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LIFE-TIME OF ACOUSTIC MAGNETIC PLASMONS IN AN ELECTRON GAS AND DETECTION BY POLARIZED NEUTRONS

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Abstract

A magnetic plasmon (MP) is an acoustic mode of the degenerate electron gas which exhibits spin modulation but not charge modulation. It is driven by the exchange interaction. Its energy lies within the continuum of one particle excitations. Here we estimate the life-time of the MP and its cross section for the inelastic scattering of spin polarized neutrons. We conclude that its detection might be feasible in simple metals with large electron densities such as Aluminum.

Key-words: Plasmons; Electron gas; Neutron scattering.

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INTRODUCTION

When a degenerate electron gas is considered as a two-component plasma, one with spin-up and the other with spin-down electrons, it exhibits two collective excitation modes. One of them is the well known optical plasmon, whose frequency in the long wave-length limit is given by $\omega_p = (4\pi e^2 N/m)^{1/2}$, where e and m are the charge and mass of the electrons and N the total electron density. The other mode is an acoustical excitation.[1] While the optical plasmon[2] describes a charge modulation, the acoustical plasmon describes a neutral magnetic modulation where the charge modulation of the up and down-spin gases compesate each other. Thus, it was called magnetic plasmon (MP) in Ref. 1 (hereafter referred to as (I)). For a one-dimensional electron gas these two types of modes describe the complete spectrum of excitations; this is not the case in three dimensions[3], where single particle excitations are also possible.

Acoustic plasmons are modes common to multicomponent degenerate Fermi systems.[4,5] For example, they were observed in photoexcited electron-hole plasmas in $GaAs$[6]. Their existence in MOS structures as a result of carriers in different subbands was theorestically studied with application to $GaAs$.[7] Acoustic plasmons in two[8] and one[9,10] dimensional systems have also been discussed. We want to emphasize that the MP here considered is entirely different in nature from those just mentioned, as it results from plasma components of two spin states and not on the existence of particles with different effective masses or belonging to different subbands.

The energy dispersion of the optical and magnetic plasmons are shown in Fig. 1, together with the one particle excitation spectrum, in the case of non-magnetic electron gas. Since the frequency of the MP lies within the continuous spectrum of individual particle excitations, no further attention was given to this mode. It was believed to be quickly Landau damped, as was explicitly stated in Ref. 1. Nonetheless, it is clear from the hydrodynamic treatment of the electron liquid that the optical plasmons alone cannot give a complete description of the collective modes of this system. The transformation to collective modes requires also the magnetic plasmons for the description of electrons with spin. To the best of our knowledge the existence of these modes was no more discussed in the literature, most probably due to the belief that they are overdamped. In closer scrutinizing the problem, however, we became aware that the MP is only driven by the exchange interaction which, in some cases, could be weak enough to allow the realization of the MP. We thus decided to evaluate the life time of the MP using the simplest, albeit widely used, model of replacing the exchange interaction by an adequately parameterized effective local potential.

In this work two basic points are discussed. First, it is shown that within traditionally accepted approximations for the exchange interaction the concept of MP is meaningful in the sense that its life-time is larger than its reciprocal frequency. Secondly, since the MP is an electrically neutral excitation possessing magnetic modulation it could be excited by polarized neutrons through the magnetic dipole-dipole interaction. The corresponding cross section is calculated.

LANDAU DAMPING OF THE MAGNETIC PLASMON

The life-time of the magnetic plasmon is limited mainly by its interaction with the individual particle excitations (conduction electrons) since its energy dispersion falls entirely within the one-particle spectrum (Fig. 1). The interaction of magnetic plasmons with conduction electrons is, however, weaker than that of optical plasmons since only the spin dependent exchange coupling can act.

For simplicity we consider the case of a non-magnetic metal, although the following treatment can be extended to a magnetized electron gas as well.

