Notas de Física - Volume V - nº 18

NOTE ON THE EFFECTS OF POLARIZATION OF AN INCIDENT PHOTON BEAM ON THE AZIMUTHAL ANGULAR DISTRIBUTION OF THE (γ,ν) REACTION PRODUCTS

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ABSTRACT

The purpose of this note is to point out some simple features in the azimuthal angular distribution in photonuclear reactions induced by a partially plane polarized photon beam. The use of such a beam can be made of special interest as a check to the electric dipole character of the photon absorption in the giant resonance region.

I. The angular distributions in photonuclear reactions.

The R-matrix formalism has been greatly discussed for the general case of a nuclear reaction $a + x \rightarrow y + b$, in which particle a collides with nucleus x [1]. After the collision, particle b is emitted in a direction defined by the polar angle θ and the azimu-

1. See for example, J.M.Blatt and L.C.Biedenharn, Rev.Mod.Phys. 24, 258, (1952) where a complete list of references can be found.

thal angle φ, the direction of reference being the axis of the beam and all quantities measured in the center-of-mass system. The recoil nucleus y is projected in the opposite direction. This formalism can be extended to include the case where a photon interacts with a nucleus [2]. Then the vector potencial $\vec{A}$ plays the rôle of the wave function for the incident particle If the incident photon beam travels along the z-axis and is polarized along the x-axis, the multipole expansion of the vector potencial $\vec{A}_z^x$ is given by

$$\vec{A}_z^x \propto \sum_{\pi} \sum_{P} \sum_{l=1} i^L (2L + 1)^{\frac{1}{2}} (i\ P)^{\sigma} \ |\pi, LP\rangle \qquad (1)$$

where π is an index giving the parity of the multipole and the connection between the $|\pi, LP\rangle$ symbol and the irreducible tensor $T^P_{L,\ell}$ of rank L and parity $(-)^\ell$ used by Rose [3] in the definition of the multipoles is easily established. σ is zero for magnetic multipoles and one for electric multipoles. The symbol $|LP\rangle$ shows that the photon may have angular momentum components about its direction of propagation equal to ± 1 only, since P is a polarization index that can assume these two values when the waves are left or right circularly polarized respectively. A basis of circularly polarized waves was used and the sum over P expresses the fact that a plane polarized wave can be constructed by superposing two circularly polarized waves [3].

The multipole angular momentum L and the spin of the bombarded nucleus $\vec{J}_x$ combine to give the total angular momentum of the sys

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2. See for example A. Agodi, Il Nuovo Cimento, Vol. V, 21 (1957) where a complete list of references can be found.

3. M.E. Rose - Multipole Fields (J. Wiley and Sons, Inc., New York, 1955).

tem $\vec{J}$. The spin of the emitted photoparticle is $\vec{s}$, the spin of the recoil nucleus is $\vec{J_y}$ and the orbital angular momentum in the final state is $\vec{\ell}$. By vector coupling $\vec{\ell}$ with $\vec{s}$ we get $\vec{j}$ and repeating the operation we combine $\vec{j}$ with $\vec{J_y}$ to get the same total angular momentum $\vec{J}$.

From general considerations it can be easily shown that the differential cross section $d\sigma$ for the emission of a particle within the solid angle element $d\Omega$ defined by the pair of angles θ, φ in a photoreaction induced by a polarized beam on an unpolarized target (sum over final spin performed), is a linear combination of functions

$$F^{L\pi,\ L'\pi'}_{\ell\ell'\ell'',jj',s} = (-)^{1-L+j'}\ Z(\ell' j' \ell j;\ s\,\ell'')\ \Big\{ i^{(L-L'-\ell'')+(\sigma-\sigma')} \left[1 + (-)^{L+L'-\ell''+(\sigma-\sigma')}\right] (L'L,\ 1\ -1|\,\ell''0)\,\mathcal{P}^{0}_{\ell''}(\cos\theta) + $$
$$+\ i^{(L-L'-\ell'')-(\sigma+\sigma)} \left[\exp(2i\varphi) + (-)^{L+L'-\ell''-(\sigma+\sigma')}\exp(-2i\varphi)\right](L'L,11|\ell''2)\ \mathcal{P}^{2}_{\ell''}(\cos\theta)\Big\}$$

