# Photofissility at 1 GeV for Nuclei throughout the Periodic Table

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A new approach to evaluate nuclear photofissilities at incident photon energies above the pion photoproduction threshold has been recently developed and proved to describe successfully the fissilities of $^{\rm nat}$Pb and $^{232}$Th target nuclei at energies $\sim 0.2 - 4.0$ GeV. The method is merely a simple, semiempirical description of the photofission reactions in which fissility, $f$, is governed by two basic quantities, namely, i) the first-chance fission probability, $\bar{f}_1$, for the average cascade residual, and ii) the slope, $\bar{s}$, of the chance-fission probability associated with the average evaporative sequence of fissionable residuals. In the present work we have extended this approach to analyse photofissity data that have been accumulated over the past fourty years or so, measured at 1 GeV, for nearly fourty target nuclei extending from Ti up to Np. Results have shown that the variation of fissility with $Z^2/A$ could be described quite satisfactorily by the proposed model.

Recently, a new approach to evaluate nuclear photofissilities at incident photon energies $E_\gamma \gtrsim 200$ MeV has been developed to overcome the calculational difficulties imposed by the available Monte Carlo codes when the target nuclei are expected to exhibit low, or very low, fissility-values ($\lesssim 1\%$), as is the case for pre-actinide, intermediate-mass, and less massive nuclei [1]. The method is a simple, semiempirical approach that has proved to work quite well in describing the fissilities of $^{\rm nat}$Pb and $^{232}$Th target nuclei induced by $\sim 0.2 - 4.0$ GeV photons, which have been measured at the Thomas Jefferson Laboratory by Cetina et al [2,3].

Since photofissility values for actinide targets have been already analysed to some detail in the framework of Monte Carlo calculations [3,4] we decided, in the present work, to extend this approach [1] to analyze photofissility data that have been accumulated over the past fourty years or so, measured at 1 GeV, for nearly fourty target nuclei extending from Ti up to Np. Results have shown that the variation of fissility with $Z^2/A$ can be described quite satisfactorily by the proposed model. We remark that the trend of fissility is seen as an inverse reflection of the behaviour of the height of the fission barrier with $Z^2/A$ [5,6].

Monte Carlo calculations are, at present, the main tool to describe quantitatively both the cascade and fission-evaporation competition processes, as well as to obtain the total fission probabilities (photofissility values) for a number of photofission reaction cases. However, for cases where the target nucleus is expected to have low fissility-values (pre-actinide, intermediate-mass, and less massive nuclei), the available codes may reveal themselves very time-consuming in obtaining a calculated fissility-curve of acceptable uncertainty over a large energy-interval such as $\sim 0.2 - 4.0$ GeV. This fact led us to develop an alternative method to calculate nuclear photofissility-values semiempirically. This approach is based on the current, two-step model for intermediate-energy photonuclear reactions, i.e. a photon-induced intranuclear cascade followed by a fission-evaporation competition process for the excited, post-cascade nucleus.

The distributions of atomic number, mass number, and excitation energy of the cascade residuals which would result from Monte Carlo calculations are here replaced by their respective average value $\left(\overline{Z}^*, \overline{A}^*, \overline{E}^*\right)$. These have been defined as functions of the incident photon energy by means of simple expressions in which the neutron and proton cut-off energies ($E_c^n$ and $E_c^p$, respectively) play a fundamental role.

Accordingly, the above quantities have been evaluated as [1]

$$\overline{Z}^* \approx Z - \frac{E_\gamma}{2}\frac{Z}{A}\frac{1}{E_c^p}, \tag{1}$$

$$\overline{A}^* \approx A - \frac{E_\gamma}{2}\left[\frac{Z}{A}\frac{1}{E_c^p} + \left(1 - \frac{Z}{A}\right)\frac{1}{E_c^n}\right], \tag{2}$$

$$\overline{E}^* \approx \begin{cases} \frac{E_\gamma}{2} + \frac{1}{4}\left(E_c^n + E_c^p\right), E_\gamma < B \\ \frac{B}{2} + \frac{1}{4}\left(E_c^n + E_c^p\right), E_\gamma \geq B, \end{cases} \tag{3}$$

where $B$ is the total nuclear binding energy for $(Z, A)$. These average quantities are thought as the substitutes of their respective distribution functions in the sense that the average cascade residual is produced with probability equal to unity.

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The second stage of the photofission reaction is here described by a fission-evaporation competition process starting from the average initial, excited residual nucleus $(\overline{Z}^*, \overline{A}^*, \overline{E}^*)$. Neutron, proton, and alpha particle emissions are considered the modes of de-excitation which may compete more significantly with each other and with the fission mode for all subsequent residuals formed at each step along the evaporative sequence.

