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The Static Confining Potential for Q.C.D. in the Mandelstam Model

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Abstract

We evaluate explicitly the quark-antiquark static potential on Quantum Chromodynamics Q.C.D. ($SU(3)$) by using the Dimensional Regularization scheme in the context of the Mandelstam approximation for the Gluonic interaction. We obtain its charge confining behavior in opposition to the expected result of a screnning charge dynamics.

Key-words: Static potential; Mandelstam propagator; Path integral.

1 Introduction

One of the still unsolved problem in the Gauge theory for strong interactions as given by Quantum Chromodynamics with gauge group $SU(3)$ is to produce arguments for the ethernal color charge confinement of the related field excitations ([1]).

A long time ago ([2]), it was argued by S. Mandelstam through a somewhat intrincate non-perturbative analysis of the Q.C.D. Schwinger-Dhyson equations the one should use as a first approximation for the small momenta infrared regime non-abelian quantum Yang-Mills path measure, including its non-perturbative aspects, an effective (somewhat phenomenological) purely abelian Gluonic action but with a free effective propagator already including the sum of a certain class of relevant Feynman diagrams for Gluons color-charge exchange. It was conjectured that the use of this scheme would be suitable if such an effective dynamics led directly to the color confinement.

It is the purpose of this paper to evaluate the static potential between two statics charges with opposite signal on the above mentioned Mandelstam effective Gluon theory and show exactly its envisaged color-charge confining property; a basic physical requirement to use directly continuum Q.C.D. with improved Mandelstam-Feynman diagramatics, at least on the level of Dyhson-Schwinger equations as earlier proposed on ref [2] by S. Mandelstam.

This study is presented on section 2.

In the section 3, we solve exactly by means of the chiral path integral bosonization a two-dimensional version of the Mandelstam model (a Schwinger model with higher derivative coupling). Finally on section 4 we study the charge screening phenomena in presence of conducting plates on the Mandelstam model through a Wilson Loop evaluation as done in section 2.

2 The Wilson Loop in the Mandelstam Model

We start our analysis by considering the (Euclidean) Effective Mandelstam Gluonic action written in terms of a path-integral in a ν-dimensional space-time R^ν

$$Z = \int D^F[A_\mu(x)] \exp\left\{-\frac{1}{2}\int d^\nu x\, d^\nu y\; A_\mu(x)\, D_m(x-y) A^\mu(y)\right\} \tag{1}$$

where the Mandelstam (free) propagator with logarithmic term is given explicitly by the Fourier transform on the (Tempered) Schwartz distributional space

$$D_m(x-y) = \frac{1}{(2\pi)^\nu}\int d^\nu p\; e^{ip(x-y)} \frac{\ell\, g(|p|^{2\alpha})}{|p|^4} \tag{2}$$

with α a positive model resummation constant (including factor index groups, etc...). (see ref [2])

The static potential between a quark and an anti-quark in the Feynman picture for particle propagations in the space-time is given by the vacuum Gluonic energy as given by eq.(1), but in presence of the above spatial-static charges. This vacuum energy of such charges separated by a space-like distance R is computed by evaluating the temporal (ergodic) limit ([1],[3]).

$$V(R) = \lim_{T\to\infty} -\frac{1}{T}\ell g\left[\left\langle \exp\, i\, e \oint_{C_{(R,T)}} A_\mu dx_\mu \right\rangle_A\right] \tag{3}$$

where the rectangle $C_{(R,T)}$ is the Feynman trajectory of the neutral pair in the space-time and the Mandelstam Gluonic normalized average as represented by the operation $\langle\ \rangle_A$ is given explicitly by the Gaussian path integral eq. (1).

