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Renormalization Of Nonabelian Gauge Theories With Tensor Matter Fields.

by

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Abstract

The renormalizability of a nonabelian model describing the coupling between antisymmetric second rank tensor matter fields and Yang-Mills gauge fields is discussed within the BRS algebraic framework.

Key-words: Renormalizability; BRS symmetry; Anomalies.

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1 Introduction

Antisymmetric tensor gauge fields have been introduced since many years. They are known to possess rather peculiar properties as, for instance, a highly nontrivial quantization and the possibility of a topological interpretation which allows to compute topological invariants which generalize the so called linking number [1].

In 1994 L. V. Avdeev and M. V. Chizhov [2] achieved the construction of a renormalizable [3] four dimensional abelian gauge model in which the antisymmetric second rank tensor fields are introduced as matter fields rather than gauge fields [1]. The model, formulated in Minkowski flat space-time, exihibits several interesting features allowing for many applications, both from phenomenological [5] and theoretical [6] point of view. This motivated ourselves to pursuing further investigations in order to generalize the model to the nonabelian case and to obtain a better understanding of its geometrical properties. In particular, we have been able to show [7] that the Avdeev-Chizhov Lagrangian can be recovered in a very simple way from a φ^4-like theory in which φ is a complex tensor fields satisfying the Minkowski self-dual condition

$$\varphi_{\mu\nu} = i\tilde{\varphi}_{\mu\nu} \ , \qquad \tilde{\varphi}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\varphi^{\rho\sigma} \ , \tag{1.1}$$

$\varepsilon_{\mu\nu\rho\sigma}$ being the totally antisymmetric Levi-Civita tensor normalized as $\varepsilon_{1234} = 1$ and

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\rho\sigma\lambda\omega} = -2(\delta^\mu_\lambda\delta^\nu_\omega - \delta^\nu_\lambda\delta^\mu_\omega) \ . \tag{1.2}$$

As proven in ref. [7], the complex self-dual condition (1.1) fixes uniquely the Lorentz contractions of the tensor φ^4-Lagrangian, reproducing thus the action of Avdeev-Chizhov. In addition, the formulation of the model as a kind of φ^4-theory gave us a straightforward way of obtaining its classical nonabelian generalization.

The aim of this work is to study the quantum properties of the classical nonabelian model previously proposed. In particular we shall be able to prove that, as it happens in the abelian case, the nonabelian generalization of the tensor matter field Lagrangian turns out to be renormalizable. The proof will be done in a regularization independent way by means of the algebraic BRS renormalization technique [8], the use of which in the present case being motivated by the explicit presence of the Levi-Civita tensor $\varepsilon_{\mu\nu\rho\sigma}$ in the Lagrangian interaction vertices as well as in the BRS transformations of the fields.

The paper is organized as follows. In Sect. 2 we briefly recall the construction of the nonabelian tensor matter Lagrangian and its BRS quantization. In Sect. 3 we discuss the stability of the model under radiative corrections. Sect. 4 is devoted to the study of the possible gauge anomalies. In particular it will be proven that, besides the well known nonabelian gauge-anomaly, the tensor matter fields do not give rise to a new anomaly.

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[1] See also ref. [4] for the supersymmetric extension.

2 The invariant action for tensor matter fields and its quantization: classical aspects

Following the construction of ref. [7], the invariant classical action describing the coupling between tensor matter fields and nonabelian Yang-Mills gauge fields is given by the following φ^4-like tensor Lagrangian:

$$\begin{aligned} S_{inv} = \; & -\frac{1}{4g^2}\int d^4x \, F^a_{\mu\nu}F^{a\mu\nu} \\ & -\int d^4x \left((D_\mu\varphi^{\mu\nu})^i (D_\sigma\varphi^\sigma{}_\nu)^{\dagger i} + \frac{q}{8}(\varphi^{\dagger i\mu\nu}\varphi^i_{\nu\sigma}\varphi^{\dagger j\sigma\beta}\varphi^j_{\beta\mu}) \right) , \end{aligned} \tag{2.1}$$

where (g,q) are coupling constants, $\varphi^i_{\mu\nu}$ denotes a complex antisymmetric tensor field constrained by the self-dual condition

$$\varphi^i_{\mu\nu} = i\tilde{\varphi}^i_{\mu\nu}, \qquad \tilde{\varphi}^i_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\varphi^{i\rho\sigma} \ , \tag{2.2}$$

and $(D_\sigma\varphi_{\mu\nu})^i$ is the covariant derivative

$$(D_\sigma\varphi_{\mu\nu})^i = \partial_\sigma\varphi^i_{\mu\nu} - iA^a_\sigma(\lambda^a)^{ij}\varphi^j_{\mu\nu} \ , \tag{2.3}$$

$(\lambda^a)^{ij}$ denoting the hermitian generators of a semisimple gauge group G taken in a complex representation specified by the indices (ij).