Fissionable evaporation residuals can be thought as being formed in generations. Let $n$ be the order of a generation of residuals: $n = 1$ corresponds to the cascade residual, i.e., the first residual $\left(\overline{Z}^*, \overline{A}^*, \overline{E}^*\right)$, and the partial fission probability is simply the first chance-fission probability $P_1^p = f_1$. For $n = 2$, formation of three evaporation residuals may occur, and the partial fission probability due to the chance-fission of the residuals in the second generation is

$$P_2^p = n_1 f_{2n} + p_1 f_{2p} + \alpha_1 f_{2\alpha}. \tag{4}$$

Similar expressions can be written for the successive generations of residuals. Each term in (4) represents the chance-fission probability of the respective residual nucleus formed. The number of fissionable residuals which may be formed in the generation of order $n$ is $3^{n-1}$, and the total fission probability of the cascade residual is, therefore, given by

$$P_f^t\left(\overline{Z}^*, \overline{A}^*, \overline{E}^*\right) = \sum_{n=1}^{n_g} P_n^p. \tag{5}$$

The maximum number of generations of residuals is estimated as $n_g \approx \overline{E}^*/\overline{E}_{ev}$, where $\overline{E}_{ev}$ represents the average total energy removed from the system per particle evaporated.

The routine calculation for the evaporation-fission competition process, i.e. the calculation of the probability values for the neutron, proton, and alpha particle emission modes and the fission mode, as well as the successive evaporation residuals formed, has already been detailed in [1]. However, new expressions to calculate $r = a_f/a_n$ (ratio of the level density parameter at the fission saddle point, $a_f$, to that of the residual nucleus after neutron evaporation, $a_n$) have been introduced here following an updated systematic analysis on level density parameter as reported in [7]. The new expressions for $r$ read

$$r = 1 + p/E^{*q}, \tag{6}$$

$$p = \begin{cases} \exp[0.257(217 - A)], & 210 < A \leq 232 \\ \exp[0.150(222 - A)], & 140 < A \leq 210 \\ \exp[0.0388(176 + A)], & 30 < A \leq 140 \end{cases} \tag{7}$$

$$q = \begin{cases} 0.0352(235 - A), & 140 < A \leq 235 \\ 0.0129(119 + A), & 30 < A \leq 140. \end{cases} \tag{8}$$

In calculating the fission and particle emission probabilities of the different residuals, the particle separation energies, total nuclear binding energy, ground-state fission barrier heights, level density parameters, and nuclear radii have been taken from the current tables or updated systematics [5-10].

A chance-fission probability, $q_{ni}$, is a quantity defined by the product of the formation probability of residual $i$ in the generation $n$ times the fission probability of this residual, $f_{ni}$. To make this definition clear, we recall that each term in Eq.(4) represents, for instance, the chance-fission probability of the residuals eventually formed in the second generation. The number of residuals which may be formed in generation $n$ is $N = 3^{n-1}$, and, therefore, the partial fission probability is

$$P_n^p = \sum_{i=1}^{N} q_{ni}. \tag{9}$$

A direct calculation of the total fission probability for the cascade residual (i.e., the target nucleus fissility) has been performed by taking into account all intermediate chance-fission probabilities of residuals eventually formed throughout the evaporation chain. This calculation has been simplified in view of the remarkable pattern exhibited by the chance-fission probability values, according to which the chances for fission are shown to lie between two rather linear (in log-scale) trends, one for the most probable, and another one for the least probable sequences of fissionable residuals. As an example, the case for $^{93}$Nb is shown in Fig.1. Calculations have also been performed at 1 GeV for other target nuclei, therefore producing different average cascade residuals (see Table 1). We remark that the same pattern like the one exhibited in Fig 1 is apparent in all cases studied [1]. Fluctuations of calculated points around the linear directions (Fig.1) are mainly due to pairing plus shell effects.

The very interesting results reported above suggest parametrizing of the chance-fission probabilities by an equation of the form

$$q_{ni} = f_1 \mathrm{e}^{-(n-1)s_i}, \tag{10}$$

in which $s_i$ denotes (in ln-scale) the slope of the straight lines (Fig.1). $i = 1$ corresponds to the *most* probable evaporation sequence of residuals ($s_1 = s_m$), and $i = N$ corresponds to the *least* probable evaporation path of residuals ($s_N = s_\ell$) (see Fig.1).