In order to evaluate the static potential given by eq.(3) it is convenient to re-write the Wilson loop inside eq.(3) by means of an external current $J_\mu(x; C_{(R,T)})$ circulating around the pair finite-time propagation space-time trajectory $C_{(C,R)} = \{x_\mu(s)\}$, namely ([3])

$$J_\mu(x; C_{(R,T)}) = i\, e \oint_{C_{(R,T)}} \delta^{(\nu)}(x_\mu - x_\mu(s)) dx_\mu(s) \tag{4}$$

The Gaussian path integral eq.(3) can be exactly evaluated and yielding the following result

$$V(R) = \lim_{T\to\infty} -\frac{1}{T}\ell g\left[\exp\left\{+\frac{1}{2}\int d^\nu x d^\nu y J_\mu(x; C_{(R,T)})\, D_m(x-y) J^\mu(y, C_{(R,T)})\right\}\right] \tag{5}$$

The evaluation of eq.(5) can be accomplished by writing it in momentum space

$$V(R) = \lim_{T\to\infty} -\frac{1}{T}\left[\int \frac{d^\nu p}{(2\pi)^\nu} f_\mu(p_\alpha, C_{(R.T)}) \times \frac{\alpha \ell g(p^2)}{p^4} \times f^\mu(-p_\alpha, C_{(R,T)})\right] \tag{6}$$

with the contour form factors

$$f_\mu(p_\alpha, C_{(R,T)}) = ie \int_{C_{(R,T)}} e^{-i\, p_\mu x_\mu(s)} dx_\mu(s) \tag{7}$$

A simple evaluation of eq.(7) provides the solutions

$$f_0(p, C_{(R,T)}) = -\frac{4e}{p_0} \sin\left(\frac{p_0 T}{2}\right) \sin\left(\frac{p_1 R}{2}\right) \tag{8}$$

and

$$f_1(p, C_{(R,T)}) = +\frac{4e}{p_1} \sin\left(\frac{p_0 T}{2}\right) \sin\left(\frac{p_1 R}{2}\right) \tag{9}$$

After inserting the contour form factors eq.(8), eq.(9) into eq.(6), we obtain as a result

$$\begin{aligned} V(R) = \lim_{T\to\infty} \frac{1}{T} \Bigg\{ & 16e^2\alpha \int_{-\infty}^{+\infty} \frac{dp_1}{(2\pi)} \frac{\sin^2\left(\frac{p_1 R}{2}\right)}{p_1^2} \\ & \times \left[\int_{-\infty}^{+\infty} \frac{d^{\nu-2}\hat{p}}{(2\pi)^{\nu-2}} \left(\int_{-\infty}^{+\infty} \frac{dp_0}{(2\pi)} \frac{(p_0^2+p_1^2)}{p_0^2} \sin^2\left(\frac{p_0 T}{2}\right) \times \frac{\ell g(p_0^2+p_1^2+\hat{p}^2)}{(p_0^2+p_1^2+\hat{p}^2)^2}\right)\right]\Bigg\}. \end{aligned} \tag{10}$$

Note that we have considered the pair spatial-static trajectory $C_{(R,T)}$ contained is a two-dimensional sub-space of the (Euclidean) space-time R^ν in a such way that we can decompose the vector $\vec{p} \in R^\nu$ as $\vec{p} = p_0\vec{e}_0 + p_1\vec{e}_1 + \hat{p}$, where $\hat{p}$ denotes the projection of $\hat{p}$ over the sub-space perpendicular to the sub-space $\{\vec{e}_0, \vec{e}_1\}$ containing the square $C_{(R,T)} = \{(x_0, x_1); -\frac{T}{2} \le x_0 \le +\frac{T}{2}; -\frac{R}{2} \le x_1 \le +\frac{R}{2}\}$.

The ergodic limit of $T \to \infty$ and the p_0-integration is easily evaluated through the use of the Distributional limit

$$\lim_{T\to\infty} \frac{\sin^2\left(\frac{p_0 T}{2}\right)}{p_0^2 T} = 2\pi\ \delta(p_0) \tag{11}$$

As a consequence we get the result

$$V(R) = 16e^2\alpha \left[\int_{-\infty}^{+\infty} \frac{dp_1}{(2\pi)} \cdot \frac{\sin^2\left(\frac{p_1 R}{2}\right)}{p_1^2} \times \int \frac{d^{\nu-2}\hat{p}}{(2\pi)^{\nu-2}} \cdot \frac{\ell g(p_1^2+\hat{p}^2)}{(p_1^2+\hat{p}^2)^2}\right] \tag{12}$$

Let us analyze the $(D-2) - \hat{P}$ dimensional integration. In order to evaluate such integral, we use the well-known formulae (from I.S. Gradshteyn & I.M. Ryzhik table of integrals – page 558 – eq.(14) – Academic Press – 1980.

