Electron-electron Bound States in Parity-Preserving QED$_3$

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By considering the Higgs mechanism in the framework of a parity-preserving Planar Quantum Electrodynamics, one shows that an attractive electron-electron interaction may dominate. The e^-e^- interaction potential emerges as the non-relativistic limit of the Møller scattering amplitude and it results attractive with a suitable choice of parameters. Numerical values of the e^-e^- binding energy are obtained by solving the two-dimensional Schrödinger equation. The existence of bound states is a strong indicative that this model may be adopted to address the pairing mechanism of high-T$_c$ superconductivity.

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In the latest 10 years, Planar Quantum Electrodynamics - QED_3- has shown to be an appropriate theoretical framework for discussing the low-energy limit of some Condensed Matter systems. Recent applications of this theory to underdoped high-T_c superconductors [1] has again caught attention for its theoretical possibilities. The history of the relation between QED_3 and superconductivity goes back to the final 80s, when the anyonic model was established by the works of Laughlin [2], and others [3]. Despite its initial success, it was afterwards demonstrated that anyonic model supports the superconducting phase only at zero temperature [4]. An alternative approach, also based on the QED_3 framework, began to be adopted by Kogan [14] to explain the formation of electron-electron bound states. Into the domain of the QED_3, there exits the necessity of yielding a mass to the gauge field in order to circumvent the appearance of a confining potential associated to the long-range Coulombian interaction. The Maxwell-Chern-Simons (MCS) term [13] is then introduced as the generator of (topological) mass for the photon, implying a screening on the Coulomb interaction. This MCS-QED_3 model was used by some authors [14], [16] as basic tool for evaluation of the Möller scattering amplitude at tree-level, whose Fourier transform (in the Born approximation) yields the e^-e^- interaction potential. In a general way, these works furnish the same result: the e^-e^- interaction comes out attractive when the topological mass (ϑ) surpasses the electronic mass (m_e), that is, $\vartheta > m_e$. This condition prevents the applicability of the MCS model to superconducting systems, since the existence of a physical excitation with so large energy in the domain of a Condensed Matter system is entirely unlikely. It is possible to argue that the introduction of the Higgs mechanism in the context of the MCS electrodynamics [5], [6] brings out a negative contribution to the scattering potential that will make feasible a global attractive potential regardless the condition $\vartheta > m_e$.

In this letter, we start from a parity-preserving QED_3 Lagrangian (without MCS term) [6], [7], [8] with spontaneous symmetry breaking (SSB). A Higgs boson and a massive photon appear in the spectrum in the broken phase. These two particles mediate the Möller scattering, whose amplitude leads to a Bessel-K_o interaction potential. It can be attractive (independent of the electron polarization) whenever the negative contribution stemming from the Higgs scalar interchange dominates over the repulsive gauge interaction. Relying on the non-relativistic approximation, the K_o- potential is inserted into the Schrödinger equation. Its numerical solution provides as with values of the e^-e^- pairing energy, which are exhibited in Table I.

We start with a parity-preserving QED_3 action (with SSB) [7], [8], built up by two polarized fermionic fields (ψ_+, ψ_-) [7], [9], a gauge potential (A_μ) and a complex scalar field (φ):

$$S = \int d^3x \Big\{ -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \overline{\psi}_+(i\not{D} - m_e)\psi_+ + \overline{\psi}_-(i\not{D}+m_e)\psi_- - y(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)\varphi^*\varphi + D^\mu\varphi^* D_\mu\varphi - V(\varphi^*\varphi) \Big\}, \tag{1}$$

