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A Rotating Quantum Vacuum and the Depolarization Problem in Storage Rings

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Abstract

We investigate the consequences of using a "Lorentz-like" transformation to connect measurements between a inertial and a rotating frame of reference. We obtain a new rotating vacuum (of a massless scalar field) different from the Minkowski one. After this we consider a monopole detector interacting with the field. The radiative processes are discussed from a rotating and inertial frame point of view. Finally using this formalism the polarization effects of electrons in circular accelerators is discussed.

Key-words: Rotating vacuum; Radiative processes.

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1 Introduction

It is a well known experimental fact that electrons (positrons) experience a gradual polarization when orbiting in a storage ring. This mechanism lead to the emission of spin-flip synchroton radiation [1]. Although the amount of spin-flip radiation is extremely small compared with the non-flip radiation one would expect asymptotically a total polarization. An open question [2]: *is why the polarization is not complete after the system reach the equilibrium ?*

Bell and Leinaas [3] studied the depolarization problem in accelerators trying to use the idea of a Unruh-Davies effect [4]. The electron in a accelerated ring is a magnetic version of the monopole detector, since there is a linear coupling between the magnetic field and the magnetic moment of the electron. Although the situation is similar to the Rindler's case where the detector goes to excited state by absorption of Rindler's particles, there is a fundamental difference. In the Rindler's case there is a past and future horizont. Part of information which would have an inertial observer is inaccessible for accelerated observers. Although the Minkowski vacuum $|0, M >$ is a pure state, for accelerated observers it must be described by a statistical operator. This is the origin of the thermal distribution of particles. As was noted by Bell and Leinaas, in the case of circular motion, the measurements of the polarization *does not agree* with the calculations if we interpret the polarization by thermal effects.

A central question is: which mapping we have to assume to compare measurements made in an inertial and a rotating frame of reference? If we assume a Galilean transformation it is well known that the rotating vacuum defined by these transformations is just the Minkowski vacuum [5]. The introduction of an apparatus device which gives information about the particle content of some state (the particle detector) raised a fundamental question. If we prepare a detector in the ground state and the field in the Minkowski vacuum there is a non-null probability to find the rotating detector in an excited state. How is possible a rotating detector be excited in the Minkowski vacuum? Another way to formulate the puzzle is: our physical intuition says that a rotating particle detector prepared in its ground state interacting with the field in the rotating vacuum must stays in the ground state. Nevertheless, assuming the Galilean transformation the Minkowski vacuum is just the rotating vacuum and the rate of excitation of the rotating detector in the rotating vacuum is different from zero. One would expect the rotating detector not be excited by the rotating vacuum [6]. Doing to these interpretational difficulties associated with the Galilean transformation we have attemptt to discuss the consequences of assuming a Lorentz-like transformation to connect measurements between an inertial and a rotating frame of reference. A natural consequence is that, this rotating vacuum is not the Minkowski one. These results leads to an alternative solution for the depolarization problem. In our approach, depolarization is related with the fact that the Minkowski vacuum is a many particle state of rotating particles. The polarization can not be complete since the process of absorption of a rotating particle from the Minkowski vacuum with spin-flip has always probability different from zero. We would like to stress that we tried to avoid many technical difficulties to emphasize only fundamental results.

The paper is organized as follows. In section 2 we canonical quantize a massless scalar field in a rotating frame of reference showing that the rotating vacuum is different from the Minkowski one. In section 3 the polarization problem is studied. Conclusions are given in section 4. In this paper we use $\hbar = c = 1$.