$$\begin{aligned}
&\int \frac{d^{\nu-2}\hat{p}}{(2\pi)^{\nu-2}} \cdot \frac{\ell g(p_1^2+\hat{p}^2)}{(p_1^2+\hat{p}^2)^2} \\
&= \Pi^{\frac{\nu-2}{2}} \left\{ \frac{\Gamma\left(\frac{6-\nu}{2}\right)}{\Gamma(2)} (|p_1|)^{\nu-6} \right\} \\
&\times (\psi(2) - \psi(3-\frac{\nu}{2}) + 2\,\ell n(|p_1|)
\end{aligned} \tag{13}$$

For the evaluation of the final p_1-integration we use the well-known Gelfand results of the Fourier Transform of Tempered (Finite-part) Distributions ([4]).

$$\sin^2\left(\frac{k_1 R}{2}\right) = -\frac{1}{4}(e^{k_1 R} + e^{-k_1 R} - 2) \tag{14}$$

and

$$\int_{-\infty}^{+\infty} e^{ip_1 R} |p_1|^\beta \; dp_1 = -2\sin\left(\frac{\beta\pi}{2}\right)\Gamma(\beta+1)|p|^{-\beta-1} \tag{15}$$

with

$$\begin{aligned}
&\int_{-\infty}^{+\infty} e^{ip_1 R}|p_1|^\beta \ell n(|p_1|) dp_1 \\
&= i\, e^{i\beta\frac{\pi}{2}} \left\{ \left[\Gamma'(\beta+1) + \frac{i\pi}{2}\Gamma(\beta+1)\right] (|R|+i\varepsilon)^{-\beta-1} - \Gamma(\beta+1)(|R|+i\varepsilon)^{-\beta-1}\cdot \ell n(|R|+i\varepsilon)\right\} \\
&- i\, e^{-i\beta\frac{\pi}{2}} \left\{ \left[\Gamma'(\beta+1) - \frac{i\pi}{2}\Gamma(\beta+1)\right] (|R|-i\varepsilon)^{-\beta-1} - \Gamma(\beta+1)(|R|-i\varepsilon)^{-\beta-1}\cdot \ell n(|R|-i\varepsilon)\right\}
\end{aligned} \tag{16}$$

By passing to the Physical limit of $\nu \to 4$ and noting that the pole of the Gamma function canceals out either with the sinus zero for $\nu \to 4$, namely.

$$\begin{aligned}
&\lim_{\nu\to 4} \sin\left(\frac{\pi}{2}(\nu-6)\right)\Gamma(\nu-4-1) \\
&\sim -\frac{1}{(\nu-5)}\Gamma(\nu-4)\cdot \sin\left(\frac{\pi}{2}(\nu-4)\right) \\
&= +\pi
\end{aligned} \tag{17}$$

We obtain, thus, the finite result for the static quark-antiquark potential in the Mandelstam Gluonic effective theory on the physical space-time R^4.

$$V(R) = (e^2\alpha)\cdot \overline{c} \cdot |R|(1+\ell n(|R|)) \tag{18}$$

Here $\overline{c}$ denotes a positive constant which depends on the Fourier Transform normalization factors, etc...

We see, thus, that the Effective Gluonic Mandelstam theory leads in a very natural way to a quark-antiquark confining potential and not to a dynamics of change color screening as it would be expected in a first analysis ([1]). This is the main result of this section.

At this point, it is worth remark that if one has added to the logarithmic propagator eq.(2) a pure quartic term of the following form

$$\tilde{D}_m(x-y) = \frac{1}{(2\pi)^\nu}\int d^\nu p\; e^{ip(x-y)}\cdot\frac{1}{|p|^4} \tag{19}$$

one obtains the same result as given by eq.(18) without the logarithmic term.

