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Exact Scale Invariance of the BF-Yang-Mills Theory in Three Dimensions

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Abstract

The "extended" BF-Yang-Mills theory in 3 dimensions, which contains a minimally coupled scalar field, is shown to be ultraviolet finite. It obeys a trivial Callan-Symanzik equation, with all β-functions and anomalous dimensions vanishing. The proof is based on an anomaly-free trace identity valid to all orders of perturbation theory.

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1 Introduction

Original inspiration for topological gauge field theories (TGFT) came from mathematics. Due to their peculiar properties, they are at the origin of a large number of interesting results and the object of continuous investigations.

The topological Yang-Mills theory and Chern-Simons theory are examples of two distinct classes of TGFT which are sometimes classified as being of "Witten type" and of "Schwarz type", respectively (see [1] for a general review and references).

Besides the Chern-Simons there exists another TGFT of Schwarz type, namely the topological BF theory. The latter represents a natural generalization of the Chern-Simons theory since it can be defined on manifolds of any dimensions whereas a Chern-Simons action exists only in odd-dimensional space-times. Moreover, the Lagrangian of the BF theory always contains the quadratic terms needed for defining a quantum theory, whereas a Chern-Simons action shows this feature only in three dimensions.

On the other hand, the Yang-Mills gauge theory has recently been re-interpreted as a deformation of a pure BF theory [2, 3, 4, 5, 6]. In its "extended" version [5], this BF-Yang-Mills (BFYM) theory is described by the action

$$\Sigma_{\mathrm{BFYM}} = \int d^n x \; \left\{ \varepsilon^{\mu_1 \cdots \mu_n} B^a_{\mu_1 \cdots \mu_{n-2}} F^a_{\mu_{n-1}\mu_n} + \left(B^a_{\mu_1 \cdots \mu_{n-2}} + D_{[\mu_1} \eta^a_{\mu_2 \cdots \mu_{n-2}]} \right)^2 \right\} \;, \tag{1.1}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ is the Yang-Mills field strength, $D_\mu \eta^a_{\mu_1 \cdots \mu_{n-3}} \equiv \partial_\mu \eta^a_{\mu_1 \cdots \mu_{n-3}} + g f^{abc} A^b_\mu \eta^c_{\mu_1 \cdots \mu_{n-3}}$ is the covariant derivative, $B^a_{[\mu_1 \cdots \mu_{n-2}]}$ an auxiliary field and $\eta^a_{[\mu_1 \cdots \mu_{n-3}]}$ a pure gauge field.

The action (1.1) possesses two kinds of symmetries: the standard gauge symmetry

$$\delta_\alpha A^a_\mu = -D_\mu \alpha^a, \quad \delta_\alpha B^a_{\mu_1 \cdots \mu_{n-2}} = -g f^{abc} B^b_{\mu_1 \cdots \mu_{n-2}} \alpha^c, \quad \delta_\alpha \eta_{\mu_1 \cdots \mu_{n-3}} = -g f^{abc} \eta^b_{\mu_1 \cdots \mu_{n-3}} \alpha^c \;, \tag{1.2}$$

and, thanks to the presence of the field η, the "topological" symmetry

$$\delta_\Lambda A^a_\mu = 0, \quad \delta_\Lambda B^a_{\mu_1 \cdots \mu_{n-2}} = -D_{[\mu_1} \Lambda^a_{\mu_2 \cdots \mu_{n-2}]}, \quad \delta_\Lambda \eta_{\mu_1 \cdots \mu_{n-3}} = \Lambda^a_{\mu_1 \cdots \mu_{n-3}} \;. \tag{1.3}$$

It is very easy to verify that if one uses the gauge condition $\eta^a_{\mu_1 \cdots \mu_{n-3}} = 0$ and the equation of motion $\delta \Sigma_{\mathrm{BFYM}} / \delta B^a_{\mu_1 \cdots \mu_{n-2}} = 0$, one recovers the usual form of the Yang-Mills action.

The quantum equivalence between the pure Yang-Mills theory and the BFYM theory has been discussed in three dimensions by [5], and in four dimension by [6, 7]. In both cases this question has been answered positively, thus confirming the interpretation of the pure YM theory as a perturbation of a topological theory [8].

The most peculiar property that topological field theories exihibit is their ultraviolet finiteness [9, 10, 11, 12]. This property relies on the existence of a topological vector supersymmetry [1, 13, 14], whose origin is manifest in the case one chooses the Landau gauge. Actually, such a symmetry turns out to be a general feature of the topological theories, and its explicit realization is extremely simple in the Landau gauge. Moreover, topological vector supersymmetry plays a crucial role in the construction of the explicit solutions to the BRS cohomology modulo d, giving therefore a systematic classification of all possible anomalies and physically relevant invariant conterterms [15].

However, there is no such vector supersymmetry in the present case since the BFYM theory is not a topological one *stricto sensu*: in particular, the energy-momentum tensor is not BRS-exact [5], a fact which is incompatible with vector supersymmetry [14].

The main purpose of this paper is, starting from the established renormalizability of the BFYM theory in three dimensions [5], to give a general proof of its ultraviolet finiteness, in all orders of perturbation theory. The proof will use the techniques developed in [16]. It is based on the validity of an anomaly-free trace identity for the energy-momentum tensor – i.e. of exact scale invariance. Actually, we believe that this technique is particularly suitable in a situation such as our's, where all the power of the topological vector supersymmetry is lost.

Finally, it should be stressed that the superrenormalizability of the model shows to be determinant in the proof of the exact scale invariance of the BFYM theory in three dimensions. Indeed, it leaves the Chern-Simons action – which has been introduced in order to regularize the infrared singularity of the theory – as the only possible invariant counterterm. Eventually, the fact that the integrand of the Chern-Simons action is not gauge invariant leads to the anomaly-free trace identity.

