CBPF-NF-015/99

The hyperon–nucleon interaction potential in the bound state soliton model: the ΛN case

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Abstract

We develop the formalism to study the hyperon-nucleon interaction potential within the bound state approach to the $SU(3)$ Skyrme model. The general framework is illustrated by applying it to the diagonal ΛN potential. The central, spin-spin and tensor components of this interaction are obtained and compared with those derived using alternative schemes.

Key-Words: hyperon–nucleon interaction, ΛN potential, $SU(3)$ Skyrme model, bound state soliton model

PACS: 13.75.Ev, 12.39.Fe, 12.39.Dc

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1 Introduction

The long standing interest in the hyperon-nucleon two-body interaction is motivated by several reasons. For example, such interaction is the fundamental building block for a microscopic understanding of hypernuclei [1]. In addition, the inclusion of the strangeness degrees of freedom calls for the extension of the models of the nucleon-nucleon potential so as to provide a unified coherent picture of the baryon-baryon interaction. Nowadays quantum chromodynamics (QCD) is considered to be the fundamental theory for the strong interactions. However, the low momentum- and energy-transfer region, which is the relevant one for nuclear physics, is dominated by non-perturbative processes. Consequently, a first principles evaluation of the baryon-baryon interaction in terms of quarks and gluons has not been possible up to now. In this situation one has to resort to models. In the phenomenological one boson exchange (OBE) model [2] the nucleon-nucleon interaction is described through the exchange of different mesons, supplemented by a short range repulsion. In the case of the hyperon-baryon interaction, the Nijmegen [3] and Jülich [4] potentials are obtained by extending the OBE model to include the degrees of freedom that carry strangeness. In this way the quite limited ΛN and ΣN scattering data can be described within a unified picture at the price of introducing a rather important number of adjustable parameters. In fact, since the experimental data are sufficiently crude they can be reproduced using various sets of parameters, out of which two representative examples are those which define the models D and F of Ref.[3].

Here we will follow an alternative approach which is based on the Skyrme model [5, 6]. This model relies on the fact that for a large number of colours N_c, QCD becomes equivalent to a local field theory of mesons [7] where the nucleons emerge as chiral topological solitons [8] of the meson effective field theory. The Skyrme model is the simplest choice of such a theory. It provides a reasonable good description of the $SU(2)$ baryon properties and has already been implemented for the construction of the nucleon-nucleon potential, as reviewed in [9, 10]. To include strangeness in this scheme we will consider the bound state approach (BSA)[11, 12] extension of the Skyrme model to flavour $SU(3)$. In this way we complement the work of Refs.[13, 14], where the hyperon-nucleon potentials have been studied in the collective coordinates approach (CCA) to the $SU(3)$ Skyrme model. Differently from the CCA, where strangeness appears as a collective rotational excitation, in the BSA hyperons are described as bound states of kaons in the background field of a $SU(2)$ soliton. It is worthwhile to point out that these soliton models have both the merit of describing the different baryonic sectors ($B = 1$ and $B = 2$, B being the baryon number) in a single comprehensive framework. Moreover, the corresponding predictions are essentially parameter free since, in principle, all the parameters in the effective action can be fixed by the meson phenomenology in the $B = 0$ sector.

The present work constitutes a first step towards a general discussion of the hyperon–nucleon potential within the bound state model. It is similar in spirit to previous NN potential calculations done in the $SU(2)$ Skyrme model, although technically much more involved. The interaction potential can be written in the general form

$$V_{HN}(\vec{r}) \quad = \quad V_C(r)\ \mathcal{O}_C + V_S(r)\ \mathcal{O}_S + V_T(r)\ \mathcal{O}_T \tag{1}$$

where $\mathcal{O}_C = I^H_{2\times2} I^N_{2\times2}$, $\mathcal{O}_S = \vec{\sigma}_H \cdot \vec{\sigma}_N$, $\mathcal{O}_T = 3\ \vec{\sigma}_H \cdot \hat{r}\ \vec{\sigma}_N \cdot \hat{r} - \vec{\sigma}_H \cdot \vec{\sigma}_N$ and $V_C(r)$, $V_S(r)$, $V_T(r)$

are the central , spin-spin and tensor parts of the interaction, respectively. They can be decomposed into an isospin independent contribution V^+ and an isospin dependent one V^-, so that

$$V_{C,S,T}(r) = V^+_{C,S,T}(r) + V^-_{C,S,T}(r)\ \vec{\tau}_H \cdot \vec{\tau}_N \ . \tag{2}$$

The contributions depending on angular momentum, like the spin-orbit coupling, are not shown because they remain unaccessible within the approximations used in our calculation. The new feature of the $SU(3)$ case is the strangeness exchange interaction mediated by kaons. In the language of the OBE models these interactions are of first order in terms of kaon exchanges, but as well as the direct contributions include higher orders (more than one boson exchange) in the $SU(2)$ sector. The general framework is illustrated by the explicit calculation of the diagonal ΛN potential, which is the simplest one due to the absence of isospin interactions.

The article is organized as follows. In Sec.2 we review briefly the BSA and in Sec.3 we describe the general procedure to obtain the interaction Lagrangian. In Sec.4 we show how to obtain the explicit form of the interaction potential in the ΛN case. In Sec.5 we present our numerical results. Finally, in Sec.6 our conclusions are given. Some useful expressions needed for the evaluation of the collective part of the matrix elements appearing in the potential can be found in the Appendix.

2 The bound state soliton model

The bound state soliton model has been discussed in detail in the literature [11, 12]. Therefore, only a brief outline of its main features will be presented here. The starting point is an effective $SU(3)$ chiral action which includes an explicit symmetry breaking term. It has the form

$$\Gamma = \int d^4x \Big(\mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_{\rm SB} \Big) + \Gamma_{\rm WZ} \ , \tag{3}$$

where $\mathcal{L}_2$ is the well-known non linear σ-model lagrangian density,

$$\mathcal{L}_2 = -\frac{f_\pi^2}{4} \, {\rm Tr} \left[L_\mu L^\mu \right] \ , \tag{4}$$

and $\mathcal{L}_4$ is the Skyrme stabilizing term,

$$\mathcal{L}_4 = \frac{1}{32\epsilon^2} \, {\rm Tr} \Big[[L_\mu, L_\nu][L^\mu, L^\nu] \Big] \ , \tag{5}$$

with the left current L_μ expressed in terms of the $SU(3)$ valued chiral field $U(x)$ as $L_\mu = U^\dagger \partial_\mu U$. Here f_π is the pion decay constant and ϵ is the so-called Skyrme parameter.

The non-local Wess-Zumino action $\Gamma_{\rm WZ}$ is given by

$$\Gamma_{\rm WZ} = -\frac{iN_c}{240\pi^2} \int_{D5} d^5x \ \varepsilon^{\mu\nu\alpha\beta\gamma} \, {\rm Tr} \left[L_\mu L_\nu L_\alpha L_\beta L_\gamma \right] \ , \tag{6}$$

where the domain of integration is a five dimensional disk D_5 whose boundary is space–time. The symmetry breaking term $\mathcal{L}_{\rm SB}$ takes into account the difference between the mass of the kaon m_K and the mass of the pion m_π as well as the difference between f_π and the kaon decay constant f_K. It is given by

$$\mathcal{L}_{\rm SB} = \frac{f_\pi^2 m_\pi^2 + 2 f_K^2 m_K^2}{12} \,{\rm Tr}\left[U + U^\dagger - 2\right] + \sqrt{3}\frac{f_\pi^2 m_\pi^2 - f_K^2 m_K^2}{6}\,{\rm Tr}\left[\lambda_8\left(U + U^\dagger\right)\right] - \frac{f_K^2 - f_\pi^2}{12}\,{\rm Tr}\left[(1-\sqrt{3}\lambda_8)\left(U L_\mu L^\mu + U^\dagger R_\mu R^\mu\right)\right] , \tag{7}$$

with λ_8 being the eighth Gell-Mann matrix and R_μ the right current $R_\mu = U \partial_\mu U^\dagger$.