where the $\mathcal{P}_{\ell}^{m}(\cos\theta)$ are the normalized Legendre's associated function of the first kind and $(\ell_1\ell_2, m_1m_2|\ell_3m_3)$ is a Clebsh-Gordan coefficient for vector coupling. From the definition of the Z-coefficient it is clear that $F = 0$ unless $\ell + \ell' + \ell'' =$ even, as a consequence of the conservation of parity.

For the analysis of the θ-distribution it is fundamental to consider separately the case where only a multipole contributes to the transition. In this case ℓ'' will be even and there will be symmetry about $\theta = \pi/2$. Moreover, a detailed knowledge of the mechanism of the reaction is not necessary to get the shape of the angular distribution, since only one term of the linear combination contributes. When radiation is unpolarized we must average over φ and the shapes of angular distribution for electric and magnetic multipoles become indistinguishable since the averaging operation removes the second term of the function F where there is the phase factor $(i)^{-2\sigma} = (-)^{\sigma}$. When more than one multipole contributes to the transition then we lose the symmetry about $\theta = \pi/2$ since odd powers of $\cos\theta$ can appear. Now the shape of the angular distribution depends on the R_{-} reduced matrix elements and it is necessary to use some model to describe the photoreaction.

In contrast, the φ-distribution is always of a simple form, proportional to $1 + \eta \cos 2\varphi$ (or proportional to $1 + \mu \cos^2 \varphi$), since the φ-independent term only vanishes when either $\sigma = \sigma'$ and $L + L' - \ell''$ is odd or $\sigma \neq \sigma'$ and $L + L' - \ell''$ is even, and the dependence in the most simple cases is in $\cos 2\varphi$ as can be seen in the following table:

			Parity of $\ell + \ell'$ (Parity of ℓ'')	Parity of $L + L' - \ell''$	Parity of $L + L' - \ell'' - (\sigma + \sigma')$
Only one multipole involved with transition	$\sigma = \sigma'$		even	even	even
Two multipoles involved in the transaction	$\sigma = \sigma'$	E+, E+ E−, E− M+, M+ M−, M−	even	even	even
		E+, E− M+, M−	odd	even	even
	$\sigma \neq \sigma'$	E+, M+ E−, M−	even	odd	even
		E−, M+ E+, M−	odd	odd	even

II) The use of a polarized photon beam.

It is well known that the most striking feature of the low energy photonuclear reactions is the giant resonance. There is good confidence that the reaction in the giant resonance region proceeds mostly through electric dipole absorption. The main evidences are the angular distribution of the fast photoparticles and the fact that the integrated cross section over the giant resonance nearly exhausts the sum rule for electric dipole radiation.

The emitted photoparticles may be classified in two groups. The first one includes all particles in the energy interval until about 3 Mev. Particles in this group (in most cases neutrons), are emitted by an evaporation process and present an energy distribution of the Maxwell type and a generally isotropic angular distribution. Most particles belong to this group. However, there are many particles emitted with energy greater than 4 Mev in spite of the vanishingly small probability of a compound nucleus, in which the excitation energy is about 20 Mev, evaporating particles into this energy interval. A direct process was proposed by Courant [4] to explain this second group. In a direct interaction the nucleus has no time to "forget" the way by which radiation was absorbed and this is reflected in the angular distribution of the fast photoparticles.

When we observe the high energy tail of the photoemitted neutrons (for the sake of simplicity we will consider only (γ ,n) reactions), the θ- angular distribution is generally expressed as A + + B $\sin^2 \theta$, just the angular distribution expected for electric di

4. E. D. Courant , Phys. Rev. 82, 703, (1951).

pole absorption.