with the scalar self-interaction potential, V, responsible for the SSB, taken as $V(\varphi^*\varphi) = \mu^2\varphi^*\varphi + \frac{\zeta}{2}(\varphi^*\varphi)^2 + \frac{\lambda}{3}(\varphi^*\varphi)^3$. The mass dimensions of the parameters μ, ζ, λ, y are respectively 1, 1, 0, 0, and the covariant derivatives, $\not{D}\psi_\pm \equiv (\not{\partial} + ie_3\not{A})\psi_\pm$, $D_\mu\varphi \equiv (\partial_\mu + ie_3A_\mu)\varphi$, state the minimal coupling between $\psi_\pm, A_\mu$, and φ. It is important to point out that the $U(1)-$symmetry coupling constant in $(1+2)$-dimensions, e_3, has dimension of $(\text{mass})^{\frac{1}{2}}$. We are interested only on a stable vacuum, for which the following conditions on the potential parameters have to be fulfilled: $\lambda > 0$, $\zeta < 0, \mu^2 \leq \frac{3}{16}\frac{\zeta^2}{\lambda}$. Denoting $\langle\varphi\rangle = v$, one readily gets that: $\langle\varphi^*\varphi\rangle = v^2 - \zeta/2\lambda + \sqrt{(\zeta/2\lambda)^2 - \mu^2/\lambda}$, whereas the condition for minimum reads as $\mu^2 + \zeta v^2 + \lambda v^4 = 0$. In the broken phase, the complex scalar field is parametrized by $\varphi = v + H + i\theta$, where θ is the would-be Goldstone boson and H is the Higgs scalar, both with vanishing v.e.v.'s. By replacing this parametrization relation into the action (1), adopting the 't Hooft gauge [11] ($S_{\rm gf} = \int d^3x \Big\{ -\frac{1}{2\xi}(\partial^m A_m - \sqrt{2}\xi M_A\chi)^2 \Big\}$), and finally taking only the bilinear and Yukawa interaction terms, one has:

$$S^{\rm SSB} = \int d^3x \Big\{ -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}M_A^2 A^\mu A_\mu + \overline{\psi}_+(i\not{\partial} - m_{\rm eff})\psi_+ + \overline{\psi}_-(i\not{\partial} + m_{\rm eff})\psi_- + \partial^\mu H\partial_\mu H - M_H^2 H^2 + \\ + \partial^\mu\theta\partial_\mu\theta - M_\theta^2\theta^2 - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 - 2yv(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)H - e_3\left(\overline{\psi}_+\not{A}\psi_+ + \overline{\psi}_-\not{A}\psi_-\right) \Big\},$$

where ξ is a dimensionless gauge parameter and the mass generated by the SSB are: $M_A^2 = 2v^2e_3^2$ (Proca mass), $M_H^2 = 2v^2(\zeta + 2\lambda v^2)$ (Higgs mass), $m_{\rm eff} = m_e + yv^2$(effective electron mass), and $M_\theta^2 = \xi M_A^2$. The latter corresponds to non-physical poles in the gauge and θ-field propagator. Their effects are mutually canceled as already known from the study of the unitarity in the 't Hooft gauge [12].

In the low-energy limit (Born approximation), the two-particle interaction potential is given by the Fourier transform of the two-particle scattering amplitude [17]. It is important to stress that in the case of the non-relativistic Möller scattering, one should consider only the t-channel (direct scattering) [17] even for distinguishable electrons, since in this limit they recover the classical notion of trajectory. From the action (1), there follow the Feynman rules for the interaction vertices: $V_{\pm H\pm} = \pm 2iyv; V_{\psi A\psi} = ie_3\gamma^\mu$, so that the e^-e^- scattering are written as bellow:

$$-i\mathcal{M}_{\pm H\pm} = \overline{u}_\pm(p_1^{'})(\pm 2ivy)u_\pm(p_1)\,[\langle HH\rangle]\,\overline{u}_\pm(p_2^{'})(\pm 2ivy)u_\pm(p_2); \quad (2)$$
$$-i\mathcal{M}_{\pm H\mp} = \overline{u}_\pm(p_1^{'})(\pm 2ivy)u_\pm(p_1)\,[\langle HH\rangle]\,\overline{u}_\mp(p_2^{'})(\pm 2ivy)u_\mp(p_2); \quad (3)$$
$$-i\mathcal{M}_{\pm A\pm} = \overline{u}_\pm(p_1^{'})(ie_3\gamma^\mu)u_\pm(p_1)\,[\langle A_\mu A_\nu\rangle]\,\overline{u}_\pm(p_2^{'})(ie_3\gamma^\mu)u_\pm(p_2); \quad (4)$$
$$-i\mathcal{M}_{\pm A\mp} = \overline{u}_\pm(p_1^{'})(ie_3\gamma^\mu)u_\pm(p_1)\,[\langle A_\mu A_\nu\rangle]\,\overline{u}_\mp(p_2^{'})(ie_3\gamma^\mu)u_\mp(p_2); \quad (5)$$

where $\langle HH\rangle$ and $\langle A_\mu A_\nu\rangle$ are the Higgs and massive photon propagators. Expressions (2) and (3) represent the scattering amplitudes for electrons of equal and opposite polarizations mediated by the Higgs particle, whereas Eqs. (4) and (5) correspond to the massive photon as mediator.

The spinors $u_\pm(p)$ stand for the positive-energy solution of the Dirac equation $(\not p \mp m)\,u_\pm(p) = 0$. We adopt the following conventions $\eta_{\mu\nu} = (+,-,-)$, $[\gamma^\mu,\gamma^\nu] = 2i\epsilon^{\mu\nu\alpha}\gamma_\alpha$, $\gamma^\mu = (\sigma_z, -i\sigma_x, i\sigma_y)$, whence one obtains

$$u_+(p) = \frac{1}{\sqrt{N}}\begin{bmatrix} E+m \\ -ip_x - p_y \end{bmatrix}, \quad u_-(p) = \frac{1}{\sqrt{N}}\begin{bmatrix} ip_x - p_y \\ E+m \end{bmatrix}, \quad (6)$$

with $N = 2m(E+m)$ being the normalization constant that assures $\overline{u}_\pm(p)u_\pm(p) = \pm 1$. Working in the center-of-mass frame [5], [6], the scattering amplitudes $\mathcal{M}_{higgs} = -2v^2y^2\left(\overrightarrow{k}^2 + M_H^2\right)^{-1}$, $\mathcal{M}_{gauge} = +e_3^2\left(\overrightarrow{k}^2 + M_A^2\right)^{-1}$ reveal to be independent of the spin polarization. Evaluating now the Fourier transform of the total amplitude scattering ($\mathcal{M}_{total} = \mathcal{M}_{higgs} + \mathcal{M}_{gauge}$), the following interaction potential comes out:

$$V^{CM}(r) = -\frac{1}{2\pi}\Big[2v^2y^2K_o(M_H r) - e_3^2K_o(M_A r)\Big]. \quad (7)$$

Considering equal Higgs and Proca masses $\left(M_H = M_A \Longleftrightarrow e_3^2 = \zeta + 2\lambda v^2\right)$, the potential (7) takes the form

$$V^{CM}(r) = CK_o(M_A r), \text{ with: } C = -\frac{1}{2\pi}\Big[2v^2y^2 - e_3^2\Big]. \quad (8)$$

It becomes attractive whenever $C < 0$, that is, $2v^2y^2 > e_3^2$.

Having determined the interaction potential, one must now look for the numerical evaluation of the binding energy associated to the e^-e^- pairs. In the non-relativistic limit, the complete two-dimensional Schrödinger equation (supplemented by the Bessel-K_0 potential)

$$\frac{\partial^2\varphi(r)}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi(r)}{\partial r} - \frac{l^2}{r^2}\varphi(r) + 2\mu_{\rm eff}[E - CK_o(M_A r)]\varphi(r) = 0 \quad (9)$$

yields the energy of the two interacting particles. Here, $\mu_{\rm eff} = \frac{1}{2}m_{\rm eff}\ \varphi(r)$ is the effective reduced mass of the e^-e^- system and φ represents the (relative) spatial part of the complete antisymmetric 2-electron wavefunction: $\Psi(r_1, s_1, r_2, s_2) = \psi(\mathbf{R})\varphi(\mathbf{r})\chi\,(s_1, s_2)$, while $\psi(\mathbf{R})$, $\chi\,(s_1, s_2)$ stand for the center-of-mass and the spin wave functions.

