use $\hbar = c = 1$.

2 Radiative processes of the monopole detector

Let us consider a system (a detector) endowed with internal degrees of freedom defining two energy levels with energy ω_g and ω_e, ($\omega_g < \omega_e$) and respective eigenstates $|g\rangle$ and $|e\rangle$. This system is weakly coupled with a hermitian massive scalar field $\varphi(x)$ with interaction lagrangian

$$L_{int} = \lambda_1 d(\tau)\varphi(x(\tau)), \tag{1}$$

where $x^\mu(\tau)$ is the world line of the detector parametrized using the proper time τ, $d(\tau)$ is the monopole operator of the detector and λ_1 is a small coupling constant between the detector and the scalar field.

In order to discuss radiative processes of the whole system (detector plus the scalar field), let us define the Hilbert space of the system as the direct product of the Hilbert space of the field $\mathbf{H_F}$ and the Hilbert space of the detector $\mathbf{H_D}$

$$\mathbf{H} = \mathbf{H_D} \otimes \mathbf{H_F}. \tag{2}$$

The Hamiltonian of the system can be written as:

$$H = H_D + H_F + H_{int}, \tag{3}$$

where the unperturbed Hamiltonian of the system is composed by the noninteracting detector Hamiltonian H_D and the free massive scalar field Hamiltonian H_F. We shall define the initial state of the system as:

$$|\mathcal{T}_i\rangle = |j\rangle \otimes |\Phi_i\rangle\,, \tag{4}$$

where $|j\rangle$, $(j = 1, 2)$ are the two possible states of the detector ($|1\rangle = |g\rangle$ and $|2\rangle = |e\rangle$) and $|\Phi_i\rangle$ is the initial state of the field. In the interaction picture, the evolution of the combined system is governed by the Schrodinger equation

$$i\frac{\partial}{\partial\tau}\left|\mathcal{T}\right\rangle = H_{int}\left|\mathcal{T}\right\rangle, \tag{5}$$

where

$$\left|\mathcal{T}\right\rangle = U(\tau, \tau_i)\left|\mathcal{T}_i\right\rangle, \tag{6}$$

and the evolution operator $U(\tau, \tau_i)$ obeys

$$U(\tau_f, \tau_i) = 1 - i\int_{\tau_i}^{\tau_f} H_{int}(\tau')U(\tau', \tau_i)d\tau'. \tag{7}$$

In the weak coupling regime, the evolution operator can be expanded in power series of the interaction Hamiltonian. To first order, it is given by

$$U(\tau_f, \tau_i) = 1 - i\int_{\tau_i}^{\tau_f} d\tau' H_{int}(\tau'). \tag{8}$$

The probability amplitude of the transition from the initial state $|\mathcal{T}_i\rangle = |j\rangle \otimes |\Phi_i\rangle$ at the hypersurface $\tau = 0$ to $\left|j'\right\rangle \otimes |\Phi_i\rangle$ at τ is given by

$$\left\langle j'\Phi_f\right| U(\tau, 0)\left|j\Phi_i\right\rangle = -i\lambda_1\int_0^{\tau} d\tau'\left\langle j'\Phi_f\right| d(\tau')\varphi(x(\tau'))\left|j\Phi_i\right\rangle, \tag{9}$$

with $|\Phi_f\rangle$ an arbitrary state of the field and $\left|j'\right\rangle$ is the final state of the detector.

The probability of the detector being excited at the hypersurface τ, assuming that the detector was prepared in the ground state is:

$$P_{eg}(\tau) = \lambda_1^2|\left\langle e\right| d(0)\left|g\right\rangle|^2\int_0^{\tau} d\tau'\int_0^{\tau} d\tau'' e^{iE(\tau''-\tau')}\left\langle\Phi_i\right|\varphi(x(\tau'))\varphi(x(\tau''))\left|\Phi_i\right\rangle, \tag{10}$$

where $\omega_e - \omega_g = E$ is the energy gap between the eigenstates of the detector.

Note that we are interested in the final state of the detector and not that of the field, so we sum over all the possible final states of the field $|\Phi_f\rangle$. Since the states are complete, we have

$$\sum_f |\Phi_f\rangle\langle\Phi_f| = 1. \tag{11}$$

Eq.(10) shows us that the probability of excitation is determined by an integral transform of the positive Wightman function.

Before starting to analyze radiative processes we would like to point out that a more realistic model of detector must also have a continuum of states. This asumption allows us to use a first order perturbation theory without taking into account higher order corrections. Although we will use in this paper the two-state model, the case of a mixing between a discrete and a continuum eigenstates deserves further investigations.

The radiative processes of the uniformly accelerated detector discussed from the inertial frame point of view was analysed by Kolbenstvedt and also Grove[8]. Let us analyze the radiactive processes of the uniformly accelerated Unruh-DeWitt detector from the point of view of a observer in a frame comoving with the detector. For the sake of simplicity we study the two-dimensional case ($D = 2$).

