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Gluon Confinement

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Abstract

In this paper we present a new model for a gauge field theory such that self-interacting spin-one particles can be confined in a compact domain. The necessary conditions to produce the confining potential appear already in the properties of the eikonal structure generated by the particular choice of the dynamics.

Key-words: Gluon confinement; Yang-Mills; Effective geometry.

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It is a common knowledge that the necessary conditions for a field theory to produce confinement is to be non-linear. Although this seems to be a good requirement, it is not enough. Since such a confinement, for instance, in the SU(3) non-Abelian gauge theory containing an octet of massless vector gluons, is still not available [1], a natural question appears: is there something missing in the Yang-Mills theory? The aim of this letter is to exhibit in a simple version a pattern that answers affirmatively to this question.

In its classical version, the confinement of massless spin-one particles can be interpreted as the deformation of their surfaces of propagation, the corresponding *light cones*, in such a way that the gluons encounter an unsurmountable barrier that forbids them to get outside the confinement region. In other terms, it appears, for the external world, as a situation that can be described equivalently in terms of the formation of a horizon. Such a deformation does not occur in the Minkowskian structure of spacetime. How to create a similar effect in a scenario containing just a set of spin-one self-interacting fields?

To obtain the required property that the information carried on by the gluons operate in a different way than the Minkowski null cones, the first step is to change the dynamics. The reason for this is related to the fact that massless Yang-Mills (YM) particles travel along the Minkowski null cone[1]. This property is the same as in the Abelian case and has its origin as a direct consequence on the construction of the YM model that makes all non-linearity of this theory to be restricted uniquely to the algebraical dependence of the field $F^a_{\mu\nu}$ on the potential A^a_μ. To change this in the realm of the gluon interaction, conserving the colour covariance, one should modify conveniently the Lagrangian. This is precisely the case which we will consider here and constitutes the basis for the actual new ingredient of our model. We shall see that a slight modification of the YM theory[2] concerning its Lagrangian seems to be the key to the understanding of the confinement problem.

We start by considering a set A^a_μ of colour multiplet that constitutes a Yang-Mills field of a non-Abelian theory[3] with $F^a_{\mu\nu}$ as the corresponding field. Let $\mathbf{F}$ be the invariant under space-time[4] and internal colour coordinates, defined by

$$\mathbf{F} \equiv \vec{F}^{\mu\nu}.\,\vec{F}_{\mu\nu} = F^{a\,\mu\nu}\,F_{a\,\mu\nu}. \tag{1}$$

The dynamics set up in the Yang-Mills approach mimetizes Maxwell electrodynamics by the identification of the Lagrangian to such quantity, that is, $L_{\text{YM}} = \mathbf{F}$. Should this be taken as an irretrievable paradigm? Does a change on this hypothesis, in the hadron world, yield the desirable consequences? Before answering to this, let us make a small comment on the classical description.

From a broad principle the Lagrangian should have the general non-linear form

$$L = L(\mathbf{F}). \tag{2}$$

Although one can go further without being necessary to specify the form of such a functional, in order to have a definite model that exhibits in a simple manner the main aspects

[1]The need for this modification comes from the property of the structure of geodesics in Minkowski space, that imposes that any particle that follows null cones cannot be bounded in a compact region.

[2]We remark that the major part, and by far the most important one, of Yang-Mills theory is maintained.

[3]We have in mind, for instance, the standard $SU(3)$ non-Abelian QCD model.

[4]The Minkowski metric is $\eta_{\mu\nu} = diag(+,-,-,-)$.

