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DOES THERE EXIST A COSMOLOGICAL HORIZON PROBLEM?

by

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Abstract: We show that for photons non-minimally coupled to gravity the so called cosmological horizon problem does not exist.

Key-words: Cosmology; Horizon; Non-minimal coupling.

1. Introduction

There are two fundamental unsolved difficulties in the usual description of the universe given by relativistic cosmology[1]:

(i) The presence of an initial singularity

(ii) The horizon problem.

Taking for granted that the average global properties of space-time are described by a Friedmann-like metric,

$$ds^2 = dt^2 - R^2(t)\ d\sigma^2 \quad , \tag{1}$$

problem (i) means that the function R(t) vanishes in a past finite distance from us, implying the divergence of some invariants of the curvature (e.g. $R^{ABCD}R_{ABCD} = \infty$) and of the energy density.

Problem (ii) means that for any observer there exists a stringent limit upon its domain of observability at any time t_0, given by

$$d_{H_0} = R(t_0) \int_0^{t_0} \frac{dt}{R(t)} \quad . \tag{2}$$

The fact that d_{H_0} is not identified with the whole space means that an arbitrary observer cannot have a complete knowledge of the global properties of the universe, neither in principle.

The purpose of this letter is to point out that both of these obstacles are removed at once as a consequence of the non-minimal coupling (NMC) of photons with gravity.

Although in the last years elementary particle physicists have explored deeply the NMC of scalar fields $\phi(x)$ with gravity (motivated for instance, by the requirements of the principle of conformal invariance), the same did not occur with vector fields (which received very little attention up to some few papers on this subject (see 2,3,4,5,7)). The claimed reason for this is that the Minimal Coupling Principle (MCP) in the case of electromagnetic phenomena, seems to be supported by observation. However, observable net consequences of the NMC should appear just as corrections to the MCP results - and thus its importance is restrained to those regions with very high values of the curvature of space-time, thus preserving the well established weak field behaviour.

All of the effects we are interested in here are precisely due to very strong gravitational fields, which dominate the scene of the two main problems of standard cosmology, i.e. the singularity and the horizon.

The most general form of the NMC of electrodynamics and gravity is given by the Lagrangian

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{4} F^{\mu\nu}F_{\mu\nu} + a_1 R W_\mu W^\mu + a_2 R_{\mu\nu} W^\mu W^\nu + a_3 R F_{\mu\nu} F^{\mu\nu} + a_4 R_{\mu\nu} F^{\mu\alpha} F_\alpha{}^\nu + a_5 R_{\mu\nu\alpha\rho} F^{\mu\nu} F^{\alpha\rho} + a_6 R F_{\mu\nu} \overset{*}{F}{}^{\mu\nu} + a_7 R_{\mu\nu\alpha\rho} F^{\mu\nu} \overset{*}{F}{}^{\alpha\rho} \qquad (3)$$

in which the star operator (*) means taking the dual, $\overset{*}{F}_{\mu\nu} = \frac{1}{2}\sqrt{-g}\,\varepsilon_{\mu\nu\alpha\rho} F^{\alpha\rho}$. Except for a_0 and a_1 (which are dimensionless), constants a_k have dimension (length)^2. Any theory which has its roots on fundamental principles should set $a_k \sim L^2_{Planck}$ (= G in natural units) since the Planck length is the unique quantity with dimension length provided naturally by current concepts without any appeal to fortuitous values of specific properties of matter. From a more restrictive point of view, it has been argued[4] that NMC is an inequivocal consequence of the presence of virtual electron loops, whose net result is to give a non-null radius for the photon of the order of $(m_e)^{-1}$. This induces a modification of the effective action of the electromagnetic field due to the addition of non-minimal curvature-dependent terms to it. The weak point of this argument is that it yields a length for the interaction process which depends - through vacuum polarization - on the peculiarities of the electron. Hence, if one is searching for a more fundamental principle to drive the adoption of that coupling (which is the case here) then one should look for a more universal ingredient.