In a $D = 2$ dimensional spacetime, the Rindler coordinates (η, ξ) are given by

$$x^0 = \xi sinh\eta \qquad -\infty < \eta < \infty \tag{12}$$

$$x^1 = \xi cosh\eta \qquad 0 < \xi < \infty. \tag{13}$$

where x^0 and x^1 are the cartesian coordinates used by inertial observers. The line element in Rindler spacetime is given by

$$ds^2 = \xi^2 d\eta^2 - d\xi^2. \tag{14}$$

The Rindler edge is globally hyperbolic and posesses a timelike Killing vector which generates a boost about the origin. Therefore it is possible to define positive and negative modes in a unambiguous way. These modes that form a complete set, basis in the space of the solutions of the massive Klein-Gordon equation are given by

$$\phi_\nu(\eta, \xi) = \frac{1}{\pi}(sinh\pi\nu)^{1/2} e^{-i\nu\eta} K_{i\nu}(m\xi), \tag{15}$$

$$\phi^*_\nu(\eta, \xi) = \frac{1}{\pi}(sinh\pi\nu)^{1/2} e^{i\nu\eta} K_{i\nu}(m\xi), \tag{16}$$

where $K_{i\nu}$ is the Bessel function of imaginary order or the Macdonald's function [9], and m is the mass of the quantum of the field.

In order to canonical quantize the scalar field in the Rindler spacetime, let us follow Fulling [10] and Sciama, Candelas and Deutsch [11]. Therefore we expand the field operator in the form

$$\varphi(\eta, \xi) = \int_0^\infty d\nu [b(\nu)\phi_\nu(\eta, \xi) + b^\dagger(\nu)\phi^*_\nu(\eta, \xi)], \tag{17}$$

where the anihilation operator for Rindler's particle satisfies

$$b(\nu)\,|0, R\rangle = 0 \qquad \forall \nu. \tag{18}$$

Let us suppose that in the surface $\eta_i = cte$, the state of the system is $|g\rangle \otimes |0, M\rangle$. The probability amplitude for the system to go to $|e\rangle \otimes |\Phi_f\rangle$ at η_f (where $|\Phi_f\rangle$ is any final state of the field) is

$$A_{|g>\otimes|0,M>\to|e>\otimes|\Phi_f>} = -i\lambda_1 d_{eg}(\eta_i) \int_{\eta_i}^{\eta_f} d\eta e^{iE\eta} \langle \Phi_f| \varphi(\eta, \xi) |O, M\rangle\,. \tag{19}$$

For a better understanding of the radiative processes, let us split the field operator in the positive and negative parts, i.e.,

$$\varphi(\eta, \xi) = \varphi^{(+)}(\eta, \xi) + \varphi^{(-)}(\eta, \xi), \tag{20}$$

where $\varphi^{(+)}(\eta, \xi)$ and $\varphi^{(-)}(\eta, \xi)$ are given by:

$$\varphi^{(+)}(\eta,\xi) = \int_0^\infty d\nu b(\nu)\phi_\nu(\eta,\xi) \tag{21}$$

and

$$\varphi^{(-)}(\eta,\xi) = \int_0^\infty d\nu b^\dagger(\nu)\phi^*_\nu(\eta,\xi). \tag{22}$$

Substituting eq.(21) and (22) in eq.(19) yields:

$$\begin{aligned} A_{|g>\otimes|0,M>\rightarrow|e>\otimes|\Phi_f>} = -i\lambda_1 d_{eg}(\eta_i)\int_{\eta_i}^{\eta_f} d\eta e^{iE\eta} \quad [& \langle\Phi_f|\,\varphi^{(+)}(\eta,\xi)\,|O,M\rangle \\ + & \langle\Phi_f|\,\varphi^{(-)}(\eta,\xi)\,|O,M\rangle]. \end{aligned} \tag{23}$$

The first term is an absorption process and since contains a factor of the form $e^{i(E-\nu)\eta}$, it will allow energy to be conserved in the asymptotic limit $\eta_i \rightarrow -\infty, \eta_f \rightarrow \infty$. The second one is an emission process and lead to integrand of the form $e^{i(E+\nu)\eta}$. For $E > 0$ and $\nu > 0$ it is rapidly oscilating, giving a negligible contribution. This expresses the fact that the detector asymptotically can only suffer a transition to the excited state absorbing a Rindler particle from the Minkowski vacuum $|O,M\rangle$. In other words, the detector registers the occupation number of Rindler particles in the Minkowski vacuum. This result allows us to assume the rotating wave approximation (RWA) to discuss the radiative processes[12]. This kind of detector is called in the literature a square-law detector. If we assume the normally ordered field correlation function with respect to the cartesian time, the rate at which quanta of the field are detected by an inertial detector in the Minkowski vacuum state vanishes. since we are interested to study the Unruh-Davies effect from the point of view of the accelerated observer we assume the RWA with respect to the Rindler's time. Using this approximation the probability amplitude becomes:

$$A_{|g>\otimes|0,M>\rightarrow|e>\otimes|\Phi_f>} \approx -i\lambda_1 d_{eg}(\eta_i)\int_{\eta_i}^{\eta_f} d\eta e^{iE\eta}\,\langle\Phi_f|\,\varphi^{(+)}(\eta,\xi)\,|O,M\rangle\,. \tag{24}$$