To describe the $B = 1$ soliton sector we introduce the ansatz [11]

$$U = \sqrt{U_\pi}\, U_K \,\sqrt{U_\pi} , \tag{8}$$

where

$$U_K = \exp\left[i \frac{\sqrt{2}}{f_\pi} \begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix} \right] , \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} , \tag{9}$$

and U_π is the soliton background field written as a direct extension to $SU(3)$ of the $SU(2)$ field u_π, i.e.,

$$U_\pi = \begin{pmatrix} u_\pi & 0 \\ 0 & 1 \end{pmatrix} , \tag{10}$$

with u_π being the conventional hedgehog solution $u_\pi = \exp[i\vec{\tau}\cdot\hat{r}F(r)]$.

Assuming that the chiral symmetry breaking along the strangeness direction is strong enough, the effective action is expanded up to the second order in the kaon field. The resulting lagrangian density can be written as the sum of a pure $SU(2)$ term depending on u_π only and an effective lagrangian density describing the interaction between the soliton and the kaon fields. The soliton profile $F(r)$ is obtained by minimizing the corresponding classical $SU(2)$ energy. On the other hand, the kaon field satisfies an eigenvalue equation which describes its dynamics in the presence of the soliton background field. In this picture, low–lying strange hyperons arise from the bound state solutions of this equation. In particular, the octet and decuplet hyperons are obtained by populating the lowest kaon bound state which carries the quantum numbers $\Lambda = 1/2,\ l = 1$. Here, Λ is the grand–spin defined by the coupling of angular momentum and isospin and l is the kaon angular momentum. The splitting among hyperons with different spin and/or isospin is given by the rotational corrections, which can be obtained after introducing time-dependent rotations as $SU(2)$ collective coordinates. This approach has been shown to be successful in describing the hyperon spectrum [12] as well as other baryon properties such as the magnetic moments [15].

3 The hyperon-nucleon interaction Lagrangian

In order to obtain the hyperon-nucleon potential we will approximate the $B = 2$ configuration by the product of two $B = 1$ solutions, one centered at $\vec{x}_1$ and the other one centered at $\vec{x}_2$. This is known as the product ansatz and is a rather good approximation for studying the medium and long distance behaviour of the potential. As well known, for short distances this approximation breaks down and the exact solution has a torus-like shape [16]. The kaon dynamics in the presence of the torus-like $B = 2$ soliton configuration has been investigated in Ref.[17]. This is relevant for the study of strange exotics such as the H–particle.

Within the product ansatz approximation the $B = 2$ field is written as

$$U_{B=2}(\vec{x};\vec{x}_1,\vec{x}_2) = U_{B=1}(\vec{x}-\vec{x}_1)U_{B=1}(\vec{x}-\vec{x}_2) \equiv U_1U_2 \quad , \tag{11}$$

where the indices $1, 2$ indicate the dependence on the coordinates of each individual soliton.

Substituting the ansatz (11) and subtracting one–particle contributions, the resulting interaction lagrangian density coming from the quadratic term $\mathcal{L}_2$ is

$$\mathcal{L}_2^{(int)} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[L^1_\mu R^\mu_2 + L^2_\mu R^\mu_1\right] \ . \tag{12}$$

Here and in what follows we have performed an explicit symmetrization in the indices $1, 2$ to ensure the invariance of the interaction under the exchange of the two particles.

The quartic term leads to

$$\begin{aligned}\mathcal{L}_4^{(int)} = \frac{1}{32\epsilon^2}\mathrm{Tr}\Big[&-4L^1_\mu L^1_\nu L^\mu_1 R^\nu_2 &+& \ 2L^1_\mu L^\mu_1 L^1_\nu R^\nu_2 + 2L^1_\mu L^1_\nu L^\nu_1 R^\mu_2 \\ &-4L^1_\mu R^2_\nu R^\mu_2 R^\nu_2 &+& \ 2L^1_\mu R^\mu_2 R^2_\nu R^\nu_2 + 2L^1_\mu R^2_\nu R^\nu_2 R^\mu_2 \\ &+4L^1_\mu L^1_\nu R^\mu_2 R^\nu_2 &-& \ 2L^1_\mu L^\mu_1 R^2_\nu R^\nu_2 - 2L^1_\mu L^2_\nu R^\nu_2 R^\mu_2 \\ &+2L^1_\mu R^2_\nu L^\mu_1 R^\nu_2 &-& \ L^1_\mu R^\mu_2 L^1_\nu R^\nu_2 - L^1_\mu R^2_\nu L^\nu_1 R^\mu_2 \\ &+(1\leftrightarrow 2)\Big] \quad ,\end{aligned} \tag{13}$$

while from the symmetry breaking term we obtain

$$\begin{aligned}\mathcal{L}_{\mathrm{SB}}^{(int)} &= \frac{f_\pi^2 m_\pi^2 + 2f_K^2 m_K^2}{24}\mathrm{Tr}\left[(U_1-1)(U_2-1) - 2 + h.c.\right] \\ &+\sqrt{3}\frac{f_\pi^2 m_\pi^2 - f_K^2 m_K^2}{12}\mathrm{Tr}\left[\lambda_8\left((U_1-1)(U_2-1)-1\right) + (1\leftrightarrow 2) + h.c.\right] \\ &-\frac{f_K^2 - f_\pi^2}{24}\mathrm{Tr}\left[U_2(1-\sqrt{3}\lambda_8)U_1\left(L^1_\mu - R^2_\mu\right)\left(L^\mu_1 - R^\mu_2\right) + (1\leftrightarrow 2) + h.c.\right]\end{aligned} \tag{14}$$

where $h.c.$ stands for hermitean conjugate and $(1 \leftrightarrow 2)$ for the exchange of the indices 1 and 2. The contribution from the Wess-Zumino term is

$$\mathcal{L}_{\mathrm{WZ}}^{(int)} = -\frac{iN_c}{96\pi^2}\mathrm{Tr}\left[L_1^3R_2 - R_2^3L_1 - \frac{1}{2}L_1R_2L_1R_2 + (1\leftrightarrow 2)\right] \tag{15}$$

where the absence of Lorentz indices indicates that we have used the one form notation, i.e., $L_1^3 R_2 = \varepsilon_{\mu\nu\alpha\beta} L_1^\mu L_1^\nu L_1^\alpha R_2^\beta$.

The use of the ansatz (8) for the individual chiral fields and the subsequent expansion up to second order in the kaon components lead us to an interaction Lagrangian that can be written as the sum of three different types of contributions. Namely,

$$\mathcal{L}^{(int)} = \mathcal{L}^{\pi d} + \mathcal{L}^{kd} + \mathcal{L}^{ke} . \tag{16}$$

A schematic representation of these interactions is shown in Fig.1. Fig.1.a represents the N_c^1 direct–term $\mathcal{L}^{\pi d}$ which is a pure $SU(2)$ contribution, the bound kaon acting as a spectator, and Figs.1.b-c the two N_c^0 terms $\mathcal{L}^{kd}$ and $\mathcal{L}^{ke}$ where K fields are present in direct and exchange interactions respectively. The $\mathcal{L}^{\pi d}$ and $\mathcal{L}^{kd}$ are both direct interactions in the sense that the final particles are not exchanged with respect to the initial state. The $\mathcal{L}^{kd}$ interaction corresponds to processes where the kaon degrees of freedom are excited (and thus subleading in $1/N_c$), while the $\mathcal{L}^{ke}$ interaction involves the exchange of a kaon between the particles and is called exchange contribution for short.

