Criticality in a Simple Model for Brain Functioning

Ladário da Silva[1], Andrés R. R. Papa[1]* and André M. C. de Souza[2]

[1]Centro Brasileiro de Pesquisas Físicas - DMF/CBPF/CNPq

Rua Dr. Xavier Sigaud, 150

22290-180 - Rio de Janeiro-RJ, Brasil

[2]Instituto de Física, Universidade Federal de Sergipe,

Rua das Laranjeiras 1838, 49055-380 SE,

Aracajú, Brazil

ABSTRACT

We introduce a simple model for a set of interacting idealized neurons. The model presents a self-organized state in which avalanches of all sizes are observed and activity is detected in all the extension of the simulated system without a typical length scale. The basic elements of the model are endowed with the main features of neuron function. On this basis it is speculated that the collective system that they conform, *i.e.*, the brain, should display self-organized criticality.

Key-words: Self-organized criticality; Neural networks; Punctuated equilibrium

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*Also at International Centre for Theoretical Physics, Trieste, Italy.

Brain function [1] is largely based on a complex system of connections between its basic components, neurons and synapses. Schematically, each neuron has an input terminal (*the dendritic arbor*) a processing center (*the soma*) and a transmission terminal (*the axon*). The dynamics of neurons and synapses follows a relatively simple sequence: i) the axon, under the order of its neuron (*presynaptic neuron*), propagates an electric signal (*spike* or *action potential*). The amplitude of the spike is of the order of tens of millivolts. ii) through some complicated electrochemical reaction this signal arrives to the soma of another neuron (*postsynaptic neuron*) where the inputs from all the presynaptic neurons connected to it are summed. In the average each neuron is connected to $10^3 - 10^4$ other neurons. The amplitude of each of the input signals at the soma is about one millivolt. These inputs may be either excitatory, hence favoring the likelihood of the appearance of a spike (in the axon of the neuron, that now becomes a presynaptic neuron) to be transmitted to its postsynaptic neurons or inhibitory, reducing the likelihood of firing. If the sum of excitatory signals within a short period of time surpasses some threshold, the probability for the emission of a spike, which is the manifestation of the instability, becomes significant. This threshold is tens of millivolts high and hence a great number of excitatory inputs are necessary in order to allow a spike. After the traumatic event, that the emission of a spike represents, the neurons require a period of time to recover. During the 1-2 millisecond following the emission of a spike, the neuron, without mattering how large the excitatory input may be, is unable of emitting a second spike. This period is called the *absolute refractary period* of the neuron. It allows a maximal frequency of about 500-1000 spikes per second. In some higher cortical areas this frequency is lowered to values not higher than 30-40 spikes per second, accusing a wide range of possible absolute refractary periods. On the other hand the brain as a whole is a system able of auto modifications. Here we present for the first time (to the best of our knowledge) a model with some of the main characteristics described above and that, at the same time, presents self-organized criticality.

By self-organized criticality we understand the tendency of certain collectively organized systems to reach a steady state without necessity of external tuning of parameters. The

lack of a typical size for bursts of activity or avalanches (except the own size of the system) and the lack of a relevant time scale are the main features of the above mentioned steady state. Self-organized criticality appears in systems and models that, at a first glance, could appear very unlike. Models for interface growth and pinning [2,3], models for biological evolution [4,5] and models for earthquakes, sand piles and rice piles [6], among others, present self-organized criticality. Experimental evidence has also been found in earthquakes [6], superconducting vortex avalanches [7] and sand piles [8].