For a better understanding of the RWA and to present the difference in the probability transition if we assume or not this approximation, let us first study the probability of transition without the RWA. In this case, let us define $F(e,\eta_i,\eta_f)$ such that

$$P(E,\eta_i,\eta_f) = \lambda_1^2|d_{eg}(\eta_i)|^2 F(e,\eta_i,\eta_f). \tag{25}$$

$F(E,\eta_i,\eta_f)$ is the response function, i.e., the probability of transition normalized by the $\lambda_1^2|m_{eg}(\eta_i)|^2$ term. Thus

$$F(E,\eta_i,\eta_f) = \int_{\eta_i}^{\eta_f} d\eta\int_{\eta_i}^{\eta_f} d\eta^{'} e^{iE(\eta-\eta^{'})}\,\langle 0,M|\,\varphi(\eta^{'},\xi^{'})\varphi(\eta,\xi)\,|0,M\rangle\,. \tag{26}$$

Instead of using $F(E,\eta_i,\xi,\eta_f,\xi')$, we will simplify the notation using $F(E,\eta_i,\eta_f)$ throughout the paper. Nevertheless in the calculations we are not restricted to constant proper acceleration of the detector but we are taking into account the general case of non-constant proper acceleration of the detector during the measure process.

Following Svaiter and Svaiter [2], let us define

$$\eta - \eta' = \tau \tag{27}$$

and

$$\eta_f - \eta_i = \Delta T. \tag{28}$$

We would like to stress that Levin, Peleg and Peres [13] also used the same technique to study radiative processes in finite observation time. Substituting eqs.(27) and (28) in eq.(26) we have

$$F(E, \eta_i, \eta_f) = \int_{-\Delta T}^{\Delta T} d\tau (\Delta T - |\tau|) e^{iE\tau} \langle 0, M| \varphi(\eta', \xi') \varphi(\eta, \xi) |0, M\rangle . \tag{29}$$

Let us define the rate $R(E, \Delta T)$, i.e., the probability transition per unit time, as :

$$R(E, \Delta T) = \frac{dF(E, \Delta T)}{d(\Delta T)} \tag{30}$$

Consequently we have:

$$R(E, \Delta T) = \int_{-\Delta T}^{\Delta T} d\tau e^{iE\tau} \langle 0, M| \varphi(\eta', \xi') \varphi(\eta, \xi) |0, M\rangle . \tag{31}$$

This important result shows that asymptotically the rate of excitation of the detector is given by the Fourier transform of the positive frequency Wightman function. This is exactly the quantum version of the Wiener-Khintchine theorem which asserts that the spectral density of a stationary random variable is the Fourier transform of the two point-correlation function.

The rate rigorously is:

$$\begin{aligned} R(E, \Delta T) = \int_{-\Delta T}^{\Delta T} d\tau e^{iE\tau} \quad & [\quad \langle 0, M| \varphi^{(+)}(\eta', \xi') \varphi^{(+)}(\eta, \xi) |0, M\rangle \\ & + \quad \langle 0, M| \varphi^{(-)}(\eta', \xi') \varphi^{(-)}(\eta, \xi) |0, M\rangle + \langle 0, M| \varphi^{(-)}(\eta', \xi') \varphi^{(+)}(\eta, \xi) |0, M\rangle \\ & + \quad \langle 0, M| \varphi^{(+)}(\eta', \xi') \varphi^{(-)}(\eta, \xi) |0, M\rangle]. \end{aligned} \tag{32}$$

The last matrix element can be writen as

$$\begin{aligned} \langle 0, M| \varphi^{(+)}(\eta', \xi') \varphi^{(-)}(\eta, \xi) |0, M\rangle \quad &= \quad \langle 0, M| \varphi^{(-)}(\eta, \xi) \varphi^{(+)}(\eta', \xi') |0, M\rangle \\ &+ \quad [\varphi^{(+)}(\eta', \xi'), \varphi^{(-)}(\eta, \xi)]. \end{aligned} \tag{33}$$

The commutator is a c-number independent of the initial state of the field. Many authors in quantum optics claim that this contribution has no great physical interest. So the matrix elements determining the detection of quanta of the field are of the form

$$\langle 0, M| \varphi^{(-)}(\eta', \xi') \varphi^{(+)}(\eta, \xi) |0, M\rangle + \langle 0, M| \varphi^{(-)}(\eta, \xi) \varphi^{(+)}(\eta', \xi') |0, M\rangle . \tag{34}$$

Such approximation is called the rotating-wave approximation (RWA). The last expression said that the excitation of the detector corresponds to the absorption of a Rindler particle from the Minkowski vacuum $|0, M\rangle$. This is a consequence of the absorptive nature of the detector. Nevertheless there are subtleties in the process. As it was shown by Unruh and Wald [14], the process is followed by the emission of a Rindler particle in a causally disconected region of the spacetime. (In the appendix we discuss this result). This result can be understood since the Minkowski vacuum $|0, M\rangle$ can be expressed into a set of EPR type of Rindler particles [15]. Thus a coherent state with respect to the annihilation operator of Minkowski particles appears to be squeezed with respect to the annihilation operator of Rindler particles. Note that this fact has been investigated in the literature in the context of quantum optics. We can imagine macroscopic detectors different from the square law detector. For example we can use the process of stimulated emission as a basis for detection. In this case the field operator would occur in anti-normal order. Different operator ordering has been also sugested. Wilkens and Lewenstein [16] proposed a photodetection scheme based on an interference between emission and absorption processes. In this paper we are assuming the absorptive nature of the detector.