Another important point to be called the reader´s atention is that if one tries to evaluate the self-energy of the guark propagator with the effective Mandelstam propagator eq.(2), namely

$$\begin{aligned}\Sigma(p) &\sim e^2\int\frac{d^\nu k}{(2\pi)^\nu}\left(\frac{\gamma_\mu(\not p-\not k)\gamma_\mu}{(p-k)^2}\right)\frac{\ell g(k^2)}{k^4}\\ &= 3\int_0^1 dx(1-x)\left\{\int\frac{d^\nu k}{(2\pi)^\nu}\left[\frac{((1-x)\;\not p-\not k)\ell g((k+xp)^2)}{\{k^2+x(1-x)p^2\}^3}\right]\right\}\end{aligned} \tag{20}$$

with the power series expansion for the logarithmic term in eq.(20) as given below

$$\ell g(k^2+x^2p^2+2(x\cdot p)x) = \ell g(k^2)+\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\left[\frac{(2x(k\cdot p)+x^2p^2)}{(k)^2}\right]^n \tag{21}$$

one should arrives at the standard Mandelstam behavior after tedious calculations.

$$\Sigma(p)\sim\not p\left[\frac{A+B\ell g(p^2)}{p^4}\right] \tag{22}$$

with A and B constant p-independent, (including possible divergences at $\nu\to 4$!).

As a consequence one see that the quark-antiquark propagator should have a behavior of the form (in the Euclidean world)

$$\begin{aligned}G_{ij}^{\mu\nu}(x-y) &= \left\langle 0\middle| T(\psi_i^\mu(x)\,\overline{\psi}_j^\nu(y))\middle|0\right\rangle_{\text{Eucl.}}\\ &\sim\int d^4p\;\frac{(p^2)\;\not p e^{ip(x-y)}}{p^4+B\ell g(p^2)+A}\end{aligned} \tag{23}$$

signaling again that at $p^2\to 0^+$ (the L.S.Z´s asymptotic limit) we find branch-cuts instead of mass-physical poles. This indicates again that it is a completely ill-defined process to apply L.S.Z´s framework to Quarks and Gluons since the quark field excitations are not physically-quantum mechanical observable. This leads one to consider only composite operators from the very beginning, as Mandelstam did in ref. [2], in order to apply correctly the L.S.Z' Quantum Field Methods, even at the small momenta region.

3 The Two-dimensional Mandelstam-Schwinger model: its Chiral path-integral bosonization

It is well-known that two-dimensional models has proved to be a useful theoretical laboratory to understand difficult dynamical features expected to be present in four-dimensional quantum chromodynamics. It is the purpose of this section to complement the analysis of confining of four-dimensional dynamical fermions in the infrared leading approximate Mandelstam model of Section 2 by means of a higher-derivative exactly soluble two-dimensional model.

Let us start this section by writing the (Euclidean) Hermitian Lagrangean of our proposed higher-derivative two-dimensional model

$$\mathcal{L}_\mu(\psi,\overline{\psi},A_\mu) = (\psi,\overline{\psi}) \begin{Bmatrix} 0 & (\not{D}_A\,\not{D}_A^*)^\mu\,\not{D}_A \\ \not{D}_A^*(\not{D}_A^*\,\not{D}_A)^\mu & 0 \end{Bmatrix} \begin{pmatrix} \psi \\ \overline{\psi} \end{pmatrix} + \frac{1}{2}\,F_{\mu\nu}^2(A) + (\psi,\overline{\psi}) \begin{pmatrix} \eta \\ \overline{\eta} \end{pmatrix} \tag{24}$$

where $(\psi,\overline{\psi})$ denotes the (independent Euclidean fermion fields two-dimensional) quarks; A_μ the usual (confining) two-dimensional eletromagnetic field with a quartic propagator on the Landau Gauge (see below) and $\not{D}_A$ is the (Euclidean) Dirac operator in the presence of this 2D quantum Gauge field. The Dirac γ matrices algebra we are using satisfy the relations

$$\{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu}\ ,\ \ \gamma_\mu\gamma_5 = i\varepsilon_{\mu\nu}\gamma_\nu\ ;\ \ \gamma_5 = i\gamma_0\gamma_1 \tag{25}$$

Note that this γ-matrices algebra is choosen in a such way that the Dirac operator $\not{D}_A$ may by written in the chiral-phase form when one consider the general Hodge decomposition of the two-dimensional eletromagnetic field.

$$A_\mu = \varepsilon_{\mu\nu}\ \partial_\nu\ \varphi + \partial_\mu\ \rho \tag{26}$$

$$\not{D}_A = e^{ig\rho}\ e^{ig\gamma_s\varphi}\cdot(\not{\partial})e^{-ig\rho}\ e^{ig\gamma_s\varphi} \tag{27}$$

Here μ is a free-parameter ranging on the interval $[1,\infty)$.