Research on brain functioning range from multichannel squids measures in real brains to modeling through neural networks. Self-organization (some times called self-tuning) has become popular between the scientists that model the brain function ([9–13]). It has been used in general problems as, for example, the cortical organization [13], as well as in the modeling of specific activities as, for instance, the breath regulation [10]. In those recent works a wide variety of technics were introduced or further developed. Among others, the study of Potts-like neurons in a competitive mechanism [11] and the minimization of cost functions for the dynamical self-adaptation of the learning rate [12] were explored. In all these cases the implemented algorithm adapts itself locally to a cost function landscape. In our model we do not attempt to give a detailed description of the elements of the brain. Instead, we represent each neuron with a barrier that characterize its instantaneous probability of releasing a spike, which is the measure of the instability of the neuron. At the end of the paper it would become clear that the system we are modeling is not only self-organized but also critical.

In the landscape framework previously mentioned, the barrier height of a given neuron separates its current state (characterized by its local probability of firing) from other more stable states. The barrier height is a relaxed measure of the number of stimulatory inputs that the neuron must receive to fire. Firing by low-barrier neurons often occurs but high-barrier neurons are difficult to fire. An equal barrier (equal firing probability) population would decay exponentially in time. When the barrier is high, firing is difficult unless accumulative activity of related neurons lower the barrier enough; the fire would only happen at

later times. The barriers are our measure of stability. The modification of a barrier can be thought as either the result of the release of a spike by the own neuron or as the consequence of a received signal that changes the stability of the neuron.

Since the smaller barriers are unstable, a collection of non-interacting neurons would converge towards a deeply frozen "dead" state with the highest barriers.

However the fundamental driving mechanism for brain functioning is the high connectivity between its parts, and inside those parts. When a neuron fires it changes the stability of its neighbors. A neuron with a low probability of firing (high barrier) could be favored to fire in subsequent time steps by a firing coming from a related neuron, causing a reduction of the barrier. At the same time, an inhibitory stimulus of a related neuron could cause an increase in the barrier height retarding any possible fire action.

While developing our model we have profited the previous knowledge about critical models gained, for example, in the paper of Zaitzev [2] about pinning and in that of Bak and Sneppen [4] for evolution. Like them, we have ignored the consequences that external actions could cause on the sample under study.

Our model, that intends to represent the main characteristics of all that was explained above, is defined by following the simulation sequence: N neurons are distributed on a ring (a line with periodic boundary conditions). A random barrier, B_i, between 0 and 1, is assigned to each neuron. The lowest barrier is detected and the corresponding neuron is fired (this is, at the same time, our updating algorithm and our definition of time step) by assigning a new random number between 0 and 1 to the barrier and by assigning to its first neighbors new random numbers between 0 and 1. Last, the site that suffers a change in its barrier as a consequence of firing itself is prohibited of firing again during a period of time T_r (the refractary period). Its neighbors are free to fire at any moment if they fulfill the condition of being the lowest. If after a certain time interval $t \leq T_r$ a nearest neighbor is fired, the barrier of the temporarily "frozen" neuron is also changed but it continues to be prohibited to produce a spike until a time $t > T_r$ has elapsed.

Note that with a simplified model, as the one we have introduced, it should not be

expected a detailed description of the system. However, we believe that it is sufficient to show some general features, in particular, the kind of universal behavior displayed by the real system that it represents.

If we begin with an arbitrary distribution of barrier heights, subsequent fire activities would be completely uncorrelated but, as time goes on (and consequently, the mean height of the barriers increases owing to the rule of selecting and changing the lowest), it would become more and more likely that near neighbors are next to fire (respecting the constraint of the refractary period). After a transient, the system reaches a stationary state boldly characterized by a step-like distribution for the barrier heights and a threshold B_c for the distribution of the lower barriers (Figure 1). The spike activity takes place in neurons with barriers below a self-organized critical value B_c. The distribution of the lowest barriers in the critical state vanishes at and above the self-organized threshold. For refractary periods greater than one the threshold remains at the same value but, the barrier distribution becomes less abrupt and in the limit $T_r \to \infty$ it becomes a uniform distribution (of zero amplitude); in this limit the distribution of the lowest barrier also vanishes except at the zero value.