Another point which is important to stress is the fact that a logarithmic ultraviolet divergence will appear in the response function, as was discussed by Svaiter and Svaiter [2]. In order to circumvent the problem of the divergence, Higuchi et al [17] considered a detector switched on and off continuously with the field. These authors claim that the ultraviolet logarithmic divergence in the excitation probability that Svaiter and Svaiter [2] found in the time dependent perturbation theory can be circumvented. Defining the probability of transition per unit proper time as $\frac{dF(E,\Delta T)}{d\Delta T}$, normalized by the selectivity of the detector, it is possible to obtain the rate of spontaneous excitation after a finite observation or switching time. Although the rate of spontaneous excitation of an unaccelerated atom (assuming that the state of the field is the Minkowski vacuum state and the state of the detector is the ground state) is negative and diverges below the uncertainty region, i.e. for $\Delta T \leq \frac{1}{E}$, there is no problem in such behavior. This happens because it is possible to consider measurements of finite duration only for $\Delta T > \frac{1}{E}$. Over shorter time intervals, it is not even possible to say what level our two-level system is in, or even to define this two-level system. Note that we are not interested in discussing the subtle problem of how to decode the information stored in the system and to convert it into a classical signal. Only with the latter the measurement process is complete. Without this mechanism it is convenient to call the first step as a "pre-measurement", but because we are not interested in discussing this controversial issue, we will still call the first step as a measurement. Back to our problem, we conclude that measurement is meaningless for intervals obeying $\Delta T \leq \frac{1}{E}$. As it was shown by Ford, Svaiter and Lyra [4], the asymptotic regime is achieved after a very short transient period. We conclude that there is no problem with the rate, concerning non positive definite rate, or divergent in intervals above the uncertainty region.

Of course, this divergence is expected, in the sense that the response function is an integral transform of the positive Wightman function, which becomes singular in one point of the spacetime. Since the field is a distribuition, the square of such object in one point of spacetime is ill-defined. This is the fundamental problem of the interacting field theory in flat and curved spacetime. If we adopt the point of view that we are interested in *measurements*, everything is in order and therefore any renormalization procedure is required. Note that we can use another physical interpretation to the mathematical calculations, as the result of making two measurements in the system separated by the time interval $\Delta\tau$, instead of the usual switching interpretation. For a short time interval, we would expect a large disturbance in the system. It is clear that the effect of a smooth switching on and off, sugested by Higuchi and collaborators (as the more realistic procedure for modeling detectors) is to prevent the region $\Delta\tau \leq \frac{1}{E}$. Again we are not interested in discussing another controversial issue: the uncertainty relation for time and energy and the different interpretations of such inequality and we are adopting the Landau and Peierls approach [18]. In this case the energy uncertainty introduced by the switching is less than the level separation of the system and the measurement can be defined.

The conclusion is that the transient terms related with the switching or finite observational time will vanish in the asymptotic limit. A straightforward calculation gives the probability of transition per unit proper time (normalized by the selectivity of the atom) of an inertial atom interacting with a massless field in the Minkowski vacuum (withouth assume the RWA):

$$R(E,\Delta T) = \frac{1}{2\pi}\Big(-E\Theta(-E) + \frac{cos E\Delta T}{\pi\Delta T} + \frac{|E|}{\pi}(Si|E|\Delta T - \frac{\pi}{2})\Big), \tag{35}$$

where $Si(z)$ is the sine integral function defined by $Si(z) = \int_0^\infty \frac{sint}{t}dt$ [9]. For large values of the argument we have $Si(\infty) = \frac{\pi}{2}$, thus in the asymptotic limit, we will obtain for the rate

$$lim_{\Delta T\to\infty} R(E,\Delta T) = \frac{-E}{2\pi}\Theta(-E). \tag{36}$$

The transition rate is proportional to the energy level gap of the atom, and in the asymptotic limit only spontaneous decay is allowed.

It is possible to repeat the calculations for the uniformly accelerated detector. In the asymptotic limit the expression that takes into account the two processes; spontaneous and induced decay or induced excitation is given by:

$$lim_{\Delta\tau\to\infty} R(E,\Delta\tau) = \frac{|E|}{2\pi}\Big(\Theta(-E)(1 + \frac{1}{e^{2\pi\alpha|E|}-1}) + \Theta(E)\frac{1}{e^{2\pi\alpha E}-1}\Big). \tag{37}$$

were $\frac{1}{\alpha}$ is the proper acceleration of the atom.