Let us consider the associated path-integral expression for the 2D-quantum higher derivative model eq.(30) in the fermion sector.

$$Z[\eta,\overline{\eta}] = \frac{1}{Z(0,0)}\int D^F\psi D^F\overline{\psi}\ DA_\mu \times \exp\left(-\int d^2x\mathcal{L}_\mu(\psi,\overline{\psi},A_\mu)(x)\right) \tag{28}$$

In order to solve exactly the two-dimensional path-integral eq.(28) by means of the Gauge invariant Bosonization technique, we consider the change of variable on the field dynamics

$$A_\mu(x) = (\varepsilon_{\mu\nu}\,\partial_\nu)\varphi(x) \tag{29}$$

$$\psi(x) = e^{-ig\gamma_5\varphi(x)}(-\Delta)_x^{-\mu}\,\chi(x) \tag{30}$$

$$\overline{\psi}(x) = \overline{\chi}(x)\,e^{-ig\gamma_5\varphi(x)} \tag{31}$$

It is worth call the reader attention that in the Euclidean world $\overline{\psi}(x)$ is an independent field of $\psi(x)$, opposite in Minkowisky space where $\overline{\psi}(x) = (\psi^*(x))^T\gamma^0$. That is the reason about the difference between eq.(30) and eq.(31).

At the quantum level of the path measures we have the non-trivial jacobians (see ref [5]) physically related to the dynamical breaking of the models axial (chiral) symmetry, namely

$$D^F[A_\mu(x)] = \det(-\Delta)\cdot D^F[\varphi(x)] \tag{32}$$

$$\begin{aligned}
D^F[\psi(x)]\,D^F[\overline{\psi}(x)] &= \frac{\det[(\not{D}_A\cdot\not{D}_A^*)^\mu\,\not{D}_A]}{\det[\not{\partial}]}D^F[\chi(x)]D^F[\overline{\chi}(x)] \\
&= \frac{\det[(\not{D}_A\,\not{D}_A^*)^\mu(\not{D}_A\,\not{D}_A^*)^{1/2}]}{\det(\not{\partial})}D^F[\chi(x)]D^F[\overline{\chi}(x)] \\
&= \left\{\frac{\det[(\not{D}_A\,\not{D}_A^*)^{\mu+\frac{1}{2}}]}{\det[\not{\partial}\,\not{\partial}^*]^{\mu+\frac{1}{2}}}\times\det(\not{\partial}\,\not{\partial}^*)^\mu\right\}D^F[\chi(x)]D^F[\overline{\chi}(x)] \\
&= \left\{\left(\det\left[\frac{\not{D}_A\,\not{D}_A^*}{\not{\partial}\,\not{\partial}^*}\right]\right)^{\mu+\frac{1}{2}}\times(\det(-\Delta))^\mu\right\}D^F[\chi(x)]D^F[\overline{\chi}(x)]
\end{aligned} \tag{33}$$

After implementing equations (29) - (33) on the fermionic generating functional eq.(28), we obtain the Bosonized associated model, where one can evaluate exactly all the models correlation field functions.

$$\begin{aligned}
Z[\eta,\overline{\eta}] &= \frac{1}{Z(0,0)}\int D[\varphi(x)]D^F[\chi(x)]D^F[\overline{\chi}(x)] \\
&\times\exp\left\{-\frac{g^2}{\pi}(\mu+\frac{1}{2})\int d^2x\left(\frac{1}{2}(\partial\varphi)^2\right)(x)\right\} \\
&\times\exp\left\{-\frac{1}{2}\int d^2x((\partial^2\varphi)^2)(x)\right\} \\
&\times\exp\left\{-\frac{1}{2}\int d^2x\left((\chi,\overline{\chi})\begin{bmatrix}0 & \vec{\not{\partial}} \\ \overleftarrow{\not{\partial}}^* & 0\end{bmatrix}\begin{pmatrix}\chi\\ \overline{\chi}\end{pmatrix}\right)(x)\right\} \\
&\times\exp\left\{\int d^2x\left[\left(\overline{\eta}\,e^{-ig\gamma_5\varphi}(-\Delta)^{-\mu}\chi\right)(x)+(\overline{\chi}e^{-ig\gamma_5\varphi}\eta)(x)\right]\right\}
\end{aligned} \tag{34}$$