When a polarized photon beam is used, the φ-angular distribution can be studied besides the θ- distribution. If the fast neutrons are selected their angular distribution must be of the form $A + 2B \sin^2\theta \cos^2\varphi$, as expected for electric dipole transition. The factor 2 was chosen to cancel the average value of $\cos^2\varphi$ when the beam is unpolarized.

A polarized photon beam can be obtained from some nuclear reactions or even from a betatron. It is well known that the bremsstrahlung photons produced when an electron strikes a target, are polarized. The unpolarization of the bremsstrahlung beam from a betatron is a consequence of the thickness of the target. Multiple scattering of electrons before bremsstrahlung occurs becomes more serious due to the fact that electrons traverse the target many times. An especial arrangement can be made in such way that the accelerated electrons are allowed to traverse the betatron target only once. With this arrangement much is lost in intensity but the depolarization is greatly reduced. On the other hand, to observe any effect of polarization it is necessary to leave the axis of the photon distribution (where the intensity is the greatest), and to work in some direction with $\beta \neq 0$ (β is the angle between the direction of the electron beam and the direction of the photon, i.e., the angle of observation of the photons). The best choice depends on how much intensity is to be lost and how high is the desired polarization. Miller[5] gives curves with which it is possible to plot po-

5. J. Miller, Rapport C.E.A. nº 6559. Centre d'Etudes Nucleaires de Saclay.

larization against the photon energy with the observation angle β and the thickness of the target, m, as parameters, that is to say, to construct the polarization function $\pi(E\gamma)$. Therefore it is possible to decompose the energy spectrum of the betatron $N(E_\gamma)$ in a polarized part $P(E_\gamma) \propto N(E_\gamma)\pi(E_\gamma)$ and an unpolarized part $U(E_\gamma)$ for some prescribed pair of parameters β and m.

When a target is bombarded by this partially polarized beam, each part of it acts separately and produces a number n_p and n_u of fast photoparticles respectively. The fraction of photoparticles emitted by the completely polarized part of the incident beam is $\rho = n_p/(n_p + n_u)$ and the angular distribution of the fast photoneutrons becomes

$$I(\theta, \varphi) \propto 1 + [2\lambda_p \rho \cos^2\varphi + (1-\rho)\lambda_u] \sin^2\theta \qquad (3)$$

The unpolarized beam $U(E_\gamma)$ produces direct photoneutrons with an angular distribution proportional to $1 + \lambda_u \sin^2\theta$. An unpolarized beam with the same energy distribution as $P(E_\gamma)$ would produce direct photoneutrons with an angular distribution proportional to

$$1 + 2\lambda_p \sin^2\theta \langle\cos^2\varphi\rangle_{Av} \equiv 1 + \lambda_p \sin^2\theta .$$

Since it is impossible to obtain experimentally an unpolarized beam with the same energy spectrum $P(E_\gamma)$ we cannot determine λ_p by a direct measurement. Nevertheless, it is easily determined by measuring the θ - angular distribution in two different φ-planes:

$$\begin{aligned} I(\theta, \varphi = 0) &\propto 1 + \lambda_0 \sin^2\theta \\ I(\theta, \varphi = \pi/2) &\propto 1 + \lambda_{\pi/2} \end{aligned} \qquad (4)$$

where $\lambda_0 = 2\lambda_p\rho + (1-\rho)\lambda_u$ and $\lambda_{\pi/2} = (1-\rho)\lambda_u$

then $\lambda_u = \frac{\lambda_{\pi/2}}{1-\rho}$ and $\lambda_p = \frac{\lambda_o - \lambda_{\pi/2}}{2\rho}$

If the degree of polarization is not too small a considerable difference is to be expected in the parameter λ of the Θ-distribution for the planes $\varphi = 0$ and $\varphi = \pi/2$. Indeed, being $\frac{\lambda_o}{\lambda_{\pi/2}} = 1 + \frac{2\rho}{1-\rho}\frac{\lambda_p}{\lambda_u}$ and since in most usual experimental circumstances $\lambda_p/\lambda \sim 1$, for ρ as small as .20, $\frac{\lambda_o}{\lambda_{\pi/2}}$ will be as large as 1.5.