2 The BF-Yang-Mills Theory in Curved Space-Time

Following [16], we write the BF-Yang-Mills action on a curved manifold, as long as its topology remains that of flat $\mathbf{R}^3$, with asymptotically vanishing curvature. It is the latter two restrictions which allow us to use the general results of renormalization theory, established in flat space.

The classical BF-Yang-Mills theory in a curved manifold $\mathcal{M}$ is defined by the action

$$\Sigma_{\mathrm{BFYM}} = \int d^3x \, \left\{ \varepsilon^{\mu\nu\rho} B^a_\mu F^a_{\nu\rho} + e \, \left(B^a_\mu + D_\mu \eta^a \right)^2 \right\} \ , \tag{2.1}$$

where e denotes the determinant of the dreibein field e^m_μ. All fields lie in the adjoint representation of the gauge group, a general compact Lie group with algebra $\left[T^a, \, T^b\right] = i \, f^{abc} T^c$.

Following [5], we add to the action (2.1) a – parity breaking – Chern-Simons term [18]

$$\Sigma_{\mathrm{CS}} = \int d^3x \, m \, \varepsilon^{\mu\nu\rho} \left(A^a_\mu \partial_\nu A^a_\rho + \frac{g}{3} f^{abc} A^a_\mu A^b_\nu A^c_\rho \right) \ , \tag{2.2}$$

where m is a topological mass. This will safe the theory from IR divergences. The zero limit, which formally recovers the massless theory, is argued to be smooth for resummed quantities [5]. Moreover, the Chern-Simons term does not change neither the algebraic structure nor the form of the operators entering the algebraic analysis.

In the Landau gauge, the gauge-fixing term $\Sigma_{\rm gf}$ reads

$$\Sigma_{\rm gf} = -\, s \int d^3x \; e \, g^{\mu\nu} \left(\partial_\mu \bar{c}_a A^a_\nu + \partial_\mu \bar{\phi}_a B^a_\nu \right) \ . \tag{2.3}$$

The action

$$\Sigma_{\rm BFYM} + \Sigma_{\rm CS} + \Sigma_{\rm gf} \ , \tag{2.4}$$

is invariant under the nilpotent s-operator defined as follows:

$$\begin{aligned}
sA^a_\mu &= -D_\mu c^a \ , \\
sc^a &= \frac{1}{2} g \, f^{abc} c^b c^c \ , \\
s\bar{c}^a &= b^a \, , \\
sb^a &= 0 \ , \\
sB^a_\mu &= -D_\mu \phi^a - g \, f^{abc} B^b_\mu c^c \ , \\
s\eta^a &= \phi^a - g \, f^{abc} \eta^b c^c \ , \\
s\phi^a &= g \, f^{abc} \phi^b c^c \ , \\
s\bar{\phi}^a &= h^a \, , \\
sh^a &= 0 \, .
\end{aligned} \tag{2.5}$$

In order to express the BRS invariance as a Slavnov-Taylor identity we couple the nonlinear variations of the quantum fields to antifields (or external sources) $A^{*\mu}_a$, $B^{*\mu}_a$, c^*_a, η^*_a, ϕ^*_a:

$$\Sigma_{\rm ext} = \int d^3x \sum_{\Phi = A^a_\mu, B^a_\mu, c^a, \eta^a, \phi^a} \Phi^* s \Phi \, . \tag{2.6}$$

The total action,

$$\Sigma = \Sigma_{\rm BFYM} + \Sigma_{\rm CS} + \Sigma_{\rm gf} + \Sigma_{\rm ext} \ , \tag{2.7}$$

obeys the Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) \; = \; \int d^3x \sum_{\Phi = A^a_\mu, B^a_\mu, c^a, \eta^a, \phi^a} \frac{\delta \Sigma}{\delta \Phi^*} \frac{\delta \Sigma}{\delta \Phi} + b \, \Sigma = 0 \, , \quad \text{with} \;\; b = \int d^3x \left(b^a \, \frac{\delta}{\delta \bar{c}^a} + h^a \, \frac{\delta}{\delta \bar{\phi}^a} \right) . \tag{2.8}$$

For later use, we introduce the linearized Slavnov-Taylor operator

$$\mathcal{B}_\Sigma \; = \; \int d^3x \sum_{\Phi = A^a_\mu, B^a_\mu, c^a, \eta^a, \phi^a} \left(\frac{\delta \Sigma}{\delta \Phi^*} \frac{\delta}{\delta \Phi} + \frac{\delta \Sigma}{\delta \Phi} \, \frac{\delta}{\delta \Phi^*} \right) + b \, . \tag{2.9}$$

$\mathcal{S}$ and $\mathcal{B}$ obey the algebraic identity

$$\mathcal{B}_{\mathcal{F}} \mathcal{B}_{\mathcal{F}} \mathcal{F}' + (\mathcal{B}_{\mathcal{F}'} - b) \, \mathcal{S}(\mathcal{F}) = 0 \, , \tag{2.10}$$

[5] According to [19] the observables shoud be independent of the mass m.