Actually, the stationary state is quite richer than that. As explained above, in the stationary state the events become correlated in space. Following Sneppen and Jensen [14] we present in Figure 2 a log-log plot of the measure of the distribution $C(x)$ of distances x between subsequent firing activities for a refractary period $T_r = 1$. The straight line manifests a power law distribution indicating that the system is critical. The possibility of the existence of a power law for related models was advanced by Bak and Sneppen [4]. However the exponent α for the law $C(x) = x^\alpha = x^{-2.32 \pm 0.05}$ does not coincide with the one they obtained (-3.15 ± 0.05) indicating that our model belongs to some other universality class. Further, our model provides the possibility of "tuning" the universality class (the exponent α depends on the refractary period T_r). As in previous models, the result does not depend on the initial conditions, so the critical state is a global atractor for the dynamics, hence it is self-organized. In Figure 3 we show the dependence of the exponent α on the

refractory period T_r. As T_r increases α decreases in absolute value and, in the limit $T_r \rightarrow \infty$, that corresponds to a gradually frozen system (each barrier is updated just once), the exponent α becomes zero.

In the critical state each barrier suffers bursts of activity alternated with long periods of calm. The *hanging garden of Babylon* shaped picture in Figure 4 presents the instantaneous values of a single barrier, during a time interval, when the system is at the critical state. It seems to present a fractal character: if we change the time scale the appearance continues to be essentially the same as in the magnification shown. The bounds for this fractal character come from the shortest time interval we can consider ($\Delta t = 1$) and from the system size we are exploring. Figure 4 is the evidence of punctuated equilibrium at the critical state; note that above the threshold the density of black points is greater.

Now, if at the critical state all the events become correlated in space it is not too difficult to realize that if we look, for a sufficiently long period of time, at the subsequent values of a single barrier, we will have some "comprehension" of what is usually called *avalanche*. We define the size s of what we prefer to call *anti-avalanche* (the name will become clear now) as the number of consecutive time steps during which the observed barrier remains constant. In Figure 4 the horizontal segments are the evidence of large anti-avalanches (the avalanches present in the system during those time intervals are far away from our barrier of interest or are small enough to, even being in the neighborhood of the barrier, not affect it). The vertical punctuated strips are time intervals of "intense" activity, *i.e.*, periods of time during which our barrier is submersed in the sea of an avalanche.

Figure 5 shows the distribution of anti-avalanches in the critical state for a single barrier and for a refractory period $T_r = 1$. The power law reveals the existence of anti-avalanches of all scales. The exponent β in the anti-avalanche distribution $A(s) = s^{\beta}$ is $\beta = -1.60 \pm 0.04$ for $T_r = 1$. Contrary to what was observed in the exponent α for the distribution of subsequent firing activity, the exponent β has a very weak dependence and we have not detected a sensible change in the value of it when going from $T_r = 0$ (the Bak-Sneppen model for biological evolution) to $T_r = 1$.

Different refractary periods (and consequently, different exponents in the distribution for jumps and for avalanches) could characterize different time scale features of the brain: short, rapidly adaptive ones (as for example the breath control) and slower long term ones (as for example, the conscience).

We have studied our model in some other variants [15] including, among others, ranges of interactions beyond the first neighbor barriers and concomitant values for the refractary period. In all cases, the system suffers self-organized criticality indicating that the model is robust, as it should be if intended to represent any real situation [4]. Similarly to Bak and Sneppen we have found the interesting feature that systems with many connections present lower barriers increasing the speed of the collective dynamics. Other interesting observation is that in systems with several concomitant refractary periods the critical behavior is dominated by the largest refractary period (the lowest exponent, in absolute value, for the distribution of consecutive spikes). For brain functioning this implies that higher cortical areas determine, in last instance, the criticality of the whole brain. As part of the extended study we have explored a system where neurons were modeled in a more detailed fashion [16]; the system presented always a self-organized critical state.