2 The Lorentz-like Transformations and a New Rotating Vacuum

The rotating detector puzzle raised a question: which mapping we have to assume to compare measurements made in an inertial frame and in a rotating frame of reference [7]? If we assume a Galilean transformation it is a well know result that the rotating vacuum defined by these transformation is just the Minkowski vacuum. Doing to interpretational difficulties associated the Galilean transformation in this paper we have attempt to discuss the consequences of assuming a "Lorentz-like" transformations to connect measurements in both frame of references. These transformations between the inertial coordinate system ($x'^{\mu} = \{t', r', \theta', z'\}$) and the rotating coordinate system ($x^{\mu} = \{t, r, \theta, z\}$) was presented by Trocheries and also Takeno [8] and are given by

$$t = t' \cosh \Omega r' - r'\theta' \sinh \Omega r', \tag{1}$$

$$r = r', \tag{2}$$

$$\theta = \theta' \cosh \Omega r' - \frac{t'}{r'} \sinh \Omega r', \tag{3}$$

$$z = z'. \tag{4}$$

Assuming this mapping to connect measurements made in the rotating frame and those made in the inertial frame, in the rotating coordinate system the line element assume a non-stationary form.

It is possible to write the transformations defined by eqs. (1) and (3) making a analogy with the Lorentz transformations. Let us define $l = r\theta$ and $\gamma = (1 - v^2)^{-\frac{1}{2}}$. It is straightforward to show that the above transformations becomes

$$t = \gamma(t' - vl'), \tag{5}$$

$$l = \gamma(l' - vt'). \tag{6}$$

The transformations defined by Trocheries and Takeno are "Lorentz-like". The fundamental difference is that in this case the velocity is $v = \tanh \Omega r$. It has been suggested by many authors that the only way to have results consistents with the Sagnac's effect is to use a Galilean transformation [9]. We would like to stress that the arguments used by these authors does not establish conclusively that we have to use the Galilean transformations. Direct supports of Lorentz-like transformation between both frames are suplied by the rotating detector puzzle and the depolarization effect of electrons in a circular accelerator.

To prove that there is a rotating vacuum different from the Minkowski one, we have to canonical quantize a massless scalar field assuming the Takeno and Trocheries mapping given by eqs.(1-4). It is an human impossible task to solve exactly the Klein-Gordon equation in this coordinate system. Making a Taylor expansion for $\cosh \Omega r$ and $\tanh \Omega r$

and retaining terms up to first order in Ωr, the line element becomes

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - 4r\Omega\theta dr dt - dz^2. \tag{7}$$

We point out that although we will consider only the case $\Omega r < 1$, the low-velocity limit of Trocheries and Takeno transformation does not give the "Galilean" transformation since we have $t = t' - \Omega r^2 \theta'$. We would like to mention that although the Takeno-Trocheries's transformation constitute a continuous group with a parameter Ω in the low velocity limit this propriety is lost. In this approximation the metric is stationary by not static. This means that although there is a timelike Killing vector field K, the spatial sections putting $t = constant$ are not orthogonal to the time lines putting r, θ, z constants, i.e., the Killing vector K is not orthogonal to the spatial section. This line element describe a physical situation in which world lines infinitesimally close to another one are spatially rotating with respect this world line. In this simplified case, the Kein-Gordon equation reads

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} - 4\Omega\theta\frac{\partial}{\partial t} - \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - 4\Omega\theta r\frac{\partial^2}{\partial r\partial t} - \frac{\partial^2}{\partial z^2}\right)\varphi(t, r, \theta, z) = 0. \tag{8}$$

The solution can be found using partial separation of variables

$$\varphi(t, r, \theta, z) = T(t)Z(z)f(r, \theta). \tag{9}$$

Assuming

$$Z(z) = e^{ik_z z} \tag{10}$$

and

$$T(t) = e^{-i\omega t}, \tag{11}$$

and finally, defining $\omega^2 = k_z^2 + q^2$ we obtain the equation for $f(r, \theta)$

$$\left[\frac{\partial^2}{\partial r^2} + (\frac{1}{r} - 4i\omega\Omega\theta r)\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + (q^2 - 4i\omega\Omega\theta)\right] f(r, \theta) = 0. \tag{12}$$