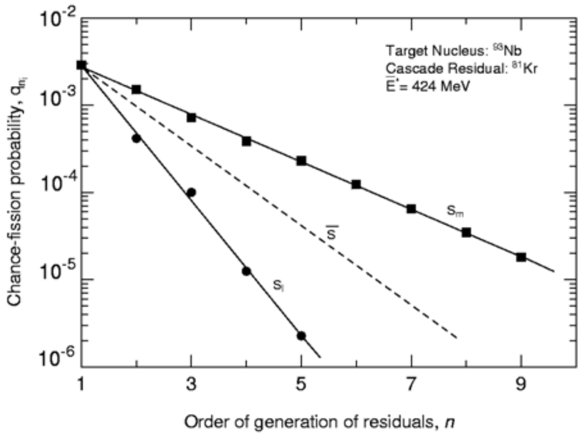


FIG. 1: Calculated chance-fission probabilities (squares and circles) versus order of generation of evaporation residuals, $n$, for 1-GeV incident photons on $^{93}$Nb target. The full lines represent the most (slope $s_m$) and least (slope $s_\ell$) evaporation paths, and the dashed one (slope $\bar{s}$) is the average evaporation path on which the equivalent, average residuals of each generation are located.

For a given generation of evaporation residuals ($n$ fixed) the partial fission probability is the summation (9). We can obtain an estimation of the $P_n^{p\prime}$s by taking simply the product of the number of residuals which may be formed in generation $n$ times a certain average chance-fission probability, $\overline{q}_n$, i.e.,

$$P_n^p = \sum_{i=1}^{n} q_{ni} \approx \overline{q}_n \times 3^{n-1}. \qquad (11)$$

The $\overline{q}_n$-values are, in turn, obtained from a certain average slope-value, $\overline{s}(s_m < \overline{s} < s_l)$, such that

$$\overline{q}_n = f_1 \mathrm{e}^{-(n-1)\overline{s}}. \qquad (12)$$

Parameter $\overline{s}$ defines the slope of an average sequence of evaporation residuals which lies between the most ($s_m$) and least ($s_l$) probable sequences of residuals (dashed line in Fig. 1). In other words, the sum of the $3^{n-1}$ chance-fission probabilities as given by (9,10) represents, for each generation of residuals, the chance-fission probability of an average, equivalent evaporation residual located on the $\overline{s}$-sequence. Parameter $\overline{s}$ is to be considered the adjustable parameter of the model, and thus its value is to be found semiempirically.

Finally, the average nuclear fissility is given by the total fission probability of the average cascade residual, and it is calculated as

$$\overline{f}_c(E_\gamma) = P_f^t(\overline{Z}^*, \overline{A}^*, \overline{E}^*) = \sum_n P_n^p = \overline{f}_1 \sum_n 3^{n-1} \mathrm{e}^{-(n-1)\overline{s}} \qquad (13)$$

which gives

$$\overline{f}_c = \frac{\overline{f}_1}{1 - 3\mathrm{e}^{-\overline{s}}}. \qquad (14)$$

In this way, for each incident photon energy on a target nucleus (i.e., an average cascade residual), fissility can be easily calculated provided the values of $\overline{f}_1$ and $\overline{s}$ are known.

Table 1: First-chance fission probability, $\overline{f}_1$, for 1 GeV incident photons in various nuclei.