As it has been discussed in ref. [7], the self-dual complex condition (2.2) completely fixes the Lorentz structure of the action (2.1). This means that, in spite of the various possible Lorentz contractions which one could expect due to the tensorial nature of the fields φ, the condition (2.2) singles out a unique term both in the kinetic and in the self-interaction sector, yielding thus a unique action. The latter is precisely given by the expression (2.1) which, of course, is left invariant by the gauge transformations

$$\begin{aligned} \delta A^a_\mu & = \partial_\mu\omega^a + f^{abc}A^b_\mu\omega^c = (D_\mu\omega)^a, \\ \delta\varphi^i_{\mu\nu} & = i\omega^a(\lambda^a)^{ij}\varphi^j_{\mu\nu} \ , \end{aligned} \tag{2.4}$$

ω being an infinitesimal parameter. The complex self-dual condition can be solved straightforwardly, implying that the tensor field $\varphi^i_{\mu\nu}$ can be parametrized as

$$\varphi^i_{\mu\nu} = T^i_{\mu\nu} + i\tilde{T}^i_{\mu\nu} \ , \qquad \tilde{T}^i_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}T^{i\rho\sigma} \ , \tag{2.5}$$

$T^i_{\mu\nu}$ being an arbitrary real antisymmetric field. Plugging now eq.(2.5) into the expression (2.1), for the invariant action one gets

$$
\begin{aligned}
S_{inv} =& -\frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu}F^{a\mu\nu} + \int d^4x \left(\frac{1}{2}(\partial_\sigma T_{\mu\nu})^2 - 2(\partial_\mu T^{\mu\nu})^2\right) \\
& -2\int d^4x\, A^a_\mu\Big((\partial_\sigma T^{\sigma\nu})\lambda^a_R \tilde{T}^\mu{}_\nu - (\partial_\sigma \tilde{T}^{\sigma\nu})\lambda^a_R T^\mu{}_\nu + (\partial_\sigma T^{\sigma\nu})\lambda^a_I T^\mu{}_\nu + (\partial_\sigma \tilde{T}^{\sigma\nu})\lambda^a_I \tilde{T}^\mu{}_\nu\Big) \\
& +\int d^4x\, A^a_\mu A^b_\sigma \Big(T^{\mu\nu}\lambda^a_I\lambda^b_I T^\sigma{}_\nu + T^{\mu\nu}\lambda^a_I\lambda^b_R \tilde{T}^\sigma{}_\nu - \tilde{T}^{\mu\nu}\lambda^a_R\lambda^b_I T^\sigma{}_\nu - \tilde{T}^{\mu\nu}\lambda^a_R\lambda^b_R \tilde{T}^\sigma{}_\nu\Big) \\
& +\int d^4x\, A^a_\mu A^b_\sigma \Big(\tilde{T}^{\mu\nu}\lambda^a_I\lambda^b_I \tilde{T}^\sigma{}_\nu - \tilde{T}^{\mu\nu}\lambda^a_I\lambda^b_R T^\sigma{}_\nu + T^{\mu\nu}\lambda^a_R\lambda^b_I \tilde{T}^\sigma{}_\nu - T^{\mu\nu}\lambda^a_R\lambda^b_R T^\sigma{}_\nu\Big) \\
& -\frac{q}{4}\int d^4x \left(2(T_{\mu\nu}T^{\nu\rho})^2 - \frac{1}{2}(T_{\mu\nu}T^{\mu\nu})^2\right) .
\end{aligned}
\tag{2.6}
$$

The quantities λ^a_R, λ^a_I in the expression (2.6) denote respectively the real and the imaginary part of the complex hermitian generators λ^a, i.e.