The perturbative solution of this equation can be found Ref. [10] and is given by

$$\begin{aligned} f(y, \theta) &= e^{i\mu\theta}\left[J_\mu(y) + l e^{i\lambda\theta} J_{\mu+\lambda}(y)\right] \\ &+ \frac{l}{2}\int d\theta' \int dy' G(y, \theta; y', \theta')\theta' \left[y'^3 J_{\mu-1}(y') + 2y'^2 J_\mu(y') - y'^3 J_{\mu+1}(y')\right], \end{aligned} \tag{13}$$

where $l = 4i\Omega\omega/q^2$, $y = qr$ and $G(y, \theta; y', \theta')$ is defined by the follow equation:

$$\left[\left(y^2\frac{\partial^2}{\partial y^2} + y\frac{\partial}{\partial y} - \mu^2 + y^2\right) + \frac{\partial^2}{\partial\theta^2} + 2i\mu\frac{\partial}{\partial\theta}\right] G(y, \theta; y', \theta') = \delta(y - y')\delta(\theta - \theta').$$

Although the spatial part of the solution of eq.(8) is extremely complicated, there is not ambiguity in the definition of positive and negative rotating modes since the temporal part is given by eq.(11) and the world line of the detector is an integral curve of the Killing vector $K = \partial/\partial t$ that generates a one-parameter group of isometries.

We must now turn to the question of single valuedness of $f(y, \theta)$. This situation is very similar to the $(2+1)$ dimensional gravity [11]. In our situation we have two different possibilities: the first is to assume that $f(y,\theta)$ is a single value function. When θ increases from 0 to 2π for a constant y, t jumps by Ωr^2 given a time helical structure. It is possible to show that this solution acquire a phase $e^{i\omega\Omega r^2}$. Using Dirac's arguments and also the Mazur ideas [12] the energy of the modes must be quantized

$$\omega = n\left(\frac{2\pi}{\Omega r^2}\right),\ n = \text{integer}.$$

Note that at principle we can choose that Ω is quantized to eliminate the phase problem. With this choice the Bogoliubov coefficients $\beta_{k\nu}$ are zero. The second one is do not assume that $f(y,\theta)$ is a single value function, and in this case the Bogoliubov coefficients $\beta_{k\nu}$ are differents from zero and proportional to $\omega\Omega^2 r^2$. It is important to realize that if we assume that $f(y,\theta)$ is single value the Minkowski and the rotating vacuum are the same only in the small velocity approximation. If we go further retaing terms of order $\Omega^2 r^2$ in the Takeno's coordinates transformation the Bogoliubov coefficients must be different from zero.

Going back to the line element given by eq.(7) we see that the line element is stationary and there is no ambiguity to define the rotating vacuum $|0,R>$ in such a way that:

$$b_{q\mu k_z}|0,R>=0 \qquad \forall\ q,\mu,k_z, \tag{14}$$

where

$$\varphi(t,r,\theta,z) = \sum_\mu \int q dq dk_z \left[b_{q\mu k_z} v_{q\mu k_z}(t,r,\theta,z) + b^\dagger_{q\mu k_z} v^*_{q\mu k_z}(t,r,\theta,z)\right]. \tag{15}$$

By sake of simplicity let use the following notation:

$$\varphi(t,r,\theta,z) = \sum_\nu b_\nu v_\nu(t,r,\theta,z) + b^\dagger_\nu v^*_\nu(t,r,\theta,z). \tag{16}$$