For a numerical solution of the Schrödinger equation, we employ the variational method. In this respect, we take as starting point the choice of a wave function that stands for the generic features of the e^-e^-

state: the trial function, whose definition must observe some conditions, such as the asymptotic behavior at infinity, the analysis of its free version and its behavior at the origin. With the help of the transformation $\varphi(r) = \frac{1}{\sqrt{r}}\, g(r)$, Eq.(9) is transformed into

$$\frac{\partial^2 g(r)}{\partial r^2} - \frac{l^2 - \frac{1}{4}}{r^2} g(r) + 2\mu_{\rm eff}[E - CK_o(M_A r)]g(r) = 0, \tag{10}$$

whose free version ($V(r) = 0$) for zero angular momentum ($l = 0$) state simplifies to

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{4r^2} + k^2\right] u(r) = 0\ . \tag{11}$$

Its general solution is given by $u(r) = B_1\sqrt{r}J_0(kr) + B_2\sqrt{r}Y_0(kr)$, with B_1 and B_2 being arbitrary constants and $k = \sqrt{2\mu_{\rm eff}E}$. In the $r \to 0$ limit, the solution to Eq.(11) approaches $u(r) \longrightarrow \sqrt{r} + \lambda\sqrt{r}\ln r$. Since the second term in Eq.(11) behaves like an attractive potential, $-1/4r^2$, this implies the unphysical possibility of obtaining a bound state ($E < 0$) even for $V(r) = 0$ [10]. Among the infinite number of self-adjoint extensions of the differential operator $-d^2/dr^2 - 1/4r^2$, the only physical choice corresponds to the Friedrichs extension ($B_2 = 0$), which behaves like $\sqrt{r}$ at the origin, indicating this same behavior for $u(r)$. The complete equation, $V(r) \neq 0$, will preserve the self-adjointness of free Hamiltonian, if the potential is "weak" in the sense of the Kato condition: $\int_0^\infty r(1 + |\ln(r)|)|V(r)|dr < \infty$. This condition also sets up a finite number of bound states (discrete spectrum) and the semi-boundness of the complete Hamiltonian. Provided that the Bessel-K_0 potential, given by Eq. (8), satisfies the Kato condition, the self-adjointness of the total Hamiltonian is assured and the existence of bound states is allowed. On the other hand, at infinity, the trial function must vanish asymptotically in order to fulfill square integrability. Therefore, a good and suitable trial function (for $l = 0$) could be taken by

$$g(r) = \sqrt{r}\exp(-\beta r)\ , \tag{12}$$

where β is a free spanning parameter to be numerically fixed in order to minimize the binding energy.

