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MAGNETIC PROPERTIES OF Ce(Fe $_{1-x}$ A1 $_x$ ) FOR x $\leq$ 0.20 RUNNING TITLE: MAGNETIC PROPERTIES OF Ce(Fe $_{1-x}$ A1 $_x$ ) $_2$ 

by

D.F. Franceschini and S.F. da Cunha

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq Rua Dr. Xavier Sigaud, 150 22290 - Rio de Janeiro, RJ - Brasil

#### **ABSTRACT**

Magnetization measurements of the system  $Ce(Fe_{1-x}Al_x)_2$  for  $x \le 0.20$  show that the substitution of Fe by Al in  $CeFe_2$  has not a simple dilution effect. Together with a accentuated decrease in the iron mean magnetic moment and in Curie temperatures was observed, above a certain concentration, a transition from ferromagnetism to a canted spin phase upon decreasing temperature. For the higher concentration range the system exhibits magnetic freezing effects. A preliminary magnetic phase diagram was proposed.

Key-words: Magnetization of Laves phase; Ce(FeAl) compounds; Canted spin phase; Magnetic phase diagram.

#### 1 - INTRODUCTION

Pseudo-binary systems  $R(Fe_{1-x}A1_x)_2$  ( $R=Rare\ Earth$  metal, Y, Zr or Sc) have been investigated by several authors [1,2,3]. The end compounds  $RFe_2$  and  $RA1_2$  in almost all cases crystalize in the Cubic Laves Phase structure  $MgCu_2$  type. In the pseudo-binary systems the structure for the intermediate concentration range belongs to the  $MgZn_2$  hexagonal type. When R is magnetic, the effect of iron substitution on the magnetic behavior is to reduce the strength of the magnetic interactions, which induces the decrease of the magnetic transition temperatures  $T_c$  and the iron mean magnetic moments  $\mu_{Fe}$ . When R is non-magnetic such effect is much more intense, giving rise to the lack of long range magnetic order at a critical concentration  $x_c$ . Thus, an interesting feature of those systems is to investigate the onset of magnetism, recently reviewed by Hilscher [2].

The transition from ferromagnetic order to a non-magnetic one in  $R(Fe_{1-x}Al_x)_2$  when R is non-magnetic, occurs in two different ways. For Zr[2] and Sc[3] systems, iron substitution induces only magnetic dilution, leading to a decrease of both  $T_c$  and  $\mu_{Fe}$  until vanishing at  $x_c$ . For  $Y(Fe_{1-x}Al_x)_2$  system [2], the transition from magnetically ordered to disordered state is intermediated by a concentration range which shows mictomagnetic or spin glass behaviour. This kind of behaviour gives rise to a magnetic phase diagram analogous to that observed in qualitatively

different systems like  $\underline{Au}$ Fe or  $(Eu_{1-x}Sr_x)S$ . This feature is an indication that there is not only magnetic dilution, but an exchange competition is also taking place.

The ferromagnetic compound CeFe, is a special case among the  $AFe_2$  compounds (A = Y, Zr, Sc, Lu). Its Curie temperature  $T_c$  and the iron mean magnetic moment  $\mu_{Fe}(T_c = 230K)\mu_{Fe} =$ 1.24  $\mu_{\mbox{\footnotesize{B}}})$  are somewhat smaller than those of other similar compounds. This difference has been attributed to some transfer of the Ce 4f electron to the conduction band [4], and would make CeFe2 more sensitive upon iron substitution. Concerning Ce(Fe,A1), only a few preliminary studies are reported. Dwight et al. [5] have studied the crystal structure of several R(Fe,Al) but paid little attention to the case of Ce. In their work there is only a comment about the small solubility range at the iron-rich side. Da Cunha et al.[6] and Takeuchi et al.[7] have respectively studied the behaviour of the magnetization and electrical resistivity in  $Ce(Fe_{0.8}Al_{0.2})_2$ . Those works report a well stabilished spin glass behaviour at low temperature and at a higher temperature, a sharp maximum in the magnetization versus temperature curve, which was not completely explained.

The aim of this work is to extend the study of the magnetic properties of the pseudobinary compounds  $Ce(Fe_{1-x}Al_x)_2$  trying to elucidate some unusual features observed in [6] and [7]. Here we report and discuss the temperature and field dependences of the magnetization for some concentrations  $0 \le x \le 0.20$ . Electrical resistivity measurements on the same samples will appear in [8].

### 2 - EXPERIMENTAL

The samples were prepared by melting stoichiometric amounts of the components in an arc furnace under a purified argon atmosphere. The nominal purity of the starting elements are of 99.99% for Ce (Rare Earth Products Ltd.), 99.99% for Fe and 99.995%, for Al (Johnson Matthey Inc.).

The samples are remelted three or four times and annealed for a week at 7000C in an argon atmosphere. After, they were quenched in liquid nitrogen. The crystal structure and lattice parameter were determined by X-ray diffraction analysis on powdered samples. The X-ray patterns for the  $x \le 0.10$  samples exhibit a single phase characteristic of the  $MgCu_2$  type structure. For  $x \ge 0.125$  the X-ray patterns show together with  ${\rm MgC\acute{u}}_2$  lines, extra lines of foreign phases (CeAl and other unidentified lines). Different stoichiometries and thermal treatments did not eliminate those foreign The lattice parameters a were derived by least square analysis using Nelson-Riley's extrapolation. The concentration dependence of the lattice parameter is shown in fig 1, where we can see a linear dependence for  $x \le 0.10$  and a rought constant value for  $x \ge 0.125$ . This kind of behaviour — together with the presen ce of extra lines in the X-ray patterns —suggests that the solubility limit to form the pure  $MgCu_2$  type structure for  $Ce(Fe_{1-x}AI_x)_2$ may be placed near x = 0.125. The comparison between the linear portion of the  $a_0$  vs x plot and the Vegard's law line for the CeFe<sub>2</sub> -CeAl<sub>2</sub> system (dot-dashed line)suggests that the Ce ions must

be in the same valence state of the CeFe<sub>2</sub> compound up to the solubility limit as explained in [8].

Magnetization was measured using a P.A.R. vibrating sample magnetometer operating at a frequence of 80 Hz and over a temperature range of 3-300K in magnetic fields up to 13 kOe. Some magnetization measurements were made using a superconducting coil with fields up to 80 kOe or an extraction magnetometer (SNCI-GRENOBLE) with fields up to 150 kOe. Temperature sensors of carbon, chromel-Au-Fe and Cu-constantan thermocouples were used. The absolute accuracy of the temperature determination was about + 1 K.

# 3 - EXPERIMENTAL RESULTS AND DISCUSSION FOR x≤0,10

### 3.1 - Results

The temperature dependence of the magnetization for  $x \le 0.07$  in a magnetic field of 1k0e is shown in fig. 2 and of 10 k0e in fig 3. The curves were obtained after the samples had been cooled in zero field from room temperature and measured increasing the temperature and then remeasured during the cooling process.