$\mathcal{F}$ and $\mathcal{F}'$ denoting arbitrary functionals of ghost number zero. From the latter follows

$$\mathcal{B}_{\mathcal{F}}\,\mathcal{S}(\mathcal{F}) = 0 \ , \quad \forall\,\mathcal{F} \quad , \tag{2.11}$$

$$\left(\mathcal{B}_{\mathcal{F}}\right)^2 = 0 \quad \text{if} \quad \mathcal{S}(\mathcal{F}) = 0\,. \tag{2.12}$$

2.1 Renormalizability

The renormalizability of the theory (absence of anomalies and stability under the quantum corrections) follows[6] from the cohomological study of [5, 6]. In these papers it is shown indeed that the BRS cohomology – in the presence of the antifields – is empty for what concerns the anomalies, and corresponds to a possible renormalization of the physical parameters g, m and of the field amplitudes, for what concerns the counterterms[7]. Hence the generating functional

$$\Gamma = \Sigma + O(\hbar)$$

of the vertex functions (one particle-irreducible Green functions), considered as a formal power series in $\hbar$, obeys the same Slavnov-Taylor identity (2.8) as the classical action Σ:

$$\mathcal{S}\left(\Gamma\right) = 0 \ . \tag{2.13}$$

In addition to the Slavnov-Taylor identity, the vertex functional Γ obeys the following additional constraints[8]:

- the Ward identity for the diffeomorphisms

$$\mathcal{W}_{\rm diff}\Gamma = \int d^3x \ \sum_{\Phi}\delta^{(\varepsilon)}_{\rm diff}\Phi\frac{\delta\Gamma}{\delta\Phi} = 0\,, \tag{2.14}$$

  with

$$\begin{aligned}
\delta^{(\varepsilon)}_{\rm diff}F_\mu &= \varepsilon^\lambda\partial_\lambda F_\mu + \left(\partial_\mu\varepsilon^\lambda\right)F_\lambda\,, \quad F_\mu = \left(A^a_\mu,\ B^a_\mu,\ e^m_\mu\right)\,, \\
\delta^{(\varepsilon)}_{\rm diff}\Phi &= \varepsilon^\lambda\partial_\lambda\Phi\,, \quad \Phi = \left(b^a,\ c^a,\ \bar c^a,\ h^a,\ \phi^a,\ \bar\phi^a,\ \eta^a\right)\,, \\
\delta^{(\varepsilon)}_{\rm diff}F^{*\mu} &= \partial_\lambda\left(\varepsilon^\lambda F^{*\mu}\right) - \left(\partial_\lambda\varepsilon^\mu\right)F^{*\lambda}\,, \quad F^{*\mu} = \left(A^{*\mu}_a,\ B^{*\mu}_a\right)\,, \\
\delta^{(\varepsilon)}_{\rm diff}\Phi^* &= \partial_\lambda\left(\varepsilon^\lambda\Phi^*\right) \quad , \quad \Phi^* = \left(c^{*a},\ \phi^{*a},\ \eta^{*a}\right);
\end{aligned}$$

- the Ward identity for the local Lorentz transformations

$$\mathcal{W}_{\rm Lorentz}\Gamma = \int d^3x \ \sum_{\Phi}\delta^{(\lambda)}_{\rm Lorentz}\Phi\frac{\delta\Gamma}{\delta\Phi} = 0\,, \tag{2.15}$$

  where

$$\delta^{(\lambda)}_{\rm Lorentz}\Phi = \frac{1}{2}\lambda_{mn}\Omega^{mn}\Phi\,, \quad \Phi = \text{any field}\,,$$

  with $\Omega^{[mn]}$ acting on Φ as an infinitesimal Lorentz matrix in the appropriate representation;

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[6]The authors of [5, 6] in fact consider the theory in flat space only, but the generalization to curved space is straightforward (see for instance [16] for a similar case).

[7]We shall come back to the question of the counterterms in the next Subsection.

[8]See for instance [15], or in [16] for a proof in a similar context.

- the Landau gauge conditions

$$\frac{\delta\Gamma}{\delta b_a} = \partial_\mu \left(eg^{\mu\nu}A^a_\nu\right),$$
$$\frac{\delta\Gamma}{\delta h_a} = \partial_\mu \left(eg^{\mu\nu}B^a_\nu\right); \tag{2.16}$$

- the ghost equations of motion

$$\mathcal{G}^a_{(\mathrm{I})}\Gamma = \frac{\delta\Gamma}{\delta\bar{c}_a} + \partial_\mu\left(eg^{\mu\nu}\frac{\delta\Gamma}{\delta A^{*\nu}_a}\right) = 0\,,$$
$$\mathcal{G}^a_{(\mathrm{II})}\Gamma = \frac{\delta\Gamma}{\delta\bar{\phi}_a} + \partial_\mu\left(eg^{\mu\nu}\frac{\delta\Gamma}{\delta B^{*\nu}_a}\right) = 0\,; \tag{2.17}$$

- the antighost equations, (peculiar to the Landau gauge [17])

$$\bar{\mathcal{G}}^a_{(\mathrm{I})}\Gamma \;=\; \int d^3x \left(\frac{\delta}{\delta c^a} + g\,f^{abc}\bar{c}_b\frac{\delta}{\delta b^c} + g\,f^{abc}\bar{\phi}_b\frac{\delta}{\delta h^c}\right)\Gamma \;=\; \Delta^a_{\mathrm{cl(I)}}\,,$$
$$\bar{\mathcal{G}}^a_{(\mathrm{II})}\Gamma \;=\; \int d^3x \left(\frac{\delta}{\delta \phi^a} + g\,f^{abc}\bar{\phi}_b\frac{\delta}{\delta b^c}\right)\Gamma \;=\; \Delta^a_{\mathrm{cl(II)}}\,, \tag{2.18}$$

with

$$\Delta^a_{\mathrm{cl(I)}} = g\int d^3x\, f^{abc}\left(A^{*\mu}_b A_{c\mu} + B^{*\mu}_b B_{c\mu} + \eta^*_b\eta_c - c^*_b c_c - \phi^*_b\phi_c\right)\,,$$
$$\Delta^a_{\mathrm{cl(II)}} = \int d^3x\,\left(gf^{abc}\left(B^{*\mu}_b A_{c\mu} - \phi^*_b c_c\right) - \eta^{*a}\right)\,;$$

(The right-hand sides of (2.18) being linear in the quantum fields will not get renormalized.)