The decay rate of a magnetic plasmon of wave vector $\mathbf{q}$, in the first Born approximation, is given by

$$\tau_{\mathbf{q}}^{-1} = (2\pi/\hbar)|M_{\mathbf{q}}|^2 I(\mathbf{q}) \tag{1}$$

where

$$I(\mathbf{q}) = \frac{2}{(2\pi)^3} \int dk^3 f(E_{\mathbf{k}})[1 - f(E_{\mathbf{k+q}})]\delta(E_{\mathbf{k+q}} - E_{\mathbf{k}} - \hbar\Omega_{\mathbf{q}}) \tag{2}$$

is an integral over the available phase space. $f(E)$ is the Fermi distribution function and $E_{\mathbf{k}} = (\hbar\mathbf{k})^2/(2m)$ the conduction electron energy dispersion. A normalization volume $V = 1$ is used. The factor two in front comes from the contribution of the two spin states of the electron. The dispersion of the acoustical magnetic plasmon is given by $\Omega_{\mathbf{q}} = v_0 q$, with[11]

$$v_0 = v \left(1 - \frac{Ge^2(N/2)^{1/3}}{mv^2}\right)^{1/2} \tag{3}$$

where $v = (3/5)^{1/2}(\hbar k_F/m)$, $k_F = (3\pi^2 N)^{1/3}$ and the constant $G = (6/\pi)^{1/3} = 1.24$ is a parameter of the exchange interaction.[12] The integral in (2) is straighforward; we obtain

$$I(\mathbf{q}) = \frac{m^2 v_0}{2\pi^2\hbar^3} \qquad \text{for} \qquad q/(2k_F) < 0.48 \tag{4}$$

$M_{\mathbf{q}}$ is the matrix element for the scattering of an electron by the magnetic plasmon. It does not depend on $\mathbf{k}$ as a consequence of the local approximation of the exchange interaction by an effective potential. As a result $M_{\mathbf{q}}$ and $I(\mathbf{q})$ are decoupled. This matrix element is here calculated using the electron-plasmon interaction Hamiltonian H_{ep} derived in I:

$$H_{ep} = \sum_{\mathbf{p},\mathbf{k},\sigma} a^+_{\mathbf{p}-\mathbf{k},\sigma} a_{\mathbf{p},\sigma} \left(\frac{4\pi e^2}{k^2}(\rho_{\mathbf{k},\uparrow} + \rho_{\mathbf{k},\downarrow}) - \frac{Ge^2}{3N_\sigma^{2/3}}\rho_{\mathbf{k},\sigma}\right) \tag{5}$$

Here $a^+_{\mathbf{p}}$ and $a_{\mathbf{p}}$ are the creation and annihilation electron operators and $\rho_{\mathbf{q},\sigma}$ are density fluctuation operators which are given in terms of creation and annihilation plasmon operators in Eq. (28) of I. N_σ is the density of electrons with spin σ. In the case of a non magnetic metal considered here $N_\sigma = N/2$. We obtain

$$|M_{\mathbf{q}}|^2 = \frac{\hbar q^2}{2mN\Omega_{\mathbf{q}}}\left(\frac{Ge^2(N/2)^{1/3}}{3}\right)^2 . \tag{6}$$

The concept of magnetic plasmon is meanigful if the quantity $P = (\tau_{\mathbf{q}}\Omega_{\mathbf{q}})^{-1}$ is considerably smaller than one. From Eqs. (1), (3) and (4) we obtain, with $E_F = (\hbar k_F)^2/(2m)$,

$$P = \frac{\sqrt{15}\pi}{8E_F^2}\left(\frac{Ge^2(N/2)^{1/3}}{3}\right)^2\left(1 - \frac{Ge^2(N/2)^{1/3}}{mv^2}\right)^{-1/2} . \tag{7}$$

Values of P, v_0 and k_F for several metals are given in Table I. For values of $k_F < 1.1$ Å^{-1} the MP cannot exist, its velocity v_0, given by Eq. (3), becomes imaginary. P is independent of q within the range $q/k_F < 0.96$ and goes approximately like k_F^{-2}.