Firstly we can observe for  $0.0 \le x \le 0.035$  that at least in some temperature range a ferromagnetic behaviour is present. The curve obtained for x = 0.02 under an applied field of 1k0e shows for T < 50K, a small but continuous decrease in the magnetization upon decreasing temperature acompained by thermomagnetic irreversible effects; under 10k0e such effects are absent and the curve is characteristic of a ferromagnetic behaviour.

For x=0.035 the decrease in magnetization below about 70K and the thermomagnetic irreversible effects are more accentuated as compared to x=0.02 even for a 10 k0e applied field and the temperature for which these effects do appear depends on the applied field. In the concentration x=0.07 the decrease in magnetization with decreasing temperature takes place at a higher temperature and a maximum appears at 145K; we have a monotonic behaviour at low temperatures. The x=0.10 concentration shows a similar behaviour to x=0.07.

In fig 4 high field magnetization curves up to 80 kOe at 4.2K for  $0.00 \le x \le 0.07$  are shown.

For x = 0.0 and 0.02 the curves are characteristic of ferromagnetic behaviour, showing knee and saturation. For x = 0,035 the lower field portion of the curve exhibit a marked curvature, showing tendency to saturation; the higher field portion however shows a magnetization step at a critical field of about 35 kOe, above which the magnetization begins to saturate. This curve shows also the development of a large histeresis loop which is, together with the magnetization step, characteristic of metamagnetism. For x = 0.07 the behaviour seems to be reminiscent of the x = 0.035 one, although with a much higher critical field ( $H_C > 80$  kOe) and a negligeable curvature in the lower field portion of the magnetization curve.

Hysteresis loops for  $x \le 0.035$  at low temperature for fields between - 13 and 13 kGe were observed. For x = 0.0 and x = 0.02 very narrow hysteresis loops were obtained. For x = 0.035 (fig. 5), we have different behaviour if the sample is

cooled in zero field or in 13 k0e although the two loops are centred at the origin.

Isothermal magnetization versus field curves up to 13 kOe were obtained for various temperatures. For x=0.02 (fig. 6), we observe at low temperatures a dependence of the initial slope with temperature. For x=0.035 (fig. 7), in the temperature range below 70K of decreasing magnetization, we have hysteresis loops with no magnetic remanence for H=0 and for  $T \ge 70K$  typical ferromagnetic behaviour was observed (fig. 8). For x=0.07 ([fig. 9) the observed upturns of the  $M_g$  vs H curves disappear at temperatures near the maximum of the  $H_g$  vs H curves.

### 3.2 - Discussion

CeFe $_2$  is a known ferromagnetic intermetallic with Curie temperature  $T_c=230 K.$  For x=0.02 the low temperature decrease in the  $M_g$  vs T curve is related to the variation of the inicial slope of the  $M_g$  vs H curves for T < 50K. For T > 50K a normal ferromagnetic behaviour is observed; the spontaneous magnetization at 4.2K is consistent with that of CeFe $_2$ , the low temperature anomalies been explained in terms of the difficult motion of domain wall. Also, the coercive field for x=0.02 is larger than for x=0.0.

The sharp decrease in the isofield magnetization curves for x=0.035 reveals a substantial reduction of the spontaneous magnetization, thus characterizing a change in the type of the magnetic structure. The observed thermo-remanent effects could

be ascribed to a ferromagnetic-spin glass like transition, similar to that observed in Y(Fe,Al) $_2$  [9]. However, the low field histeresis loops obtained at 4.2K (fig.5) do not confirm this hypotesis since the loops are centred at the origin. So, the iron magnetic moments must be ordered in a non collinear manner, i.e. the magnetization has a ferromagnetic plus an antiferromagnetic component say a canted spin structure. The high field magnetization curve at 4.2K (fig.4) and the low field isothermal curves (fig.7) do corroborate this point. These results show a turning point in the magnetization curves at a critical field  $H_{\rm C}$ , characteristic of a magnetic transition to a more ordered state along the field. We can see that critical fields  $H_{\rm C}$  decrease with increasing temperature, until vanishing. At the vanishing point of  $H_{\rm C}$ , the isothermal magnetization curves show clearly a ferromagnetic behaviour.

The results for x = 0.07 indicate a very similar behaviour to that of x = 0.035. Appart from the higher critical fields  $H_c$  and much smaller difference between the transition temperatures of the canted -ferro and ferro-para transitions, the magnetization curves can be explained in the same way. The high field magnetization curve at 4.2K looks like those obtained for x = 0.035 with low field and is showed in fig.7. The  $M_g$  vs  $H_c$  curves for T >77K (fig 9a) show a temperature dependent critical field  $H_c$ , which vanishes at a temperature very close to the peak. In fig.9b we can see that the magnitude of the magnetization is about the same order as the obtained for x = 0.02 and x=0.035 at temperatures near  $H_c$ . So, this result indicates a probable ferromagnetic ordering of iron magnetic moments. Fig.10

shows the date of figs.9a and 9b plotted as  $M_g$  vs T curves. These curves show an effect of the increasing applied field similar to that observed for x=0.035.

The Curie temperature  $T_c$  and the iron mean magnetic moment  $\mu_{Fe}$  were determined for all samples with  $0,0 \le x \le 0.07$  and are listed in table 1. Curie temperatures were determined from  $\left(M_g\right)^2$  vs T plots of low field (150 0e) magnetization data. An example is shown in the insert of fig.10. The iron moments  $\mu_{Fe}$  were determined at 4,2K from magnetization data obtained for magnetic field of 80 k0e. The room temperature  $M_g$  vs H curves for this sample show however a small but not negligeable, magnetic impurity contribution. The magnetization due to these impurities was subtracted from the obtained extrapolation value at T = 4,2K. As in other systems, both magnetic moment and Curie temperatures decrease sharply upon increase of Al concentration as show in fig.11. Data for Y  $\left(Fe_{1-x}Al_x\right)_2$  taken from [10] are plotted in the same figure for comparison.

# 4 - EXPERIMENTAL RESULTS AND DISCUSSION FOR x>0.10

### 4.1 - Results

The temperature and field dependences of the magnetization were measured in samples with composition  $x \ge 0.125$ . The  $M_g$  vs T curve for x = 0.15 measured with H = 1 k0e and zero field cooled is shown in figure 12 and is characteristic of the range  $0.125 \le x \le 0.20$ . At high temperatures we can observe a peak, the same behaviour as observed for x = 0.07. However, at low tempe-

ratures, a second peak appears at  $T_{\mathbf{f}}$  for all the three compositions at temperatures which vary from 10K to 15K. To investigate the origin of the low temperature peak, as in [6] we have performed measurements after the samples had been cooled in zero field from high temperature to 3K, and then measured in monotonical increase of temperature up to about 30K with an applied field of 300 Oe. These curves presents irreversible effects for T<T, but reversibility accurs for T>T, Afterwards the magnetization has been measured in a field H=300 Oe but now during the cooling process from 30K to 3K. Once the lowest temperature is achieved the magnetization is re-measured for increasing temperature. The variation is observed to be identical to that measured during cooling with a constant applied field of 300 Oe thus been reversible within the time scale of the experiment (a few minutes per experimental point). An example of these measurements is shown in figure 13. For 1 kOe applied fiels the same characteristic behaviour is observed but with a broader and lower intensity peak and with a slight shift of Tf.