It is important to remark that we have used the basic identity below to arrive at eq.(34) with α a real positive parameter and used throughout on the formulae

$$
\begin{aligned}
(\not{D}_A\, \not{D}_A^*)^\alpha\, \not{D}_A &= e^{ig\gamma_5\varphi}[(\not\partial\, \not\partial^*)^\alpha\, \not\partial]e^{ig\gamma_5\varphi} \\
&= e^{ig\gamma_5\varphi}((-\Delta)^\alpha \not\phi)e^{ig\gamma_5\varphi} \qquad (35)
\end{aligned}
$$

It is important point out that the part of the Lagrangean with Fermions sources in the new field parametrization are not symmetric in its form as that of eq.(24) in the old field parametrization as a consequence of our asymmetric change of variable in the (independent in the Euclidean world!) two-dimensional quarks fields.

Finally we have the explicitly expression for our Fermion propagator in terms of the free-propagators of the Bosonized theory

$$
\begin{aligned}
&\left\langle \psi(x)\,\overline{\psi}(y)\right\rangle = (-\Delta)_x^\mu \left\{ \left\langle \chi(x)\overline{\chi}(y)\right\rangle^{(0)} \times \right. \\
&\left. \exp\left\{ -\frac{1}{2}g^2 \left[\frac{\pi}{g^2(\mu+\frac{1}{2})} \left((-\partial^2)^{-1}(x.y) - (-\partial^2 + \frac{g^2}{\pi}(\mu+\frac{1}{2}))^{-1}(x,y) \right) \right] \right\} \right. \qquad (36)
\end{aligned}
$$

Here

$$
\left\langle \chi_\alpha(x)\,\overline{\chi}_\beta(y)\right\rangle^{(0)} = \frac{1}{2\pi}(\gamma_\mu)_{\alpha\beta}\frac{(x_\mu - y_\mu)}{|x-y|^2} \qquad (37)
$$

and

$$
(-\partial^2)^{-1}(x,y) = -\frac{1}{2\pi}\ell g|x-y| \qquad (38)
$$

$$
(-\partial^2 + \frac{g^2}{\pi}(\mu+\frac{1}{2}))^{-1}(x.y) = \frac{1}{2\pi}K_0\left(\sqrt{\frac{g^2}{\pi}(\mu+\frac{1}{2})}|x-y| \right) \qquad (39)
$$

Note that we have used the general decomposition in eq.(41)

$$
(a(-\partial^2)^2 + b(-\partial^2))^{-1}(x,y) = \frac{1}{b}\left\{ -\frac{1}{2\pi}\ell g|x-y| - \frac{1}{2\pi}K_0\left(\sqrt{\frac{b}{a}}|x-y| \right) \right\} \qquad (40)
$$

The short-distance behavior of the fermion propagator is strong than the usual free case by a μ-power derivative (strong asymptotic freedom).

$$
\lim_{|x-y|\to 0} \left\langle \psi(x)\overline{\psi}(y)\right\rangle \sim (-\Delta)_x^\mu \left\langle \chi_\alpha(x)\overline{\chi}_\beta(y)\right\rangle \qquad (41)
$$

The long-distance behavior by its turn is exactly given by

$$
\begin{aligned}
&\lim_{|x-y|\to\infty} \langle \psi(x)\overline{\psi}(y)\rangle \sim \\
&\lim_{|x-y|\to\infty} \left\{ (-\Delta)_x^\mu \left[\left\langle \chi(x)\,\overline{\chi}(y)\right\rangle^{(0)} \times |x-y|^{|4\mu+2|} \right] \right\} \qquad (42)
\end{aligned}
$$

which shows an anomalous behavior in the infra-red limit and signaling the impossibility to use L.S.Z interpolating fields for the 2D fermion fields as similar phenomenon in the Mandelstam model of section 2.

Anyway it is a straightforward procedure the exactly computation of all fermionic correlation function of the higher derivative model eq.(34) as in last references of ref. [5].

4 Color Charge Screening on the Mandelstam Model

Sometimes it is argued that it is important to realize that the absence of coulored states in the expected nuclear strong force theory of Quantum Chromodynamics may not be equivalent to the ethernal quark-qluon confinement as showed by us in the Effective Abelian Gluon Mandelstam model analyzed in section 1 by an explicitly Wilson Loop evaluation.