Let us use the RWA to obtain the probability of transition per unit time in the general case of a massive field. Going back to the eq.(32) using the RWA we have to the rate:

$$\begin{aligned} R(E,\Delta T) = \int_{-\Delta T}^{\Delta T} d\tau e^{iE\tau} \quad & [\quad \langle 0,M|\,\varphi^{(-)}(\eta',\xi')\varphi^{(+)}(\eta,\xi)\,|0,M\rangle \\ & + \quad \langle 0,M|\,\varphi^{(-)}(\eta,\xi)\varphi^{(+)}(\eta',\xi')\,|0,M\rangle]. \end{aligned} \tag{38}$$

Defining $v(\nu,\xi)$ and the complex conjugate $v^*(\nu,\xi)$ such that

$$\phi_\nu(\eta,\xi) = e^{-i\nu\eta}v(\nu,\xi) \tag{39}$$

and

$$\phi_\nu^*(\eta,\xi) = e^{i\nu\eta}v^*(\nu,\xi), \tag{40}$$

we have

$$R(E,\Delta T) = \int_{-\Delta T}^{\Delta T} d\tau e^{iE\tau} \int d\sigma d\nu \, \langle 0, M|\, b^{\dagger}(\sigma) b(\nu)\, |O, M\rangle$$
$$[\quad e^{i(\sigma\eta'-\nu\eta)} v^{*}(\sigma,\xi') v(\nu,\xi) + e^{i(\sigma\eta-\nu\eta')} v^{*}(\sigma,\xi) v(\nu,\xi')]. \tag{41}$$

Using the fact that

$$\langle 0, M|\, b^{\dagger}(\sigma) b(\nu)\, |O, M\rangle = \frac{\delta(\sigma-\nu)}{e^{2\pi\sigma}-1}, \tag{42}$$

we have to the rate

$$R(E,\xi',\xi) = \int_0^{\infty} d\nu \quad [\quad \frac{1}{e^{2\pi\nu}-1} v^{*}(\nu,\xi') v(\nu,\xi) \int_{-\Delta T}^{\Delta T} d\tau e^{i(E-\nu)\tau}$$
$$+ \quad \frac{1}{e^{2\pi\nu}-1} v^{*}(\nu,\xi) v(\nu,\xi') \int_{-\Delta T}^{\Delta T} d\tau e^{i(E+\nu)\tau}]. \tag{43}$$

Note the absence of the first term of eq.(37) in eq.(43). The spontaneous emission term does not appear since we adopted the RWA and disregard the commutator $[\varphi^{+}(\eta',\xi'), \varphi^{-}(\eta,\xi)]$ in the rate $R(E,\Delta T)$. The conclusion is that this term (which is independent of the state of the field) is responsible for the spontaneous emission processes (this radiative process is induced by vacuum fluctuations). It is well known that in a semiclassical theory where a quantum mechanical system interacts with a classical field only stimulated emission and absorption are predicted. The use of the RWA or the Glauber[19] correlation function possesses a simple physical interpretation of a classical field with classical correlation functions interacting with the detector.

As it has been pointed out by Milonni and Smith [20] and Ackerhalt, Knight and Eberly [21], there is a different approach to study radiative processes without using perturbation theory, but using the Heisenberg equations of motion. In this approach it is possible to obtain nonperturbative expressions for radiative processes where the radiation reaction appears in a very simple way: the part of the field due to the atom (detector) that drives the Dicke operators [22]. In this approach it is possible to identify the role of radiation reaction and vacuum fluctuations in spontaneous emission. We would like to stress the fact that the contribution of vacuum fluctuations and radiation reaction can be chosen arbitrarily, depending on the order of the Dicke and field operators. As it was discussed by Dalibard, Dupont-Roc and Cohen-Tannoudji [23], there is a preferred ordering in such a way that the vacuum fluctations and radiation reaction contribute equally to the spontaneous emission process. More recently this approach was developed by Audretsch and Muller and also Audretsch, Muller and Holzmann [24] to study the Unruh effect. These authors constructed the following picture of the Unruh effect. The effect of vacuum fluctuations is changed by the acceleration, although the contribution of radiation reaction is unaltered. Due to the modified vacuum fluctuation contribution, transition to an excited state becomes possible even in the vacuum. Note that different results appear in the literature. Barut and Dowling [26] concluded that the thermal response of the detector does not arise through an interaction with "real" particles, but from the spectrum of its self-field which has become altered by the change to a non-inertial frame. The discrepancy between above cited results comes from the arbitrariness in the operator ordering when one constructs the interaction Hamiltonian.

Going back to Eq.(43), it is possible to generalize the result for switching detectors. Let us rewrite the rate $R(E,\xi',\xi)$ introducing a switching function $g(\tau)$. Thus

$$\begin{aligned} r(E,\xi',\xi) = \int_0^\infty d\nu \quad & [\quad \frac{1}{e^{2\pi\nu}-1} v^*(\nu,\xi')v(\nu,\xi)\int_{-\infty}^{\infty} d\tau g(\tau) e^{i(E-\nu)\tau} \\ & + \quad \frac{1}{e^{2\pi\nu}-1} v^*(\nu,\xi)v(\nu,\xi')\int_{-\infty}^{\infty} d\tau g(\tau) e^{i(E+\nu)\tau}]. \end{aligned} \tag{44}$$

The Fourier transform of $g(\tau)$ is defined as

$$G(s) = \int_{-\infty}^{\infty} d\tau g(\tau) e^{-2\pi i\tau s}. \tag{45}$$

Therefore we can write eq.(41) as

$$\begin{aligned} r(E,\xi',\xi) = \int_0^\infty d\nu \quad & [\quad \frac{1}{e^{2\pi\nu}-1} v^*(\nu,\xi')v(\nu,\xi) G(\frac{\nu-E}{2\pi}) \\ & + \quad \frac{1}{e^{2\pi\nu}-1} v^*(\nu,\xi)v(\nu,\xi') G(\frac{\nu+E}{2\pi})]. \end{aligned} \tag{46}$$