Let us consider, just to exhibit a simple example, for the time being, the theory in which all a's vanish except a_1. This case has been largely examined in the literature (Novello-Salim[2], Novello-Heintzmann[3], Novello-Romero[5], Davies-Toms[8], etc.).

Novello and Salim have found in 1979 an Eternal Universe without singularity generated by space-time dependent "massive" photons ($m \propto \sqrt{R}$), constrained by Lagrangian L_1. The geometry is given by a Friedmann model $ds^2 = dt^2 - A^2(t)d\sigma^2$, and the radius of the Universe takes the value $A(t)=(t^2+Q^2)^{1/2}$, in which Q is a constant dependent only on the value of the intensity of the field. It is worth while remarking that this Universe can be interpreted in terms of perturbations of Minkowski space-time, to which it reduces in the past and future infinities[5]. Let us just mention that the very possibility of the existence of such a solution is related to the violation of the condition $R_{\mu\nu}v^{\mu}v^{\nu} < 0$, for time-like curves with tangent vector given by v^{μ}.

We do not extend on this any longer (see ref. [7]) but just quote it here as a reminder of how NMC could avoid singularities.

Let us now pass on to the main point of this letter.

2. The False Horizon ?

The so-called cosmological horizon problem appears as a direct consequence of the propagation of electromagnetic disturbances in a singular expanding universe. Since the early days of general relativity the MCP of electrodynamics with gravity has been widely accepted, from which follows that photons propagate through null geodesics. Such property acquired a sound support by the evidence of the bending of light near a massive object (a star) and by the red-shift observations. Any theory of light must take both these results into account.

Let us now examine the new observational consequences of NMC. In this letter we shall concentrate our analysis on the dynamics corresponding to set $a_1 = a_2 = 0$ (which eliminates gauge non-invariance of the theory) and $a_6 = a_7 = 0$ (this assumes parity conservation) in (3).

If the Lagrangian (3) under the above conditions describes correctly the behaviour of electrodynamics in curved background space-time, does it follow that the photon propagates through null geodesics ? In the case of MC the standard procedure which deals with geometric optic approximation gives a definite yes as the answer to that question. We shall see now that this is not the case, in general, for NMC.

Instead of using the high frequency approximation of the geometric optic technique, we follow Lichnerowicz[6] and look for the propagation of arbitrary discontinuities of the electromagnetic field.

Let the symbol $[\psi]_S$ represent the discontinuity of any function ψ through a given hypersurface S. For the electromagnetic field $F_{\mu\nu}$ we study the situation in which we have:

$$[F_{\mu\nu}]_S = 0 \tag{4a}$$

$$[F_{\mu\nu;\lambda}]_S = [F_{\mu\nu,\lambda}]_S = \phi_{\mu\nu}k_\lambda \quad . \tag{4b}$$

The first equality in (4b) comes from the continuity of the geometry; the second one is nothing but Hadamard's condition, which guarantees that there exists a function $\phi_{\mu\nu}$ which makes (4b) be true. The characteristic surface S is represented by the equation $\phi(x) = 0$ and has its normal given by $k_\mu = \nabla_\mu\phi$.

From

$$\overset{*}{F}{}^{\mu\nu}{}_{;\nu} = 0 \tag{5}$$

and (4b) we obtain the cyclic condition

$$\phi_{\mu\nu}k_\lambda + \phi_{\nu\lambda}k_\mu + \phi_{\lambda\mu}k_\nu = 0 \quad . \tag{6}$$

The equations of motion derived from Lagrangian (3) in the case we are considering reduce to

$$F^{\mu\nu}{}_{;\nu} - \left[4a_3RF^{\mu\nu}+2a_4\{R^\mu{}_\alpha F^{\alpha\nu}-R^\nu{}_\alpha F^{\alpha\mu}\} + 4a_5R^{\mu\nu\alpha\rho}F_{\alpha\rho}\right]_{;\nu}= 0 \tag{7}$$

which yields the following equation for the discontinuity

$$\phi^{\mu\nu}k_\nu - \left[4a_3R\phi^{\mu\nu}+2a_4\{R^\mu{}_\alpha \phi^{\alpha\nu}-R^\nu{}_\alpha \phi^{\alpha\mu}\} + 4a_5R^{\mu\nu\alpha\rho}\phi_{\alpha\rho}\right]k_\nu= 0 \tag{8}$$