| Target nucleus | Cascade residual $\bar{Z}^*$ | $\bar{A}^*$ | $\bar{f}_1$ | Target nucleus | Cascade residual $\bar{Z}^*$ | $\bar{A}^*$ | $\bar{f}_1$ |
|---|---|---|---|---|---|---|---|
| $^{237}$Np | 89 | 226 | 1.70x10$^{-1}$ | $^{165}$Ho | 63 | 154 | 0.78x10$^{-3}$ |
| $^{233}$U | 88 | 222 | 1.58x10$^{-1}$ | $^{163}$Dy$^a$ | 62 | 152 | 0.82x10$^{-3}$ |
| $^{235}$U | 88 | 224 | 1.78x10$^{-1}$ | $^{159}$Tb | 61 | 148 | 5.20x10$^{-4}$ |
| $^{238}$U | 88 | 227 | 1.51x10$^{-1}$ | $^{157}$Gd$^a$ | 60 | 146 | 5.27x10$^{-4}$ |
| $^{232}$Th | 86 | 221 | 0.98x10$^{-1}$ | $^{150}$Sm$^a$ | 58 | 139 | 4.56x10$^{-4}$ |
| $^{209}$Bi | 79 | 198 | 1.43x10$^{-2}$ | $^{144}$Nd$^a$ | 56 | 133 | 5.65x10$^{-4}$ |
| $^{207}$Pb$^a$ | 78 | 196 | 1.43x10$^{-2}$ | $^{140}$Ce$^a$ | 54 | 129 | 5.84x10$^{-4}$ |
| $^{208}$Pb | 78 | 197 | 1.13x10$^{-2}$ | $^{139}$La | 53 | 128 | 5.12x10$^{-4}$ |
| $^{204}$Tl$^a$ | 77 | 193 | 1.06x10$^{-2}$ | $^{128}$Te$^a$ | 48 | 117 | 0.89x10$^{-3}$ |
| $^{201}$Hg$^a$ | 76 | 190 | 0.94x10$^{-2}$ | $^{122}$Sb$^a$ | 46 | 110 | 1.55x10$^{-3}$ |
| $^{197}$Au | 75 | 186 | 5.50x10$^{-3}$ | $^{119}$Sn$^a$ | 46 | 108 | 1.67x10$^{-3}$ |
| $^{195}$Pt$^a$ | 74 | 184 | 5.02x10$^{-3}$ | $^{115}$In$^a$ | 44 | 104 | 1.82x10$^{-3}$ |
| $^{190}$Os$^a$ | 72 | 179 | 2.43x10$^{-3}$ | $^{112}$Cd$^a$ | 44 | 101 | 1.47x10$^{-3}$ |
| $^{186}$Re$^a$ | 71 | 175 | 2.08x10$^{-3}$ | $^{108}$Ag$^a$ | 42 | 96 | 1.58x10$^{-3}$ |
| $^{184}$W$^a$ | 70 | 173 | 1.69x10$^{-3}$ | $^{96}$Mo$^a$ | 37 | 85 | 2.30x10$^{-3}$ |
| $^{181}$Ta | 69 | 170 | 1.38x10$^{-3}$ | $^{93}$Nb | 36 | 81 | 2.88x10$^{-3}$ |
| $^{178}$Hf$^a$ | 68 | 167 | 1.26x10$^{-3}$ | $^{65}$Zn$^a$ | 25 | 53 | 1.55x10$^{-2}$ |
| $^{175}$Lu$^a$ | 67 | 164 | 1.07x10$^{-3}$ | $^{63}$Cu$^a$ | 23 | 51 | 2.02x10$^{-2}$ |
| $^{173}$Yb$^a$ | 66 | 162 | 1.17x10$^{-3}$ | $^{59}$Ni$^a$ | 23 | 47 | 3.70x10$^{-2}$ |
| $^{174}$Yb | 66 | 163 | 0.99x10$^{-3}$ | $^{56}$Fe$^a$ | 21 | 44 | 5.34x10$^{-2}$ |
| $^{169}$Tm | 65 | 158 | 0.94x10$^{-3}$ | $^{48}$Ti$^a$ | 17 | 37 | 1.54x10$^{-1}$ |

[a]Mean mass number of the naturally occurring isotopes.

The average first-chance fission probability, $\overline{f}_1$, can be evaluated as usually [1]. The values of $\overline{s}$ are, in turn, determined from the experimental average fissility-values measured at 1 GeV for various target nuclei, where the average cascade residuals are those of both $\overline{Z}^*$ and $\overline{A}^*$ integer (Eqs. (1) and (2), with excitation energy given by Eq. (3)). The final values of $\overline{s}$ are then obtained from a smooth trend of $\overline{s}$ versus $Z^2/A$.

We have applied the present, phenomenological, semiempirical photofission approach to analyse the photofissility data obtained at 1 GeV for a number of pre-actinide, intermediate-mass, and less massive nuclei [3, 11-24]. Some data for actinide targets [3,11, 14,25-28] are also presented. Photofissility is defined as the ratio of the photofission cross section, $\sigma_f$, to the total nuclear photoabsorption cross section, $\sigma_a^T$, both quantities being measured at the same incident photon energy value, i.e.

$$f = \frac{\sigma_{\gamma,f}}{\sigma_a^T} = \frac{\sigma_{\gamma,f}}{A\sigma_{\gamma,N}} = \frac{\sigma_{\gamma,f}/A}{\sigma_{\gamma,N}}. \quad (15)$$

Among all target nuclei here investigated, $^{237}$Np does exhibit the best chance for fission (a fact which has been demonstrated experimentally [3]). Besides, the photofission cross section for $^{237}$Np represents pratically its total nuclear photoabsorption cross section. From the measurement of $\sigma_{\gamma,f}$ taken at 1 GeV for $^{237}$Np by Cetina et al. [3], it results that the average photon-nucleon interaction cross section equals to $\sigma_{\gamma,N} = 165 \pm 2\ \mu b$.