- the Ward identities for the rigid gauge invariances

$$\mathcal{W}^a_{\mathrm{rigid(I)}}\Gamma = \int d^3x \sum_{\Phi\,=\,\text{all fields}} f^{abc}\Phi_b\frac{\delta}{\delta\Phi^c}\Gamma = 0\,, \tag{2.19}$$

$$\mathcal{W}^a_{\mathrm{rigid(II)}}\Gamma = \int d^3x\,\left(f^{abc}\left(A^b_\mu\frac{\delta}{\delta B^c_\mu} + c^b\frac{\delta}{\delta\phi^c} + \bar{\phi}^b\frac{\delta}{\delta\bar{c}^c} + h^b\frac{\delta}{\delta b^c} + B^{*b}_\mu\frac{\delta}{\delta A^{*c}_\mu} + \phi^{*b}\frac{\delta}{\delta c^{*c}}\right) - \frac{\delta}{\delta\eta^a}\right)\Gamma = 0\,,$$

following from (2.8) and (2.18) and from the "anticommutation relations"

$$\bar{\mathcal{G}}^a_{(\mathrm{A})}\mathcal{S}(\mathcal{F}) + \mathcal{B}_{\mathcal{F}}\left(\bar{\mathcal{G}}^a_{(\mathrm{A})}\mathcal{F} - \Delta^a_{\mathrm{cl(A)}}\right) = \mathcal{W}^a_{\mathrm{rigid(A)}}\mathcal{F}\,,\quad \mathrm{A} = \mathrm{I},\mathrm{II}\,,\ \forall\,\mathcal{F}\,. \tag{2.20}$$

3 Superrenormalizability and Counterterms

The possible invariant counterterms which can be freely added to the action at each order of perturbation theory are of the trivial or of the nontrivial type [15]. The latter belong to the cohomology of the linearized Slavnov-Taylor operator (2.9) for the integrated insertions and may be found in [5, 6]. It consists of the two gauge invariant terms, $\int F^a_{\mu\nu}F^{\mu\nu}_a$ and the Chern-Simons action. The trivial counterterms are all possible variations of field polynomials of dimension 3 at most and of ghost number -1. Ultraviolet dimension, ghost number and Weyl dimension[9] are displayed in Table 1.

[9]See Section 4.

	A_μ	B_μ	η	b	h	c	$\bar{c}$	ϕ	$\bar{\phi}$	$A^{*\mu}$	$B^{*\mu}$	c^*	η^*	ϕ^*	e^μ_μ
d	1/2	3/2	1/2	3/2	1/2	-1/2	3/2	1/2	1/2	5/2	3/2	7/2	5/2	5/2	0
$\Phi\Pi$	0	0	0	0	0	1	-1	1	-1	-1	-1	-2	-1	-2	0
d_W	-1/2	1/2	1/2	3/2	1/2	-1/2	3/2	1/2	1/2	1/2	-1/2	1/2	-1	-1	-1

Table 1: Ultraviolet dimension d, ghost number $\Phi\Pi$, and Weyl dimension d_W.

There are however restrictions due to the superrenormalizability of the theory. The latter stems from the coupling constant-dependent power-counting formula [20, 16]

$$d\,(\gamma) = 3 - \sum_{\Phi} d_\Phi N_\Phi - \frac{1}{2} N_g \ , \tag{3.1}$$

for the degree of divergence of a 1-particle irreducible Feynman graph, γ. Here N_Φ is the number of external lines of γ corresponding to the field Φ, d_Φ is the dimension of Φ as given in Table 1, and N_g is the power of the coupling constant g in the integral corresponding to the diagram γ.

This formula implies that the dimension of the counterterms is bounded by three, the dimension 1/2 of the coupling constant g being included in the computation. But, since they are generated by loop graphs, they are of order 2 in g at least. Hence, not taking now into account the dimension of g, we see that their real dimension is bounded by 2. Imposing also the restrictions due to the gauge conditions (2.16), ghost equations (2.17) and antighost equations (2.18), as well as rigid gauge invariance (2.19), diffeomorphism invariances (2.14) and local Lorentz invariance (2.15), we easily see that only the Chern-Simons action survives as a possible counterterm. This means that the possible radiative corrections can be reabsorbed through a redefinition of the topological mass m only.

4 Quantum Scale Invariance

Here, the argument is similar to the one presented in [16]. It relies on a local form of the Callan-Symanzik equation. This will allow us to exploit the fact that the integrand of the Chern-Simons term in the action is not gauge invariant, although its integral is so. Such a local form of the Callan-Symanzik equation is provided by the "trace identity".

