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Confining Strings Revisited – A Short Comment

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Abstract

We show that the Polyakov's confining string Nucl. Phys. B486, (1997) 23, is the author's previously proposed self-avoiding extrinsic strings (Luiz C.L. Botelho, Rev. Bras. Fis. 16, 279, (1986); CALTECH–preprint 68, 1444, (1987); J. Math. Phys. 30 (9), (1989), 2160).

The more important problem in the present days of theoretical and mathematical physics is how to quantize correctly Non-Abelian Gauge Field Theories defined on the physical continuum space-time. The only result in this direction still remains an formal Ansatz from the experimental and theoretical point of view: the use of the Higgs mechanism in order to prevent (already at the classical level of the Weinberg-Scalar theory), the "infrared disaster" of the on-shell theories scattering matrix by given classically mass parameters for gauge bosons. Probably, this Ansatz is wrong from a strict quantum field theoretic point of view since it makes heavier use of a probably trivial $\lambda\phi^4$-field theory in four dimensions and of the somewhat unphysical gauges of t'Hooft for the Yang-Mills Fields ([1]) (see the comments on pag. 38 the J.C. Taylor book "Gauge theories of weak interactions – Cambride Monographs on Mathematical Physics). However, it was realized by K. Wilson ([2]) that in the Ising like euclidean path integral crude approximation framework (Lattice Gauge Theory) theses non-abelian gauge field theories in the lattice at a bare strong coupling regime are naturally expressed in terms of the *Euclidean* ([3]) Wilson Loops defined by the matter content trajectories $C = \{X_\mu(\sigma);\ 0 \leq \sigma \leq 1;$ σ =proper-time parameters

$$W[C] = Tr\ I\!P \left\{ exp \left[- \oint_C \ A_\mu dX^\mu \right] \right\} \tag{1}$$

It is worth remark that eq. (1) is not invariant under the reverse trajectories reparametrization $X_\mu(\sigma) = X_\mu(-\sigma)$ in the Euclidean world, although $W[C]W[-C] = 1$ still holds true. (Note that typical interaction energy densities, such as $\bar\psi\psi$, $\bar\psi\gamma^s\psi$, $\bar\psi\gamma^\mu\psi A_\mu$ which are real function (distributions) in the Minkowski space-time are complex on the Euclidean world).

It was argued on ref. [4] by A.M. Polyakov, a euclidean string functional integral Ansatz for eq. (1) based on a coupling of a abelian rank-two antisymmetric tensor field $B_{\mu\nu}(x)$ (the Polyakov's axion field) with the string orientation area tensor previously proposed by this author (see [5]) but with an important difference: This rank-two antisymmetric tensor field B has a non trivial dynamic content. Namelly (see eq. ((12)-(15))

- ref. [4])

$$W[C] = \frac{\int D^F[B_{\mu\nu}]e^{-S[B_{\mu\nu}]}e^{\left(i\int_{\sum_c} Bd\sigma\right)}}{\int D^F[B_{\mu\nu}]e^{-S[B_{\mu\nu}]}} \tag{2}$$

where the axion action is given by

$$S(B) = \frac{1}{4e^2}\int d^2x \left(B^2_{\mu\nu} + dB\cdot arsen\,\frac{dB}{m^2} - \sqrt{m^4-(dB)^2}\right) \tag{3}$$

At this point we point out that the functional integral weight eq. (3) makes sense only for those field configurations which makes eq. (3) a *real number*, namelly: $\sup\limits_{x\in R^\nu} |dB(x)| \le m^2$.

Unfortunately this bound on the kinetic energy of the axion field is impossible for those distributional fields configurations making the domain of the axion functional integral eq. (2), unless $m^2 \to \infty$ [6] and comments below eq. (40) of ref. [4]. (A quantum field may be bounded but not its kinetic energy!).

In this situation, eq. (19) of ref. [4], A.M. Polyakov must turn into a pure White-Gaussian action for the axion field B

$$S[B] = \frac{1}{4e^2}\int B^2(x)d^2x \tag{4}$$

One has, thus, the following correct formulae instead of those of ref. [4]

$$W[C] \sim exp[-F(C;\sum_C)] \tag{5}$$

where the surface functional weight is given by the self-avoiding extrinsic action firstly proposed in a minimal area context solution for the Q.C.D-Loop wave equation in second ref. of [5]

$$F(C,\sum_C) = \int_{\sum} d\sigma_{\mu\alpha}(x)(\delta^{\mu\lambda}\delta^{\alpha\rho}\delta^\nu(x-y))d\sigma_{\lambda\rho}(y) \tag{6}$$

It is straightforward to see that for fixed constant e^2, the limit $m^2 \to \infty$ leads to a pure Nambu-Goto action strongly coupled ([4]) (see refs. [8] and Appendix for a different result)

$$F(C,\sum_C) \sim \lim_{m^2\to\infty} c_1(e^2m)\int d^2\xi\sqrt{g}(\xi) + \lim_{m^2\to\infty} c_2(e^2/m)\int d^2\xi(\nabla t_{\mu\nu})^2\sqrt{g}$$
$$+O\left(\frac{1}{m}\right) \sim \frac{1}{2\pi\alpha'}\int d^2\xi\ \sqrt{g}(\xi) \tag{7}$$

Here the Regge slope effective parameter is given by $\frac{1}{2\pi\alpha'} = c_1(e^2m)$.