In fig.14 we present typical isothermal magnetization versus field curves for fields up to 150 k0e. These curves show the same metamagnetic behaviour as that of x=0.035, but with no indication of saturation.

## 4.2 - Discussion

As it was explained in section 2, the samples for  $x \ge 0.125$  exhibit, besides of the cubic C15 structure, the presence of foreign phases like CeAl. This compound is antiferromagnetic with a Néel temperature of lOK [1], 12]. Our expe-

rimental results for x≥0.125 at low temperatures, are typical of freezing phenomena characterised by different susceptibility behaviours as samples were cooled with or without field. Other results that corroborate this idea are the variation of the susceptibility with temperature for different values of frequency as we can see in figure 15, and also the displaced hysteresis loops at low temperature, figure 16.

The fact that the lattice parameter  $a_0$  and the residual resistivity  $\rho_0^{[8]}$  becomes roughly constant in this concentration range suggest that possibly the MgCu<sub>2</sub> dilution limit of the CeFe<sub>2</sub> - CeAl<sub>2</sub> system for the iron rich side is attained.

The low temperature maximum of the magnetization observed in these samples cannot be atributed to the CeAl phase. Since there are other unidentified phases in that composition range, we by no means can be sure that the observed spin glass behaviour is intrinsic to the  ${\rm MgCu_2}$   ${\rm Ce(Fe_{1-x}Al_x)_2}$  phase. So, more detailed work near the probable solubility limit must be done to elucidate this matter.

The high field magnetization data for x=0.15 show characteristic features of magnetic behaviour at the canted spin temperature range. In fig. 14 we can see the strong variation of  $H_C$  and the width of the histeresis loop as the temperature changes. The higher critical field  $H_C=110$  kOe at 4.2K - as compared to that observed for x=0.035, confirms the tendency insinuated in x=0.07 composition.

#### 5 - CONCLUSION

Magnetization measurements of the pseudobinary system  $Ce(Fe_{1-x}Al_x)_2$  for  $x \le 0.20$  show that the substitution of Fe by Al in  $CeFe_2$  has not a simple dilution effect. Together with the usual decreases in the iron mean magnetic moment  $\mu_{Fe}$  and in the ferromagnetic Curie temperature  $T_c$ , we observe a transition from ferromagnetism to a non-collinear order, upon decreasing temperature, above a certain concentration. For the higher concentration range the system exhibits magnetic freezing effects. As in  $Y(FeAl)_2$  or Fe-Al the disorder produced by the Al adition gives rise to a negative exchange competing with the positive, ferromagnetic one [13].

For x<0,10 ferromagnetic order is present in some temperature range and for  $0.035 \le x \le 0.10$  changes in magnetic structure take place at low temperature with moment ordered probably in a non-collinear way. For x=0 and 0.02 only ferromagnetic behaviour was observed. For  $0.035 \le x \le 0.10$  the transition from a ferromagnetic to a non-collinear order was indicated by jumps in the Mg vs T curves. This low temperature magnetic phase exhibits metamagnetic behaviour, which was indicated by steps and large histeresis loops in the Mg vs H curves.

The electrical resistivity measured in the same samples  $x \ge 0.10$  as a function of temperature [8] exhibits two minima, the low temperature one is expected to be related to a spin-glass state. The higher temperature minimum corresponds to the temperature at which  $H_c = 0$  or to the jump in  $M_g$  vs T curves if

measurements have been made in zero field. In general, the shape of the  $M_g$  vs T curves may be explained in the following way. If the temperature  $T_o$ , where the critical field  $H_c$  equalizes the applied field in measurements of  $M_g$  vs T, is much lower than  $T_c$  (Curie temperature), ferromagnetic behaviour will dominate in some range of temperature as in x=0.035. On the contrary, if  $T_o \sim T_c$ , the  $M_g$  vs T curves will exhibits a narrow peak as observed in x=0.07. So the high temperature resistivity minimum [8] was found to be related to this transition and may be ascribed to an extra scattering mechanism introduced by the disorder in the magnetic structure. For  $x \ge 0.125$  we observe a well characterized spin-glass behaviour at low temperature.

Finally, we propose a preliminary magnetic phase diagram for the iron rich side of the  $\mathrm{Ce}(\mathrm{Fe}_{1-x}\mathrm{Al}_x)_2$  system in fig.17. The diagram was constructed by plotting against composition  $\mathrm{T}_c$ ,  $\mathrm{T}_f$  and  $\mathrm{T}_o$  (temperature associated to be inflexion point in the jump of the temperature dependence of the magnetization).

The almost constant behaviour observed for x≥0.10 is an aditional indication of the limited solubility of the system at the iron rich side. Concerning this matter, a more detailed investigation is being performed in the composition range near the probable solubility limit; the idea is to investigate if the spin glass behaviour is or not intrinsic to the MgCu<sub>2</sub> phase.

### **Acknowledgements**

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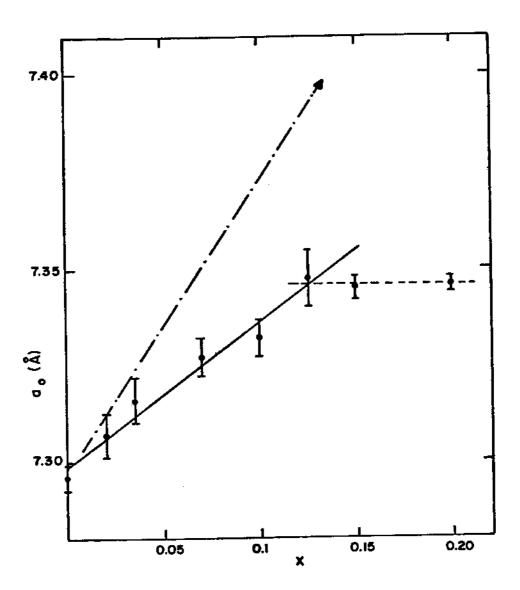
#### FIGURE CAPTIONS

- Fig. 1 Variation of the lattice parameter  $a_0$  with Al composition x for  $Ce(Fe_{1-x}Al_x)_2$ . The dot-dashed line represents the Vegard's law for the  $CeFe_2$ - $CeAl_2$  system.
- Fig. 2 Magnetization vs temperature for x = 0.02, 0.035 and 0.07 under 1 k0e applied field:
  - (\*) zero field cooled sample, data taken with increasing temperature; (x) sample cooled under measuring field, data taken with decreasing temperature.
- Fig. 3 Magnetization vs temperature for x =0.02 and 0.035 under 10 k0e applied field:

  (①) zero field cooled sample, data taken with increasing temperature; (△), (x) sample cooled under measuring field, data taken with increasing and decreasing temperature respectively.
- Fig. 4 Magnetization vs field at T=4.2K for  $CeF_2(*)$ ; x=0.02(0); x=0.035(+) and x=0.07(x). The dashed line in the x=0.035 plot corresponds to the first magnetization process. The insert contains the x=0.07 data plotted in a expanded scale.
- Fig. 5 Histeresis loops for x = 0.035 at 3K with -13<H<13 k0e:

  (\*\*) zero field cooled sample; (x) 13 K0e field cooled sample. The insert shows a expanded view of the central region of both loops.
- Fig. 6 Isothermal magnetization vs field curves for x =0.02. The dashed-line plotted curves corresponds to the variating initial slope temperature range.
- Fig. 7 Isothermal magnetization vs field curves for x =0.035 and  $T \le 77K$ .