As illustrated by the power law distribution of anti-avalanches, at the critical state, the neurons are connected at all scales. All neurons belonging to a single avalanche should remain in activity at the "same" time period, thus, they might be considered as a single domain of the brain. We thus have a hierarchical organization of neurons up to and including the whole brain and we can speculate that the whole brain acts as a single interconnected neuron; it suffers bursts of activity as a consequence of excitations coming, in some way, from other neurons. A situation similar to this was obtained in the model of Bak and Sneppen and was previously suggested by Lovelock [17] for biological evolution. It is, very probably, a general feature of some classes of self-organized critical models and systems.

Koch [18] has called the attention on the difficulty to store in the genoma all the information required to initialize a network of neurons without the intervention of an external programmer. There are in the brain of the order of 10^{14} synapses to be initialized. Maybe

the present work adresses the way to liberate the genoma of such burdensome responsabil-ity. In his work Koch presented a result provided by W. Newsome and K. Britten which measured the firing response of a single neuron (as we have done in Figure 4) in a monkey visual cortex. It is clearly seen in our case as well as in their, the alternance of periods of high activity (firings) and periods of calm. The results strongly resemble each other but, for a quantitative comparation larger experimental measurements would be required. The main conclusion of this work is that, if the basic elements of some system display the properties we have worked on throughout the paper, then the global system, *i.e.* the brain (and each part of it) should show self-organized criticality. Corroboration of this result would be an amazing example of self-organized criticality and at the same time will demonstrate the high universality expected in what is believed to be one of the later evolution products in the nervous system [1]: the brain.

REFERENCES

[1] D.J. Amit, *Modeling Brain Function* (Cambridge University Press, Cambridge, 1989).

[2] S.I. Zaitzev, Physica A **189**, 411 (1992).

[3] K. Sneppen, Phys. Rev. Lett. **69**, 3539 (1992).

[4] P. Bak and K. Sneppen, Phys. Rev. Lett. **71**, 4083 (1993).

[5] J. Suzuki and K. Kaneko, Physica D **75**, 328 (1994).

[6] P. Bak and K. Chen, Sci. Am. **264**, 26 (1991).

[7] S. Field, J. Witt, F. Nori and X. Ling, Phys. Rev. Lett. **74**, 1206 (1995).

[8] G.A. Held et al., Phys. Rev. Lett. **65**, 1120 (1990).

[9] A. Utsugi, Comp. Neural Systems **7**, 727 (1996).

[10] C-S. Poon, Neural Networks **9**, 1367 (1996).

[11] C-H. Liou and J-M. Wu, Neural Networks 9, 671 (1996).

[12] R. Salomon and J.L. van Hemmen, Neural Networks **9**, 589 (1996).

[13] S. Becker, Comp. Neural Systems **7**, 7 (1996).

[14] K. Sneppen and M.H. Jensen, Phys. Rev. Lett. **71**, 101(1993).

[15] L. da Silva, A.R.R. Papa and A.M.C. de Souza (in preparation).

[16] After the release of a spike, the threshold for the emission of a subsequent spike becomes larger than originally. After that and gradually, it returns back to its original value. The time during which the activation threshold remains greater than the original is called *relative refractory period* and is normally larger than the absolute refractory period. Since in the network of neurons, the excitatory potentials are determined exclusively by the mutual interactions of the neurons (do not depend on arbitrary external inputs)

this makes unlike that a neuron will fire during its relative refractary period. On the other hand, a neuron that has not released an spike for a sufficiently long time because the sum of external inputs has not reached the threshold loses gradually its probability of firing. For a recent account of the state-of-knowledge about neuron functioning see reference [18].

[17] J.E. Lovelock, *Gaia, A New Look at Life on Earth* (Oxford University Press, Oxford, 1979).

[18] C. Koch, Nature **385**, 207 (1997).

FIGURE CAPTIONS

Figure 1.-Distribution of barriers in the critical state (circles) and of the minimum barrier (squares). The system size used was $N = 2048$.