$$
\lambda^a = \lambda^a_R + i\lambda^a_I \ . \tag{2.7}
$$

From the commutation relations

$$
\left[\lambda^a, \lambda^b\right] = i f^{abc}\lambda^c \ , \tag{2.8}
$$

we get

$$
\begin{aligned}
\left[\lambda^a_R, \lambda^b_R\right] - \left[\lambda^a_I, \lambda^b_I\right] &= -f^{abc}\lambda^c_I \ , \\
\left[\lambda^a_R, \lambda^b_I\right] + \left[\lambda^a_I, \lambda^b_R\right] &= f^{abc}\lambda^c_R \ ,
\end{aligned}
\tag{2.9}
$$

and from the hermiticity condition $\lambda^a = \lambda^{a\dagger}$ it follows

$$
(\lambda^a_R)^{ij} = (\lambda^a_R)^{ji} \ , \qquad (\lambda^a_I)^{ij} = -(\lambda^a_I)^{ji} \ . \tag{2.10}
$$

The quantities $T\lambda^a_I\lambda^b_I T$, etc.... in the expression (2.6) have to be understood in matrix notation, i.e. as $T^i(\lambda^a_I)^{ij}(\lambda^b_I)^{jk}T^k$, etc....

The gauge transformations (2.4), when written in terms of $T^i_{\mu\nu}$, read

$$
\begin{aligned}
\delta A^a_\mu &= \partial_\mu\omega^a + f^{abc}A^b_\mu\omega^c = (D_\mu\omega)^a, \\
\delta T^i_{\mu\nu} &= -\omega^a\Big((\lambda^a_R)^{ij}\tilde{T}^j_{\mu\nu} + (\lambda^a_I)^{ij}T^j_{\mu\nu}\Big) \ , \\
\delta \tilde{T}^i_{\mu\nu} &= \omega^a\Big((\lambda^a_R)^{ij}T^j_{\mu\nu} - (\lambda^a_I)^{ij}\tilde{T}^j_{\mu\nu}\Big) \ .
\end{aligned}
\tag{2.11}
$$

Notice that, due to the fact that the tensor matter representation is a complex representation [7] ($\lambda^a_R \neq 0$), the transformations (2.11) mix the components of the field $T_{\mu\nu}$ with those of the dual tensor field $\tilde{T}_{\mu\nu}$, generalizing thus the chiral abelian transformations given in [2].

Expressions (2.6) and (2.11) will be taken as the starting point for the discussion of the quantum properties of the model. Remark that the Levi-Civita tensor $\varepsilon_{\mu\nu\rho\sigma}$ is present in both the cubic ATT and the quartic $AATT$ interaction terms of the invariant action (2.6) as well as in the gauge transformations (2.11).

The quantization of the tensor matter action (2.6) can be done straightforwardly by means of the standard BRS procedure. Adopting a Landau gauge, the quantized BRS invariant action reads

$$S = S_{inv} + S_{gf} \ , \tag{2.12}$$

the gauge fixing term S_{gf} being given by

$$S_{gf} = \int d^4x \, s\left(\bar{c}^a \partial^\mu A^a_\mu\right) = \int d^4x \, \left(b^a \partial^\mu A^a_\mu - \bar{c}^a \partial^\mu (D_\mu c)^a\right) \tag{2.13}$$

where the fields $(c, \bar{c}, b)$ denote respectively the ghost, the antighost and the lagrangian multiplier and s is the nilpotent BRS operator defined as

$$\begin{aligned} sA^a_\mu &= (D_\mu c)^a, \\ sT^i_{\mu\nu} &= -c^a\left((\lambda^a_R)^{ij}\tilde{T}^j_{\mu\nu} + (\lambda^a_I)^{ij}T^j_{\mu\nu}\right) \ , \\ sc^a &= -\frac{1}{2}f^{abc}c^bc^c \ , \\ s\bar{c}^a &= b^a \ , \qquad sb^a = 0 \ . \end{aligned} \tag{2.14}$$