- Fig. 8 Isothermal magnetization vs field curves for x = 0.035 and  $T \ge 7.7K$ .
- Fig. 9 Isothermal magnetization vs field curves for x = 0.07 with (a) 143.2>T>77K; (b) T>146.8.
- Fig.10 Isofield magnetization curves for x =0.07 (the data is the same of fig.9, replotted against temperature). Insert shows  $\text{M}^2 \text{vsT}$  dependence at temperatures near  $\text{T}_{\text{C}}$  for x =0.07.
- Fig.11 Iron mean magnetic moment and ferromagnetic Curie temperatures normalized to x = 0 for  $Y(Fe_{1-x}Al_x)_2(x)$  (reference [10]) and for  $Ce(Fe_{1-x}Al_x)_2$  (\*). (See text).
- Fig.12 Magnetization vs temperature for x = 0.15 under 1 k0e applied field (zero field cooled sample).
- Fig.13 Susceptibility (Mg/H) vs temperature for x =0.15 under 0.3 and 1.0 kOe: ( $\triangle$ ) and ( $\square$ ) field cooled sample, ( $\triangle$ ) and ( $\square$ ) ZFCS.
- Fig.14 Isothermal magnetization vs field curves (field up to 150 kOe) for x=0.15.
- Fig.15 AC susceptibility for x = 0.20: (x) 155 Hz; ( $\bullet$ ) 1550 Hz.
- Fig.16 Histeresis loops for x=0.15 at 4.2K:(•) zero field cooled sample; (x) sample cooled in 13 k0e field.
- Fig.17 Proposed magnetic phase diagram: (♠) T<sub>c</sub>; (x) T<sub>o</sub>; (△) T<sub>f</sub>;
  P-paramagnetic; C- canted spin ;SG spin glass;
  F-Ferromagnetic.