It is straighforward to show that the Minkowski vacuum can be expressed as a many rotating-particles state. By comparing the expansion of the field operator using the inertial modes and the rotating modes it is possible to obtain the expression comparing both vacua, i.e $|0,M>$ and $|0,R>$:

$$|0,M>= e^{\frac{i}{2}\sum_{\mu,\nu} b^\dagger(\mu)V(\mu,\nu)b^\dagger(\nu)}|0,R> \tag{17}$$

where

$$V(\mu,\nu) = i\sum_k \beta^*_{\mu k}\alpha^{-1}_{k\nu}, \tag{18}$$

and the Bogoliubov coefficients are given by $\beta_{\nu k} = -(v_\nu, \psi^*_k)$ and $\alpha_{\nu k} = (v_\nu, \psi_k)$. It is clear that the number of rotating particles in a specific mode in the Minkowski vacuum is given by

$$<0,M|N_R(\nu)|0,M>= \sum_k |\beta_{k\nu}|^2. \tag{19}$$

Let us choose the hypersurface $t' = constant$ to find the Bogoliubov coefficients i.e.,

$$\beta_{\nu k} = i \int_0^{2\pi} d\theta' \int_{-\infty}^{\infty} dz \int_0^{\infty} r dr \, [v_\nu(x')(\partial_{t'}\psi_k(x')) - (\partial_{t'}v_\nu(x'))\psi_k(x')] \, . \tag{20}$$

The Bogoliubov coefficients $\beta_{\nu k}$ must be non-zero since the positive and negative frequency rotating modes are mixture between positive and negative inertial modes. The important conclusion from the above arguments is that the Minkovski $|0, M >$ and this rotating vacuum $|0, R >$ are not the same.

To analyse the rotating particle content of the Minkowski vacuum and to make a parallel with electrons and positrons in magnetic field it is possible to introduce a monopole detector with a linear coupling with the scalar field [13]. This system is endowed with internal degrees of freedom defining two energy levels with energy ω_1 and ω_2, $(\omega_1 < \omega_2)$ and respective eigenstates $|1\rangle$ and $|2\rangle$ $(\omega_2 - \omega_1 = E)$. The important result is that asymptotically the rate of excitation (decay) of the detector is given by the Fourier transform of the positive frequency Wightman function. This is exactly the quantum version of the Wiener-Khintchine theorem which asserts that the spectral density of a stationary random variable is the Fourier transform of the two point-correlation function. We would like to stress that also it is possible to find non-asymptotic results [14].

Let us analyse the process of transition of the detector from the rotating frame of reference point of view. We will have three different processes: $R_{1\to 2}(E, \Delta T)$ is the rate of absorption of rotating particles from the Minkowski vacuum, $R_{2\to 1}(E, \Delta T)$ is the rate of induced emission of a rotating particle and $A_{2\to 1}(E, \Delta T)$ is the rate of spontaneous emission of a rotating particle (stimulated emission induced by the $|0, R >$ vacuum fluctuations. A straighforward calculation shows that the rate of excitation have two different terms: the first is not related with any measurement of the particle content of any state (vacuum fluctuation) and a second one which will be proportional to the number of rotating particles in the Minkowski vacuum.

The above described situation raises a central question. Where does the energy of excitation come from if we analyse the process from the point of view of the inertial observer? The non-inertial observer does not meet any difficult. At some initial time we prepare the detector in the ground state and the field in the Minkowski vacuum. Since the Minkowski vacuum is a many rotating-particles state, the detector goes to excited state absorbing a positive energy particle. For large time intervals energy conservation holds. For the point of view of the inertial observer the field is in the empty state. How is possible the excitation? A natural answer is to say that it is necessary an external accelerating agency to suplly energy to keep the detector in the rotating world-line. It is possible to show that the detector goes to excited state with the emission of a Minkowski particle. In the next section we will perform the second quantization of the detector Hamiltonian to analyse the absorption and emission processes from the inertial point of view.

3 Second Quantization of the Total Hamiltonian and Polarization Effects on Electrons and Positrons in Storage Rings

In this section we will prove that the process: absorption (emission) of positive energy rotating particle with excitation (decay) of the detector (from the non-inertial point of view) is interpreted as a emission of a Minkowski particle with excitation (decay) of the detector from the inertial point of view. We will try to answer this question applying the ideas developed by us in the preceding sections. Before start the second quantization of the detector and interaction Hamiltonian let us remember the main results of the section 2 and 3.