Coupling now the nonlinear transformations of (A, T, c) to a set of external BRS invariant sources (Ω, η, τ)

$$S_{ext} = \int d^4x \left(\Omega^a_\mu sA^\mu + \tau^a sc^a + \frac{1}{2}\eta^i_{\mu\nu} sT^{i\mu\nu}\right) \ , \tag{2.15}$$

it is easily verified that the complete action

$$\Sigma = S_{inv} + S_{gf} + S_{ext} \ , \tag{2.16}$$

obeys the following classical Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) = \int d^4x \left(\frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta\Sigma}{\delta\Omega^{a\mu}} + \frac{\delta\Sigma}{\delta\tau^a}\frac{\delta\Sigma}{\delta c^a} + \frac{1}{2}\frac{\delta\Sigma}{\delta T^i_{\mu\nu}}\frac{\delta\Sigma}{\delta\eta^{i\mu\nu}} + b^a\frac{\delta\Sigma}{\delta\bar{c}^a}\right) = 0 \ . \tag{2.17}$$

The ultraviolet dimensions and the ghost number of all the fields and external sources are displayed in the following table.

	A_μ	$\bar{c}$	c	b	$T_{\mu\nu}$	$\eta_{\mu\nu}$	Ω_μ	τ
dim.	1	2	0	2	1	3	3	4
gh. num.	0	-1	1	0	0	-1	-1	-2

Table Ultraviolet dimensions and ghost numbers.

Besides the Slavnov-Taylor identity, the classical complete action is characterized by two additional useful identities [8], namely the gauge fixing condition

$$\frac{\delta\Sigma}{\delta b^a} = \partial_\mu A^{a\mu} \ , \tag{2.18}$$

and the ghost identity, usually valid in the Landau gauge [9]

$$\mathcal{G}_a \Sigma = \Delta_a^{\rm cl} \ , \tag{2.19}$$

with $\mathcal{G}_a$ and $\Delta_a^{\rm cl}$ given respectively by

$$\mathcal{G}_a = \int d^4x \left(\frac{\delta}{\delta c^a} + f^{abc}\bar{c}^b \frac{\delta}{\delta b^c} \right) \ , \tag{2.20}$$

and

$$\Delta_a^{\rm cl} = -\int d^4x \left(f^{abc}(\tau^b c^c + \Omega_\mu^b A^{c\mu}) - \frac{1}{2}\eta^i_{\mu\nu}(\lambda_R^a)^{ij}\widetilde{T}^{j\mu\nu} - \frac{1}{2}\eta^i_{\mu\nu}(\lambda_I^a)^{ij}T^{j\mu\nu} \right) \ , \tag{2.21}$$

We remark that the breacking term $\Delta_a^{\rm cl}$ in the right hand side of equation (2.19), being linear in the quantum fields, is a classical breacking, i.e it is not affected by the quantum corrections. Finally, commuting the gauge fixing condition (2.18) and the ghost equation (2.19) with the Slavnov-Taylor identity (2.17) we get two more conditions [8], the familiar antighost equation

$$\frac{\delta\Sigma^a}{\delta\bar{c}} + \partial_\mu \frac{\delta\Sigma}{\delta\Omega^{\mu a}} = 0 \ , \tag{2.22}$$

and the Ward identity expressing the rigid invariance of the classical action (2.16), i.e.

$$\mathcal{W}^a_{\rm rig}\Sigma = 0 \ , \tag{2.23}$$

with

$$\begin{aligned} \mathcal{W}^a_{\rm rig} &= \int d^4x\, f^{abc}\Big(A^b_\mu \frac{\delta}{\delta A^c_\mu} + \Omega^b_\mu \frac{\delta}{\delta \Omega^c_\mu} + \tau^b \frac{\delta}{\delta\tau^c} + c^b \frac{\delta}{\delta c^c} + \bar{c}^b \frac{\delta}{\delta \bar{c}^c} \Big) \\ &+ \frac{1}{2}\int d^4x\, (\lambda^a_R)^{ij} \Big(\widetilde{T}^i_{\mu\nu} \frac{\delta}{\delta T^j_{\mu\nu}} - \widetilde{\eta}^i_{\mu\nu} \frac{\delta}{\delta \eta^j_{\mu\nu}} \Big) \\ &- \frac{1}{2}\int d^4x \left((\lambda^a_I)^{ij} \left(T^i_{\mu\nu} \frac{\delta}{\delta T^j_{\mu\nu}} - \eta^i_{\mu\nu} \frac{\delta}{\delta \eta^i_{\mu\nu}} \right) \right. \ . \end{aligned} \tag{2.24}$$

In summary, the complete classical action Σ is characterized by: the Slavnov-Taylor identity (2.17), the Landau gauge fixing condition (2.18), the ghost and antighost equations (2.19), (2.22) and the rigid gauge invariance (2.23).