INELASTIC CROSS SECTION FOR THE SCATTERING OF SPIN POLARIZED NEUTRONS BY MAGNETIC PLASMONS

The general formalism for neutron diffraction[13,14] yields the following formula for the inelastic scatering cross section of a neutron with creation of a magnetic plasmon of momentum $\mathbf{q}$:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{p'}{p}\left(\frac{M_n}{2\pi\hbar^2}\right)^2 |< \mathbf{p}', s_z', \mathbf{q}|H|\mathbf{p}, s_z|^2 \ \delta(\mathsf{E}_\mathbf{p} - \mathsf{E}_{\mathbf{p}'} - \hbar\Omega_\mathbf{q}) \tag{8}$$

Here M_n is the neutron mass, $\mathbf{p}$ and s_z are the momentum and z-spin component of the neutron in the initial state and $\mathbf{p}'$ and s_z' the corresponding values in the final state. $\mathbf{q} = \mathbf{p} - \mathbf{p}'$ is the momentum of the created magnetic plasmon. $\mathsf{E}_\mathbf{p}$ is the neutron energy. The Hamiltonian H describes the dipole-dipole magnetic interaction between the neutron and the magnetic plasmon:

$$H = \int d^3R \ \Psi^+(\mathbf{R})H_n(\mathbf{R})\Psi(\mathbf{R}) \tag{9}$$

where

$$\Psi(\mathbf{R}) = \sum_{\mathbf{p},s_z} c_{\mathbf{p},s_z} e^{i\mathbf{p}.\mathbf{R}}|s_z\rangle \tag{10}$$

is the neutron field operator with $c_{\mathbf{p},s_z}$ the neutron annihilation operator, $\mathbf{R}$ the neutron position coordinate and $|s_z\rangle$ and eigenstate of the neutron spin operator S_z.

$$H_n(\mathbf{R}) = -\int d^3r \ \vec{\mu}_{pl}(r).\mathbf{B}_S(\mathbf{r} - \mathbf{R}) \tag{11}$$

where

$$\vec{\mu}_{pl}(r) = g_e\mu_B\rho(r)\mathbf{z} \tag{12}$$

is the density of magnetic moment of the MP, $\mathbf{z}$ is a unit vector in the z-direction, g_e is the electron gyromagnetic factor, μ_B the Bohr magneton and

$$\rho(r) = \sum_\mathbf{q}(\rho_{\mathbf{q},\uparrow} - \rho_{\mathbf{q},\downarrow})e^{i\mathbf{q}.\mathbf{r}} \tag{13}$$

is the MP spin density at the site $\mathbf{r}$.

$$\mathbf{B}_S(\mathbf{r} - \mathbf{R}) = curl_\mathbf{r}\left(\frac{\vec{\mu}_n \times (\mathbf{r} - \mathbf{R})}{|\mathbf{r} - \mathbf{R}|^3}\right) \tag{14}$$

is the magnetic field operator.[14] It corresponds to the magnetic moment operator of the neutron at position $\mathbf{R}$:

$$\vec{\mu}_n = \frac{g_n|e|\hbar}{M_n c}\,\mathbf{S} \tag{15}$$

which acts at site $\mathbf{r}$ through the dipole-dipole interaction. Here $g_n = -1.91$ is the neutron g-factor, c the velocity of light and $\mathbf{S}$ the neutron spin operator.

Replacing Eqs. (12) and (13) into Eq. (11), we are left with a Fourier transform of $B_S(\mathbf{r}-\mathbf{R})$ which can be carried out analytically. Finally, the integral in the space $\mathbf{R}$ in Eq. (9) leads to a delta function of momentum conservation, namely $\delta(\mathbf{p}-\mathbf{p}'-\mathbf{q})$. The integrals can be performed easily because Eq. (9) is clearly a Fourier transform of a convolution. We thus obtain