Fig, 1

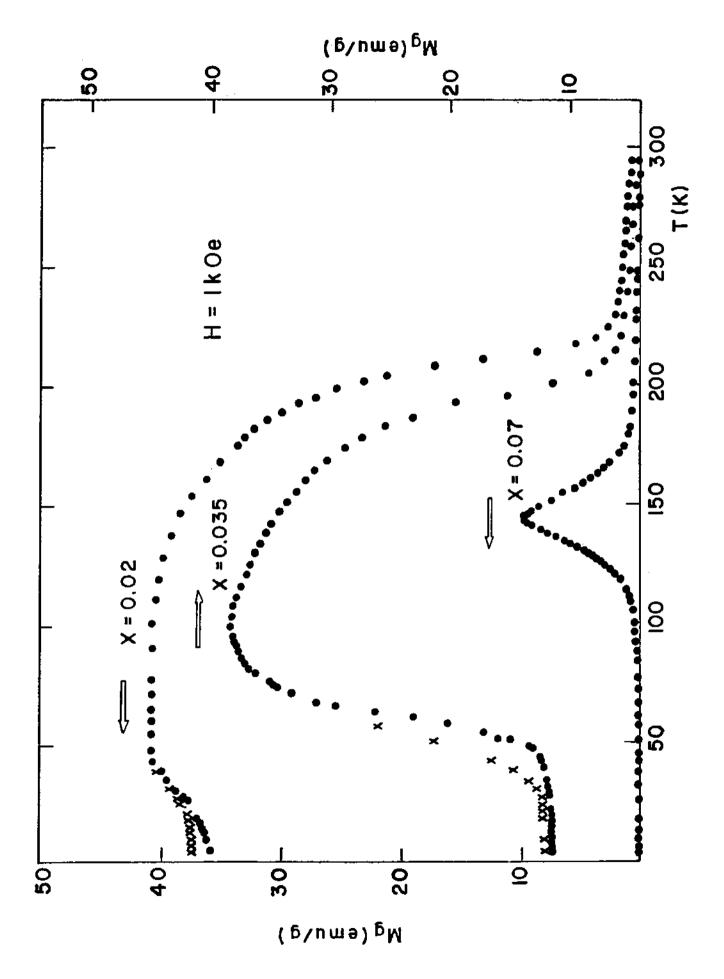


Fig. 2

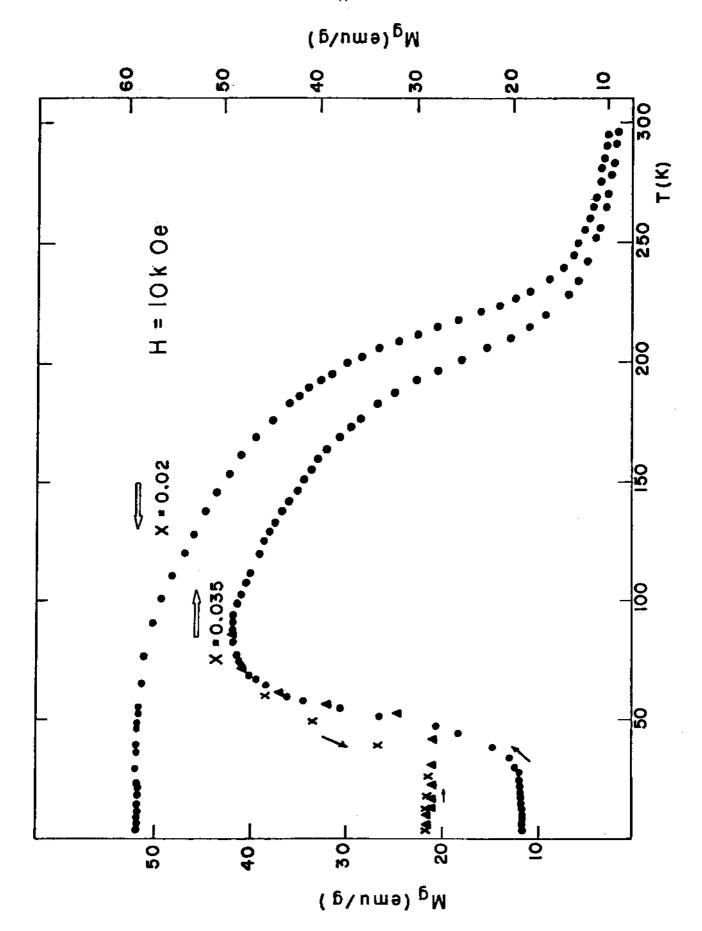


Fig. 3

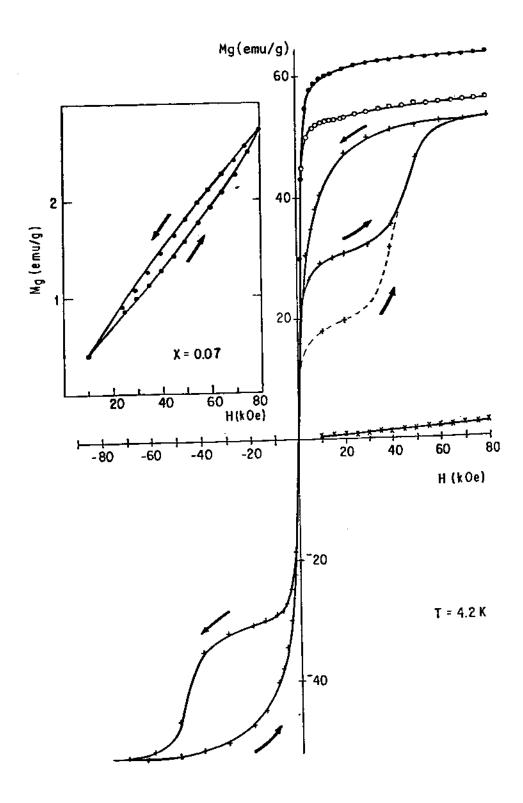


Fig. 4

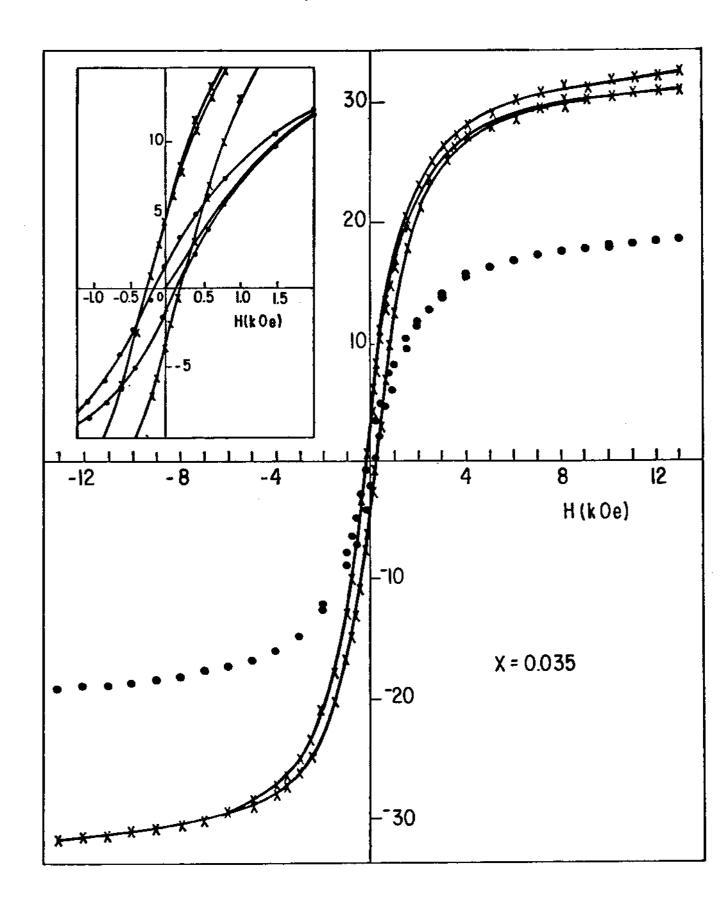


Fig. 5

