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Response of military bridges to an armor column and infantary platoon under disordered retreat - a stochastic approach

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Abstract

We analyze the military bridge response to a random loading.

1 Introduction

The random stresses in military bridges ([1], specially under retreat (a common military procedure in south america armies: remember Falkland Islands!)), are always caused by the irregular composition and speed of random motions. In the stochastic approach, the bridge is assumed to be a mechanical system of given elastic properties. The main problem is, thus, how to determine the stochastic characteristics of the deflections and stress of the bridge under the random excitation (a disordered retreat!) with the objective to determine the first-passage stochastic bridge deflection, the main parameter in the bridge design in order to prevent the bridge colapses (see Appendix 1) and the marines and the armour-cars on the bridge fall in the jungle's river (igarapé) full of piranhas, anacondas and giant alligators (jacarés-açús) when in retreat.

The differential equation governing the bridge deflection $U(x,t)$ will take to be the Darboux equation

$$\begin{aligned} &\frac{\partial^4 U(x,t)}{\partial^4 x} + \frac{\partial^2 U(x,t)}{\partial^2 t} + \frac{\nu}{(t+a)}\frac{\partial U}{\partial t}(x,t) = \rho(x,t) \\ &U(x,t_0) = f(x) \\ &U_t(x,t_0) = g(x) \end{aligned} \tag{1}$$

The external load will be denoted by $\rho(x,t)$ and it is suposed of the form

$$\rho(x,t) = \delta(x - vt)h(t) \tag{2}$$

where the concentrated force $h(t)$ is assumed to satisfy a gaussian (normal) statistic

$$\langle h(t_1)h(t_2)\rangle = C(t_1,t_2) \tag{3}$$

In what follows we suppose (at least!) that the "crowd" under retreat under enemy fire will keep a minimum of order by moving themselves with a *constant speed* v (there is always a patriotic sargent-major with a taurus-pistol 45 trying to impose order on the heard stampid!).

Let us presently in a resumed form the solution of eq. (2), (3), (4). Its solution reads

$$U(x,t) = \left(t^{-\frac{\nu}{2}}\right)\phi(x,t) \tag{4}$$

$$\phi_k(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dx\ e^{ikx}\phi(x,t) \tag{5}$$

$$\phi_k(t) = k\sqrt{t}\left\{A(k)J_{\left(\frac{\nu-1}{2}\right)}(k^2t) + B(k)Y_{\left(\frac{\nu-1}{2}\right)}(k^2t)\right\} \tag{6}$$

where $A(k)$ and $B(k)$ are adjusted by the initial conditions for $t = t_0 > 0$.

The particular solution is easily written down in terms of the homogeneous solutions eq. (6) and depending linearly on the "heard stampid" stochastic loading eq. (3).

Finally, let us present the $y(t)$ two-point out-put correlation function associated to the over-damped case of a harmonic oscillator with mass m and damping parameter ν and the imput modeled by a shot noise $S(t)$ with $h(\sigma) \in L^2(R)$ and $\{t_i\}$ are poisson distributed random times with expected arrival rate λ (a stupid parachute army brigade with their airplane hitted and jumping without command on the civilians roof's houses since the transport is on fire and going to crash!)

$$S(t) = \sum_{\{i\}} h(t - t_i) \tag{7}$$

In this case, we have the exactly formulae for the roof's two-point response

$$E[y(t_1)y(t_2)] = \langle y(t_1)y(t_2)\rangle = \left(\frac{1}{m^2w^2}\right)$$
$$exp(-\nu(t_1+t_2))\int_0^{t_1} d\sigma_1 \int_0^{t_1} d\sigma_2\ exp(\nu(\sigma_1+\sigma_2))\ [\sin(w(t_1-\sigma_1))][\sin(w(t_2-\sigma_2))] \times$$
$$\times\{\lambda\int_{-\infty}^{+\infty} d\alpha\ h(\alpha)\ h(\alpha + |\sigma_2-\sigma_1|)\} \tag{8}$$

The above written object give us directly the out-put processes spectral density in terms of the convolution of the function $h(\sigma)$ (the air-borne soldier weight!) with itself (last reference in [3]).

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Appendix A – On the Bridge Collapse Parameter

In this Appendix we point out how to obtain theoretically the parameter b associated to the level-crossing of the bridge deflection collapse $v(x,\cdot) = b$ in the simple case of a tensioned string.

Let us firstly point out the form of the complete wave equation associated to the string with a space-time density $\rho(x,t)$ and a space-time internal tension $T(x,t)$ ("Quantum Geometry of Bosonic Strings-Revisted (CBPF-NF-042/99) Luiz C.L. Botelho and Raimundo C.L. Botelho)

$$\left(\frac{\rho(x,t)}{T(x,t)}\right)\frac{\partial^2 U(x,t)}{\partial^2 t} = \frac{\partial}{\partial x}\left[arcsinh\left(\frac{\partial U(x,t)}{\partial x}\right)\right] \tag{A.1}$$

The parameter b is the threshold value of the string deflection $U(x,t)$ which makes the non-linear term of eq. (A-1) (the righ-hand side) to safely be substituted by the usual term $\frac{\partial^2}{\partial^2 x}U(x,t)$, which, by its turn, allows the existence of harmonic string excitations (and decent guitar music compositions!).

It is worth point out that *there is no harmonic solutions of eq. (A-1)* $(U(x,t) = A(x)exp(iwt))$ *in the simple case of* $\rho(x,t)/T(x,t) - \frac{1}{v^2} = c^{te}$, that may be possible explanation for the non-harmonic guitar music of the *musicists-physicists* Jorjão-Marcão of the band "Planet Boy-Uol(ás).com.br". May be they play their compositions guitar above the linear parameter b! (a reverberant noisy?) ([2]).

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