Some interesting switching functions are:

$$g_1(\tau) = \begin{cases} 1 & |\tau| < \frac{1}{2}\theta \\ 0 & |\tau| > \frac{1}{2}\theta \end{cases} \tag{47}$$

$$g_2(\tau) = \begin{cases} 1 - \frac{|\tau|}{\theta} & |\tau| < \frac{1}{2}\theta \\ 0 & |\tau| > \frac{1}{2}\theta \end{cases} \tag{48}$$

$$g_3(\tau) = e^{-\pi(\tau/\theta)^2}, \tag{49}$$

with the respective Fourier transforms:

$$G_1(\nu-E) = \frac{\theta sin\pi(\nu-E)\theta}{\pi(\nu-E)\theta}, \tag{50}$$

$$G_2(\nu-E) = \theta \left(\frac{sin\pi(\nu-E)\theta}{\pi(\nu-E)\theta} \right)^2, \tag{51}$$

$$G_3(\nu-E) = \theta e^{-\pi((\nu-E)\theta)^2}. \tag{52}$$

Substituting any of the Fourier transform in eq.(46) we obtain the rate for a switching detector. Note that we introduce the switching function in the rate and not in the Lagrangian interaction term. Although this procedure is not correct from the mathematical point of view, it will reproduce eq.(33) and eq.(47) of ref.[25]. For broadband detectors this procedure is totally justified.

Finally note that we did all the calculations using the Rindler's time η, and the proper acceleration does not appear in Eq.(46). To obtain the correct expression we have to relate the Rindler time to the detector's proper time. Using this fact we have to do the replacement

$$\nu \longrightarrow 2\pi\alpha\nu$$

where α^{-1} is the proper acceleration of the detector. With this replacement we recovered the usual result.

3 Stress Tensor Detector

In this section we will repeat all the calculations that we did in the previous section, using the following interaction Lagrangean density:

$$L_{int} = \lambda_2 d^{\mu}(\tau)\partial_{\mu}\varphi(x(\tau)). \tag{53}$$

The calculations are exactly the same as we did. The only difference is that we have the components R_{00}, R_{01} and R_{11}, given respectively by:

$$\begin{aligned} R_{00}(E,\xi',\xi) = \int_0^{\infty} d\nu \quad [& \frac{\nu^2}{e^{2\pi\nu}-1} v^*(\nu,\xi')v(\nu,\xi)\int_{-\Delta T}^{\Delta T} d\tau e^{i(E-\nu)\tau} \\ + & \frac{\nu^2}{e^{2\pi\nu}-1} v^*(\nu,\xi)v(\nu,\xi')\int_{-\Delta T}^{\Delta T} d\tau e^{i(E+\nu)\tau}], \end{aligned} \tag{54}$$

$$\begin{aligned} R_{01}(E,\xi',\xi) = \int_0^{\infty} d\nu \quad [& \frac{\nu}{e^{2\pi\nu}-1} v^*(\nu,\xi')\frac{\partial}{\partial\xi}v(\nu,\xi)\int_{-\Delta T}^{\Delta T} d\tau e^{i(E-\nu)\tau} \\ + & \frac{\nu}{e^{2\pi\nu}-1}\frac{\partial}{\partial\xi} v^*(\nu,\xi)v(\nu,\xi')\int_{-\Delta T}^{\Delta T} d\tau e^{i(E+\nu)\tau}], \end{aligned} \tag{55}$$

$$\begin{aligned} R_{11}(E,\xi',\xi) = \int_0^{\infty} d\nu \quad [& \frac{1}{e^{2\pi\nu}-1}\frac{\partial}{\partial\xi'} v^*(\nu,\xi')\frac{\partial}{\partial\xi}v(\nu,\xi)\int_{-\Delta T}^{\Delta T} d\tau e^{i(E-\nu)\tau} \\ + & \frac{1}{e^{2\pi\nu}-1}\frac{\partial}{\partial\xi} v^*(\nu,\xi)\frac{\partial}{\partial\xi'}v(\nu,\xi')\int_{-\Delta T}^{\Delta T} d\tau e^{i(E+\nu)\tau}]. \end{aligned} \tag{56}$$

Note that $R_{\sigma\rho}$ is not a tensor but μ and ν refer to directions in the detector's rest frame. The response of the detector is orientation dependent. In a $D = 2$ dimensional spacetime, the diference between the monopole and the derivativelly coupled detector in the rate is the term ν^2, $\nu\frac{\partial}{\partial\xi}v(\nu,\xi)$ and $\frac{\partial}{\partial\xi'}v^*(\nu,\xi')\frac{\partial}{\partial\xi}v(\nu,\xi)$ in the components R_{00}, R_{01} and R_{11} respectivelly. In the case $D = 4$ in R_{0i} and R_{ij} will appear the angle between the spacelike direction defined by d^i and the spacelike component of the momentum vector.