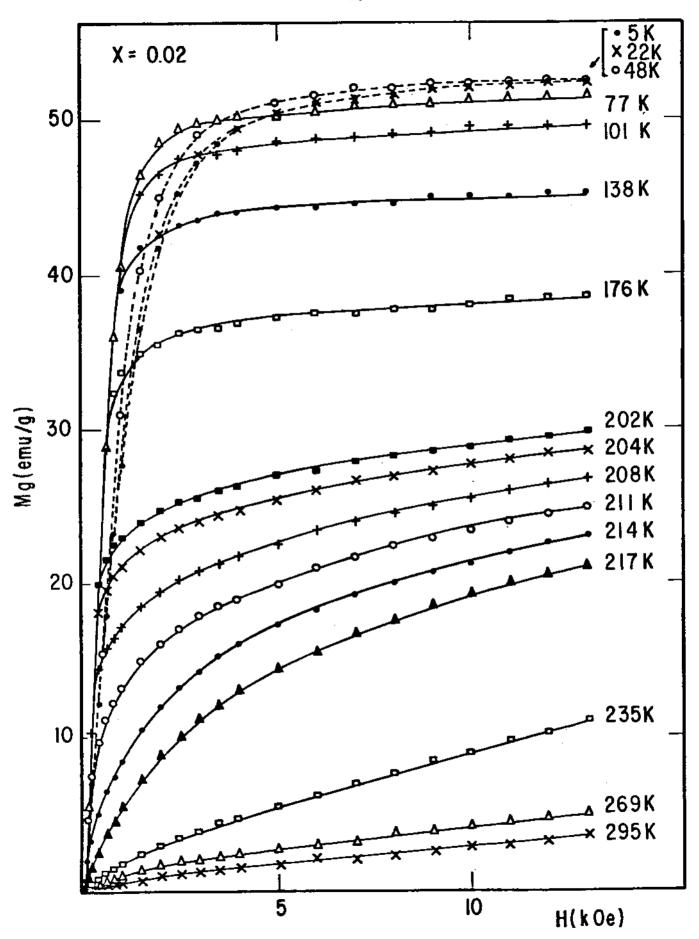


Fig. 6

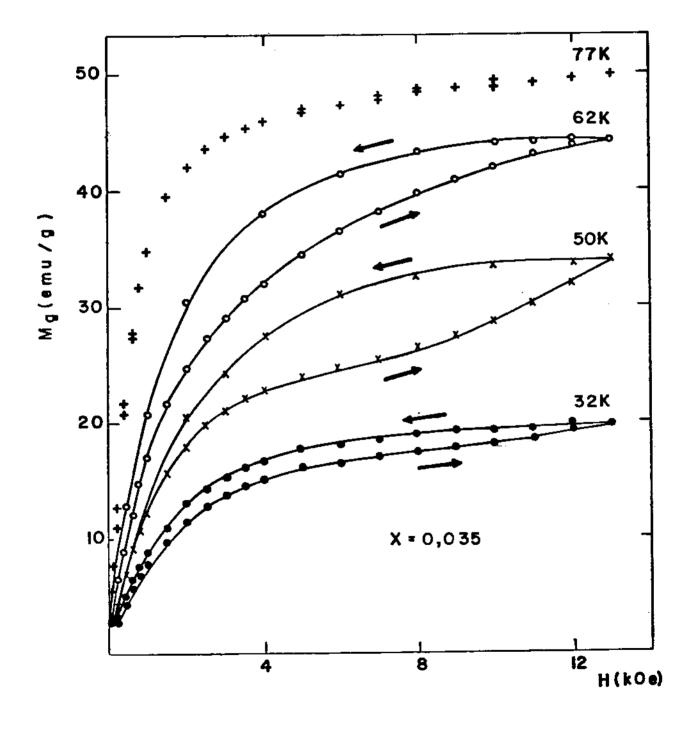


Fig. 7

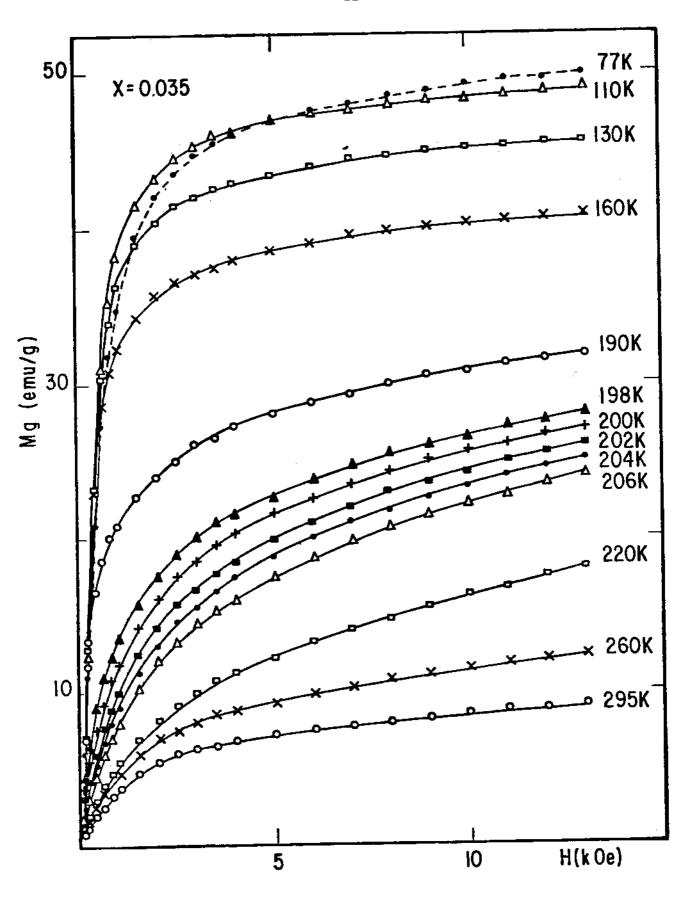


Fig. 8

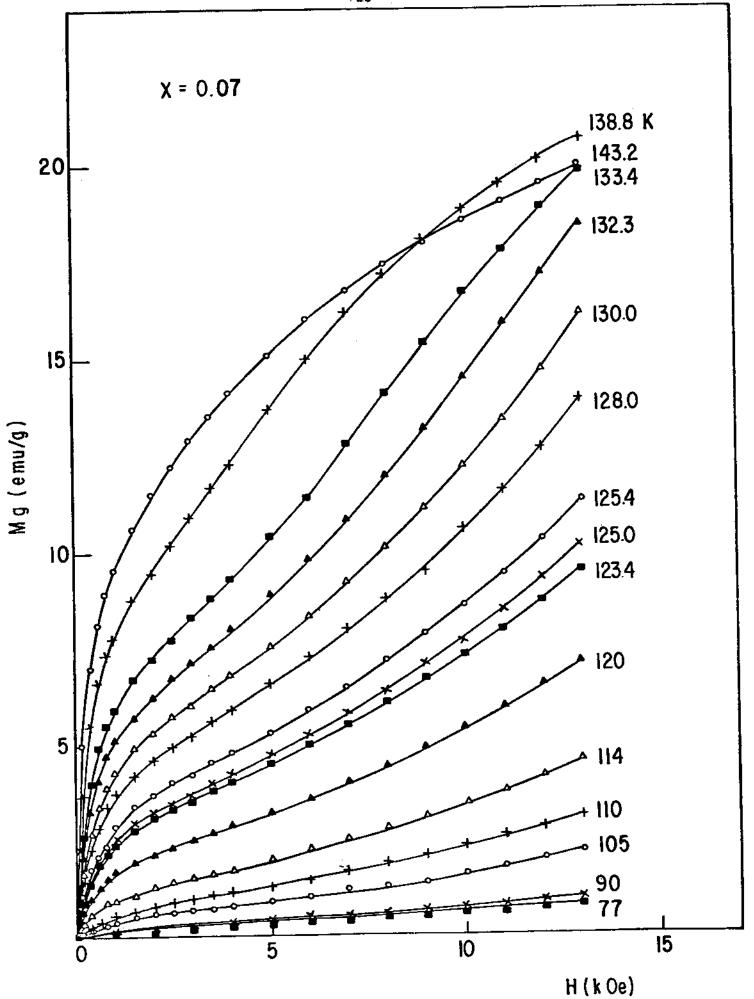


Fig. 9a