The absence of color charged states can still be a result of these color quantum numbers just screened by the quark-antiquark pairs creation on the presence of the Gluon field and leading, thus, to the physical picture that the test charges (a static pair!) are surrounded by a cloud of quark-antiquark pairs playing the role of plasmons. It is, thus, expected that the resulting Wilson loop colorless object of section 1 no longer leads to a rising linear confining potential as showed on that section, but rather to an exponentially falling potential characterizing the short range screened strong interactions like similiar screening phenomena in two-dimensional Q.E.D. (see section 2 for the case of $\mu = 0$).

In this section we intend to show such screening phenomena by an explicitly calculation in the above mentioned four-dimensional Effective Gluon Mandelstam model by considering the existence of totally reflecting walls on the point $z = 0$ and $z = a$ of the space-time which turns out to be of the cylindrical form $R^{\nu-1} \times [0, a]$. We further impose Dirichlet boundary conditions on the "effective" abelian Gluonic Mandelstam field at the walls $z = 0$ and $z = a$. Its propagator, thus, posseses the following analytical expression on momentum space by taking into account explicitly the above pointed out Boundary condition

$$G((\vec{r},z,t);(\vec{r}',z',t')) = \sum_{m=0}^{\infty}\left\{\int_{-\infty}^{+\infty}\frac{d^{\nu-2}\vec{p}}{(2\pi)^{\nu-2}}\cdot\frac{dp_0}{(2\pi)}e^{-i\vec{p}(\vec{r}-\vec{r}')}e^{+ip_0(t-t')}\right.$$
$$\left.\times\sin\left(\frac{m\pi}{a}z\right)\sin\left(\frac{m\pi}{a}z'\right)\times[p_0^2+\vec{p}^2+\left(\frac{m\pi}{a}\right)^2]^2\right\} \tag{43}$$

The static-potential of such a screened pair separated by a space-like distance R on the sub-space perpendicular to the plane z (and with a coordinate $z=\overline{z}$) is given by the temporal (ergodic) limit result (see Wilson Loop´s discussions on section 1) namely

$$V(R) = e^2\sum_{m=1}^{\infty}\left[\left(1-\cos\left(\frac{2\pi m}{a}\overline{z}\right)\right)V_m(R)\right] \tag{44}$$

with $\overline{p}=(\hat{p},p_1)\in R^{\nu-2}$

$$V_m(R) = \left\{\int_{-\infty}^{+\infty}\frac{d^{\nu-3}\hat{p}}{(2\pi)^{\nu-3}}\frac{dp_1}{(2\pi)}\frac{\sin^2\left(\frac{p_1R}{2}\right)}{p_1^2}\right.$$
$$\left.\times\lim_{T\to\infty}\left\{\int_{-\infty}^{+\infty}\frac{dk_0}{(2\pi)}\frac{\sin^2\left(\frac{p_0T}{2}\right)}{T}\left(1+\frac{p_1^2}{p_0^2}\right)\times\frac{1}{(\hat{p}^2+p_1^2+p_0^2+\left(\frac{m\pi}{a}\right)^2)^2}\right\}\right. \tag{45}$$

The evaluation of the ergodic limit on eq.(45) is similar to those analyzed in section 1 and leading to the result

$$V_m(R) = \int_{-\infty}^{+\infty}\frac{dp_1}{(2\pi)}\cdot\sin^2\left(\frac{p_1R}{2}\right)\left[\int\frac{d^{\nu-3}\hat{p}}{(2\pi)^{\nu-3}}\frac{1}{(\hat{p}^2+p_1^2+\left(\frac{m\pi}{a}\right)^2)^2}\right]$$
$$= \overline{c}(\nu)\int_{-\infty}^{+\infty}\frac{dp_1}{(2\pi)}\sin^2\left(\frac{p_1R}{2}\right)\left(p_1^2+\left(\frac{m\pi}{a}\right)^2\right)^{\frac{\nu-7}{2}} \tag{46}$$

with $\overline{c}(\nu)$ a positive constant, finite for $\nu\to 4$ and depending on the Fourier integral definition normalization factors geometrical sizes of the loop $C_{(R,T)}$, etc... which exact value will not be of our interest here, since it is convergent for $\nu\to 4$ as a function of the space-time dimensionality ν. The evaluation of the integral on eq.(46) can be easily accomplished through the useful formula