of our ideas, we limit all our considerations here to the specific model provided by:

$$L_{\rm NDE} = \epsilon_s \left(1 - e^{\frac{\mathbf{F}}{4\epsilon_s}}\right) \tag{3}$$

in which ϵ_s is a constant. The corresponding equation of motion is given by

$$D_\nu^{ac} \left\{ e^{\frac{\mathbf{F}}{4\epsilon_s}} F_c^{\mu\nu} \right\} = 0 \tag{4}$$

that is

$$[\delta_{cd}\partial_\nu + g\, c_{acd} A_\nu^a] \left(e^{\frac{\mathbf{F}}{4\epsilon_s}} F^{d\,\mu\nu} \right) = 0, \tag{5}$$

where c_{abc} are the constants of the structure of the gauge group and g is the strong interaction coupling constant. To proceed with the examination of the corresponding behavior of the classical gluons in such non-linear theory there is no better way than to consider the very high energy case through the analysis of the eikonal. In the standard Yang-Mills dynamics, the eikonal is nothing but null-cones of the Minkowski background spacetime, as in Maxwell theory. This is not the case for our Lagrangian. Indeed, there exist examples of spin-one theories in which the eikonal follows null geodesics in an effective geometry which depends not only on the background metric but also on the field properties. This has been shown in the case of pure non-linear electrodynamics [2]. This result keeps being valid in the non-Abelian gauge theory, as we will now show[5].

Let Σ be a surface of discontinuity for the gauge field. Following Hadamard's [3] condition we take the potential and the field as being continuous through Σ but having its first derivative discontinuous, that is:

$$[F^a_{\mu\nu}]_\Sigma = 0, \tag{6}$$

and

$$[\partial_\lambda F^a_{\mu\nu}]_\Sigma = f^a_{\mu\nu} k_\lambda, \tag{7}$$

in which the symbol $[J]_\Sigma$ represents the discontinuity of the function J through the surface Σ and k_λ is the normal to Σ.

Applying these conditions into the equation of motion (4) we obtain

$$f_a^{\mu\nu} k_\nu + \frac{1}{2\epsilon_s} \xi F_a^{\mu\nu} k_\nu = 0, \tag{8}$$

where ξ is defined by

$$\xi \equiv F_a^{\alpha\beta} f^a_{\alpha\beta}. \tag{9}$$

From the cyclic identity,

$$D_\lambda^{bc} F^a_{\mu\nu} + D_\mu^{bc} F^a_{\nu\lambda} + D_\nu^{bc} F^a_{\lambda\mu} = 0 \tag{10}$$

[5]At the basis of this property rests the fact that the dependence of the group connection on the potential do not contain derivatives but only an algebraic form. Standard conditions for the wave disturbances, like Hadamard's structure, imply that this sector of the non-linearity does not affect the velocity of propagation.

and using the above continuity conditions of the potential and the fields yields

$$f^a_{\mu\nu}k_\lambda + f^a_{\nu\lambda}k_\mu + f^a_{\lambda\mu}k_\nu = 0. \qquad (11)$$

Multiplying this equation by $k^\lambda\, F^{\mu\nu}_a$ yields

$$\xi k_\nu\, k_\mu \eta^{\mu\nu} + 2\, F^{\mu\nu}_a\, f^a_{\nu\lambda}\, k^\lambda\, k_\mu = 0. \qquad (12)$$

Using Eq. (8) in this expression and after some algebraic manipulations the equation of propagation of the disturbances is obtained:

$$\{\eta_{\mu\nu} + \Lambda_{\mu\nu}\}\, k^\mu k^\nu = 0 \qquad (13)$$

in which the quantity $\Lambda_{\mu\nu}$ is

$$\Lambda_{\mu\nu} \equiv -\frac{1}{\epsilon_s}\, F^{a\,\lambda}_{\mu}\, F_{a\,\lambda\nu}. \qquad (14)$$

The net effect of this modification of the non-linearity of the Yang-Mills theory can thus be summarized in the following property: *The disturbances of the gauge field controlled by the non-linear Lagrangian* $L_{\rm NDE}$ *propagate through null geodesics of the modified effective geometry given by:*

$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{\epsilon_s}\, F^{a\lambda}_{\mu}\, F_{a\,\lambda\nu}. \qquad (15)$$

Let us emphasize that this property stands only from the structural form of the dynamics of our theory. To avoid misunderstanding[6] we state:

- **This geometry modification is a pure spin-one non-linear phenomenon.**

Thus, we conclude from the above statement that gluon dynamics can be examined through the properties of null geodesics in the modified geometry. The fact that in this theory massless spin-one particles do not follow Minkowski null cone occurs as a direct consequence of the particular non-linear dynamics used in our model, which is distinct from the one contained in standard Yang-Mills theory.