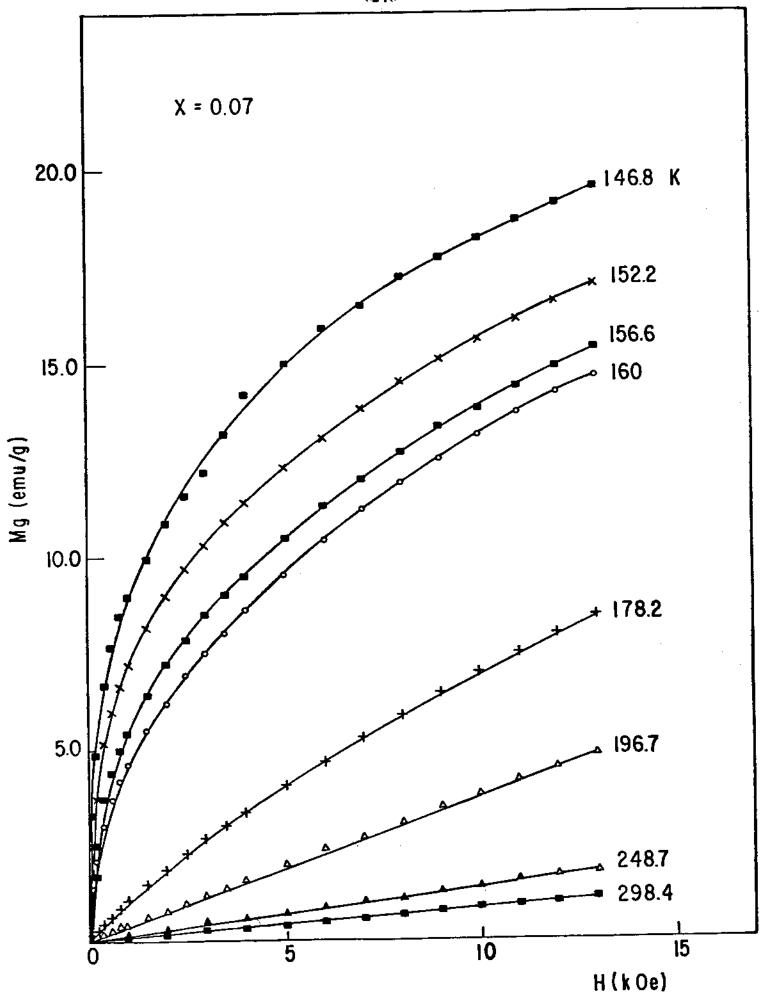


Fig. 9b

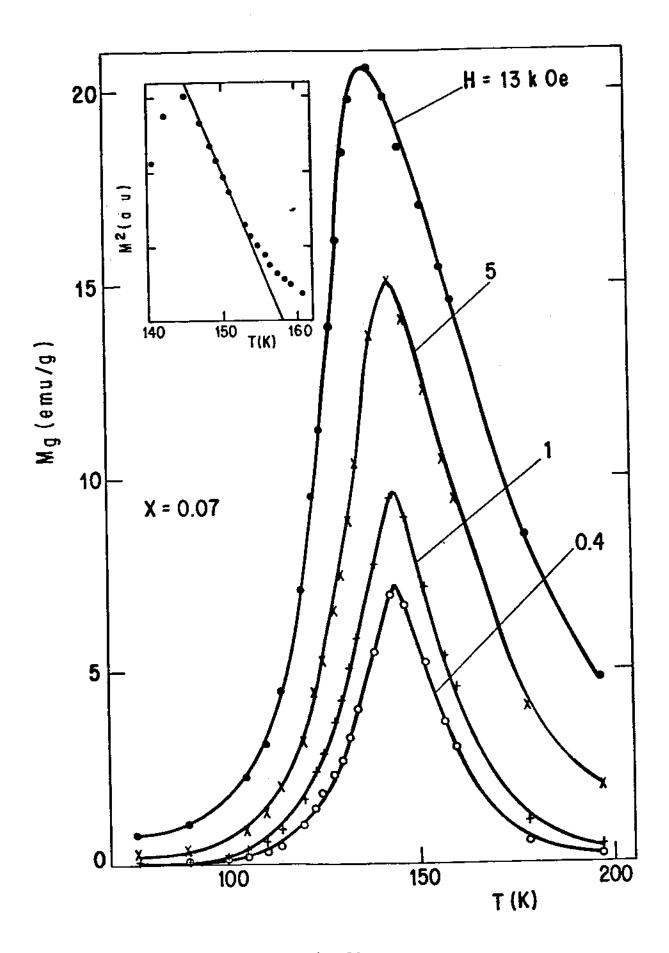


Fig. 10

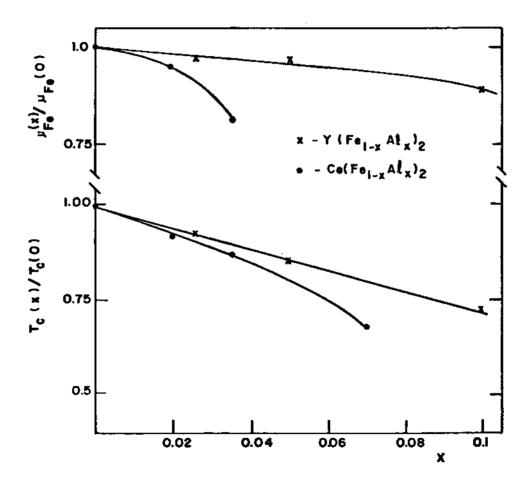


Fig. 11

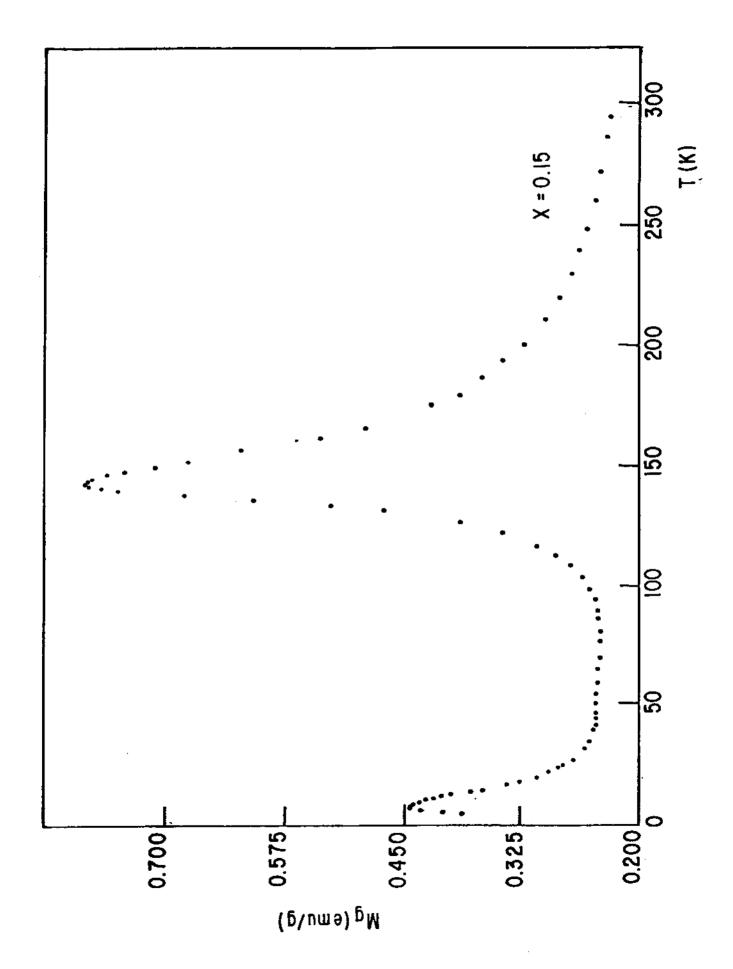
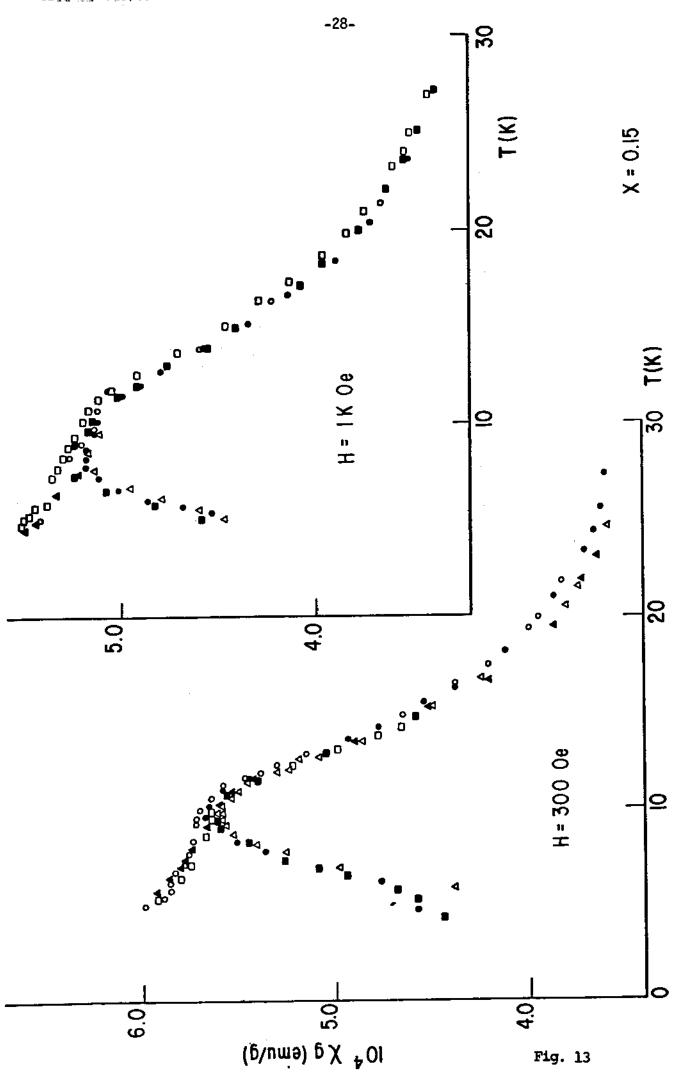