Figure 2.-Log-log plot of the dependence of $C(x)$ with x. The exponent of the power law is $\alpha = -2.32 \pm 0.05$; we used an absolute refractary period $T_r = 1$.
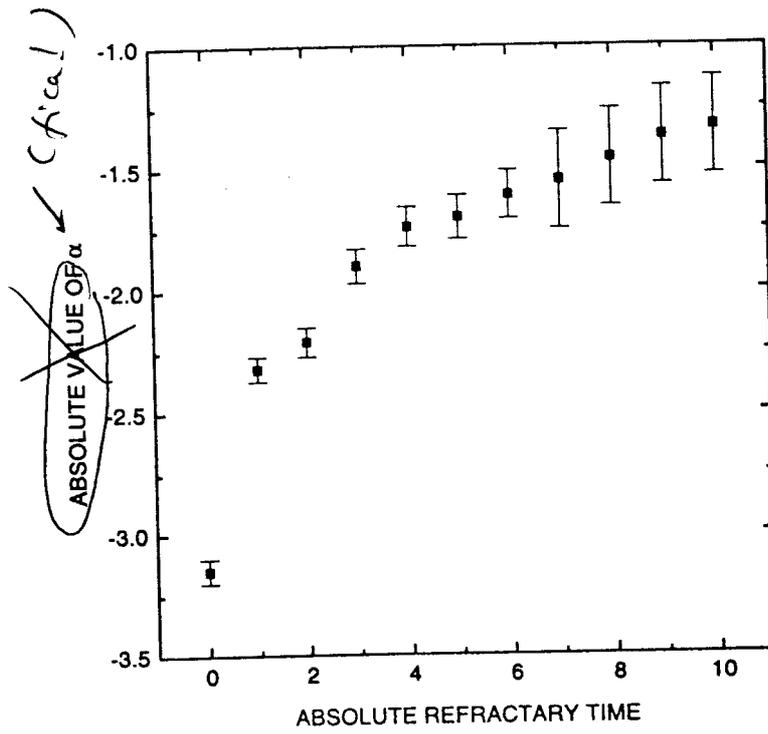
Figure 3.-Dependence of the exponent α of the power law in Figure 1 as a function of the refractary period T_r.