3 Stability under radiative corrections

In order to analyse the stability [8] under radiative corrections of the classical action Σ we perturb it by means of an integrated local polynomial $\tilde{\Sigma}^{\rm count}$ with dimension four and ghost number zero, depending on the sources, on the fields and on their derivatives;

$$\Sigma \rightarrow (\Sigma + \epsilon\tilde{\Sigma}^{\rm count}) \ , \tag{3.1}$$

and we require that, to the first order in ϵ, the perturbed action $(\Sigma + \epsilon\tilde{\Sigma}^{\rm count})$ satisfies the same set of identities obeyed by the unperturbed action Σ, i.e. the Slavnov-Taylor identity (2.17), the Landau gauge fixing condition (2.18), the ghost and antighost equations (2.19), (2.22) as well as the rigid gauge invariance (2.23). As it is well known, the perturbation $\tilde{\Sigma}^{\rm count}$ represents the most general local invariant counterterm which is compatible with the simmetries and constraints characterizing the action and which can be freely added at each order of perturbation theory [8].

Requiring then that equations (2.17), (2.18), (2.19), (2.22) and (2.23) hold to the first order in ϵ, we get the following conditions:

$$\mathcal{B}_\Sigma \tilde{\Sigma}^{\rm count} = 0 \ , \tag{3.2}$$

$$\frac{\delta\tilde{\Sigma}^{\rm count}}{\delta b^a} = 0 \ , \tag{3.3}$$

$$\mathcal{G}_a \tilde{\Sigma}^{\rm count} = \int d^4\!x \left(\frac{\delta\tilde{\Sigma}^{\rm count}}{\delta c^a} + f^{abc}\bar{c}^b \frac{\delta\tilde{\Sigma}^{\rm count}}{\delta b^c} \right) = 0 \ , \tag{3.4}$$

$$\frac{\delta\tilde{\Sigma}^{{\rm count}\, a}}{\delta\bar{c}} + \partial_\mu \frac{\delta\tilde{\Sigma}^{\rm count}}{\delta\Omega^{\mu a}} = 0 \ , \tag{3.5}$$

and

$$\mathcal{W}^a_{\rm rig} \tilde{\Sigma}^{\rm count} = 0 \ , \tag{3.6}$$

where $\mathcal{B}_\Sigma$ is the so called linearized Slavnov-Taylor operator

$$\begin{aligned} \mathcal{B}_\Sigma \ &= \int d^4\!x \left(\frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta\Omega^{a\mu}} + \frac{\delta\Sigma}{\delta\Omega^{a\mu}}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta\tau^a}\frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta\tau^a} \right) \\ &+ \int d^4\!x \, \frac{1}{2}\left(\frac{\delta\Sigma}{\delta T^i_{\mu\nu}}\frac{\delta}{\delta\eta^{i\mu\nu}} + \frac{\delta\Sigma}{\delta\eta^{i\mu\nu}}\frac{\delta}{\delta T^i_{\mu\nu}} \right) \ , \end{aligned} \tag{3.7}$$

which, as a consequence of the Slavnov-Taylor identity (2.17), turns out to be nilpotent

$$\mathcal{B}_\Sigma \mathcal{B}_\Sigma = 0 \ . \tag{3.8}$$

In order to analyse the conditions (3.2)–(3.6) let us begin with the equation (3.3) which implies that $\tilde{\Sigma}^{\rm count}$ does not depend on the lagrangian multiplier b. Moreover, from the antighost condition (3.5) it follows that the variables $\bar{c}$ and Ω enter only through the combination

$$\gamma_\mu = \Omega_\mu + \partial_\mu\bar{c} \ . \tag{3.9}$$

Turning now to the homogeneous ghost condition (3.4) and taking into account eq.(3.3), it is easily seen that $\tilde{\Sigma}^{\rm count}$ depends on the ghost field c only through its space-time derivatives. Hence $\tilde{\Sigma}^{\rm count}$ can be parametrized in the following way:

$$\tilde{\Sigma}^{\rm count} = \Sigma^c(A) + \Sigma^c(A,T) + \Sigma^c(T) + \beta \int d^4x \, \gamma^{a\mu}\partial_\mu c^a \ , \qquad (3.10)$$

where β is an arbitrary parameter and $\Sigma^c(A)$, $\Sigma^c(T)$ depend respectively only from the gauge field A and the tensor fields T, while $\Sigma^c(A,T)$ collects both contributions.