In general, each term in the effective action, Eq.(3), contributes to the three different pieces $\mathcal{L}^{\pi d}$, $\mathcal{L}^{kd}$ and $\mathcal{L}^{ke}$ in which we have split the interaction Lagrangian. For the quadratic term of the action we get

$$\mathcal{L}_2^{\pi d} = \frac{f_\pi^2}{4} \,\mathrm{Tr}\left[l_\mu^1 r_2^\mu + l_\mu^2 r_1^\mu \right] , \tag{17}$$

$$\begin{aligned}
\mathcal{L}_2^{kd} = {} & \frac{1}{4}\left(D^\mu K_2^\dagger n_2^\dagger l_\mu^1 n_2 K_2 - K_2^\dagger n_2^\dagger l_\mu^1 n_2 D^\mu K_2 \right) \\
& + \frac{1}{4}\left(D^\mu K_1^\dagger n_1 r_\mu^2 n_1^\dagger K_1 - K_1^\dagger n_1 r_\mu^2 n_1^\dagger D^\mu K_1 \right) \\
& - \frac{1}{8}\Big(K_2^\dagger n_2^\dagger r^{2\mu} l_\mu^1 n_2 K_2 + K_2^\dagger n_2^\dagger l_\mu^1 r^{2\mu} n_2 K_2 \\
& \quad + K_1^\dagger n_1 l_\mu^1 r^{2\mu} n_1^\dagger K_1 + K_1^\dagger n_1 r^{2\mu} l_\mu^1 n_1^\dagger K_1 \Big) + (1 \leftrightarrow 2) ,
\end{aligned} \tag{18}$$

$$\begin{aligned}
\mathcal{L}_2^{ke} = {} & \frac{1}{2}\left(D_\mu K_1^\dagger n_1 n_2 D^\mu K_2 + D_\mu K_2^\dagger n_2^\dagger n_1^\dagger D^\mu K_1 \right) \\
& + \frac{1}{4}\left(D_\mu K_1^\dagger n_1 r^{2\mu} n_2 K_2 + D_\mu K_2^\dagger n_2^\dagger l^{1\mu} n_1^\dagger K_1 \right) \\
& - \frac{1}{4}\left(K_1^\dagger n_1 l^{1\mu} n_2 D_\mu K_2 + K_2^\dagger n_2^\dagger r^{2\mu} n_1^\dagger D_\mu K_1 \right) \\
& - \frac{1}{8}\left(K_1^\dagger n_1 l^{1\mu} r_\mu^2 n_2 K_2 + K_2^\dagger n_2^\dagger r^{2\mu} l_\mu^1 n_1^\dagger K_1 \right) + (1 \leftrightarrow 2) ,
\end{aligned} \tag{19}$$

where $n = \sqrt{u_\pi}$ and we used the definitions

$$l_\mu = u_\pi^\dagger \partial_\mu u_\pi \;\; ; \;\; r_\mu = u_\pi \partial_\mu u_\pi^\dagger , \tag{20}$$

$$D_\mu = \partial_\mu + \frac{1}{2}(n^\dagger \partial_\mu n + n \partial_\mu n^\dagger) . \tag{21}$$

In the case of the Wess-Zumino term the pure $SU(2)$ contribution $\mathcal{L}^{\pi d}_{\rm WZ}$ vanishes. The remaining two contributions are

$$\begin{aligned}\mathcal{L}^{kd}_{\rm WZ} &= -\frac{\omega N_c}{48\pi^2 f_\pi^2}\Big[K_2^\dagger n_2^\dagger l_1 r_2 n_2 D K_2 + D K_2^\dagger n_2^\dagger l_1 r_2 n_2 K_2 \\ &\quad -\frac{1}{2} K_2^\dagger n_2^\dagger \left(l_1^3 + l_1 r_2^2 - l_1 r_2 l_1\right) n_2 K_2 \\ &\quad -(F_i \leftrightarrow -F_i) + h.c.\Big] + (1 \leftrightarrow 2) \ , \end{aligned} \tag{22}$$

and

$$\begin{aligned}\mathcal{L}^{ke}_{\rm WZ} &= \frac{\omega N_c}{48\pi^2 f_\pi^2}\Big[K_1^\dagger n_1^\dagger S n_2^\dagger D K_2 + D K_1^\dagger n_1^\dagger S n_2^\dagger K_2 \\ &\quad +\frac{1}{2} K_1^\dagger n_1^\dagger \left(S l_2 - r_1 S\right) n_2^\dagger K_2 - (F_i \leftrightarrow -F_i)\Big] + (1 \leftrightarrow 2) \ , \end{aligned} \tag{23}$$

where F_i is the profile of the ith soliton and $S = l_2^2 + r_1^2 - l_2 r_1$. When performing $(F_i \leftrightarrow -F_i \ ; \ i = 1,2)$, the replacements $l \leftrightarrow r, n \leftrightarrow n^\dagger$ should be done. Moreover, we have used that for bound antikaons $\dot{K} = i\omega K$, with $\omega > 0$.

Similar expressions can be obtained for the Skyrme and symmetry breaking terms $\mathcal{L}_4$ and $\mathcal{L}_{\rm SB}$, respectively. Since their lengthy explicit forms are not particularly instructive we are not going to display them here.

Next, the quantization of the two soliton system is performed using collective coordinates. We rotate the bound states, one independently of the other

$$\begin{aligned} u_1 &\to A_1 u_1 A_1^\dagger \quad , \quad u_2 \to A_2 u_2 A_2^\dagger \ , \\ K_1 &\to A_1 K_1 \quad , \quad K_2 \to A_2 K_2 \ , \end{aligned} \tag{24}$$

where A_1 and A_2 are $SU(2)$ matrices. This dependence on the collective coordinates will be reexpressed in terms of the relative coordinate $C = A_1^\dagger A_2$. In order to obtain the physical particles we perform projections onto states with good spin and isospin quantum numbers. The corresponding general wavefunctions of the hyperons can be found e.g. in Ref.[18].

Finally, to obtain the interaction potential we have to take matrix elements of $\mathcal{L}^{(int)}$ between the relevant two–baryon wavefunctions and integrate out the center of mass coordinate $\vec{R}$. For the latter purpose it is convenient to express the individual positions of the particles in terms of $\vec{R}$ and their relative separation $\vec{r}$,

$$\begin{aligned} \vec{x}_1 &= \vec{R} + \frac{m_2}{m_1+m_2}\,\vec{r} \ , \\ \vec{x}_2 &= \vec{R} - \frac{m_1}{m_1+m_2}\,\vec{r} \ , \end{aligned} \tag{25}$$

where m_1 and m_2 are the physical masses of the individual particles. We choose $\vec{r}$ to point in the $\hat{z}$ direction and perform the integration in $\vec{x}' = \vec{x} - \vec{R}$. In this way we obtain

$$V_{HN}^{(int)}(r) = -\int_0^\infty dR\, R^2 \int_{-1}^1 d\eta \int_0^{2\pi} d\varphi \ < \mathcal{L}^{(int)} > \tag{26}$$

where $\eta = \hat{r} \cdot \hat{R}$. By performing the analytical integration over φ one obtains an operator with a general structure that allows to identify the different components of the potential, like e.g. central component, spin-spin component, etc. The remaining integrations over R and η are to be done numerically.