In Minkowski space time it is possible to quantize a massless scalar field using the cylindrical coordinate adapted to inertial observers. Thus the scalar field can be expanded using an orthonormal set of modes

$$\varphi(x) = \sum_i a_i u_i(x) + a_i^\dagger u_i^*(x) \tag{21}$$

where

$$a_i|0, M>= 0 \qquad\qquad \forall i. \tag{22}$$

There is an inequivalent quantization using coordinates adapted to a rotating observer. The scalar field can be expanded using a second set of orthonormal modes

$$\varphi(x) = \sum_j b_j v_j(x) + b_j^\dagger v_j^*(x) \tag{23}$$

where

$$b_j|0, R>= 0 \qquad\qquad \forall j. \tag{24}$$

As both sets are complete, the non-inertial modes can be expanded in terms of the inertial ones, i.e.,

$$v_j(x) = \sum_i \alpha_{ji} u_i(x) + \beta_{ji} u_i^*(x). \tag{25}$$

Using the fact that both sets are complete and orthonormal, it is possible to write the annihilation and creation operators of non-inertial particles in the mode j as a linear combination of inertial creation and annihilation operators [15], i.e.,

$$b_j = \sum_i \alpha_{ji}^* a_i - \beta_{ji}^* a_i^\dagger. \tag{26}$$

To second quantize the detector Hamiltonian, let us use the notation introduced in chapter 3, i.e. $|g>=|1>$ and $|e>=|2>$ and introduce the Dicke operators [16]

$$S^+ = |2><1|, \tag{27}$$

$$S^- = |1><2|, \tag{28}$$

and finally

$$S_z = \frac{1}{2}(|2><2| - |1><1|). \tag{29}$$

Using the orthonormality of the energy eigenstates of the detector Hamiltonian and the Dicke operators, we can write the detector Hamiltonian as

$$H_D = ES_z + \frac{1}{2}(\omega_1 + \omega_2). \tag{30}$$

The operators S^+, S^- and S_z satisfy the angular momentum commutation relations corresponding to spin 1/2 value. It is clear that S^+ and S^- are respectively creation and annihilation operators of the detector states ($S^+|1>=|2>, S^+|2>=0, S^-|2>=|1>, S^-|1>=0$). The interaction Hamiltonian between the detector and the scalar field can be written as

$$H_{int} = \lambda[m_{21}S^+ + m_{12}S^- + S_z(m_{22} - m_{11})]\varphi(x), \tag{31}$$

where the matrix elements of the monopole operator of the detector $m(\tau)$ are given by:

$$<i|m(0)|j>= m_{ij}. \tag{32}$$

We should simplify the discussion choosing $m_{11} = m_{22}$. As we will see the part if the interaction hamiltonian with the S_z operator is responsible for the non-flip synchroton radiation. Substituting eq.(23) in eq.(31) we see that there are different processes with absorption or emission of rotating particles with excitation or decay of the detector. It is possible to show that some of these processes are virtual, and only processes with energy conservation survive in the asymptotic limit, i.e., excitation of the detector with absorption of a rotating particle (processes involving $b_j S^+$) and decay of the detector with emission of a rotating particle (processes involving $b_j^\dagger S^-$).

The first process is generated by the following operators:

$$m_{12}\sum_j v_j(x) b_j S^+. \tag{33}$$

Substituting eq.(25) and eq.(26) in eq.(33) it is clear that the above process of absorption of a rotating particle in the mode j is the following:

$$\sum_{ijk}\left(\beta^*_{ji}\alpha_{jk}u_k(x) + \beta^*_{ji}\beta_{jk}u^*_k(x)\right) a_i^\dagger S^+. \tag{34}$$

Therefore this process for the inertial observer is an excitation of the detector with creation of Minkowski particles.