It is worth remark that the expected extrinsic action will come from the existence of intrinsic fermionic fields in the non-abelian case ([7]). As a general conclusion, one can says that the charged Elfin Extrinsinc Self-Avoiding strings ([7]) still remains the only available candidate to quantize *non-abelian gauge fields at the quantum average decoupling t'Hoot limit.*

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Appendix A

In this Appendix we present some related comments on eq. (7) for the author proposal of a self-avoiding extrinsic reparametrization string for Q.C.D ($SU(\infty)|$ in R^D, namelly

$$A[X^\alpha(\xi)] = \lambda \int_\xi \sqrt{h(\xi)} \int_{\xi'} \sqrt{h(\xi')}(\zeta^{\mu\nu}(X(\xi))\zeta_{\mu\nu}(X(\xi')) - 1)\delta^{(D)}(X_\alpha(\xi) - X_\alpha(\xi')) \tag{A.1}$$

Here, the surface area tensor responsable by the extrinsic properties of Q.C.D string (and explaining the asymptotic freedom of the underline Q.C.D field theory) is given by

$$\zeta^{\mu\nu}(X(\xi)) = \varepsilon^{ab}\partial_a X^\mu \partial_b X^\nu \Big/ \sqrt{h(\xi)} \tag{A.2}$$

and

$$\sqrt{h(\xi)} = \sqrt{det\{\partial_a X^\mu \partial_b X_\mu\}}(\xi) \tag{A.3}$$

By considering eq. (A-1) in a regularized form for the delta-function interaction

$$\delta^{(D)}_\Lambda(X(\xi) - X(\xi') = \int_{|k|<\Lambda} d^D k \ exp(ik_\alpha(X_\mu(\xi) - X_\alpha(\xi')) \tag{A.4}$$

namelly,

$$A^{(\Lambda)}[X^\alpha(\xi)] \sim \sum_{p=0}^{\infty} \frac{(-1)^p}{p!2^{2p}\Gamma(p+\frac{D}{2})}\left(\frac{\Lambda^{D+2p}}{D+2p}\right)\int_\xi \int_{\xi'}$$
$$\left\{\sqrt{h(\xi)}\sqrt{h(\xi')}[(\partial_a\zeta_{\mu\nu}\partial^b\zeta_{\mu\nu})(\xi)(\xi-\xi')_a(\xi-\xi')_b + \cdots](X_\alpha(\xi) - X_\alpha(\xi')^{2p}\right\}$$

At one-loop order ($p \leq 1$) one has explicitly the bare counter-terms:

$$\begin{aligned} A_1 &\sim \beta(\Lambda)^4 \int_\xi \sqrt{h(\xi)}(\partial_a\zeta^{\mu\nu}\partial^a\zeta_{\mu\nu})(\xi) \\ A_2 &\sim \beta(\Lambda)^4 \int_\xi \sqrt{h(\xi)}[(\partial_a\zeta^{\alpha\beta})(\partial^a\zeta^{\mu\beta})(\partial_b X_\alpha)(\partial^b X_\mu)(\xi) \\ A_3 &\sim \beta(\Lambda)^0 \int_\xi \sqrt{h(\xi)}\{(\partial^2\zeta^{\alpha\beta})(\partial^2\zeta^{\alpha\beta})\}(\xi) \text{ etc...} \end{aligned} \tag{A.5}$$

Note the absence of a pure Nambu-Goto counter-term due to the fact of our "normalized" form of interaction eq. (A-1), otherwise one has a counter-term associated to it.

At this point we consider the Extrinsic U.V. divergence $X_\mu(\xi) = X_\mu(\xi')$ *but with* $\xi \neq \xi'$.

In the physical situation of line of self-intersections (where the equation $X_\mu(\xi) = X_\mu(\xi')$ always defines a sub-manifold of dimensiona 1). (Note that the Q.C.D'string world-sheet is generically described by the union of vertical surfaces cylinders locally in contact along the self-intersecting vertical lines passing through the points $\sigma_j = \{\xi_j^1, \zeta\}$ with $X_\mu(\xi_j^1, \zeta) = X_\mu(\xi_{j+1}^1, \zeta)$); the object $\zeta_{\mu\nu}(X(\sigma_j))\zeta^{\mu\nu}(X(\sigma_{j+1})) = cos\ \alpha(\xi_j^1; \xi_{j+1}^1)$; the cosinus of the constant angle between the extrinsic surface tangent plane both possessing in common the vertical line $X_\mu(\xi_j^1, \zeta)$ $(0 \leq \zeta \leq 1)$. In this situation eq. A-1 reduces to a pure (intrinsic) self-avoiding action of the cylinder surface's branche with the associated tangent plane above cited. The renormalizability is consequence of the last reference [8]. Anothert important remark to be pointed out is that the usual concepts of Differential Geometry do not apply to the Quantum Geometry of surfaces since the quantum surfaces are "distributional" and not C^k-differentiable objects. As a consequence, differential-topological concepts are meaningless here (Chern class, etc...) ([9] - chapter 13).

Related to others works, we call the reader attention that the above exposed results have nothing to do with the non-sense and wrong studies presented by H. Kleinert (Phys. Lett. 174B, 335 (1986)).

Finally, we *conjecture* (as Maldacena did!) that the simplest supersymmetric version of the self-avoiding extrinsic string "solves" the supersymmetric Q.C.D. ($SU(\infty)$) *if supersymmetry makes sense in quantum field theory.*