When electric dipole absorption takes place, the fast photoparticles present a tendency to be emitted in a direction parallel to the polarization vector. An experiment can be projected to measure the ratio between the number of particles projected in this direction and the number of particles projected in a direction orthogonal to it. In the plane $\Theta = \pi/2$ the expected ratio is

$$\frac{\parallel}{\perp}(\Theta = \pi_2) = 1 + \frac{2\lambda_p \rho}{1+(1-\rho)\lambda_u} = \frac{1+\lambda_o}{1+\lambda_{\pi/2}} \qquad (6)$$

If we measure in the same experiment the three quantities

$$\frac{\parallel}{\perp}(\Theta = \pi/2),\ \lambda_o \text{ and } \lambda_{\pi/2}$$

equation (6) is a good check to the electric dipole character of the transition. When some significant deviation is observed it is an indication that multipolarities other than electric dipole contribute to the transition even when this is not indicated clearly by Θ-distribution alone.

III) The calculation of ρ.

Let us now consider how to calculate ρ defined as the fraction of photoemitted neutrons due to the completely polarized part of the incident beam, as seen by a neutron-counter. For a crude computation we use:

$$\rho = \frac{\displaystyle\int_{(E_\alpha)_0}^{(E_\alpha)_{max}} \sigma(E_\alpha)\, e^{-\frac{R}{\Lambda(E_\alpha)}}\, T(E_\alpha)\, dE_\alpha \int_{(E_\gamma)_0}^{(E_\gamma)_{max}} N(E_\gamma)\,\pi(E_\gamma)\,\sqrt{E_\alpha - E_\gamma}\,\sigma_{\gamma_n}(E_\gamma)\, dE_\gamma}{\displaystyle\int_{(E_\alpha)_0}^{(E_\alpha)_{max}} \sigma(E_\alpha)\, e^{-\frac{R}{\Lambda(E_\alpha)}}\, T(E_\alpha)\, dE_\alpha \int_{(E_\gamma)_0}^{(E_\gamma)_{max}} N(E_\gamma)\,\sqrt{E_\alpha - E_\gamma}\,\sigma_{\gamma_n}(E_\gamma)\, dE_\gamma} \qquad (7)$$

where we consider a Fermi distribution of neutrons inside the nucleus; $\sigma_{\gamma_n}(E_\gamma)$ is the cross section for absorption of electric dipole radiation by a neutron inside the nucleus; $\Lambda(E_\alpha)$ is the mean free path for neutrons in the nuclear matter; R is the nuclear radius (the mean distance travelled by an "excited" neutron before reaching the nuclear surface); $T(E_\alpha)$ is the transmission coefficient for a step barrier where there is a sudden modification in the mass value and $\sigma(E_\alpha)$ is the curve of sensibility of the neutron counter. The limits of integration are shown in figure 1: E_F is the Fermi energy, equal to 58 Mev for an effective mass 0.78 times the neutron mass, S is the separation energy (about 8 Mev) and ϵ_0 is the energy above which the neutrons are considered directly emitted (about 4 Mev). The energy of the emitted neutrons is