Finally, it is important to compare our results with the conclusions obtained by Padmanabhan and Singh [27]. In the above mentioned article, these authors presented an example (a detector in a uniformly rotating frame) to conclude that the monopole detector is a fluctuometer [28]. In this case, although the expectation value of the number operator (in the rotating frame) in the Minkowski vacuum is zero, the power spectrum (the Fourier transform of the positive Wightman function) is not zero. They concluded that the power spectrum is not related to the existence of "real" particles. The uniformly accelerated detector goes to the excited state not because there is real Rindler particles in the Mikowski vacuum, but because the accelerating source supplies energy to the transition.

The key point of this discussion is given by Eq.(31) and Eq.(33). Assuming the RWA we disregard a vacuum piece, i.e., a contribution that is independent of the state of the field. Nevertheless we still have obtained a non-nule probability of excitation, since the Minkowski vacuum is filled by thermal Rindler particles. The Eq.(34) gives to the power spectrum contribution from the following physical process: absorption of a Rindler particle. In other words, assuming the RWA the power spectrum and the expectation value of the Rindler number operator in the Minkowski vacuum must be proportional. The inclusion of the commutator of Eq.(33) in Eq.(38) gives a vacuum piece contribution.

4 The asymptotic accelerated detector

The aim of this section is to discuss the following physical situation. How the monopole detector behaves if is traveling along a world line in such a way the detector is inertial in the infinite past and has a constant proper acceleration in the infinite future.

Let us consider the folowing transformation of coordinates between the inertial (t, x) and non-inertial coordinates (η, ξ),

$$t + x = \frac{2}{a} sinh a(\eta + \xi) \tag{57}$$

$$t - x = -\frac{1}{a} e^{a(\eta+\xi)} \tag{58}$$

This coordinate system was investigated by Kalnins and Miller[29], and by this reason we will call it as the Kalnins and Miller coordinate system.

The line element in this coordinate system can be written as:

$$ds^2 = (e^{2a\xi} + e^{-2a\eta})(d\eta^2 - d\xi^2) \tag{59}$$

The proper acceleration in the world line $\xi = cte$ is given by

$$lim_{\eta\to-\infty} \alpha(\eta, \xi_0) = 0 \tag{60}$$

and

$$lim_{\eta\to+\infty} \alpha(\eta, \xi_0) = a e^{-a\xi_0} = \alpha_\infty \tag{61}$$

The discussion of these issues can be found in ref.[30]. Eqs.(60) and (61) show that $\xi = cte$ is the world line of an uniformly accelerated observer. Note that the hypersurface $\eta = \eta_0$ is a Cauchy surface for the region $t - x < 0$.

The massive Klein-Gordon equation in the Kalnins-Miller manifold reads

$$[\frac{\partial^2}{\partial\eta^2} - \frac{\partial^2}{\partial\xi^2} + m^2(e^{-2a\eta} + e^{2a\xi})]\varphi(\eta,\xi) = 0. \tag{62}$$

The situation is very different from the Rindler coordinate system. The metric is not static and there is an ambiguity in defining positive and negative frequencies modes. A straightforward calculation reveals that there are two well behaved complete set $(\phi_\nu(x), \phi^*_\nu(x))$ and $(\varphi_\nu(x), \varphi^*_\nu(x))$, basis in the space of the solutions of the massive Klein-Gordon equation in the Kalnins-Miller manifold. These two complete sets are given by

$$\phi_\nu(\eta,\xi) = \frac{1}{2}\left(\frac{\nu(1-e^{-2\pi\nu})}{\pi a}\right)^{\frac{1}{2}} H^{(1)}_{i\nu}(\frac{m}{a}e^{-a\eta})K_{i\nu}(\frac{m}{a}e^{a\xi}) \tag{63}$$

$$\phi^*_\nu(\eta,\xi) = \frac{1}{2}\left(\frac{\nu(1-e^{-2\pi\nu})}{\pi a}\right)^{\frac{1}{2}} H^{(2)}_{i\nu}(\frac{m}{a}e^{-a\eta})K_{i\nu}(\frac{m}{a}e^{a\xi}) \tag{64}$$

and

$$\varphi_\nu(\eta,\xi) = (\frac{\nu}{\pi a})^{\frac{1}{2}} J_{i\nu}(\frac{m}{a}e^{-a\eta})K_{i\nu}(\frac{m}{a}e^{a\xi}) \tag{65}$$

$$\varphi^*_\nu(\eta,\xi) = (\frac{\nu}{\pi a})^{\frac{1}{2}} J_{-i\nu}(\frac{m}{a}e^{-a\eta})K_{i\nu}(\frac{m}{a}e^{a\xi}). \tag{66}$$

The modes given by eqs.(63) and (64) are the positive and the negative frequency modes in the infinite past and the modes given by eqs.(65) and (66) are the positive and negative frequency modes in the infinite future. By this reason we will call them "inertial" and "accelerated" modes respectively.