In order to show the confinement induced by such non-linear model there is no better way than to investigate the behavior of the null geodesics in this geometry. For our purposes, we restrict ourselves here on the analysis of a spherically symmetric and static solution. A direct computation shows that in the spherical coordinate system such a particular solution can be found uniquely in terms of a radial component given by:

$$F^a_{01} \;\; = \;\; f(r)\, n^a, \qquad (16)$$

in which n^a is a constant vector in the colour space and $f(r)$ is given by the relation:

$$e^{-\frac{f^2}{2\epsilon_s}}\, f = \frac{Q}{r^2}. \qquad (17)$$

[6]The reason for this additional assertion is due to the fact that modifications on the underlying geometry are traditionally supposed to be connected to gravitational forces. We would like to stress that this **is not** the case here.

The parameter Q is related to distribution of the charge $q(r)$:

$$Q = \int \mathrm{d}^3 x q(r). \tag{18}$$

From the standard definition of the energy momentum tensor we obtain:

$$T_{\mu\nu} = -\epsilon_s \left(1 - e^{\frac{\mathbf{F}}{4\epsilon_s}}\right) \eta_{\mu\nu} + e^{\frac{\mathbf{F}}{4\epsilon_s}} \vec{F}_{\mu\alpha}.\vec{F}^{\alpha}{}_{\nu}, \tag{19}$$

and for the density of energy T_{00} results:

$$T^0{}_0 = \frac{Q}{r^2}\left[\frac{\epsilon_s + f^2}{f} - \epsilon_s\right]. \tag{20}$$

To analyse the confinement property it is enough to look for the radial equation of motion of the geodesics in this solution, as a function of the proper time. The simplest way to obtain the eikonal equation is by means of the variational principle

$$\delta \int \left[(1-f^2)\dot{t}^2 - (1-f^2)\dot{r}^2 - r^2\,\dot{\theta}^2 - r^2\,\sin^2\theta\,\dot{\varphi}^2\right] ds = 0. \tag{21}$$

in which we have used the effective geometry as obtained from Eqs.(13, 14) and in which $f(r)$ is given by Eq. (17) and a dot means proper time derivative.

We obtain for the radial dependence:

$$\dot{r}^2 + V_{eff} = l_0^2 \tag{22}$$

in which the potential V_{eff} has the form:

$$V_{eff} = \frac{\epsilon_s h_0^2}{r^2\left[\epsilon_s - f(r)^2\right]} - \frac{\epsilon_s^2 l_0^2}{\left[\epsilon_s - f(r)^2\right]^2} + l_0^2, \tag{23}$$

in which h_0 and l_0 are constants of motion.

A direct inspection on the form of this potential shows that the gluons in the L_{NDE} non-linear theory behave as particles endowed with energy l_0^2, immersed in a central field of forces characterized by the potential V_{eff}. Near the critical radius defined by $f(r_c)^2 = \epsilon_s$, the net attractive power on the gluon increases enormously. This is precisely the origin of the infinite barrier that forbids a gluon to get out from such region[7].

Let us summarize what we have achieved. Massless spin-one particles (gluons) obeying Yang-Mills dynamics travel along null cones. In a Minkowski spacetime there is no way to confine such particles in a compact region, once it could be associated with the presence of a singular horizon. We are then led to a modification of the self interaction properties of the gluons. We present here a model that can be equivalently described in terms of an effective change of the background geometry. We analyse a particular example of a static, spherically symmetric solution and proceed to the exam of the corresponding null geodesics, the gluon paths, in the associated geometry. It then follows that the behavior of gluons can be examined in terms of the potential given in Eq. (23) showing, through the appearance of a horizon, the required confining feature. This result allows us to argue that the final solution of the confinement of the gluons could be found along these lines.

[7]Note that the energy density is finite at the critical radius.

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