Fig. 12



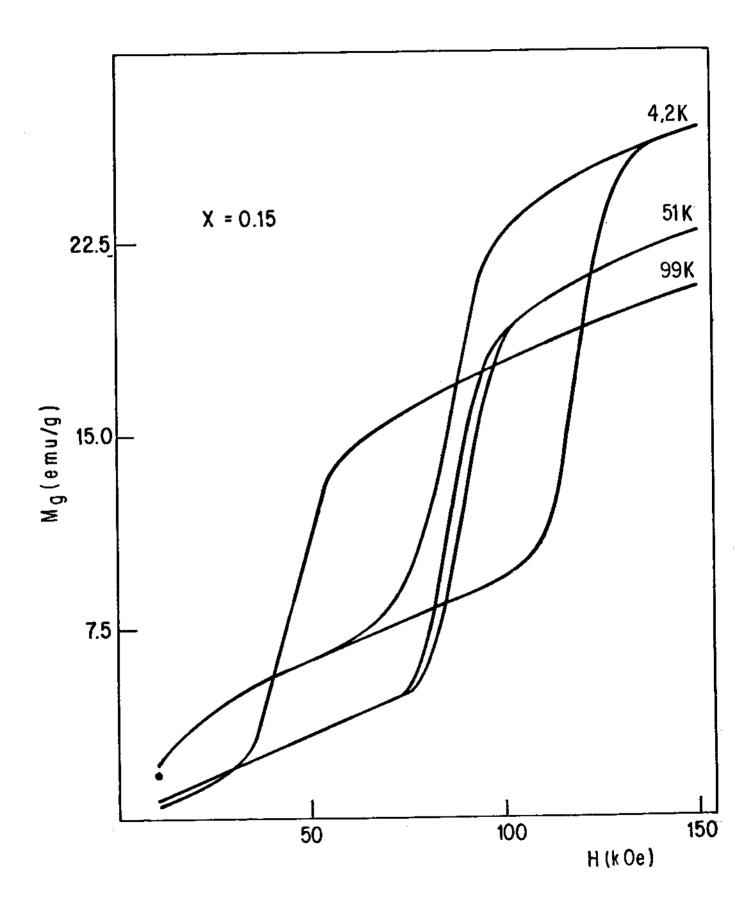


Fig. 14

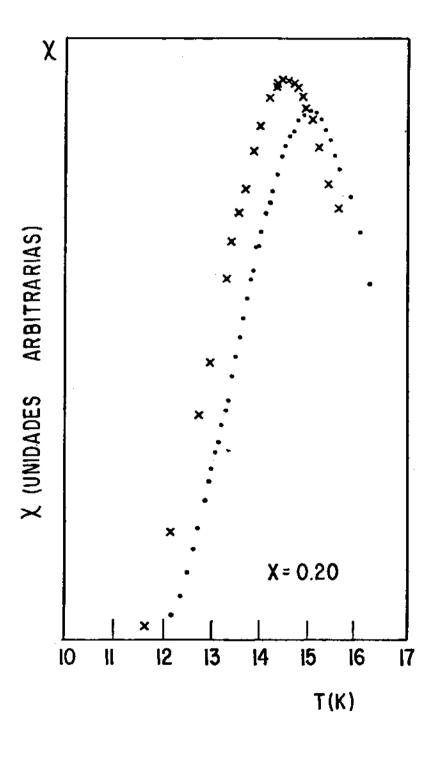


Fig. 15

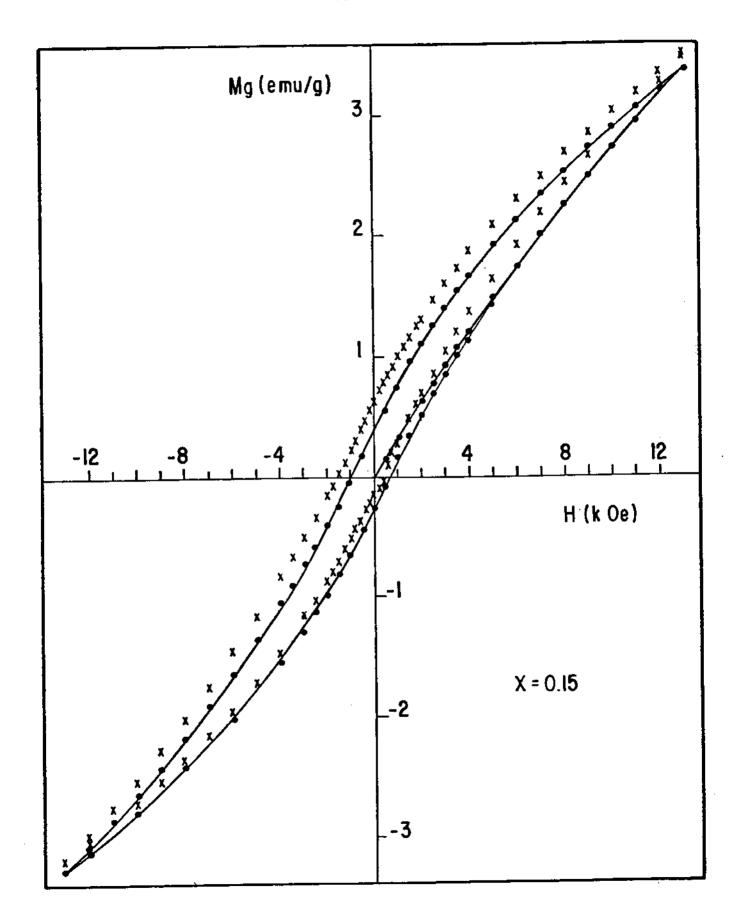


Fig. 16

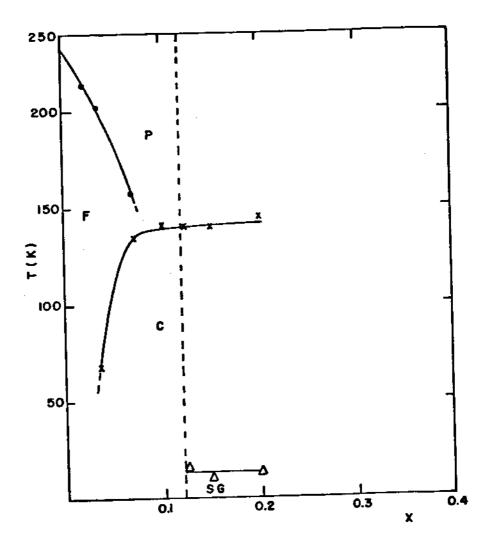


Fig. 17

TABLE 1

Х	т <sub>с</sub> (к)	μ <sub>Fe</sub> (μ <sub>B</sub> /Fe)
0.0 0.02 0.035 0.07	232.0 213.8 201.8 158.0	1.33 1.26 1.08

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