The Bogoliubov coefficients between the Minkowski modes and the inertial and accelerated modes are given respectively by :

$$|\beta_{\nu\mu}|^2_{in} = \frac{1}{2\pi^2\nu\epsilon sinh\pi\nu}(Re)^2(-i\nu)![2a(\epsilon-\mu)]^{i\nu} \tag{67}$$

and

$$|\beta_{\nu\mu}|^2_{ac} = \frac{1}{2\pi\epsilon a}\frac{1}{e^{2\pi\nu}-1} \tag{68}$$

where ϵ is the energy of the Minkowski modes. Using eq.(67) and (68) it is possible to obtain the transition rate of the detector. The problem is that it is not possible to make asymptotic measurements, but only for finite time $\Delta T = \eta_f - \eta_i$. For small ΔT it is possible to assume that the metric is static and we have two different outcomes if the measurement is made in the infinite past or in the infinite future. Using the Bogoliubov coefficients between the Minkovski modes and the $(\varphi^*(\eta,\xi), \varphi(\eta,\xi))$ and $(\phi^*(\eta,\xi), \phi(\eta,\xi))$ we obtain that if a measurement is made in the asymptotic future we have a similar result as Rindler's. Nevertheless if the measurement is made in the infinite past we obtain a non-expected result, i.e, the rate is not zero although the detector is inertial in this region. This situation deserves further investigations.

5 Conclusions

In this paper we discussed radiative processes of different detectors interacting with a massive scalar field. The probability of transition per unit proper time of the accelerated detector is obtained for the monopole Unruh-DeWitt detector and also for the derivatively coupled detector.

We used the RWA in order to simplify the calculations of a detector with nonconstant proper acceleration. We want to remark that the standard "photodetection" scheme is based on absorption of quantum of the field by the detector. Using first order perturbation theory the rate of excitation is proportional to the Fourier transform of a normal ordered product of the negative and positive parts of the field operator. Nevertheless, normal ordering with respect to the Rindler time is not normal ordered with respect to the Minkowski time. Therefore an absorption of a Rindler particle by the detector is view by an inertial observer by an emission and absorption of Minkowski particles, i.e., the annihilation and creation operators of Rindler particles contains a mixture of both positive and negative frequency parts of the field operator with respect to the inertial time.

Summarizing, the Minkowski vacuum $|0, M\rangle$ with respect to the annihilation operator of inertial particles is a squeezed state with respect to the annihilation operator of Rindler particles. In this way, the absorption of Rindler particles combine both absorption and emission of Minkowski particles in the excitation processes.

6 Appendix

In the appendix we will demonstrate that the excitation of the detector travelling in the right edge of the Rindler manifold is followed by a annihilation of a Rindler particle in the right edge of the Rindler manifold and the creation of a Rindler particle in its left edge.

For the sake of simplicity let us suppose that the mass of the quanta of the field is zero i.e. $m^2 = 0$ and define

$$^{R}u_k = (4\pi\nu)^{-1/2} e^{i(k\xi - \nu\eta)} \qquad inR, \tag{69}$$

$$^{R}u_k = 0 \qquad inL, \tag{70}$$

and

$$^{L}u_k = 0 \qquad inR, \tag{71}$$

$$^{L}u_k = (4\pi\nu)^{-1/2} e^{i(k\xi + \nu\eta)} \qquad inL. \tag{72}$$

Defining $^{L,R}v_\nu(\xi) = (4\pi\nu)^{-1/2} e^{ik\xi}$, it is possible to write the field operator as

$$\varphi(\eta, \xi) = \int_0^\infty d\nu \quad [\quad b^{(1)}(\nu)^L v(\nu, \xi) e^{i\nu\eta} + b^{(1)\dagger}(\nu)^L v^*(\nu, \xi) e^{-i\nu\eta}$$
$$+ \quad b^{(2)}(\nu)^R v(\nu, \xi) e^{-i\nu\eta} + b^{(2)\dagger}(\nu)^R v(\nu, \xi) e^{i\nu\eta}]. \tag{73}$$

Let us suppose that in $\eta_i = cte$ the state of the system is $|\mathcal{T}_i\rangle = |g\rangle \otimes |0, M\rangle$. The probability amplitude for the system to go to $|\mathcal{T}_f\rangle = |e\rangle \otimes |\Phi_f\rangle$ is:

$$\begin{aligned} A^1_{|g>\otimes|0,M>\rightarrow|e>\otimes|\Phi_f>} &= -i\lambda_1 m_{eg}(\eta_i)\int_{\eta i}^{\eta_f} d\eta \int_0^\infty d\nu[\\ e^{i(E-\nu)\eta}({}^L v^*(\nu,\xi)\,\langle\Phi_f|\, b^{(1)^\dagger}(\nu)\,|O,M\rangle &+ {}^R v(\nu,\xi)\,\langle\Phi_f|\, b^{(2)}(\nu)\,|O,M\rangle) \\ +e^{i(E+\nu)\eta}({}^L v(\nu,\xi)\,\langle\Phi_f|\, b^{(1)}(\nu)\,|O,M\rangle &+ {}^R v(\nu,\xi)\,\langle\Phi_f|\, b^{(2)^\dagger}(\nu)\,|O,M\rangle)]. \end{aligned} \tag{74}$$

In the amplitude we have two terms. The first is an absorption process in the right edge and an emission in the left edge of the Rindler manifold. The second is an absorption process in the left edge and an emission in the right edge of the Rindler manifold. Nevertheless since this second one contains the factor $e^{i(E+\nu)\eta}$ which rapidly oscillates, it gives a negligible contribution to the amplitude for $\tau >> \frac{1}{|E|}$.

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