By taking the measured value of $\sigma_{\gamma,f}$ at 1 GeV for the various nuclei studied, the photofissilities are thus obtained from (15). These data are depicted in Fig. 2 (full circles). On the other hand, the calculated values for the average first-chance fission probability, $\bar{f}_1$, are those listed in Table 1. By introducing the values of these two quantities into Eq.(14), the semiempirical $\overline{s}$-values have been obtained, resulting in a linear dependence between $\overline{s}$ and $Z^2/A$, such that

$$\overline{s} = -0.04447(Z^2/A) + 2.6968,\ Z^2/A \leq 34\ , \quad (16)$$

$$\overline{s} = 1.295 \pm 0.039, \quad Z^2/A > 34 \quad \text{(actinides)}. \quad (17)$$

Finally, the calculated average photofissility-values, $\overline{f}_c$, have been obtained from Eq.(14), and the resulting trend is shown in Fig. 2 (full line). A comparison between calculated and experimental fissilities shows that in 83% of the cases the data are reproduced within a factor lower than 2.5 (see inset graph in Fig. 2). Such an agreement can be considered very satisfactory in view of the great uncertainties present in the photofission cross section measurements, as well as uncertainties associated with the nuclear parameter-values of the model.

Figure 2 shows, in addition, that photofissility at 1 GeV varies by three orders of magnitude for nuclei extending from Ti to Np. Besides, and what is more important, the trend exhibited by photofissility is essentially an inverse reflection of the behaviour of the height of the fission barrier of nuclei throughout the periodic table, as it comes from either the liquid drop [5] or droplet [6] models of the atomic nucleus.

To conclude, as already discussed in [1], parameter $\overline{s}$ is related to the location of the "point of fission" along the evaporation-fission competition sequence. Inspection on Eq. (13) shows that fissility is reached with the cumulative, partial fission probability which increases with the order of generation of residuals, $n$, but at a rate dictated by the $\overline{s}$-value. The greater $\overline{s}$, more likely is the fission to occur near the beginning of the evaporative sequence, i.e., near the cascade residual.

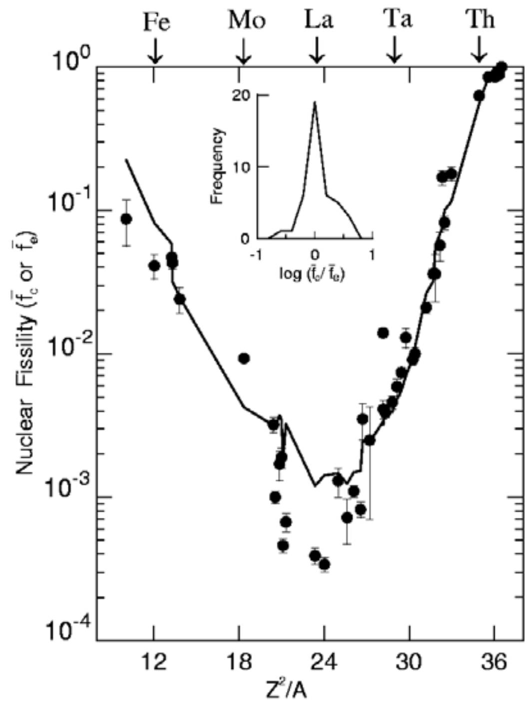


FIG. 2: Fissility versus $Z^2/A$ for 1-GeV incident photons on various target nuclei. Points represent average experimental values ($\bar{f}_e$) from data reported in [3, 11-28]. The calculated trend from the present model ($\bar{f}_c$) is shown by the full line. The inset graph shows the distribution of the quantity log ($\bar{f}_c/\bar{f}_e$) for the nuclei investigated (see table 1).

The present study has demonstrated that the variation of fissility measured at 1-GeV incident photons in a number of target nuclei throughout the periodic table could be described quite satisfactorily by a new, semiempirical approach previously developed by us to analyse intermediate-energy photofissilities [1]. The strong correlation between the general trend of the height of the fission barrier with $Z^2/A$ and the trend of fissility is clearly evidenced, specially in the region of the rare earth nuclei, where a large minimum in fissility occurs. The present model can, of course, be easily applied to nuclear photofission reactions at other incident intermediate-energy values.

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