$$\epsilon = E_\alpha - E_F - S \simeq E_\alpha - 66 \text{ Mev}.$$

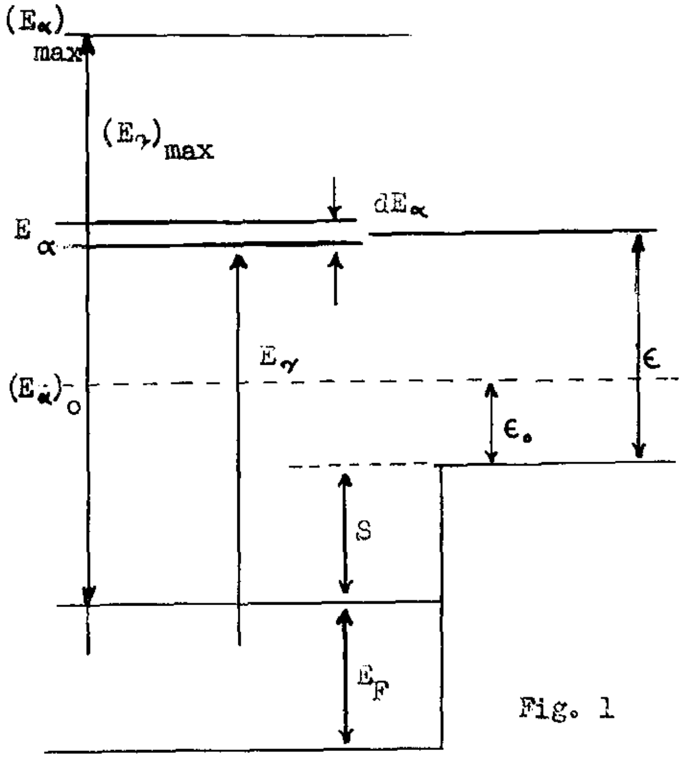

Fig. 1

A crude estimation of φ was performed with the following data: i) the Shiff's bremsstrahlung spectrum for maximum electron energy equal to 24 Mev[6]; ii) the Miller's curves [5] for $\beta = 2{,}875 \times \times 10^{-2}$ rd and $m = 4.50 \times 10^{-2} \frac{Z}{\sqrt{A}}\, t\ (mg/cm^2) = 0.6$; iii) since the ρ value is very insensitive to the form of the cross section $\sigma_{\gamma_n}(E_\gamma)$ we have used an expression of the same type as the cross-section given by Courant for the direct photoprocess [4]; iv) $\Lambda(E_\alpha)$ is related to the imaginary part of the complex potential (average over distances inside the nucleus) as given by Gomes [7]; v) the neutron counter is the reaction $Si^{28}(n, p)$ a very commonly used method of detection of photoneutrons [8].

The result obtained was $\rho = 0{,}13$ and the mean energies of the neutrons produced by the polarized part of the beam and by the unpolarized part were respectively $\bar{\epsilon}_p = 7.7$ Mev and $\bar{\epsilon}_u = 8.4$ Mev. Zatsepina et al.,[9] have shown conclusively that the parameter λ depends on the neutron energy (as clearly indicated by Wilkinson's theory[10]). However, the mean energies $\bar{\epsilon}_p$ and $\bar{\epsilon}_u$ being not too different and the peaks of the neutron spectra being very broad (half width about 4 Mev) we would expect $\lambda_p \sim \lambda_u$. (See figure 2).

A preliminary experiment was performed by Goldemberg et al.,[11]

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6. L.Schiff, Phys.Rev. 83, 252,(1951); L.Katz and A.G.W.Cameron, Canad. J. Phys. 29, 518, (1951)
7. L.C.Gomes, Notas de Física,Vol V,Nº 8, (1959)
8. See for example, F.Ferrero,L.Gonella, A.O.Hanson, R.Melvano and C.Tribuno. Il Nuovo Cim. Vol V, 242,(1957).
9. G.N.Zatsepina, L.E.Lazareva and A.N.Pospelov,Soviet Physics JETP Vol.5,21 (1957).
10. D.H. Wilkinson - Physica, Vol XXII, 1039 (1956).
11. J.Goldemberg, P.Dyal and J.O'Connel, The azimuthal angular distribution of fast photoneutrons from bismuth produced by plane polarized X-rays (1958). Unpublished paper privately communicated to us by Dr. Goldemberg.