The formalism developed so far is valid for both $H = \Lambda, \Sigma$. Nevertheless, as several steps in this procedure imply long and involved calculations, in what follows we restrict ourselves to the study of the ΛN interaction potential. Since Λ is an isoscalar particle the number of terms to be calculated is greatly reduced in this particular case.

4 The ΛN potential in the adiabatic approximation

We illustrate the general procedure to derive the interaction potential by considering some specific terms of the interaction Lagrangian. In this derivation we neglect terms depending on the collective rotational velocities (non-adiabatic terms). These terms would give rise to e.g. spin-orbit contributions and are subleading by, at least, one order in $1/N_c$ with respect to the contributions considered here. It should be noticed that, even within this approximation, the full calculation of all the terms contributing to the ΛN potential is quite long. To be confident of our results all the expressions were cross-checked by independent calculations, with the exception of the $\mathcal{L}_4$ kaonic contributions which could only be evaluated with the help of an algebraic computer code.

4.1 The direct contributions

Let us start by considering a direct interaction of the $\mathcal{L}^{\pi d}$-type. One sees that there is no $\mathcal{L}_2$ contribution of this kind to the ΛN potential. The reason is that, after the introduction of the collective coordinates, eq.(17) contains the expression

$$C^\dagger l_1^j C = l_{1a}^j C^\dagger \sigma_a C = l_{1a}^j R_{ab}(C)\sigma_b \ , \tag{27}$$

with R_{ab} a rotation operator defined by eq.(A.1). Because of eq.(A.8) this leads to a vanishing ΛN matrix element. The most important contribution comes from $\mathcal{L}_4$, which is responsible for the central repulsion. It can be easily obtained by replacing L and R from eq.(13) by their $SU(2)$ counterparts, since the kaon acts here as a spectator. After taking matrix elements and replacing in eq.(26), we obtain

$$\begin{aligned} V_C^{\pi d}(r) = \frac{2\pi}{3\epsilon^2} \int_0^\infty dR\, R^2 \int_{-1}^1 d\eta \Bigg[& (F_1' F_2')^2 + \left(F_1' \frac{s_2}{x_2}\right)^2 + \left(F_2' \frac{s_1}{x_1}\right)^2 \\ & + 3\left(\frac{s_1 s_2}{x_1 x_2}\right)^2 - \left(F_1'^2 - \left(\frac{s_1}{x_1}\right)^2\right)\left(F_2'^2 - \left(\frac{s_2}{x_2}\right)^2\right)(\hat{x}_1 \cdot \hat{x}_2)^2 \Bigg] \end{aligned} \tag{28}$$

where $s_i = \sin F_i$, $c_i = \cos F_i$ and $F_i' = dF_i/dx_i$.

As an example of the treatment of the $\mathcal{L}^{kd}$–type terms we take the contribution from the Wess–Zumino term, eq.(22). Again, after the introduction of collective coordinates, all terms containing expression (27) vanish. In this case, however, there are still two additional terms. One of these terms gives simply

$$C^\dagger l_1^3 C = -6F_1' \frac{s_1^2}{x_1^2} \ , \tag{29}$$

since l^3 is an isoscalar proportional to the $SU(2)$ contribution to the baryon density. This is the only non-vanishing contribution. The other term contains the expression

$$\begin{aligned} C^\dagger l_1 C r_2 C^\dagger l_1 C &= \varepsilon_{ijk} l_{1a}^i r_{2b}^j l_{1c}^k C^\dagger \sigma_a C \sigma_b C^\dagger \sigma_c C \\ &= \varepsilon_{ijk} l_{1a}^i r_{2b}^j l_{1c}^k \sigma_d \sigma_b \sigma_f R_{ad}(C) R_{cf}(C) \end{aligned} \tag{30}$$

and although the corresponding collective matrix element is non-zero, the total matrix element vanishes due to the antisymmetry of ε–tensor. Therefore, from eq.(29) we finally obtain

$$\int d\varphi < \Lambda_2' N_1' | \mathcal{L}_{\rm WZ}^{kd} | \Lambda_2 N_1 > \ = \ -\frac{\omega N_c}{8\pi^2 f_\pi^2} k_2^2 F_1' \frac{s_1^2}{x_1^2} \ \mathcal{O}_C \ . \tag{31}$$

It should be mentioned that the terms containing more than two $C^\dagger, C$ pairs give non-zero contributions to the direct $\mathcal{L}_4$ interactions since no ε–tensor is present in that case.

4.2 The exchange contributions

To illustrate the calculation of the exchange contributions we consider the first two terms in the symmetrized form of $\mathcal{L}_2^{ke}$. After introducing collective coordinates and neglecting non-adiabatic terms they reduce to

$$\frac{1}{2} D_\mu K_1^\dagger n_1 C n_2 D^\mu K_2 + (F_i \leftrightarrow -F_i) + h.c. \tag{32}$$

Using Eq.(A.10) the relevant matrix element reads

$$< \Lambda_1' N_2' | \frac{1}{2} D_\mu K_1^\dagger n_1 C n_2 D^\mu K_2 | \Lambda_2 N_1 > \ = \ \frac{1}{4} \delta_{I_3^N, I_3^{N'}} < J_3^{\Lambda'} | D_\mu K^\dagger n | J_3^N >_1 < J_3^{N'} | n D^\mu K | J_3^\Lambda >_2 \tag{33}$$

The individual matrix elements can be calculated using a projection theorem given in Ref.[18]. Given the explicit form of the hegdehog ansatz we obtain

$$\begin{aligned} n_2 D^0 K_2 &= \frac{\omega k_2}{2\sqrt{\pi}} \left(\not{s}_2 - i \not{c}_2 \vec\sigma_2 \cdot \hat x_2 \right) \ , \\ n_2 D^a K_2 &= \frac{1}{2\sqrt{\pi}} \left[i k_2' \not{s}_2 \ \hat x_2^a + \left(\not{c}_2 k_2' - \not{c}_2^3 \frac{k_2}{x_2} \right) \vec\sigma_2 \cdot \hat x_2 \ \hat x_2^a + \not{c}_2^3 \frac{k_2}{x_2} \ \sigma_2^a + \not{c}_2^2 \frac{k_2}{x_2} \not{s}_2 \ (\hat x_2 \times \vec\sigma_2)^a \right] \end{aligned} \tag{34}$$

where k' stands for the radial derivative of the kaon wavefunction. Moreover, we use the short-hand notation

$$\not{s} = \sin\frac{F}{2} \quad , \quad \not{c} = \cos\frac{F}{2} \ . \tag{35}$$