$$H = C \sum_{\substack{s_z,s_z' \\ \mathbf{p},\mathbf{q}}} c^+_{\mathbf{p}-\mathbf{q},s_z'}\; c_{\mathbf{p},s_z}(\rho_{\mathbf{q},\uparrow}-\rho_{\mathbf{q},\downarrow})\mathbf{G}(\mathbf{S},\mathbf{q}) \tag{16}$$

where $C = 2\pi\hbar^2 e^2 g_e g_n/(mM_n c^2)$ and

$$\mathbf{G}(\mathbf{S},\mathbf{q}) = \mathbf{S}.\mathbf{z} - \frac{(\mathbf{z}.\mathbf{q})(\mathbf{S}.\mathbf{q})}{\mathbf{q^2}}\ . \tag{17}$$

It is convenient to introduce the spin operators $S^\pm = S_x \pm iS_y$ and the variables $q^\pm = q_x \pm iq_y$. Thus,

$$\mathbf{G}(\mathbf{S},\mathbf{q}) = S_z - \frac{q_z}{q^2}\left(\frac{S^+q^- + S^-q^+}{2} + S_z q_z\right)\ . \tag{18}$$

Let us consider the case of incoming neutrons with spin polarized along the $+z$-direction (up-spin). We then have

$$W^+ = |\langle 1/2|\mathbf{G}(\mathbf{S},\mathbf{q})|1/2\rangle|^2 = \frac{1}{4}\left(1-\frac{q_z^2}{q^2}\right)^2 \tag{19}$$

for scattering without spin-flip, and

$$W^-|\langle -1/2|\mathbf{G}(\mathbf{S},\mathbf{q})|1/2\rangle|^2 = \frac{1}{4}\,\frac{q_z^2}{q^4}(q^2-q_z^2) \tag{20}$$

for scattering with neutron spin-flip. The inelastic cross section then becomes

$$\frac{d^2\sigma^\pm}{d\Omega dE'} = \frac{p'N(\hbar q g_e g_n r_0)^2}{2pm\hbar\Omega}\,W^\pm\delta(E-E'-\hbar\Omega)\ . \tag{21}$$

Here $r_0 = e^2/(mc^2)$ is the classical radius of the electron and the indices $\pm$ refer to scattering with and without neutron spin-flip. E and E' are the initial and final neutron energies respectively. In view of the finite life-time of the magnetic plasmon, it is more appropriate to replace the delta function in Eq. (21) by the lorentzian distribution[15]

$$\frac{\tau/\pi}{1+(E+E'-\hbar\Omega_{\mathbf{q}})^2\tau^2}\ . \tag{22}$$

CONCLUSIONS

Within the hydrodynamic approximation, the degenerate electron gas considered as a two component plasma, constituted by the electrons with up and down spin, displays two collective modes: the usual optical plasmon and an acoustical magnetic plasmon with dispersion $\Omega = v_0 q$ where v_0 is given by Eq. (3). The later, however, is meaningful only for sufficiently large electron densities such that $P = (\Omega\tau)^{-1}$ is considerably smaller than one, where τ is the life-time. Table I lists some metals in which the magnetic plasmon could be detected. Aluminum seems to be a good candidate. The inelastic cross section for the scattering of neutrons shows a broad resonance centered at $\Omega_{\mathbf{q}}$. The order of magnitude of the MP inelastic cross section, given by Eq. (21), is comparable to that of magnons, given by Eq. (75) of Ref. 13, with an "effective form factor" $|F_{\mathbf{q}}|^2 \simeq 2q/k_F$. In the present case, however, energy and momentum conservation require neutrons with the Fermi velocity of the electron gas, which means neutron energies of the order of $(M/m)E_F$ which is in the range of several KeV. The experiment is thus difficult. If feasible, experiments with spin polarized neutrons in which the scattered beam is both energy and spin analyzed might detect the magnetic plasmons also through the difference between the cross sections with and without spin flip and their $\mathbf{q}$-dependences.