Finally, conditions (3.2), (3.6) can be easily worked out along the same line used in the standard case of Yang-Mills coupled to ordinary spinor fields [8], yielding the following result:

$$\begin{aligned}
\Sigma^c(A) = & \ -\frac{1}{4g^2}\int d^4x \left((\rho-2\beta)F^a_{\mu\nu}F^{a\mu\nu} - 2\beta f_{abc}F^{a\mu\nu}A^b_\mu A^c_\nu\right) \\
\Sigma^c(T) = & \ -\frac{\delta}{2}\int d^4x \left(\frac{1}{2}(\partial_\alpha T_{\mu\nu})^2 - 2(\partial_\mu T^{\mu\nu})^2\right) \\
& -\frac{(\alpha-\delta)}{4}\int d^4x \left(2(T_{\mu\nu}T^{\nu\rho})^2 - \frac{1}{2}(T_{\mu\nu}T^{\mu\nu})^2\right) \\
\Sigma^c(A,T) = & \ (2\beta+\delta)\int d^4x \, A^a_\mu\Big((\partial_\alpha T^{\alpha\nu})\lambda^a_R\widetilde{T}^\mu{}_\nu - (\partial_\alpha\widetilde{T}^{\alpha\nu})\lambda^a_R T^\mu{}_\nu\Big) \\
& +(2\beta+\delta)\int d^4x \, A^a_\mu\Big((\partial_\alpha T^{\alpha\nu})\lambda^a_I T^\mu{}_\nu + (\partial_\alpha\widetilde{T}^{\alpha\nu})\lambda^a_I\widetilde{T}^\mu{}_\nu\Big) \\
& -(2\beta+\frac{\delta}{2})\int d^4x \, A^a_\mu A^b_\alpha\Big(T^{\mu\nu}\lambda^a_I\lambda^b_I T^\alpha{}_\nu + T^{\mu\nu}\lambda^a_I\lambda^b_R\widetilde{T}^\alpha{}_\nu\Big) \\
& +(2\beta+\frac{\delta}{2})\int d^4x \, A^a_\mu A^b_\alpha\Big(\widetilde{T}^{\mu\nu}\lambda^a_R\lambda^b_I T^\alpha{}_\nu + \widetilde{T}^{\mu\nu}\lambda^a_R\lambda^b_R\widetilde{T}^\alpha{}_\nu\Big) \\
& -(2\beta+\frac{\delta}{2})\int d^4x \, A^a_\mu A^b_\alpha\Big(\widetilde{T}^{\mu\nu}\lambda^a_I\lambda^b_I\widetilde{T}^\alpha{}_\nu - \widetilde{T}^{\mu\nu}\lambda^a_I\lambda^b_R T^\alpha{}_\nu\Big) \\
& -(2\beta+\frac{\delta}{2})\int d^4x \, A^a_\mu A^b_\alpha\Big(T^{\mu\nu}\lambda^a_R\lambda^b_I\widetilde{T}^\alpha{}_\nu - T^{\mu\nu}\lambda^a_R\lambda^b_R T^\alpha{}_\nu\Big) \ ,
\end{aligned} \qquad (3.11)$$

with (ρ,α,δ) arbitrary coefficients. One sees thus that the most general local BRS invariant counterterms contains four arbitrary parameters. The latters are easily seen to correspond to a renormalization of the coupling constants, of the field amplitudes and of the sources. Indeed, making the following redefinitions:

$$
\begin{aligned}
&g_0 = (1-\varepsilon\frac{\rho}{2})g \ , && q_0 = (1+\varepsilon\alpha)q \ , \\
&A_0^\mu = (1-\varepsilon\beta)A^\mu \ , && \Omega_0^\mu = (1+\varepsilon\beta)\Omega^\mu \ , \\
&\bar{c}_0 = (1+\varepsilon\beta)\bar{c} \ , && b_0 = (1+\varepsilon\beta)b \ , \\
&T_0^{\mu\nu} = (1-\varepsilon\frac{\delta}{4})T^{\mu\nu} \ , && \eta_0^{\mu\nu} = (1+\varepsilon\frac{\delta}{4})\eta^{\mu\nu} \ , \\
&c_0 = c \ , && \tau_0 = \tau \ ,
\end{aligned}
\tag{3.12}
$$

one easily checks that

$$
\Sigma + \varepsilon\tilde{\Sigma}^{\text{count}} = \Sigma(g_0, q_0, A_0, c_0, \bar{c}_0, b_0, T_0, \Omega_0, \eta_0, \tau_0) + O(\varepsilon^2) \ . \tag{3.13}
$$