Similar expressions are obtained for the operator $D_\mu K_1^\dagger n_1$. In this way, one obtains the explicit form of the matrix element, eq.(33). Next, we integrate out the center of mass coordinate. At this stage it is convenient to define the operators

$$\begin{aligned} \hat{\mathcal{O}}_C &= (I)_{\Lambda' N}(I)_{N'\Lambda} \ , \\ \hat{\mathcal{O}}_S &= (\vec{\sigma}_1)_{\Lambda' N}\cdot(\vec{\sigma}_2)_{N'\Lambda} \ , \\ \hat{\mathcal{O}}_T &= 3\ \hat{r}\cdot(\vec{\sigma}_1)_{\Lambda' N}\ \hat{r}\cdot(\vec{\sigma}_2)_{N'\Lambda} - (\vec{\sigma}_1)_{\Lambda' N}\cdot(\vec{\sigma}_2)_{N'\Lambda} \end{aligned} \tag{36}$$

and make use of the relation

$$\int_0^{2\pi} d\varphi\ \vec{\sigma}_1\cdot\hat{x}_i\ \vec{\sigma}_2\cdot\hat{x}_j = \frac{2\pi}{3}\left(x_{ij}\hat{\mathcal{O}}_S + G_{ij}\hat{\mathcal{O}}_T\right) \tag{37}$$

with $x_{ij} = \hat{x}_i\cdot\hat{x}_j$ and

$$G_{ij} = \frac{3R^2}{2x_ix_j}(\eta^2-1) + x_{ij}\ . \tag{38}$$

Using the expressions given above we obtain the complete result

$$\begin{aligned} &\int d\varphi < \Lambda_1' N_2'| \left[\frac{1}{2}D_\mu K_1^\dagger n_1 C n_2 D^\mu K_2 + (F_i \leftrightarrow -F_i) + h.c.\right] |\Lambda_2 N_1> = \\ = \quad & -\frac{1}{12}\delta_{I_3^N, I_3^{N'}}\Bigg\{\omega^2 k_1k_2\left(3\not{s}_1\not{s}_2\hat{\mathcal{O}}_C - \not{c}_1\not{c}_2 x_{12}\hat{\mathcal{O}}_S - \not{c}_1\not{c}_2 G_{12}\hat{\mathcal{O}}_T\right) \\ & +\hat{\mathcal{O}}_C\left(-3k_1'k_2'\not{s}_1\not{s}_2 x_{12}\right) \\ & +\hat{\mathcal{O}}_S\Bigg[\frac{k_1k_2}{x_1x_2}\not{c}_1^2\not{c}_2^2\left(\not{c}_1\not{c}_2(1+x_{12}^2) - 2\not{s}_1\not{s}_2x_{12}\right) \\ & \qquad + \frac{k_1k_2'}{x_1}\not{c}_1^3\not{c}_2(1-x_{12}^2) + \frac{k_1'k_2}{x_2}\not{c}_1\not{c}_2^3(1-x_{12}^2) \\ & \qquad + k_1'k_2'\not{c}_1\not{c}_2x_{12}^2\Bigg] \\ & +\hat{\mathcal{O}}_T\Bigg[-\frac{k_1k_2}{x_1x_2}\not{c}_1^2\not{c}_2^2\Big(\not{c}_1\not{c}_2(G_{11}+G_{22}-G_{12}x_{12}) - \not{s}_1\not{s}_2G_{12}\Big) \\ & \qquad + \frac{k_1k_2'}{x_1}\not{c}_1^3\not{c}_2(G_{22}-G_{12}x_{12}) + \frac{k_1'k_2}{x_2}\not{c}_1\not{c}_2^3(G_{11}-G_{12}x_{12}) \\ & \qquad + k_1'k_2'\not{c}_1\not{c}_2G_{12}x_{12}\Bigg]\quad\Bigg\}\ . \end{aligned} \tag{39}$$

In order to recover the operator structure of the potential as given in eq.(1) we still have to perform a Fierz rearrangement and write the operators $\hat{\mathcal{O}}_C, \hat{\mathcal{O}}_S, \hat{\mathcal{O}}_T$ in terms of the operators $\mathcal{O}_C, \mathcal{O}_S, \mathcal{O}_T$ appearing in such equation. We obtain

$$\alpha\ \hat{\mathcal{O}}_C + \beta\ \hat{\mathcal{O}}_S + \gamma\ \hat{\mathcal{O}}_T = \frac{1}{2}(\alpha+3\beta)\,\mathcal{O}_C + \frac{1}{2}(\alpha-\beta)\,\mathcal{O}_S + \gamma\ \mathcal{O}_T\ , \tag{40}$$

where α, β and γ are arbitrary functions depending only on the relative separation r. All the other exchange contributions to the interaction potential can be treated in exactly the same way.

5 Numerical results and discussion

In the numerical calculations we used the physical values for the different mesonic parameters appearing in the Lagrangian, that is $m_\pi = 138\ MeV$, $m_K = 495\ MeV$, $f_\pi = 93\ MeV$, $f_K/f_\pi = 1.22$ and take $\epsilon = 4.26$ in order to fit the mass difference between the nucleon and the Δ. With these values the hyperon excitation spectrum is rather well described. On the other hand, the absolute baryon masses come out too high by about 800 MeV. This is a generic problem of the topological soliton models that can be fixed by properly taking into account the quantum corrections to the soliton mass [19]. Recently, this has been explicitly shown in the case of the bound state soliton model [20].

The individual contributions of the different direct and exchange terms to the ΛN potential are shown in Fig.2 and Fig.3 , respectively. As a general feature we see that the pionic contributions are much larger (in absolute value) than the kaonic ones as expected from N_c-counting. We also notice that in the present scheme the direct terms only contribute to the central potential. In particular, although there is an attractive symmetry breaking contribution to $V_C^{\pi d}$, such part is completely dominated by the repulsive contribution coming from the quartic term. As already mentioned there are no $\mathcal{L}_2$- and $\mathcal{L}_{\rm WZ}$-contributions of this type. In the case of the direct kaonic part the quartic and WZ contributions are attractive and similar in magnitude. As seen in Fig.3 all the different terms in the Lagrangian contribute to the exchange potentials. All these contributions are attractive except for those coming from the symmetry breaking term $\mathcal{L}_{\rm SB}$ which are, in any case, quite small.

Our total predictions for the central, spin-spin and tensor components of the ΛN potential are presented in Fig.4. As anticipated from the discussion above, we observe that the spin–spin and tensor interactions are supressed by an order of magnitude with respect to the central interaction which turns out to be repulsive at any distance. Noting that V_C is dominated by the pionic contributions it is clear that such a behaviour is very much related with the well known problem of the missing central attraction at intermediate distances in the $SU(2)$ Skyrme model. As reviewed in Ref.[9] many mechanisms have been proposed to solve this problem. Whatever such solution could be, our $SU(3)$ calculations show that the inclusion of strangeness degrees of freedom are not likely to spoil it since the central kaonic contributions are attractive.

The sign of our predicted spin-spin contribution implies that there will be more attraction in the ${}^3S_1$ channel than in the ${}^1S_0$. The empirical information about the sign of the spin-spin interaction is somewhat unclear. From the existing Λp scattering data it is very difficult to draw a definite conclusion. In fact, various versions of the OBE model that fit the scattering data equally well lead to rather different predictions for the ${}^3S_1$ and ${}^1S_0$ scattering lengths (see e.g. Ref.[21]). On the other hand, the hypernuclei data tend to favor a repulsive ΛN spin-spin interaction although again the question is not completely settled. The most clear indication comes from the ${}^4_\Lambda H$ and ${}^4_\Lambda He$ doublet states. However, the analysis of these states depends on non-trivial four-body calculations. There is also some empirical information from other hypernuclei like e.g. ${}^{11}_{\Lambda}B$. There, however, the situation is even more complicated because of the role played by spin-orbit interactions. Forthcoming experiments on both hyperon-proton scattering [22] and hypernuclear γ spectroscopy [23] are expected to provide critical tests on this issue. ¿From the point of view of the Skyrme model our results are consistent with those obtained in the previous $SU(3)$ collective coordinates calculations [13, 14] in the absence of channel couplings. One might argue that since in our model the pion exchanges are taken into account beyond the OBE a good deal of the mixing with the ΣN channel is taken into account. However, our approach still allows for non-vanishing off-diagonal $N\Lambda - N\Sigma$ terms. There are indications that when such mixing with rotational excited configurations is included the sign of the spin-spin interaction in the Skyrme model might be reversed [13].

Finally, for $r > 1.2\ fm$ our prediction for the tensor component of the potential agrees well with the OBE models. For smaller distances there are large discrepances between OBE model D and F. For example, at $r \approx .9\ fm$ one has $V_T^{OBE-D} \approx -14\ MeV$ while $V_T^{OBE-F} \approx +9\ MeV$. In such region our results favor those of model D.