in the same experimental conditions as above mentioned, to measure the ratio $\frac{\parallel}{\perp}$ ($\theta = \pi/2$). For a bismuth target with four Si-detectors disposed in a cross in the plane $\theta = \pi/2$, the result was 0.97 ± 0.06. The θ - angular distribution was measured in a plane containing the photon beam and perpendicular to the direction of polarization at the off-center position of the beam and the result was $\lambda_{\pi/2} \sim 1$ in accordance with previously published results. Therefore the expected ratio $\frac{\parallel}{\perp}$ ($\theta = \pi/2$) was about 1.15 (within the approximation $\lambda_0 = \lambda_{\pi/2}$). The low value obtained for this ratio, an evidence of a possible isotropy for the φ-distribution, is not sufficiently serious to be considered as inconsistent with the theoretical estimate, especially when we consider that possible corrections tend to decrease the calculated value. The chief corrections are: i) the effect of magnetic dipole transitions; if the θ-distributions are of the same type in the electric and magnetic dipole transitions, viz. $A + B \sin^2 \theta$, the μ-parameter of the φ-distribution have opposite signs and the anisotropies tend to cancel each other (electric quadrupole transitions are important only for protons due to effective charge considerations); ii) a neutron can be excited to a high energy level by absorbing a photon and then, after a collision, be emitted with an energy still greater than ϵ_0. Writing equation (7), it was considered that all neutrons that collide before reaching the nuclear surface are removed from the high energy group of photoneutrons as well as they are not able to produce recoil neutrons in this group. A realistic appreciation of this fact becomes very important when the photon beam is monochromatic but it is not very important for the betatron spectrum [12]. A very crude estimate has indicated that for a

12. We are indebted to Dr. Gomes for pointing out this fact to us.

maximum photon energy of 24 Mev that collide before being emitted is inferior to 5 %.

It would be interesting however, to perform a new experiment where it would be possible to measure the θ-distribution for $\varphi = 0$ and $\varphi = \pi/2$, thus allowing the determination of λ_p and λ_u, as well as to measure the ratio $\frac{\parallel}{\perp}$. This would be a good check for the electric dipole character of the photon absorption in the resonance region of bismuth.