Figure 4.-Temporal dependence of the value of a barrier when the system is at the self-organized critical state. It has a fractal character: if the time scale is changed the shape continues to be the same, as shown in the zoomed area.
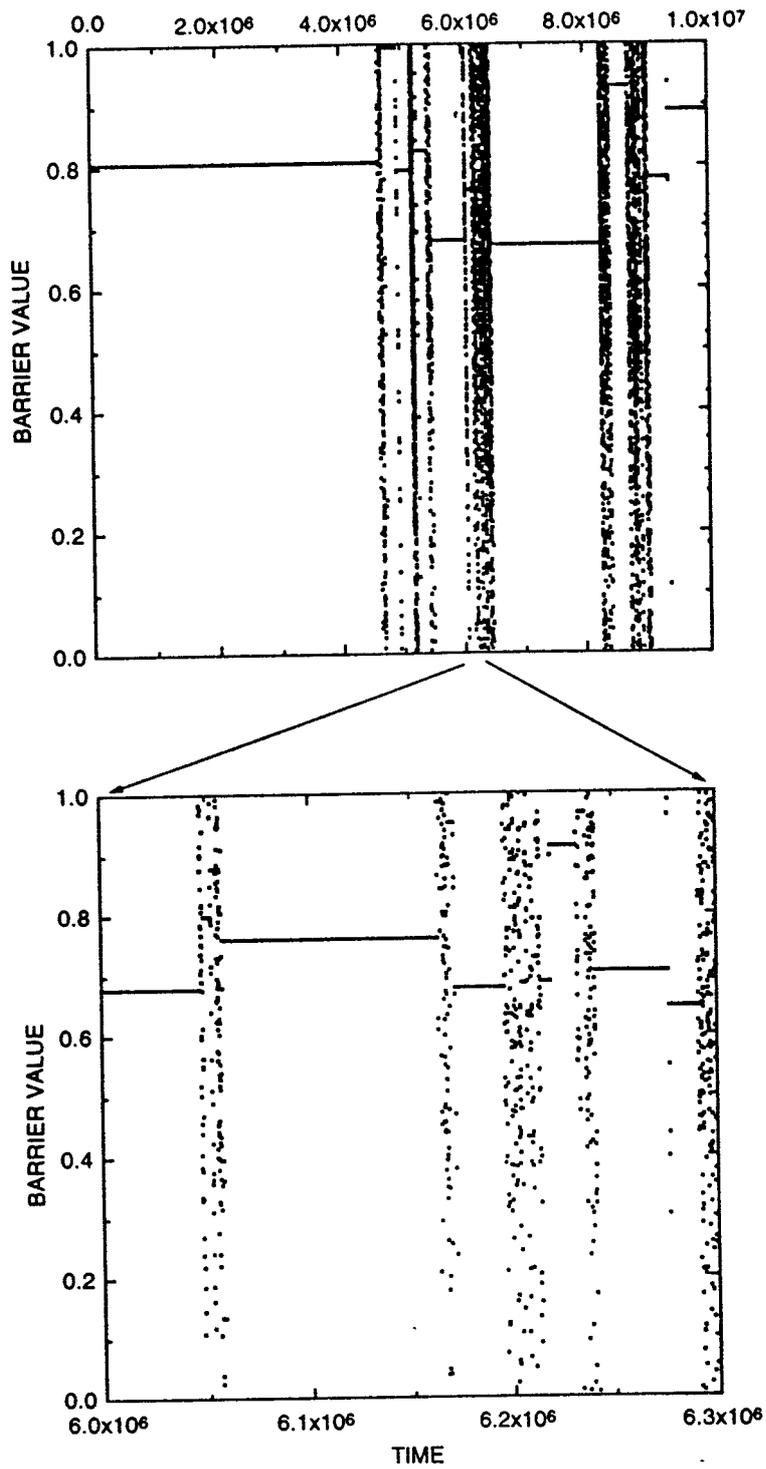
Figure 5.-Distribution of anti-avalanches for a single barrier when the whole system