The dashed lines in Fig.4 represent the results of the collective approach [14] to the $SU(3)$ Skyrme model. We see that the magnitude and sign of the potentials are similar to those of the bound state approach. However, the situation is different for the behaviour at large distances. Although in both cases V_C decays basically in the same way, the range of the V_S and V_T in the CCA is much longer. This is not difficult to understand since in the CCA the only meson that determines the fall-off of the radial functions is the pion meson. On the other hand, in the BSA the spin-spin and tensor components of the ΛN potential are given by the kaonic components. The corresponding range is, therefore, associated with the kaon mass which is several times larger than the pion mass. In the central potential these differences do not appear because of the dominance of the pionic $\mathcal{L}_4$-contribution already discussed. It should be mentioned that in all the cases the behaviour of the potentials at large distances obtained in the BSA is in good agreement with the results of the OBE models.

6 Conclusions

We have investigated the hyperon-nucleon two-body interaction in the framework of the bound state approach to the $SU(3)$ Skyrme model. We would like to stress the fact that the Skyrme model approach incorporates chiral symmetry and the large N_c expansion in an elegant way. Moreover, by relating the physics of sectors with different baryonic numbers it gives parameter free predictions for the present calculation, in contrast with more phenomenological approaches. Our studies have been based on the product ansatz for the $B = 2$ soliton field which is adequate for the medium and large separation distances discussed in this work. We have found that there are three classes of contributions within the adiabatic approximation used here. One type corresponds to order N_c^1 purely $SU(2)$ contributions in which kaons act as simply spectators. The other two are of order N_c^0 and correspond to direct and exchange kaonic interactions. Although the formalism we have followed is suitable for any diagonal or off-diagonal hyperon-nucleon potential we have concentrated on the diagonal ΛN interaction. There, important simplifications appear in the expressions for the potentials which still happen to be lengthy and cumbersome. We have found that the central potential is repulsive at any distance. This is strongly related to the missing intermediate range central attraction of the NN potential as calculated in the $SU(2)$ Skyrme model. Within our scheme, any of the suggested solutions of this problem is expected to bring in some attraction also in the ΛN case. Generally speaking our results are very similar to those of the $SU(3)$ collective coordinate approach to the Skyrme model [14]. An exception to this is the range of the spin-spin and tensor interactions. For these quantities the values obtained in the present calculation seem to be more realistic. Finally, there are some indications that the coupling to vibrations could give the missing central attraction while the coupling to rotationally excited states may change the sign of the spin-spin interaction [13]. The formalism developed in the present work provides a general framework for future investigations of such issues within the bound state soliton model.

Acknowledgements

This work was partially supported by a grant of the Fundación Antorchas, Argentina and FAPERGS and FINEP, Brasil. N.N.S. also acknowledges a grant of the ANPCYT, Argentina. The authors would like to express their gratitude to the late Prof. J.M. Eisenberg for stimulating discussions. C.L.S. and N.N.S. would also like to thank Prof. A. Gal for valuable conversations regarding the phenomenology of ΛN potentials. C.L.S. thanks M.Kruczenski for initial guidance on the use of algebraic computer codes and the CNPq-Brasil for financial support through a CLAF fellowship.

Appendix

In this appendix we present a set of formulae which are useful for calculating the matrix elements of the collective lagrangian operators.

The rotation operator written in the cartesian basis

$$R_{ab}(C) = \frac{1}{2} \mathrm{Tr} \left[\sigma_a C \sigma_b C^\dagger \right] \tag{A.1}$$

can be expressed in terms of the spherical tensor $D_{\alpha\beta}(C)$ in the following way

$$R_{ab} = \hat{e}_\alpha \cdot \hat{e}_a \; \hat{e}^*_\beta \cdot \hat{e}_b \; D_{\alpha\beta}, \tag{A.2}$$

where $\hat{e}_\alpha$, with $\alpha = +1, 0, -1$, are the usual spherical unit vectors and $\hat{e}_a$ the cartesian ones.

Using Eq.(A.2) it is not difficult to show that

$$\left(D^{(1/2)}_{mn} \right)^* D^{(1/2)}_{m'n'} = \frac{1}{2} \left(\delta_{mm'} \delta_{nn'} + \; < m' | \sigma^a | m > R_{ab} < n | \sigma^b | n' > \right) \; . \tag{A.3}$$

The evaluation of the matrix elements amounts to an integration over $SU(2)$. For products of R_{ab} we have

$$\frac{1}{2\pi^2}\int dC\ R_{ab}R_{cd} = \frac{1}{3}\delta_{ac}\delta_{bd}\ , \tag{A.4}$$

$$\frac{1}{2\pi^2}\int dC\ R_{ab}R_{cd}R_{ef} = \frac{1}{6}\epsilon_{ace}\epsilon_{bdf}\ . \tag{A.5}$$

Using explicit forms of the Λ and N wavefunctions

$$\begin{aligned} |\Lambda > &= \frac{1}{\sqrt{2}\pi}\ |\frac{1}{2}J_3^\Lambda >\ , \\ |N > &= \frac{i}{\pi}(-1)^{\frac{1}{2}+I_3^N} D^{(1/2)}_{-I_3^N,J_3^N}\ , \end{aligned} \tag{A.6}$$

together with the expressions above, the relevant collective matrix elements can be easily calculated. For the direct terms we have

$$< \Lambda_2' N_1' | \Lambda_2 N_1 > = \delta_{I_3^N,I_3^{N'}}\delta_{J_3^N,J_3^{N'}}\delta_{I_3^\Lambda,I_3^{\Lambda'}}\ , \tag{A.7}$$

$$< \Lambda_2' N_1' | R_{ab}(C) | \Lambda_2 N_1 > = 0\ , \tag{A.8}$$

$$< \Lambda_2' N_1' | R_{ab}(C) R_{cd}(C) | \Lambda_2 N_1 > = \frac{1}{3}\delta_{ac}\delta_{bd} < \Lambda_2' N_1' | \Lambda_2 N_1 > \tag{A.9}$$

and for the exchange terms

$$< \Lambda_1' N_2' | \mathcal{O}_\mu^\dagger(K_1) C_{\mu\nu} \mathcal{O}_\nu(K_2) | \Lambda_2 N_1 > = \frac{\delta_{I_3^N,I_3^{N'}}}{2} < J_3^{\Lambda'} | \mathcal{O}^\dagger(K_1) | J_3^N >< J_3^{N'} | \mathcal{O}(K_2) | J_3^\Lambda >\ , \tag{A.10}$$

$$< \Lambda_1' N_2' | \mathcal{O}_\mu^\dagger(K_1) C_{\mu\nu} \mathcal{O}_\nu(K_2) R_{ab}(C) | \Lambda_2 N_1 > = \frac{\delta_{I_3^N,I_3^{N'}}}{6} < J_3^{\Lambda'} | \mathcal{O}^\dagger(K_1)\sigma_a | J_3^N >< J_3^{N'} | \sigma_b \mathcal{O}(K_2) | J_3^\Lambda > \tag{A.11}$$

REFERENCES

[1] C.B. Dover and A. Gal, in *Progress in Particle and Nuclear Physics*, ed. D. Wilkinson (Pergamon, Oxford, 1984), Vol.12, p.171.

[2] K. Holinde, Phys. Rep. **68** (1981) 121.

[3] M.M. Nagels, T.A. Rijken and J.J. de Swart, Phys. Rev. **D12** (1975) 744; *ibid* **D15** (1977) 2547; *ibid* **D20** (1979) 1633.