The above equation states that the most general local BRS invariant counterterms can be reabsorbed through a redefinition of the parameters, of the fields and of the sources of the initial action, showing thus that the complite classical action Σ is stable under radiative corrections.

Let us conclude this section by remarking that the nonrenormalization of the ghost field c and of the related external source τ, as expressed by eqs.(3.12), is due to ghost condition (3.4), i.e to the choice of the Landau gauge [9].

4 Anomalies

We face now the problem of finding the possible anomalies wchich may affect the Slavnov-Taylor identity (2.17) at the quantum level. Taking into account that the gauge-fixing condition (2.18), the ghost equation (2.19) as well as the antighost equation (2.22) and the rigid invariance (2.23) can be easily proven to hold at the quantum level [8], it follows that the breaking Δ^1 corresponding to the extension of the Slavnov-Taylor identity (2.17) is an integrated local polynomial of dimension four and ghost number one which has to satisfies the following constraints:

$$
\frac{\delta\Delta^1}{\delta b^a} = 0 \ , \qquad \frac{\delta\Delta^{1\,a}}{\delta\bar{c}} + \partial_\mu\frac{\delta\Delta^1}{\delta\Omega^{\mu a}} = 0 \ , \tag{4.1}
$$

$$
\mathcal{G}_a\Delta^1 = 0 \ , \qquad \mathcal{W}^a_{\text{rig}}\Delta^1 = 0 \ , \tag{4.2}
$$

$$
\mathcal{B}_\Sigma\Delta^1 = 0 \ . \tag{4.3}
$$

Conditions (4.1), (4.2) imply that Δ^1 is independent from b, that the fields $\bar{c}$ and Ω^μ enter through the combination (3.9), and that Δ^1 depends on the ghost field c only through its space-time derivatives.

It follows then that, as done in the previous section, the breaking Δ^1 can be parametrized as

$$\Delta^1 = \Delta^1(A,c) + \Delta^1(T,c) + \Delta^1(A,T,c) \ , \tag{4.4}$$

where $\Delta^1(A,c)$ and $\Delta^1(T,c)$ depend respectively only from the gauge field A and the tensor field T, while $\Delta^1(A,T,c)$ collects both contributions. Of course, all the three terms of eq. (4.4) contain the Faddeev-Popov ghost c[2]. Concerning now the equation (4.3), it is easily seen to split into the three conditions

$$\int d^4x \left((D_\mu c^a) \frac{\delta \Delta^1(A,c)}{\delta A^a_\mu} - \frac{1}{2} f_{abc} c^b c^c \frac{\delta \Delta^1(A,c)}{\delta c^a} \right) = 0 \ , \tag{4.5}$$

$$\begin{aligned} \int d^4x \Bigg(\quad & \tfrac{1}{2} f_{abc} c^b c^c \frac{\delta \Delta^1(T,c)}{\delta c^a} + \partial_\mu c^a \frac{\delta \Delta^1(A,T,c)}{\delta A^a_\mu} \\ & -\tfrac{1}{2} c^a \frac{\delta \Delta^1(T,c)}{\delta T^{\mu\nu}} \left(\lambda^a_R \widetilde{T}^{\mu\nu} + \lambda^a_I T^{\mu\nu} \right) \Bigg) = 0 \ , \end{aligned} \tag{4.6}$$

$$\begin{aligned} \int d^4x \Bigg(\quad & f_{abc} A^b_\mu c^c \frac{\delta \Delta^1(A,T,c)}{\delta A^a_\mu} - \tfrac{1}{2} f_{abc} c^b c^c \frac{\delta \Delta^1(A,T,c)}{\delta c^a} \\ & -\tfrac{1}{2} c^a \frac{\delta \Delta^1(A,T,c)}{\delta T^{\mu\nu}} \left(\lambda^a_R \widetilde{T}^{\mu\nu} + \lambda^a_I T^{\mu\nu} \right) \Bigg) = 0 \ . \end{aligned} \tag{4.7}$$

The first equation (4.5) is recognized to be the well known consistency condition for the pure gauge anomaly, meaning that $\Delta^1(A,c)$ can be identified, modulo trivial BRS cocycles, with the usual nonabelian gauge anomaly [10, 11].