Let us now exemplify and examine this equation in the case of an homogeneous and isotropic Friedmann-type background geometry, with its source given by a perfect fluid with $p = \frac{1}{3}\rho$ and flat space section. These last two assumptions are reasonable, once we are interested in the corrections to photon propagation due to very high gravitational fields, which corresponds to a very remote epoch of the Universe, when it was radiation dominated and the solutions for flat, opened or closed space section had a common limit. As the Weyl tensor vanishes in this geometry, we can write

$$R^{\mu\nu\alpha\rho}\phi_{\alpha\rho}k_\nu = R^\mu{}_\alpha\phi^{\alpha\nu}k_\nu - R^\nu{}_\alpha\phi^{\alpha\mu}k_\nu - \frac{1}{3} R\phi^{\mu\nu}k_\nu \tag{9}$$

which reduces the part of eq. (8) due to a_5 to a combination of the two other ones. On the other hand a radiation gas implies $R = 0$, which eliminates the interaction corresponding to a_3. Therefore from now on we may set $a_3 = a_5 = 0$. Substituting

$$R_{\mu\nu} = -\frac{4}{3}\rho v_\mu v_\nu + \frac{1}{3}\rho g_{\mu\nu} \tag{10}$$

in equation (8) yields

$$\left\{1 - \frac{4}{3} a_5\rho\right\} \phi^{\mu\nu}k_\nu = -\frac{8}{3} a_5\rho\ k_0\ \phi^{\mu\nu}v_\nu \quad . \tag{11}$$

where $k_0 \equiv k_\nu v^\nu$. ($\phi^{\mu\nu}k_\nu v_\mu = 0$ as might be seen by substituting (10) in (8) and multiplying by v_μ). From equation (6) we obtain

$$\phi_{\mu\nu}k^2 + \phi_{\nu\lambda}k^\lambda k_\mu - \phi_{\mu\lambda}k^\lambda k_\nu = 0 \quad . \tag{12}$$

Substituting the value of $\phi_{\mu\lambda}k^\lambda$ in terms of $\phi_{\mu\lambda}v^\lambda$ given by eq. (11) into eq. (12) we obtain

$$\phi_{\mu\nu}k^2 - N(k_\mu\phi_{\nu\lambda} - k_\nu\phi_{\mu\lambda})\ v^\lambda\ k_0 = 0 \tag{13}$$

where we defined

$$N \equiv \frac{\frac{8}{3} a_4\rho}{1 - \frac{4}{3} a_4\rho} \quad .$$

Using once again eq. (6) we arrive at

$$k^2 + N\,k_0{}^2 = 0 \quad . \tag{14}$$

Remembering the solution $\rho(t) = \frac{3}{4}\,t^{-2}$ for a radiation gas, this yields for the velocity of light the expression.

$$v^2 = \frac{k_0{}^2}{|k_i k^i|} = \left|\frac{1 - \frac{\alpha G}{t^2}}{1 + \frac{\alpha G}{t^2}}\right| \quad . \tag{15}$$

where we set $a_4 = \alpha G$ and α is a pure number.

The first remarkable result is that the characteristic surfaces are not null any more. Non-minimally coupled photons do not move along null geodesic curves. Besides, in the domain of validity of the classical description of the present sceme ($t \gg t_{Planck}$) photons move closer and closer to the null cone. However, as we go towards the past there is a rapid increase of the velocity of light near t_{Planck}.[†] That is to say, at $t = t_{Planck}$ the light velocity increases without limit, inducing a "Big Flash" which sprays throughout the whole universe, generating in this way the necessary conditions for a termalisation phenomenon to occur, whose consequence is the observed spatial isotropy. Thus, in the above theory of non-minimally coupled photons the horizon problem does not exist.

[†]It is obviously too naif to pretend to apply our present knowledge to describe earlier times. Note that our conclusions depend on the sign of α . We assume $\alpha < 0$.

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