[4] B. Holzenkamp, K. Holinde and J. Speth, Nucl. Phys. **A500** (1989) 485; A. Reuber, K. Holinde and J. Speth, Nucl. Phys. **A570** (1994) 543.

[5] T.H.R. Skyrme, Proc. Roy. Soc. **A260** (1961) 127; **A262** (1961) 237.

[6] I. Zahed and G.E. Brown, Phys. Rep. **142** (1986) 1; G. Holzwarth and B. Schwesinger, Rep. Prog. Phys. **49** (1986) 825.

[7] G.'t Hooft, Nucl. Phys. **B72** (1974) 461; *ibid* **B75** (1974) 461.

[8] E. Witten, Nucl. Phys. **B160** (1979) 57.

[9] J.M. Eisenberg and G. Kälbermann, Int. J. Mod. Phys. **E5** (1996) 423; in *Proc. Int. Workshop on Hadron Physics 96*, ed. E. Ferreira *et al.* (World Scientific, Singapore, 1997) p.55.

[10] T.S. Walhout and J. Wambach, Int. J. Mod. Phys. **E1** (1992) 665.

[11] C.G. Callan and I. Klebanov, Nucl. Phys. **B262** (1985) 365.

[12] N.N. Scoccola, H. Nadeau, M.A. Nowak and M. Rho, Phys. Lett. **B201** (1988) 425; C.G. Callan, K. Hornbostel and I. Klebanov, Phys. Lett. **B202** (1988) 269; U. Blom, K. Dannbom and D.O. Riska, Nucl. Phys. **A493** (1989) 384.

[13] G. Kälbermann and J.M. Eisenberg, Phys. Lett. **B235** (1990) 6; Phys. Rev. **D46** (1992) 446.

[14] B. Schwesinger, F.G. Scholtz and H.B. Geyer, Phys. Rev. **D51** (1995) 1228.

[15] Y. Oh, D.-P. Min, M. Rho and N.N. Scoccola, Nucl. Phys. **A534** (1991) 493.

[16] J.J.M. Verbaarschot, Phys. Lett. **B195** (1987) 235.

[17] G.L. Thomas, N.N. Scoccola and A. Wirzba, Nucl. Phys. **A575** (1994) 623.

[18] C. Gobbi, D.O. Riska and N.N. Scoccola, Nucl. Phys. **A544** (1992) 671.

[19] B. Moussallam, Ann. Phys. **225** (1993) 264; F. Meier and H. Walliser, Phys. Rep. **289** (1997) 383.

[20] N.N. Scoccola and H. Walliser, Phys. Rev. **D58** (1998) 094037.

[21] Th.A. Rijken, V.G.J. Stoks and Y. Yamamoto, *nucl-th/9807082*.

[22] M. Ieiri et al. , Nucl.Phys. **A639** (1998) 21c.

[23] H. Tamura, Nucl.Phys. **A639** (1998) 83c.

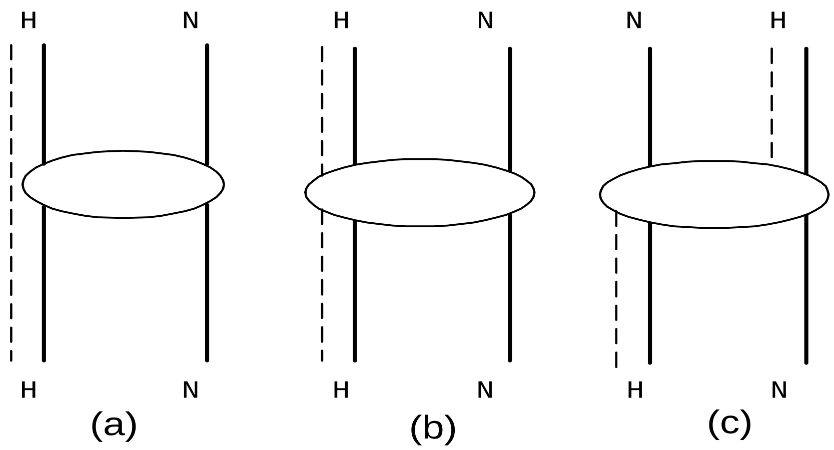

Fig.1 – Schematic representation of the different types of contributions to the hyperon-nucleon potential. (a) Direct "pionic" contributions $\mathcal{L}^{\pi d}$, (b) direct "kaonic" contributions $\mathcal{L}^{kd}$ and (c) exchange "kaonic" contributions $\mathcal{L}^{ke}$.

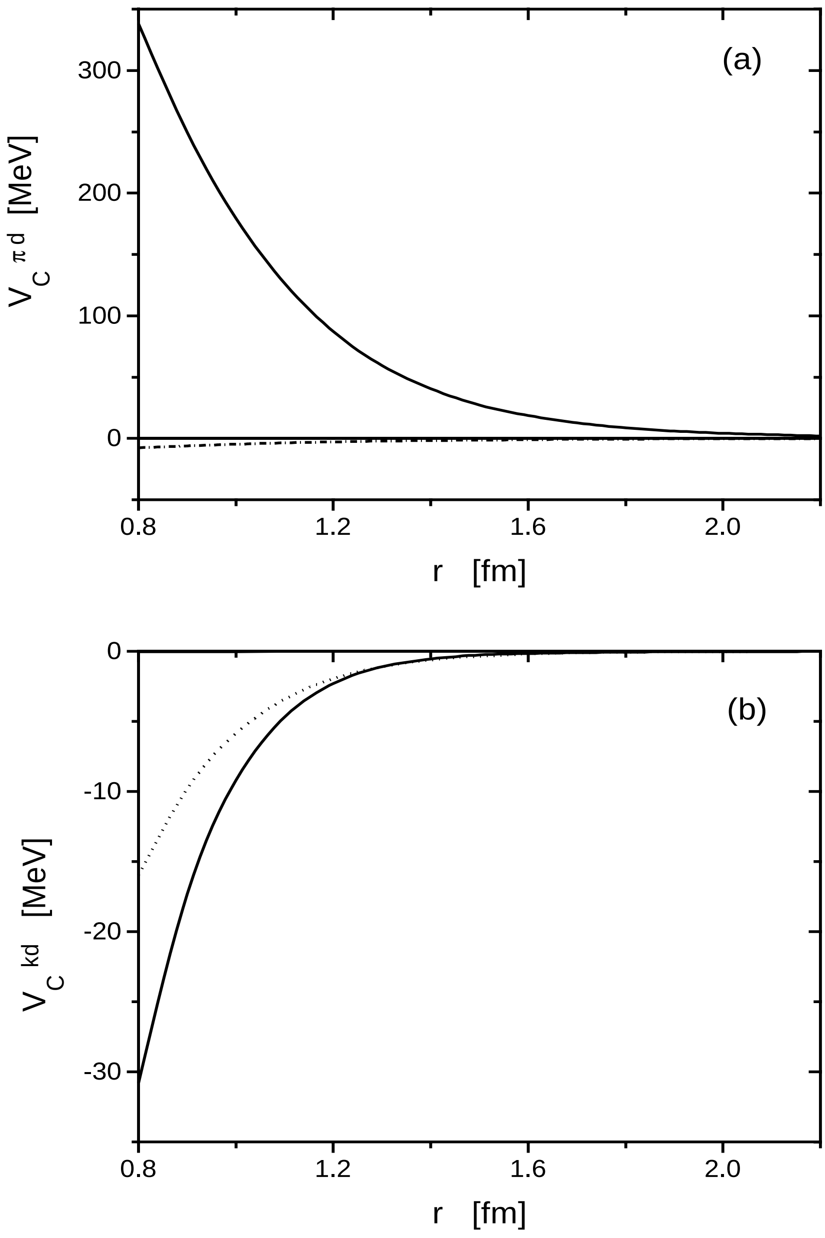

Fig.2 – (a) Central contributions to the ΛN potential coming from the direct "pionic" terms, $\mathcal{L}^{\pi d}$. (b) Central contributions to the ΛN potential coming from direct "kaonic" terms $\mathcal{L}^{kd}$. The full line represents the contributions from $\mathcal{L}_4$, the dotted line those from $\mathcal{L}_{\rm WZ}$ and the dashed-dotted line those from $\mathcal{L}_{\rm SB}$.