Finally, from the equations (4.6), (4.7) for $\Delta^1(T,c)$ and $\Delta^1(A,T,c)$ one easily gets

$$\begin{aligned} \Delta^1(T,c) + \Delta^1(A,T,c) = \quad & \int d^4x \, \partial_\mu c^a \Big(\partial^\nu \widetilde{T}_{\nu\beta} a^a T^{\beta\mu} - \partial^\nu T_{\nu\beta} a^a \widetilde{T}^{\beta\mu} + \partial^\nu T_{\nu\beta} b^a T^{\beta\mu} \\ & + \partial^\nu \widetilde{T}_{\nu\beta} b^a \widetilde{T}^{\beta\mu} + A^b_\nu T^{\mu\alpha} M^{ab}_1 T_\alpha{}^\nu + A^b_\nu \widetilde{T}^{\mu\alpha} M^{ab}_1 \widetilde{T}_\alpha{}^\nu \\ & + A^b_\nu \widetilde{T}^{\mu\alpha} M^{ab}_2 T_\alpha{}^\nu - A^b_\nu T^{\mu\alpha} M^{ab}_2 \widetilde{T}_\alpha{}^\nu \Big) \ , \end{aligned} \tag{4.8}$$

where the matrices a^a and b^a obey the relations,

$$\begin{aligned} & [\lambda^a_I, a^b] - [\lambda^a_R, b^b] - f_{abc} a^c = 0 \ , \\ & [\lambda^a_R, a^b] + [\lambda^a_I, b^b] - f_{abc} b^c = 0 \ , \end{aligned} \tag{4.9}$$

[2]The fact that the expression (4.4) does not depend on the external sources is easily seen to be a consequence of the power-counting and of the homogeneous ghost condition of eq. (4.2).

and M_1^{ab} and M_2^{ab} are given respectively by

$$
\begin{aligned}
M_1^{ab} &= \left(\lambda_R^a a^b + \lambda_I^a b^b \right) , \\
M_2^{ab} &= \left(\lambda_I^a a^b - \lambda_R^a b^b \right) .
\end{aligned}
\tag{4.10}
$$

It turns out however that the expression (4.8), although solution of the BRS consistency condition (4.3), can be actually written as a pure $\mathcal{B}_\Sigma$-cocycle, i.e.

$$
\Delta^1(T,c) + \Delta^1(A,T,c) = B_\Sigma \hat{\Delta}
\tag{4.11}
$$

with

$$
\hat{\Delta} = \int d^4x \; A^a_\mu \Big(\partial^\nu \widetilde{T}_{\nu\beta} a^a T^{\beta\mu} - \partial^\nu T_{\nu\beta} a^a \widetilde{T}^{\beta\mu} + \partial^\nu T_{\nu\beta} b^a T^{\beta\mu} + \partial^\nu \widetilde{T}_{\nu\beta} b^a \widetilde{T}^{\beta\mu} \Big) \; ,
\tag{4.12}
$$

meaning that $\Delta^1(T,c)$ and $\Delta^1(A,T,c)$ can be reabsorbed as local counterterms.

This shows that the tensor matter fields do not introduce new anomalies, the only possible nontrivial breaking being the nonabelian gauge anomaly [10, 11]. Moreover, due to the Adler–Bardeen theorem [11, 12], the latter is definitively absent if its numerical coefficient is adjusted in such a way that it vanishes at one loop order, guarantying then that the model is anomaly free.

5 Conclusion

The nonabelian generalization of the model proposed by Avdeev and Chizhov [2] has been proven to be renormalizable. This property has to be understood as a first step towards a correct physical interpretation of the tensor matter fields. Of course, many aspects of this model remain to be clarified . Let us mention, for instance, the construction of a unitary scattering operator (see also ref. [6] for a discussion on the positivity of the tensor matter Hamiltonian and on the corresponding Fock space) and the consistent introduction of the couplings among the tensor fields and the ordinary scalar or spinor matter fields [3]. We hope to report soon on these questions.

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