has attained the self-organized critical state. The value of the exponent of the power law is $\beta = -1.60 \pm 0.04$; as before the calculation was done for a refractary period $T_r = 1$.
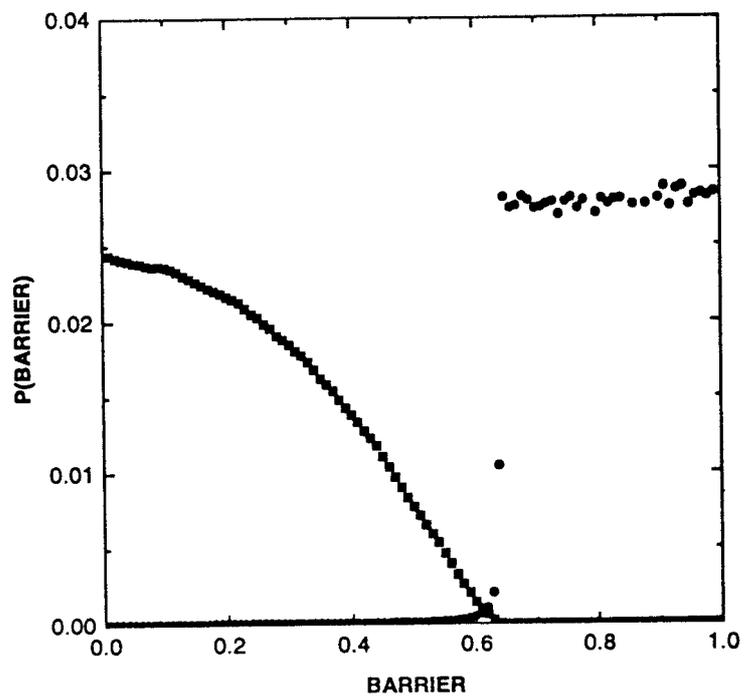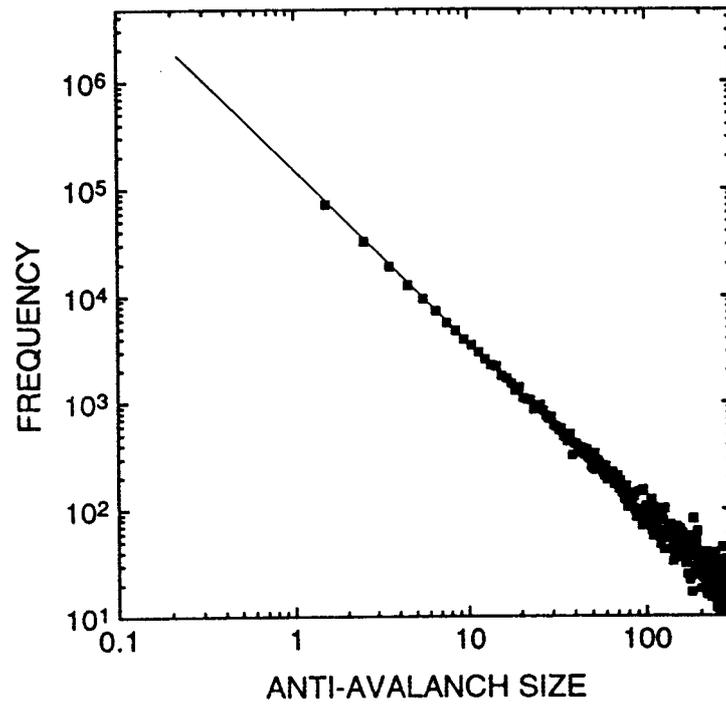
FIGURE 1
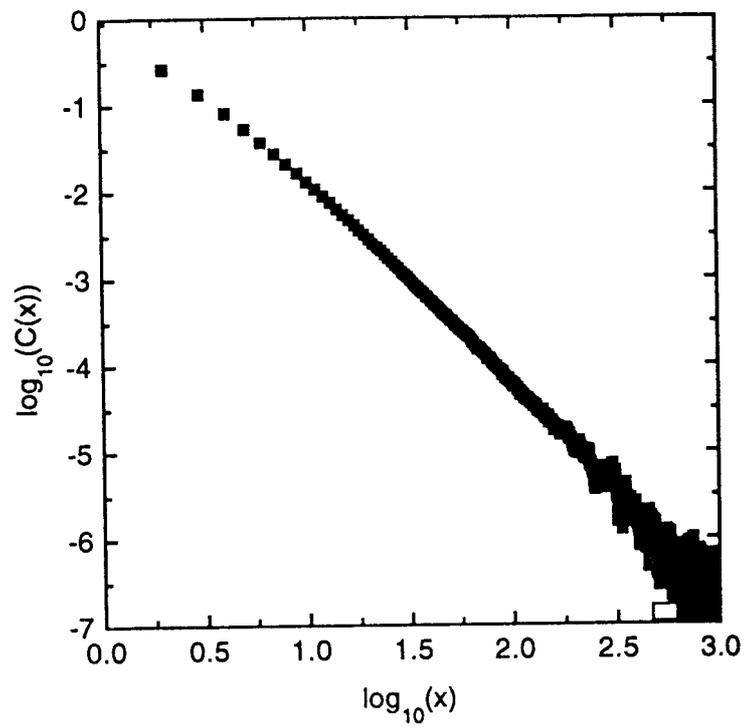
FIGURE 2

FIGURE 3

FIGURE 4

FIGURE 5