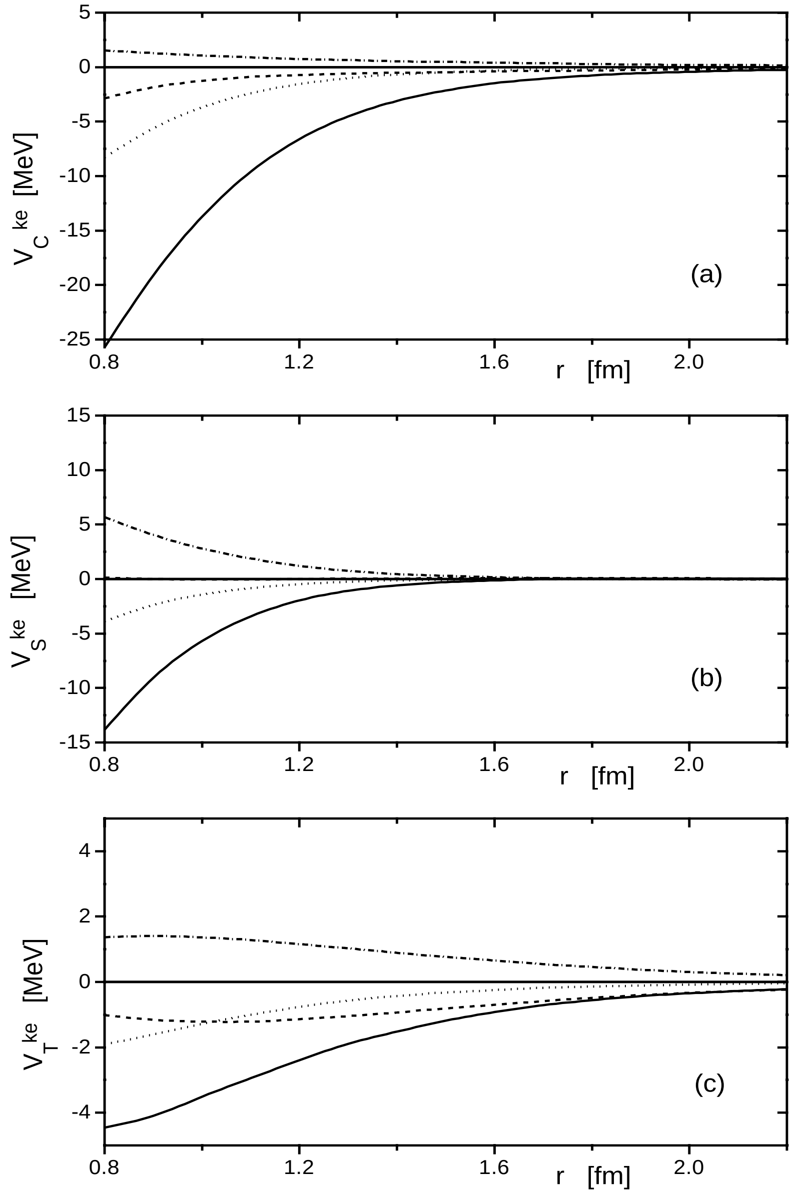

Fig.3 – Contributions from the exchange "kaonic" terms $\mathcal{L}^{ke}$ to: (a) Central component of the ΛN potential, (b) Spin-spin component of the ΛN potential, (c) Tensor component of the ΛN potential. In the three panels the dashed line represents the contributions from $\mathcal{L}_2$, the full line those from $\mathcal{L}_4$, the dotted line those from $\mathcal{L}_{\rm WZ}$ and the dashed-dotted line those from $\mathcal{L}_{\rm SB}$.

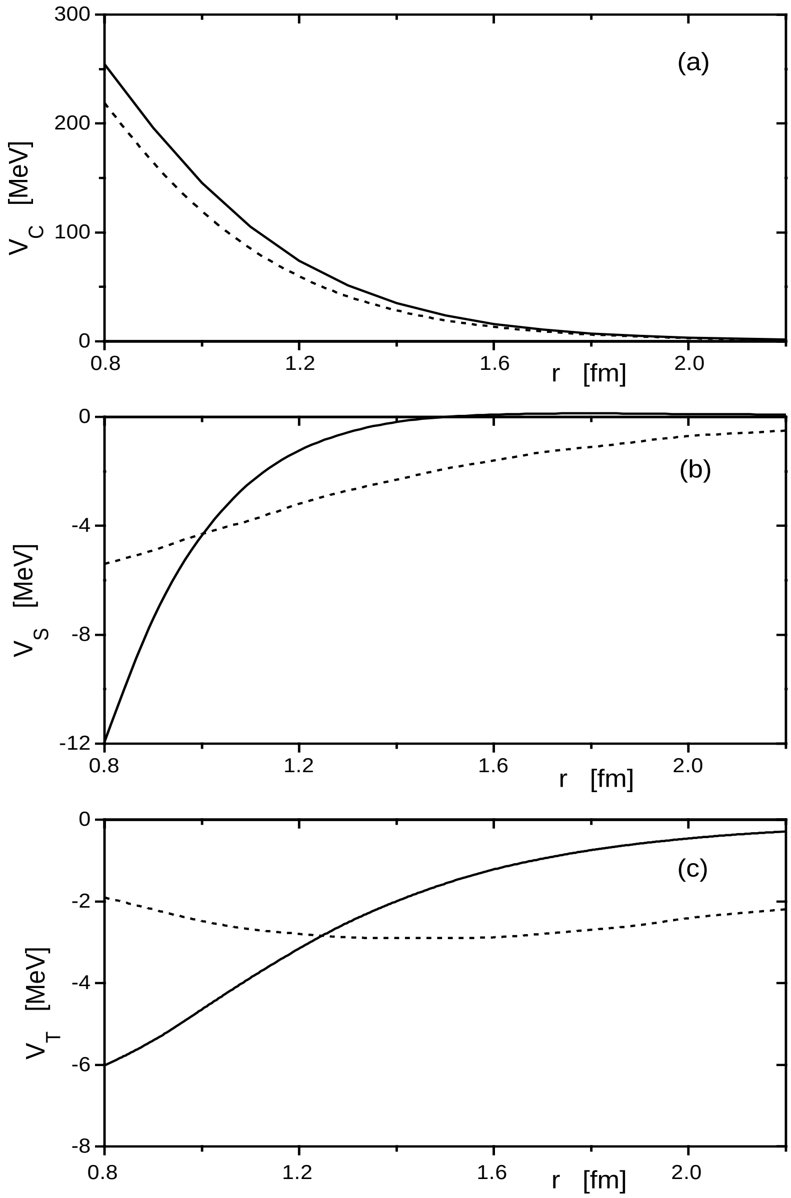

Fig.4 – Components of the ΛN potential as defined in Eq.(1): (a) central component V_C, (b) spin-spin component V_S and (c) tensor component V_T. In the three panels the full line represents the results of the present calculation and the dashed line those of the CCA as given in Ref.[14]