The second process is generated by the following operators:

$$m_{21}\sum_j v^*_j(x) b_j^\dagger S^-. \tag{35}$$

Substituting eq.(25) and eq.(26) in eq.(35) we see that the above process of emission of a rotating particle in the mode j is the following:

$$\sum_{ijk}\left(\alpha_{ij}\alpha^*_{jk}u^*_k(x) + \alpha_{ij}\beta^*_{jk}u_k(x)\right) a_i^\dagger S^-. \tag{36}$$

Therefore this process for the inertial observer is a decay of the detector with creation of Minkowski particles.

Now we are able to understand the problem of the synchroton radiation. In the emission of synchroton radiation by electrons moving along a circular orbit, there are two kinds of processes: the first is the emission of photons without spin flip of the electron and the second is emission with spin flip. We will restrict our discussion to the second case. To make a parallel with the detector's problem we have to assume that the electron trajectory is "classical" (there is no fluctuation of the electron path) or even after the photon emission there is no recoil (as was stressed by Bell and Leinaas, the results does not depend on position fluctuations of the electron trajectory). There are two differents results in the literature depending on the value of the Landé g factor of the electron. Jackson showed that the rate of transition from an initial state with the spin of the electron directed along the magnetic field (high energy state) to a state with the electron spin in opposite to the magnetic field (lower energy state) is lower than the opposite situation if the Landé g factor of the electron obeys $0 < g < 1.2$. It is important to stress that the situation is opposite of the naive description where polarization arises from spontaneous emission as the spin move from its "upper" (high energy state) to its "lower" (low energy state) in the magnetic field. For the case where $1.2 < g < 2$ Jackson and also other authors obtained that after the photon emission the electron spin will tends to orient themselves in opposite to the magnetic field (going to the lower energy state). Of course, positrons spins will have an oposite behavior. These both results are consistent with our interpretation that absorption (emission) of a rotating particle with excitation (decay) of the detector in the non-inertial frame is interpreted as emission of a Minkowski particle with excitation (decay) of the detector in the inertial frame.

To find the degree of polarization before the equilibrium situation is achieved let us define the occupation number of electrons with spins directed in oposition to the magnetic field (lower energy state) by N_1, and N_2 the number of electrons with spins directed to the magnetic field. Of course we have $N_1(t) + N_2(t) = N$, where N=$constant$ is the total numbers of electrons in the ring. We will do all the calculations in the rotating frame of reference. The degree of polarization of an ensemble of electrons in the beam is defined as

$$P(t) = \frac{N_1(t) - N_2(t)}{N_1(t) + N_2(t)}. \tag{37}$$

The equation of the evolution of N_1 and N_2 are given by

$$\frac{dN_1}{dt} = N_2\left[\rho(E)R_{2\to1}(E,\Delta T) + A_{2\to1}(E,\Delta T)\right] - N_1\left[\rho(E)R_{1\to2}(E,\Delta T)\right] \tag{38}$$

and

$$\frac{dN_2}{dt} = N_1\left[\rho(E)R_{1\to2}(E,\Delta T)\right] - N_2\left[\rho(E)R_{2\to1}(E,\Delta T) + A_{2\to1}(E,\Delta T)\right]. \tag{39}$$

Let us avoid the difficult to find $R_{1\to2}(E,\Delta T))$ and $R_{2\to1}(E,\Delta T)$ and using the following approximation i.e,

$$\rho(E)R_{2\to1}(E,\Delta T) + A_{2\to1}(E,\Delta T) = \sigma_{21} = constant \tag{40}$$

and

$$\rho(E)R_{1\to 2}(E,\Delta T) = \sigma_{12} = constant. \tag{41}$$

Then starting from a situation where there is no polarization i.e., $P(t=0)=0$ it is possible to find the polarization until the equilibrium situation is achieved. It is necessary only to integrate the above equations. A straightforward calculation gives

$$N_1(t) = \frac{N}{2}\left(\frac{\sigma_{12}-\sigma_{21}}{\sigma_{12}+\sigma_{21}}\right)e^{-(\sigma_{12}+\sigma_{21})t} + N\left(\frac{\sigma_{21}}{\sigma_{12}+\sigma_{21}}\right) \tag{42}$$

and

$$N_2(t) = -\frac{N}{2}\left(\frac{\sigma_{12}-\sigma_{21}}{\sigma_{12}+\sigma_{21}}\right)e^{-(\sigma_{12}+\sigma_{21})t} + N\left(\frac{\sigma_{12}}{\sigma_{12}+\sigma_{21}}\right). \tag{43}$$

