PURE MASSLESS SCALAR GEON

*Michele Lunetta**

Instituto de Física, Universidade Federal da Paraíba,
58000 João Pessoa - PB, Brazil

Idel Wolk and A.F. da F. Teixeira

Centro Brasileiro de Pesquisas Físicas,
22290 Rio de Janeiro - RJ, Brazil

ABSTRACT

The strongly velocity-dependent gravitational effects of the pure massless scalar field are described. To this end, geodesics are studied for a sourceless, singularity-free, new exact solution of the Einstein-scalar field equations. The static, cylindrically symmetric system is described in Weyl's canonical coordinates with cosmic time.

(*) Present address: CBPF, Av. Wenceslau Braz 71, 22290
Rio de Janeiro - Brazil

1. INTRODUCTION

Some years ago Melvin [1,2] and Thorne [3] investigated solutions of the Einstein-Maxwell's equations corresponding to electromagnetic fields without matter, charge or current. Such sourceless structures are ordinarily termed geons (Wheeler [4]). In the pure electric and magnetic geons, the electromagnetic fluxes are held together solely by their inherent gravitational pull.

It seems worthwhile to investigate the theoretical possibility of geons arising also from fields of nonelectromagnetic nature. Indeed, exact and numerical solutions of various classical fields have recently been objects of great interest, mainly due to their possible relevance to microphysics [5,6,7]. The study of the scalar field is particularly important, since it is one of the simplest quantities permitted by the stringent covariance requirements of general relativity.

In this paper we investigate a sourceless, long range scalar field in the absence of matter or any other field. Under such conditions, some subtle gravitational effects inherent to the pure scalar field can be studied in detail. As in Melvin's universe, we assume staticity and cylindrical symmetry of the system. We label the system "scalar geon", after noticing the finiteness of the physical components of the energy-momentum tensor, as well as of several gravitational invariants. We also study the motion of test particles and light rays, and thereby explain how the gravitation of the pure scalar geon acts on the different components of the velocity.

2. LINE ELEMENT AND INVARIANTS

We investigate the line element

$$ds^2 = dt^2 - (dr^2 + dz^2)e^{b^2r^2} - r^2d\phi^2 , \qquad (1)$$

where b = const. It is an exact solution of the Einstein-scalar equations [8]

$$R_{\mu\nu} = 2(\partial_\mu S)\,\partial_\nu S , \qquad (2)$$

where S = bz is a scalar field. The absence of a distributed source of S is ensured [9] by the vanishing Laplacian

$$\partial_\mu[g^{1/2}g^{\mu\nu}\partial_\nu S] = 0 . \qquad (3)$$

The scalar curvature R and other gravitational invariants are finite everywhere, and decrease monotonically with increasing radial distance:

$$R = -2b^2e^{-b^2r^2} , \quad R^\mu_\nu R^\nu_\mu = R^2 , \quad R^{\mu\nu}_{\rho\sigma}R^{\rho\sigma}_{\mu\nu} = 3R^2 . \qquad (4)$$

Similarly, all components of the energy-momentum T^μ_ν take extreme (finite) values on the z-axis and decrease radially. Taken together, these results imply nonexistence of singular sources of fields. The lines of force of the gradient of scalar field are held together by self-gravitation alone. Our static, cylindrically symmetric system is then the massless scalar counterpart of the electromagnetic geons of Melvin [1] .

3. TIMELIKE GEODESICS AND LIGHT RAYS

Scalar geons present unusual gravitational features. To see these, we initially investigate the timelike geodesics:

$$\ddot{x}^{\mu} + \{{}^{\mu}_{\nu\rho}\} \dot{x}^{\nu}\dot{x}^{\rho} = 0 \quad , \quad \dot{x}^{\mu}\dot{x}_{\mu} = 1 \, , \tag{5}$$

where a dot means d/ds. From the symmetries of the system we readily find

$$s' = (1-v^2)^{1/2} \quad , \quad z' = \xi e^{-b^2r^2} \quad , \quad \phi' = \lambda r^{-2} \quad , \tag{6,7,8}$$

where v, ξ,λ are constants and a prime means d/dt. The radial velocity satisfies

$$r'' = -b^2 r \, (r'^2 - \xi^2 e^{-2b^2r^2}) + \lambda^2 r^{-3} e^{-b^2r^2} \, , \tag{9}$$

from which we obtain the first integral

$$r'^2 = [v^2 - \lambda^2 r^{-2}] e^{-b^2r^2} - \xi^2 e^{-2b^2r^2} \, . \tag{10}$$