The energy-momentum tensor is defined as the tensorial quantum insertion obtained as the following derivative of the vertex functional with respect to the dreibein:

$$\Theta_\nu{}^\mu \cdot \Gamma = e^{-1} e_\nu{}^m \ \frac{\delta \Gamma}{\delta e_\mu{}^m} \ . \tag{4.1}$$

From the diffeomorphism Ward identity (2.14), follows the covariant conservation law:

$$e\, \nabla_\mu \left[\Theta_\nu{}^\mu (x) \cdot \Gamma \right] = w_\nu\,(x)\,\Gamma + \nabla_\mu w_\nu{}^\mu\,(x)\,\Gamma \,, \tag{4.2}$$

where ∇_μ is the covariant derivative with respect to the diffeomorphisms[10], the differential

[10]For a tensor T, such as, e.g., A_μ or $\delta/\delta A^{*\mu}$:

$$\nabla_\lambda T^{\mu\cdots}_{\nu\cdots} = \partial_\lambda T^{\mu\cdots}_{\nu\cdots} + \Gamma_\lambda{}^\mu{}_\rho T^{\rho\cdots}_{\nu\cdots} + \cdots - \Gamma_\lambda{}^\rho{}_\nu T^{\mu\cdots}_{\rho\cdots} - \cdots \,,$$

operators $w_\lambda(x)$ and $w_\lambda{}^\mu(x)$ acting on Γ representing contact terms. They are

$$w_\lambda(x) = \sum_{\text{all fields}} (\nabla_\lambda \Phi) \frac{\delta}{\delta\Phi} , \tag{4.3}$$

– becoming the translation Ward operator in the limit of flat space – and

$$\begin{aligned} w_\lambda{}^\mu(x) &= A^{*\mu}\frac{\delta}{\delta A^{*\lambda}} + B^{*\mu}\frac{\delta}{\delta B^{*\lambda}} - A_\lambda\frac{\delta}{\delta A_\mu} - B_\lambda\frac{\delta}{\delta B_\mu} - \delta_\lambda{}^\mu\left(c^*\frac{\delta}{\delta c^*} + A^{*\nu}\frac{\delta}{\delta A^{*\nu}} + \right. \\ &\quad \left. + B^{*\nu}\frac{\delta}{\delta B^{*\nu}} + \phi^*\frac{\delta}{\delta\phi^*} + \eta^*\frac{\delta}{\delta\eta^*}\right) . \end{aligned} \tag{4.4}$$

In the classical theory, the integral of the trace of the tensor $\Theta_\lambda{}^\mu$,

$$\int d^3x\, e\, \Theta_\mu{}^\mu = \int d^3x\, e_\mu{}^m \frac{\delta\Sigma}{\delta e_\mu{}^m} \equiv N_e\Sigma , \tag{4.5}$$

turns out to be an equation of motion, up to soft breakings. This follows from the identity, easily checked by inspection of the classical action:

$$N_e\Sigma = \left(\sum_{\Phi\,=\,\text{all fields}} d_{\rm W}(\Phi)\, N_\Phi + m\partial_m + \frac{1}{2}g\partial_g \right)\Sigma , \tag{4.6}$$

where N_Φ is the counting operator and $d_{\rm W}(\Phi)$ the Weyl dimensions (see Table 1) of the field Φ. We note that (4.6) is the Ward identity for rigid Weyl symmetry [21] – broken due to the dimensionful parameters m and g.

The trace $\Theta_\mu{}^\mu(x)$ therefore turns out to be vanishing, up to total derivatives, mass terms and dimensionful coupling, in the classical approximation, up to field equations, which means that (4.1) is the improved energy-momentum tensor. It is easy to check that, for the classical theory, we have in fact:

$$w(x)\Sigma \equiv \left(e_\mu^m(x)\frac{\delta}{\delta e_\mu^m(x)} - w^{\rm trace}(x) \right)\Sigma = \Lambda(x) , \tag{4.7}$$

or, equivalently:

$$e\,\Theta_\mu{}^\mu(x) = w^{\rm trace}(x)\Sigma + \Lambda(x) , \tag{4.8}$$

with

$$w^{\rm trace}(x) = \sum_\Phi d_{\rm W}(\Phi)\,\Phi\frac{\delta}{\delta\Phi} , \tag{4.9}$$

and

$$\begin{aligned} \Lambda &= \frac{m}{2}\varepsilon^{\mu\nu\lambda}A^a_\mu F^a_{\nu\lambda} - \frac{g}{2}f^{abc}\Big(\left(A^{*\mu}_a + eg^{\mu\nu}\partial_\nu\bar{c}_a\right)A^b_\mu c^c + \left(B^{*\mu}_a + eg^{\mu\nu}\partial_\nu\bar{\phi}_a\right)B^b_\mu c^c - \frac{1}{2}c^{*a}c^b c^c + \\ &\quad - \varepsilon^{\mu\nu\lambda}B^a_\mu A^b_\nu A^c_\lambda - 2\,e\,g^{\mu\nu}\left(B^a_\mu + D_\mu\eta^a\right)A^b_\nu\eta^c + \phi^{*a}\phi^b c^c\Big) + \text{total derivative terms} \end{aligned} \tag{4.10}$$

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where the $\Gamma_{\cdot\cdot}^{\cdot}$'s are the Christoffel symbols corresponding to the connexion ω_μ^{mn}. The covariant derivative of a tensorial density $\mathcal{T}$, such as, e.g., $A^{*\mu}$ or $\delta/\delta A_\mu$, is related to that of the tensor $e^{-1}\mathcal{T}$ by:

$$\nabla_\lambda \mathcal{T}^{\mu\cdots}_{\nu\cdots} = e\,\nabla_\lambda\left(e^{-1}\mathcal{T}^{\mu\cdots}_{\nu\cdots}\right) .$$

Λ being invariant under $\mathcal{B}_\Sigma$. The latter is the effect of the breaking of scale invariance due to the dimensionful parameters. The dimension of Λ – the dimensions of g and m not being taken into account – is lower than three: it is a soft breaking.