IV) Conclusions

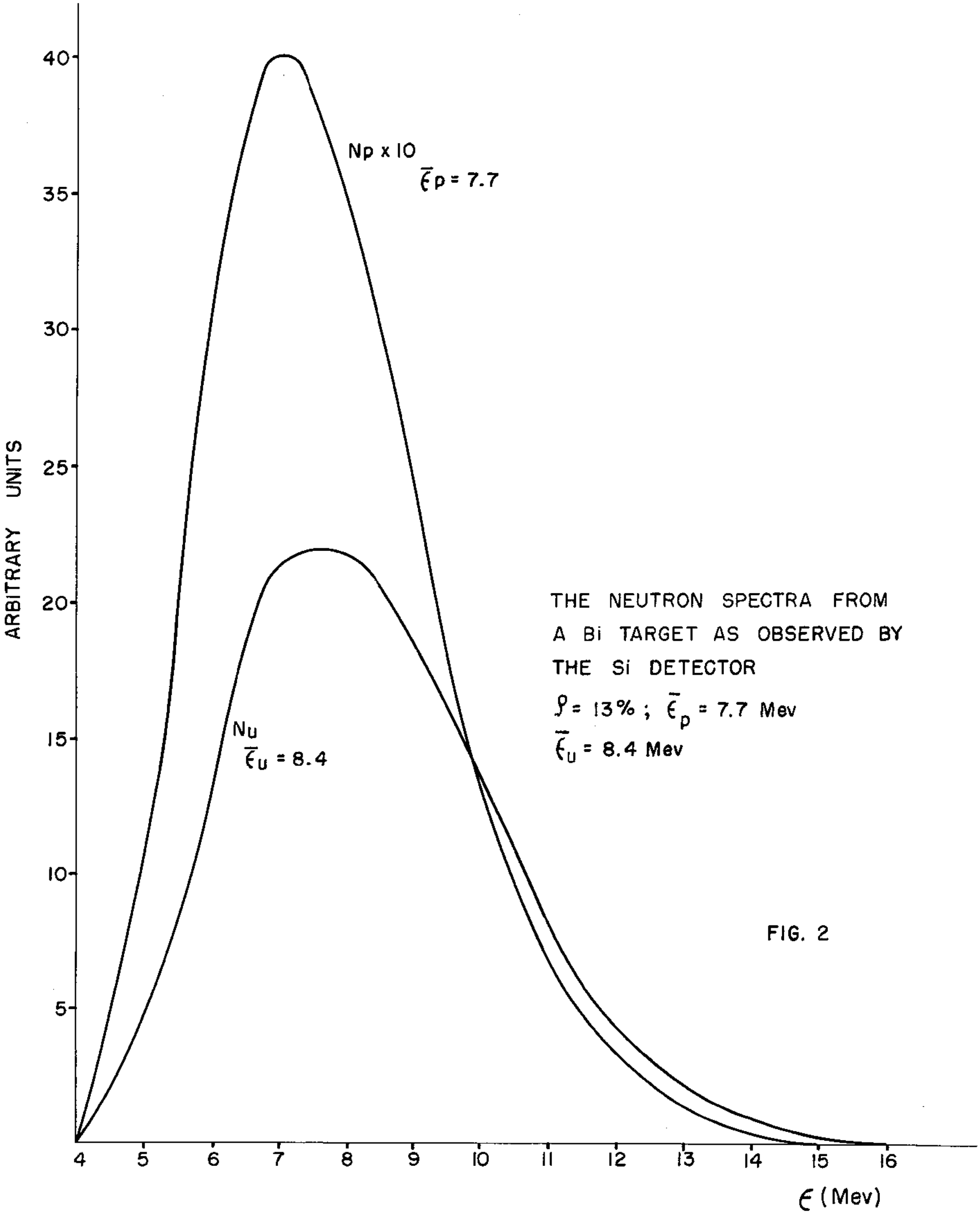
As a conclusion, it can be said that: i) the detection of a supposed linear polarization of the photons can be checked by the examination of the azimuthal angular distribution of the products of the photoreactions; necessarily an anisotropy will be present; ii) whatever may be the absorbed multipole and even when more than one multipole contributes to the transition, the φ-distribution has the simple form $1 + \mu \cos^2 \varphi$ with a polarized photon beam. For electric dipole adsorption and a partially polarized photon beam

$$\mu = \frac{2 \lambda_p \rho}{1 + (1 - \rho)\lambda_u} = \frac{\lambda_0 - \lambda_{\pi/2}}{1 + \lambda_{\pi/2}} \text{ with } \rho \text{ given by (7); iii)}$$

nevertheless the φ-distribution alone will not be sufficient to determine the character of the absorbed multipole. If we obtain in the same experiment the θ and φ-distributions, equation (6) being satisfied the transition is exclusively through electric dipole; any significant deviation indicating the participation of other multipolarities in the photoreaction for the detection of which the θ-distribution is not sufficient information.

V) Acknowledgments

The author wishes to express his thanks to Drs. J. Goldemberg and L. C. Gomes for many valuable discussions and suggestions. We are especially grateful to Dr. Goldemberg for complete information on an unpublished work done by him, O'Connell, and Dyal at the University of Illinois.



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ϵ (Mev)
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$\bar{\epsilon}_p$ = 7.7
Nu
$\bar{\epsilon}_u$ = 8.4
THE NEUTRON SPECTRA FROM
A Bi TARGET AS OBSERVED BY
THE Si DETECTOR
ρ = 13% ; $\bar{\epsilon}_p$ = 7.7 Mev
$\bar{\epsilon}_u$ = 8.4 Mev

FIG. 2