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FIGURE CAPTIONS AND TABLE

Fig. 1 - Excitation spectrum of the electron gas displaying the individual particle continuum (between the full lines), and the optical (OP) and the acoustical magnetic plasmon (MP) dispersions (dashed lines). The values are appropriate for Aluminum ($k_F = 1.75\ \AA^{-1},\ E_F = 11.6\ eV,\ \hbar\omega_p = 15.7\ eV$). Note that the acoustical plasmon dispersion falls entirely within the continuum.

Table I - Values of k_F, $P = (\Omega\pi)^{-1}$ and v_0 for several metals. Ω, τ and v_0 are the frequency, life-time and group velocity of the magnetic plasmons, respectively, and k_F is the Fermi wavector of the metal in a free electron model.

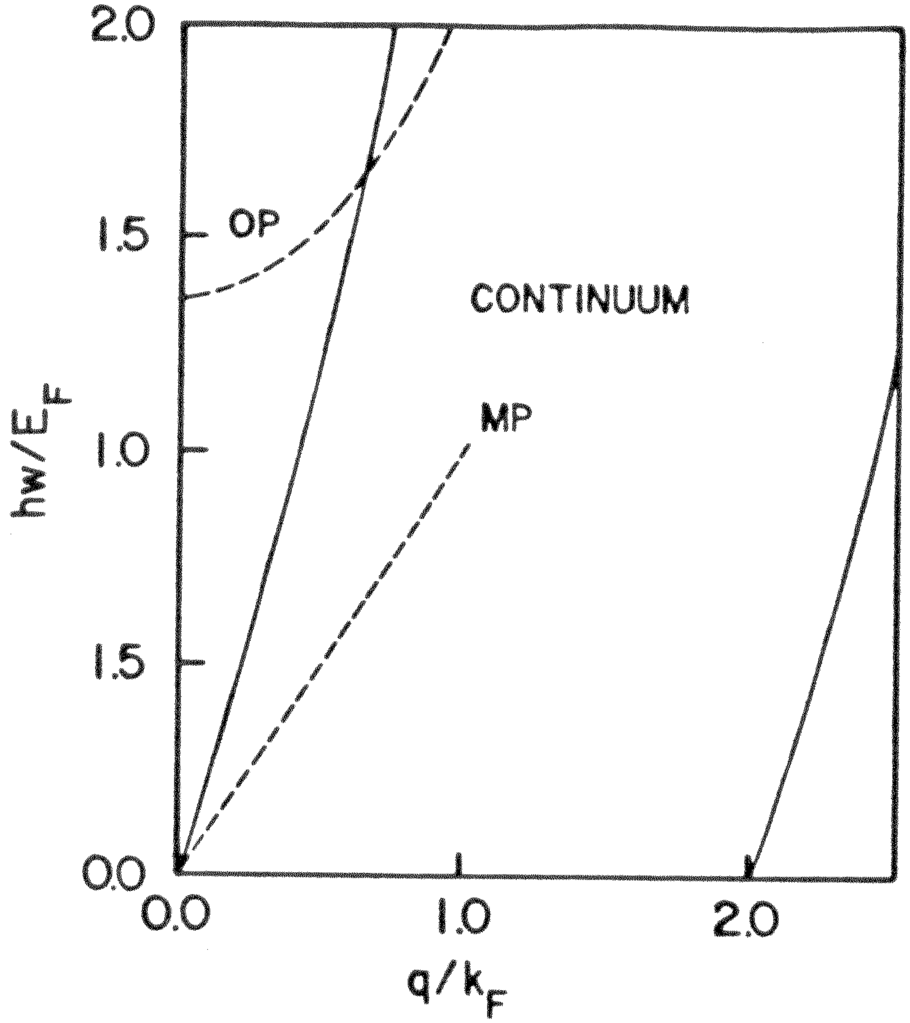

Figure 1

TABLE I

ELEMENT	$k_F(\AA^{-1})$	P	$\hbar v_0 (eV.\AA)$
Al	1.75	0.12	6.67
Sn	1.63	0.15	5.95
Pb	1.58	0.16	5.55
ln	1.50	0.19	5.06
Au	1.20	0.42	2.84

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