To promote the trace identity (4.7) or (4.8) to quantum level, we first observe that the following commutation relations hold:

$$\mathcal{B}_{\mathcal{F}} w\,(x)\,\mathcal{F} - w\,(x)\,\mathcal{S}(\mathcal{F}) = 0 \ .$$

$$\begin{gathered}
\left[\frac{\delta}{\delta\Phi\,(y)}, w\,(x)\right] = -d_{\mathrm{W}}\,(\Phi)\,\delta\,(x-y)\,\frac{\delta}{\delta\Phi\,(x)}\ , \quad \Phi = (b^a,\, h^a)\ , \\
\left[\bar{\mathcal{G}}^a_{(\mathrm{I})}, w(x)\right] = \frac{1}{2}\frac{\delta}{\delta c^a(x)}\ , \quad \left[\bar{\mathcal{G}}^a_{(\mathrm{II})}, w(x)\right] = -\left(\frac{\delta}{\delta\phi^a\,(x)} - g f^{abc}\bar{\phi}^b\frac{\delta}{\delta b^c\,(x)}\right)\ , \\
\left[\mathcal{G}^a_{(\mathrm{I})}\,(y)\,, w(x)\right] = -\frac{3}{2}\delta\,(x-y)\,\mathcal{G}^a_{(\mathrm{I})}\,(x) + \frac{3}{2}\partial_\mu\delta\,(x-y)\left(e g^{\mu\nu}\frac{\delta}{\delta A^{*\nu}_a}\right)(y)\ , \\
\left[\mathcal{G}^a_{(\mathrm{II})}\,(y)\,, w(x)\right] = -\frac{1}{2}\delta\,(x-y)\,\mathcal{G}^a_{(\mathrm{II})}\,(x) + \frac{1}{2}\partial_\mu\delta\,(x-y)\left(e g^{\mu\nu}\frac{\delta}{\delta B^{*\nu}_a}\right)(y)\ .
\end{gathered} \tag{4.11}$$

Now the relations (4.11) applied to the vertex functional Γ yield, for the insertion $w(x)\Gamma$, the properties

$$\begin{gathered}
\mathcal{B}_\Gamma w\,(x)\,\Gamma = 0\ , \\
\frac{\delta}{\delta b_a(y)}w(x)\Gamma = -\frac{3}{2}\partial_\mu\delta(x-y)\,(e\,g^{\mu\nu}A^a_\nu)\,(y)\ , \\
\frac{\delta}{\delta h_a(y)}w(x)\Gamma = -\frac{1}{2}\partial_\mu\delta(x-y)\,(e\,g^{\mu\nu}B^a_\nu)\,(y)\ , \\
\bar{\mathcal{G}}^a_{(\mathrm{I})}w(x)\Gamma = \frac{1}{2}\frac{\delta\Gamma}{\delta c_a(x)}\ , \\
\bar{\mathcal{G}}^a_{(\mathrm{II})}w(x)\Gamma = -\left(\frac{\delta}{\delta\phi^a\,(x)} - g f^{abc}\bar{\phi}^b\frac{\delta}{\delta b^c\,(x)}\right)\Gamma + g f^{abc}\left(B^{*b}_\mu A^{\mu c} - \phi^{*b}c^c\right)(x) \\
\mathcal{G}^a_{(\mathrm{I})}\,(y)\,w(x)\Gamma = \frac{3}{2}\partial_\mu\delta\,(x-y)\left(e g^{\mu\nu}\frac{\delta\Gamma}{\delta A^{*\nu}_a}\right)(y) \\
\mathcal{G}^a_{(\mathrm{II})}\,(y)\,w(x)\Gamma = \frac{1}{2}\partial_\mu\delta\,(x-y)\left(e g^{\mu\nu}\frac{\delta\Gamma}{\delta B^{*\nu}_a}\right)(y)
\end{gathered} \tag{4.12}$$

where we have used the constraints (2.16), (2.17) and (2.18).

The quantum version of (4.7) or (4.8) will be written as

$$w(x)\Gamma = \Lambda(x)\cdot\Gamma + \Delta(x)\cdot\Gamma\ , \tag{4.13}$$

where $\Lambda(x)\cdot\Gamma$ is some quantum extension of the classical insertion (4.10), subjected to the same constraints (4.12) as $w(x)\Gamma$ (see the Appendix). It follows that the insertion $\Delta\cdot\Gamma$ defined by (4.13) obeys the homogeneous constraints

$$\mathcal{B}_\Gamma\left[\Delta(x)\cdot\Gamma\right] = 0\ ,$$

$$\frac{\delta}{\delta b_a(y)}\left[\Delta(x)\cdot\Gamma\right]=0\ ,\quad \frac{\delta}{\delta h_a(y)}\left[\Delta(x)\cdot\Gamma\right]=0\ , \tag{4.14}$$
$$\bar{\mathcal{G}}^a_{(\mathrm{A})}\left[\Delta(x)\cdot\Gamma\right]=0\ ,\quad \mathcal{G}^a_{(\mathrm{A})}\left[\Delta(x)\cdot\Gamma\right]=0\ ,\quad \mathrm{A}=\mathrm{I},\,\mathrm{II}\ ,$$

beyond the conditions of invariance or covariance under $\mathcal{W}_{\rm diff}$, $\mathcal{W}_{\rm Lorentz}$ and $\mathcal{W}_{\rm rigid}$.

By power-counting the insertion $\Delta\cdot\Gamma$ has dimension 3, but being an effect of the radiative corrections, it possesses a factor g^2 at least, and thus its effective dimension is at most two due to the superrenormalizability (see (3.1)). It turns out that there is no insertion obeying all these constraints – effective power-counting selects the Chern-Simons Lagrangian density, but the latter is not BRS invariant. Hence $\Delta\cdot\Gamma=0$: there is no radiative correction to the insertion $\Lambda\cdot\Gamma$ describing the breaking of scale invariance, and (4.13) becomes

$$e\,\Theta_\mu{}^\mu(x)\cdot\Gamma=w^{\rm trace}(x)\Gamma+\Lambda(x)\cdot\Gamma\,. \tag{4.15}$$

This local trace identity leads to a trivial Callan-Symanzik equation (see e.g. Section 6 of [16]):

$$\left(m\partial_m+\frac{1}{2}g\partial_g\right)\Gamma=\int d^3x\ \Lambda(x)\cdot\Gamma\,, \tag{4.16}$$

but now with no radiative effect at all: the β-functions associated to the parameters g and m both vanish, scale invariance remaining affected only by the soft breaking Λ. There is also no anomalous dimension as well.