Once the trial function is already known, it still lacks a discussion on the physical parameters (ν^2, e_3^2, y^2) that compose the proportionality constant, C, of the Bessel potential, in such a way that numerical values may be attributed to them. The vacuum expectation value, v^2, indicates the energy scale of the spontaneous breakdown of the $U(1)-$local symmetry. This is a free parameter, being usually determined by some experimental data associated to the phenomenology of the model under investigation, as occurs in the electroweak Weinberg-Salam model, for example. On the other hand, the y parameter measures the coupling between the fermions and the Higgs scalar, working in fact as an effective constant that embodies contributions of all possible mechanisms of electronic interaction via Higgs-type (scalar) excitations, as the spinless bosonic interaction mechanisms: phonons, plasmons, and other collective excitations. This theoretical similarity suggests an identification of the field theory parameter with an effective electron-scalar coupling (instead of an electron-phonon one): $y \to \lambda_{\rm es}$. Specifically, in QED$_3$, the electromagnetic coupling constant squared, e_3^2, has dimension of mass, rather than the dimensionless character of the usual four-dimensional QED$_4$ coupling constant. This fact might be understood as a memory of the third dimension that appears (into the coupling constant) when one tries to work with a theory intrinsically defined in three space-time dimensions. This dimensional peculiarity could be better implemented through the definition of a new coupling constant in three space-time dimensions [14], [15]: $e \to e_3 = e/\sqrt{l_\perp}$, where $l_\perp$ represents a length orthogonal to the planar dimension. The smaller is $l_\perp$, smaller is the remnant of the frozen dimension, larger is the planar character of the model and the coupling constant e_3, what reveals its effective nature. In this sense, it is instructive to notice that the effective value of e_3^2 is always larger than $e^2 = 1/137$ whenever $l_\perp < 1973.26$ Å, since 1 (Å)$^{-1}$ = 1973.26 eV. This particularity broadens the repulsive interaction for small $l_\perp$ and requires an even stronger Higgs contribution to account for a total attractive interaction.

The following Table, constructed for zero angular momentum state ($l = 0$), has as input data the three parameters $(\nu^2, l_\perp, y)$, while the output parameters are: $\beta-$ the minimization parameter, $E_{e^-e^-}-$ the e^-e^- binding energy, and $\langle r\rangle-$ the average-length of the wavefunction.

$v^2(meV)$	$l_\perp(\text{Å})$	y	$C_s\ (meV)$	$M_H\ (meV)$	β	$E_{e^-e^-}\ (meV)$	$\langle r\rangle\ (\text{Å})$
120.0	15.0	2.1	-15.7	480.0	63.1	-74.7	15.6
120.0	14.0	2.1	-4.8	496.8	35.8	-19.7	27.6
120.0	13.0	2.2	-8.6	515.6	47.2	-37.8	20.9
100.0	12.0	2.6	-24.2	489.9	81.1	-120.2	12.2
100.0	12.0	2.5	-8.0	489.9	45.9	-35.2	21.5
100.0	10.0	2.7	-2.9	536.6	27.8	-11.0	35.5
100.0	10.0	2.8	-20.4	536.6	72.1	-97.6	13.7
100.0	6.0	3.5	-8.0	692.8	45.9	-32.5	21.5
80.0	6.0	3.9	-5.4	619.6	37.6	-21.4	26.2
70.0	4.0	5.1	-6.6	709.9	41.7	-26.2	23.6
60.0	8.0	3.9	-4.0	464.7	33.1	-16.7	29.8

TABLE I. Input data $(v^2, l_\perp, y)$ and output data $(E_{e-e}, \langle r\rangle)$ for the Schrödinger Equation

The numerical data of Table I show that the attractive Bessel potential, derived for a non-relativistic regime, may effectively promote the formation of e^-e^- bound states. The procedure here carried out puts in evidence that, by properly fitting the free parameters of the model, one can obtain bound states of the order of $10-100\ meV$ and wavefunction average-length in the range $10-30$ Å, which reveals the suitability of the framework employed to address the physical mechanism that underlies the constitution of Cooper pairs in the high-T$_c$ superconductors. Finally, we can assert that the photon Proca mass, generated by the SSB, plays the same role of the topological mass (ϑ) in that it determines the Coulomb interaction screening and the Meissner effect, without breaking parity-symmetry. The data exhibited in Table I concern an s-wave state: $l = 0$ and spin singlet $(\uparrow\downarrow, S = 0)$. According to the results of this letter, we conclude by stressing the fundamental role played by the Higgs mechanism in QED$_3$ as essential for the appearance of an attractive e^-e^- potential.

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