$$\int_{-\infty}^{+\infty}dx\frac{\sin^2(ax)}{(x^2+b^2)^{\mu}} = \int_0^{\infty}dx\frac{1}{(x^2+b^2)^{\mu}} - \int_0^{\infty}dx\frac{\cos(2ax)}{(x^2+b^2)^{\mu}}$$
$$= \left(\frac{b^{-2\mu+1}}{2}\right)\frac{\Gamma(\frac{1}{2})\Gamma(\mu-\frac{1}{2})}{\Gamma(\mu)} - \frac{1}{\sqrt{\pi}}\left(\frac{b}{a}\right)^{\mu+\frac{1}{2}}\cos\left(\pi(\mu+\frac{1}{2})\right)\Gamma(\mu+1)K_{-(\mu+\frac{1}{2})}(2ab) \tag{47}$$

and leading to the envisaged result for the harmonic m-potential contributing to the Fourier expansion eq.(46)

$$V_m(R) = \bar{c}(\nu)\left\{\left[\left(\frac{m\pi}{2a}\right)^{\nu-6}\frac{\Gamma(\frac{1}{2})\Gamma(\frac{6-\nu}{2})}{\Gamma(\frac{D-\nu}{2})}\right]\right.$$
$$\left.-\left[\frac{1}{\sqrt{\pi}}\left(\frac{2m\pi}{aR}\right)^{\frac{8-\nu}{2}}\cos\left(\pi\left(\frac{8-\nu}{2}\right)\right)\Gamma\left(\frac{9-\nu}{2}\right)\times K_{(\frac{\nu-8}{2})}(\frac{m\pi}{a}R)\right]\right\} \tag{48}$$

Now its straightforward to see directly from eq.(48) the Casimir vacuum-energy content of the Abelian Gluonic Mandelstam Field as given by the convergent Fourier series below

$$E_{\text{ Casimir }}(\bar{z}) = e^2\bar{c}(\nu)\sum_{m=1}^{\infty}\left[\left(1-\cos\left(\frac{2m\pi}{a}\bar{z}\right)\right)\right]\times\left[\left(\frac{m\pi}{2a}\right)^{\nu-6}\frac{\Gamma(\frac{1}{2})\Gamma(\frac{6-\nu}{2})}{\Gamma(\frac{7-\nu}{2})}\right] \tag{49}$$

The expected exponential falling at large distance R of the static potential, signaling screening of color charges for our Mandelstam Gluonic Abelian field with pure quartic propagator, is given by the second term on eq.(48)

$$V(R) \underset{R\to\infty}{\sim} (-e^2)\sum_{m=1}^{\infty}\left[\left(1-\cos\left(\frac{2\pi m}{a}\bar{z}\right)\right)\right]$$
$$\times\left[\frac{1}{\sqrt{\pi}}\left(\frac{2m\pi}{a}\right)^{\frac{8-\nu}{2}}\cos\left(\pi\left(\frac{8-\nu}{2}\right)\right)\Gamma\left(\frac{9-\nu}{2}\right)\ e^{-\frac{m\pi}{a}R}\right] \tag{50}$$

$$\sim e^{-\frac{\pi}{a}R}(-e^2)\left\{\sum_{m=1}^{\infty}\left[\left(1-\cos\left(\frac{2\pi m}{a}\bar{z}\right)\right)\right]\right.$$
$$\left.\times\left[\frac{1}{\sqrt{\pi}}\left(\frac{2m\pi}{a}\right)^{\frac{8-\nu}{2}}\cos\left(\pi\left(\frac{8-\nu}{2}\right)\right)\Gamma\left(\frac{9-\nu}{2}\right)e^{-\frac{(m-1)\pi}{a}R}\right.\right\}$$
$$\sim (-e^2)(e^{-\frac{\pi}{a}R})\overline{W}(R) \tag{51}$$

where the harmonic sum on the integers m is convergent due to the Bessel function argument (see eq.(48)).

Finally, we call the reader attention that similiar result is obtained for a propagator with a logarithmic term as that one considered on section 1.

Detailed calculations taking into account quantum corrections, finite temperature effects, etc... will appear in other paper.

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