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Appendix

We sketch here a proof that it is possible to construct the quantum extension $\Lambda(x)\cdot\Gamma$, appearing in (4.13), of the classical insertion $\Lambda(x)$ given in (4.8,4.10), such that it obeys the same constraints as $w(x)\Gamma$ in (4.12). Let us first introduce a BRS invariant external field $\rho(x)$, with dimension and ghost number zero coupled to $\Lambda(x)$, introducing the new classical action

$$\Sigma^{(\rho)}=\Sigma+\int d^3x\,\rho(x)\,\Lambda(x)\ . \tag{A.1}$$

Defining the "extended" antighost operators (see (2.18))

$$\bar{\mathcal{G}}^{(\rho)a}_{(\mathrm{I})}\ =\ \bar{\mathcal{G}}^a_{\mathrm{I}}-\frac{1}{2}\int d^3x\ \rho\frac{\delta}{\delta c^a}\ ,$$

$$\bar{\mathcal{G}}^{(\rho)a}_{\rm (II)} = \bar{\mathcal{G}}^{a}_{\rm (II)} + \int d^3x \; \rho \left(\frac{\delta}{\delta \phi^a} - g f^{abc} \bar{\phi}^b \frac{\delta}{\delta b^c} \right) \; , \tag{A.2}$$

and the new classical insertions

$$\begin{aligned} \Delta^{(\rho)a}_{\rm cl(I)} &= \Delta^{a}_{\rm cl(I)} \; , \\ \Delta^{(\rho)a}_{\rm cl(II)} &= \Delta^{a}_{\rm cl(II)} + g \int d^3x \; \rho \; \left(f^{abc} \left(B^{*b}_\mu A^{\mu c} - \phi^{*b} c^c \right) - \eta^{*a} \right) \; , \end{aligned} \tag{A.3}$$

we easily see that the fulfillment of the required constraints amounts to prove, for the new vertex functional

$$\Gamma^{(\rho)} = \Sigma^{(\rho)} + O\left(\hbar\right) \; , \tag{A.4}$$

the Slavnov-Taylor identity

$$\mathcal{S}\left(\Gamma^{(\rho)}\right) = 0 \; , \tag{A.5}$$

and the new antighost identities

$$\bar{\mathcal{G}}^{(\rho)a}_{\rm (I)} \Gamma^{(\rho)} = \Delta^{(\rho)a}_{\rm cl(I)} + O\left(\rho^2\right) \; , \quad \bar{\mathcal{G}}^{(\rho)a}_{\rm (II)} \Gamma^{(\rho)} = \Delta^{(\rho)a}_{\rm cl(II)} + O\left(\rho^2\right) \; , \tag{A.6}$$

and to impose the extended gauge conditions

$$\begin{aligned} \frac{\delta \Gamma^{(\rho)}}{\delta b_a} &= \partial_\mu \left(e g^{\mu\nu} A^a_\nu \right) + \frac{3}{2} e g^{\mu\nu} \partial_\mu \rho A^a_\nu \; , \\ \frac{\delta \Gamma^{(\rho)}}{\delta h_a} &= \partial_\mu \left(e g^{\mu\nu} B^a_\nu \right) + \frac{1}{2} e g^{\mu\nu} \partial_\mu \rho B^a_\nu \; . \end{aligned} \tag{A.7}$$

In (A.6) and later on, there are terms of order ρ^2 and more, but we are not interested in them since the constraints we are looking for are obtained by differentiating (A.5–A.7) once with respect to ρ at $\rho = 0$.

The antighost operators (A.2) forms together with the Slavnov-Taylor operator (2.8) an algebra