We next consider three simple cases:

a) *motions in planes* $z = const.$ We set $\xi = 0$ in (10) and use (8) to obtain

$$(dr/d\phi)^2 = r^2(r^2/r_0^2 - 1) \, e^{-b^2r^2} \, , \tag{11}$$

where r_0 is the minimal radial location of the particle, given by

$$\lambda^2 = r_0^2 v^2 \, . \tag{12}$$

The particle spirals inwards until it reaches $r = r_0$, then spirals outwards to radial infinity.

b) *meridian plane motions crossing the z-axis.* Setting $\lambda=0$ in (10) and using (7) gives

$$(dr/dz)^2 = e^{b^2 r^2} \sec^2\alpha - 1 \quad , \tag{13}$$

where $\alpha \neq \pi/2$ is the angle of incidence on the z-axis, given by

$$\xi^2 \sec^2\alpha = v^2 \quad . \tag{14}$$

The shape of orbit is obtained numerically: after crossing the z-axis, the particle goes to radial infinity in a direction which gradually tends to a normal of z-axis. If the angle of incidence α is less than 45^0 the particle seems repelled from the inner regions of the scalar geon; it is attracted only when the particle recedes beyond a certain radial distance. For larger angles of incidence the acceleration is everywhere directed towards the z-axis.

The special case $\alpha = \pi/2$ must be treated separately: we set $\xi = \lambda = 0$ in (10) and obtain

$$dr/dt = \pm v\ e^{-b^2 r^2/2} \quad . \tag{15}$$

The motion is rectilinear and perpendicular to the z-axis. The impression is that a Gaussian-like, Newtonian gravitational potential is operating, centered on the axis and vanishing at radial infinity:

$$N(r) = -\frac{1}{2} v^2 e^{-b^2 r^2} . \tag{16}$$

c) *meridian plane motions not crossing the z-axis.* We again set $\lambda = 0$ in (10) and use (7) to obtain

$$(dr/dz)^2 = e^{b^2(r^2-r_0^2)} - 1 \quad , \tag{17}$$

where, as before, r_0 measures the minimal radial location of the particle:

$$b^2 r_0^2 = \ell n \left[\xi^2/v^2\right] \quad . \tag{18}$$

The trajectories are now U-shaped, with vertex on r_0, and present asymptotic characteristics identical to those of case b).

Null geodesics are formally obtained from (5), provided we set $\dot{x}^\mu \dot{x}_\mu = 0$. As a consequence, trajectories for light rays are still given by equations (11), (13) and (17). Also the law of motion (15) is satisfied by rays, setting $v^2 = 1$.

4. COMMENTS AND CONCLUSION

The line element (1) is written in Weyl's [10] canonical coordinates. Cosmical time is being used, implying that a particle once at rest remains at rest [11]. The constant parameter b has dimension of inverse length, and measures the strength of the scalar field.

We could summarize the results of the previous Section as: Particles and light rays travelling under the gravitational action of the scalar geon always escape to the radial infinity.

However, it became apparent that the gravitation of the geon depends strongly on the direction of particle's velocity. Since this is a feature unknown in nonrelativistic gravity, a few complementary comments seem worthwhile. We look more closely into the acceleration equation (9), rewritten as:

$$d^2r/dt^2 = r\left[b^2(dz/dt)^2 - b^2(dr/dt)^2 + (d\phi/dt)^2 e^{-b^2r^2}\right]. \quad (19)$$

First, we see that the longitudinal term dz/dt contributes positive definitely to d^2r/dt^2. As a consequence, all particles and rays which are momentarily travelling parallel to (but not along!) the z-axis are pushed outwards. In this respect, the scalar geon greatly differs from Melvin's magnetic universe, which allows a whole bundle of longitudinal null geodesics [2,3,12].

Second, we note that the radial term dr/dt contributes negative definitely to d^2r/dt^2. However, the contribution is not sufficient to reverse the motion of any radially outgoing particle, as is seen from Eq. (15). In Melvin's universe, oppositely, "the gravitational attraction toward the center of the universe is great enough to prevent any object of nonzero rest mass from escaping to radial infinity" [3].

Third, although the contribution of the azimuthal term in $d\phi/dt$ to d^2r/dt^2 is positive definite, nevertheless the factor $\exp(-b^2r^2)$ makes this contribution smaller than the corresponding one in case of flat spacetime (b=0). That is to say, a particle momentarily performing a purely azimuthal motion is driven outwards, although only weakly. Differently from Melvin's universe [12], the scalar geon does not permit circular orbits for particles or rays at whatsoever finite radial distance.

Finally, we remark that our scalar geon is a plasm of index 2, according to Melvin's [1] nomenclature. Since pure electromagnetic universes are also index 2, it seems interesting to investigate the gravitational properties of geons containing simultaneously all such fields of long range. Studies along this line are already in progress [13].

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