The degree of polarization of the beam is

$$P(t) = \left(\frac{\sigma_{21}-\sigma_{12}}{\sigma_{12}+\sigma_{21}}\right)\left(1-e^{-(\sigma_{12}+\sigma_{21})t}\right). \tag{44}$$

We obtained that if $R_{1\to 2}(E,\Delta T)$, $R_{2\to 1}(E,\Delta T)$ and $A_{2\to 1}(E,\Delta T)$ are independent of time the asymptotic degree of polarization is constant i.e.,

$$\lim_{t\to\infty} P(t) = \left(\frac{\sigma_{21}-\sigma_{12}}{\sigma_{12}+\sigma_{21}}\right). \tag{45}$$

Experimental results show us a not complete polarization. Why there is residual depolarization? This is the puzzle stressed by Jackson [2] and also Bell and Leinaas [3]. From the former equation it is easy to see that the polarization can not be complete, since the process absorption of a rotating particle with excitation of the detector has always probability different from zero. In the asymptotic limit we have that if

$$R_{21} + A_{21} > 3R_{12}, \tag{46}$$

the lower energy state is prefered ($1.2 < g < 2$, for the Landé g factor), and if

$$R_{21} + A_{21} < 3R_{12}, \tag{47}$$

the higher energy state is prefered ($0 < g < 1.2$ for the giromagnetic factor).

We remark that the results that the polarization can not be complete was obtained in a very crude approximation where the rates $R_{1\to 2}(E,\Delta T)$, $R_{2\to 1}(E,\Delta T)$ and $A_{2\to 1}(E,\Delta T)$ does not depend on time. A more realistic result can be obtained assuming that this rates does depend on time. Defining $n_1 = N_1/N$, $n_2 = N_2/N$ and also

$$\rho(E)R_{2\to 1}(E,\Delta T) + A_{2\to 1}(E,\Delta T) = \sigma_{21}(t) \tag{48}$$

and

$$\rho(E)R_{1\to 2}(E,\Delta T) = \sigma_{12}(t), \tag{49}$$

we obtain the following equations:

$$n_1(t) + n_2(t) = 1 \tag{50}$$

and

$$\frac{dn_1(t)}{dt} + n_1(t)\left[\sigma_{12}(t) + \sigma_{21}(t)\right] = \sigma_{21}(t). \tag{51}$$

The general solution that we are looking for, involves two quadratures and it is given by

$$n_1(t) = Ce^{-\int^t [\sigma_{12}(t')+\sigma_{21}(t')]dt'} + e^{-\int^t [\sigma_{12}(t')+\sigma_{21}(t')]dt'} \int^t dt' \sigma_{21}(t') e^{\int^{t'} [\sigma_{12}(t'')+\sigma_{21}(t'')]dt''}. \tag{52}$$

With the values of $R_{2\to 1}(E, \Delta T)$, $R_{1\to 2}(E, \Delta T)$ and $A_{2\to 1}(E, \Delta T)$, it is possible to find the exact degree of polarization.

4 Conclusions

In this paper we discuss two different puzzles: the depolarization problem in circular rings and the rotating detector puzzle (actually they are the two sides of the same coin). For electrons in storage rings, a residual depolarization has been found experimentally. Bell and Leinaas investigate this effect using the idea of circular Unruh-Davies effect. We propose a alternative solution to both problems using a coordinate transformation between the inertial and the rotating frame derived by Trocheries and Takeno. With our assumption a new rotating vacuum is presented.

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