$$\begin{aligned} \bar{\mathcal{G}}^{(\rho)a}_{\rm (I)} \mathcal{S}\left(\mathcal{F}\right) + \mathcal{B}_{\mathcal{F}} \left(\bar{\mathcal{G}}^{(\rho)a}_{\rm (I)} \mathcal{F} - \Delta^{(\rho)a}_{\rm cl(I)} \right) &= \mathcal{W}^{(\rho)a}_{{\rm rigid}_{\rm ((I))}} \mathcal{F} \; , \\ \bar{\mathcal{G}}^{(\rho)a}_{\rm (II)} \mathcal{S}\left(\mathcal{F}\right) + \mathcal{B}_{\mathcal{F}} \left(\bar{\mathcal{G}}^{(\rho)a}_{\rm (II)} \mathcal{F} - \Delta^{(\rho)a}_{\rm cl(II)} \right) &= \mathcal{W}^{(\rho)a}_{{\rm rigid}_{\rm ((II))}} \mathcal{F} \; , \\ \frac{\delta}{\delta b_a} \left(\bar{\mathcal{G}}^{(\rho)a}_{\rm (I)} \mathcal{F} - \Delta^{(\rho)a}_{\rm cl(I)} \right) - \bar{\mathcal{G}}^{(\rho)a}_{\rm (I)} \left(\frac{\delta \mathcal{F}}{\delta b_a} - \partial_\mu \left(e g^{\mu\nu} A^a_\nu \right) - \frac{3}{2} e g^{\mu\nu} \partial_\mu \rho A^a_\nu \right) &= 0 \; , \\ \frac{\delta}{\delta h_a} \left(\bar{\mathcal{G}}^{(\rho)a}_{\rm (II)} \mathcal{F} - \Delta^{(\rho)a}_{\rm cl(II)} \right) - \bar{\mathcal{G}}^{(\rho)a}_{\rm (II)} \left(\frac{\delta \mathcal{F}}{\delta h_a} - \partial_\mu \left(e g^{\mu\nu} B^a_\nu \right) - \frac{1}{2} e g^{\mu\nu} \partial_\mu \rho B^a_\nu \right) &= 0 \; , \\ \bar{\mathcal{G}}^{(\rho)a}_{\rm (I)} \left(\bar{\mathcal{G}}^{(\rho)b}_{\rm (I)} \mathcal{F} - \Delta^{(\rho)b}_{\rm cl(I)} \right) + \bar{\mathcal{G}}^{(\rho)b}_{\rm (I)} \left(\bar{\mathcal{G}}^{(\rho)a}_{\rm (I)} \mathcal{F} - \Delta^{(\rho)a}_{\rm cl(I)} \right) &= 0 \; , \\ \bar{\mathcal{G}}^{(\rho)a}_{\rm (II)} \left(\bar{\mathcal{G}}^{(\rho)b}_{\rm (II)} \mathcal{F} - \Delta^{(\rho)b}_{\rm cl(II)} \right) + \bar{\mathcal{G}}^{(\rho)b}_{\rm (II)} \left(\bar{\mathcal{G}}^{(\rho)a}_{\rm (II)} \mathcal{F} - \Delta^{(\rho)a}_{\rm cl(II)} \right) &= 0 \; , \end{aligned} \tag{A.8}$$

where $\mathcal{F}$ is an arbitrary functional of ghost number zero, and

$$\mathcal{W}^{(\rho)a}_{{\rm rigid}_{\rm ((I))}} = \mathcal{W}^{a}_{{\rm rigid}_{\rm ((I))}} \; , \tag{A.9}$$

$$\mathcal{W}^{(\rho)a}_{\mathrm{rigid}_{((\mathrm{II}))}} = \int d^3x \ (1+\rho) \left(f^{abc} \left(A^b_\mu \frac{\delta}{\delta B^c_\mu} + c^b \frac{\delta}{\delta \phi^c} + \bar{\phi}^b \frac{\delta}{\delta \bar{c}^c} + h^b \frac{\delta}{\delta b^c} + B^{*b}_\mu \frac{\delta}{\delta A^{*c}_\mu} + \phi^{*b} \frac{\delta}{\delta c^{*c}} \right) - \frac{\delta}{\delta \eta^a} \right) \ .$$

The proof of the Slavnov-Taylor identity (A.5) follows from the triviality of the cohomology for the ρ-dependent cocycles of ghost number one.

The recursive proof of (A.6) also follows that of the $\rho = 0$ theory [15]. Assuming (A.6) to be valid up to the order $n-1$ in $\hbar$, we can write

$$\bar{\mathcal{G}}^{(\rho)a}_{(\mathrm{A})} \Gamma^{(\rho)} = \Delta^{(\rho)a}_{\mathrm{cl(A)}} + \hbar^n \, \Delta_{(\mathrm{A})} + O\left(\hbar^{n+1}\right) + O\left(\rho^2\right) \ , \quad \mathrm{A} = \mathrm{I}, \mathrm{II} \ , \tag{A.10}$$

where Δ_{I} and Δ_{II} are integral local field polynomials of dimension $7/2$ and $5/2$, respectively (dimension $1/2$ of g included), ghost number -1 which, due the algebra (A.8), are subjected to the constraints

$$\frac{\delta \Delta^b_{(\mathrm{A})}}{\delta b_a} = \frac{\delta \Delta^b_{(\mathrm{A})}}{\delta h_a} = 0 \ , \quad \mathcal{B}_{\Gamma^{(\rho)}} \Delta^b_{(\mathrm{A})} = 0 \ , \tag{A.11}$$

and to the integrability conditions

$$\bar{\mathcal{G}}^{(\rho)a}_{(\mathrm{A})} \Delta^b_{(\mathrm{B})} + \bar{\mathcal{G}}^{(\rho)b}_{(\mathrm{B})} \Delta^a_{(\mathrm{A})} = 0 \ . \tag{A.12}$$

Using the constraints (A.11), the integrability conditions simplify to

$$\int d^3x \ \frac{\delta}{\delta X^a_{(\mathrm{A})}} \Delta^b_{(\mathrm{B})} + \int d^3x \ \frac{\delta}{\delta X^b_{(\mathrm{B})}} \Delta^a_{(\mathrm{A})} = 0 \ , \tag{A.13}$$

with

$$X^a_{(\mathrm{I})} = \left(1 - \frac{\rho}{2}\right)^{-1} c^a \ , \quad X^a_{(\mathrm{II})} = (1+\rho)^{-1} \phi^a \ . \tag{A.14}$$

The solution of (A.14) is trivial [15]

$$\Delta^a_{(\mathrm{A})} = \int d^3x \ \frac{\delta}{\delta X^a_{(\mathrm{A})}} \hat{\Delta} \ , \tag{A.15}$$

with

$$\frac{\delta \hat{\Delta}}{\delta b_a} = \frac{\delta \hat{\Delta}}{\delta h_a} = 0 \ ,$$

which shows that the breaking $\Delta^a_{(\mathrm{A})}$ at order n in $\hbar$ can be compensated by the counterterm $-\hbar^n \, \hat{\Delta}$.

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