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Off-diagonal helicity density matrix elements for vector mesons produced in polarized e^+e^- processes

M. Anselmino[1], M. Bertini[2], F. Caruso[3,4], F. Murgia[5] and P. Quintairos[6]

[1]Dipartimento di Fisica Teorica, Università di Torino and
INFN, Sezione di Torino, Via P. Giuria 1, 10125 Torino, Italy

[2]Theoretical Physics, Lund University
Sölvegatan 14a, S-223 62 Lund, Sweden

[3]Centro Brasileiro de Pesquisas Físicas
R. Dr. Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil

[4]Instituto de Física da UERJ
Rua São Francisco Xavier 524, 20559-900 Rio de Janeiro, Brazil

[5]INFN, Sezione di Cagliari and Dipartimento di Fisica, Università di Cagliari
C.P. 170, I-09042 Monserrato (CA), Italy

[6]Instituto de Física Teórica - UNESP
Rua Pamplona 145, 01405-900 São Paulo, Brazil

Abstract

Final state $q\bar{q}$ interactions give origin to non zero values of the off-diagonal element $\rho_{1,-1}$ of the helicity density matrix of vector mesons produced in e^+e^- annihilations, as confirmed by recent OPAL data on ϕ, D^* and K^*'s. New predictions are given for $\rho_{1,-1}$ of several mesons produced at large x_E and small p_T – *i.e.* collinear with the parent jet – in the annihilation of polarized e^+ and e^-; the results depend strongly on the elementary dynamics and allow further non trivial tests of the Standard Model.

Key-words: Vector mesons; Polarization; Standard model.

1 - Introduction

In a series of papers [1]–[3] it was pointed out how the final state interactions between the q and $\bar{q}$ produced in e^+e^- annihilations – usually neglected, but indeed necessary – might give origin to non zero values of spin observables which would otherwise be forced to vanish. The off-diagonal spin density matrix element $\rho_{1,-1}(V)$ of vector mesons may be sizeably different from zero [1, 2] due to a coherent fragmentation process which takes into account $q\bar{q}$ interactions; indeed, predictions were given [3] for several spin 1 particles produced at LEP in two jet events, provided they carry a large fraction x_E of the parent quark energy and have a small intrinsic $\boldsymbol{k}_\perp$, *i.e.* they are collinear with the parent jet.

The values of $\rho_{1,-1}(V)$ are related to the values of the off-diagonal helicity density matrix element $\rho_{+-;-+}(q\bar{q})$ of the $q\bar{q}$ pair, generated in the $e^-e^+ \to q\bar{q}$ process [3]:

$$\rho_{1,-1}(V) \simeq [1 - \rho_{0,0}(V)]\, \rho_{+-;-+}(q\bar{q}) \tag{1}$$

where the value of the diagonal element $\rho_{0,0}(V)$ can be taken from data. The values of $\rho_{+-;-+}(q\bar{q})$ depend on the elementary short distance dynamics and can be computed in the Standard Model. Thus, a measurement of $\rho_{1,-1}(V)$ is a further test of the constituent dynamics, more significant than the usual measurement of cross-sections in that it depends on the product of different elementary amplitudes, rather than on squared moduli. With unpolarized e^+ and e^-

$$\rho_{+-;-+}(q\bar{q}) = \frac{1}{4N_{q\bar{q}}} \sum_{\lambda_-,\lambda_+} M_{+-;\lambda_-\lambda_+}\, M^*_{-+;\lambda_-\lambda_+}\,, \tag{2}$$

where the M's are the helicity amplitudes for the $e^-e^+ \to q\bar{q}$ process and

$$4N_{q\bar{q}} = \sum_{\lambda_q,\lambda_{\bar{q}};\lambda_-,\lambda_+} |M_{\lambda_q\lambda_{\bar{q}};\lambda_-\lambda_+}|^2\,. \tag{3}$$

At LEP energy, $\sqrt{s} = M_Z$, one has [3]

$$\rho_{+-;-+}(q\bar{q}) \simeq \overset{Z}{\rho}_{+-;-+}(q\bar{q}) \simeq \frac{1}{2}\,\frac{(g_V^2 - g_A^2)_q}{(g_V^2 + g_A^2)_q}\,\frac{\sin^2\theta}{1+\cos^2\theta}\ . \tag{4}$$

where g_V and g_A are the Standard Model coupling constants [reported for convenience in Eq. (15)] and θ is the vector meson production angle in the e^-e^+ c.m. frame.

At lower energies, where weak interactions can be neglected, one has:

$$\rho_{+-;-+}(q\bar{q}) \simeq \overset{\gamma}{\rho}_{+-;-+}(q\bar{q}) = \frac{1}{2}\,\frac{\sin^2\theta}{1+\cos^2\theta}\ . \tag{5}$$

Eq. (1) is in good agreement with OPAL Collaboration data on ϕ, D^* and K^*, including the θ dependence induced by Eq. (4) [4, 5]; however, no sizeable value of $\rho_{1,-1}(V)$ for $V = \rho, \phi$ and K^* was observed by DELPHI Collaboration [6]. Further tests are then necessary. Predictions for $\rho_{1,-1}(V)$, with $V = \phi, D^*$ or B^* produced in $NN \to VX$, $\gamma N \to VX$ and $\ell N \to \ell VX$ processes were given in Ref. [7].

We consider here again the process $e^+e^- \to VX$, assuming all possible polarization states for the initial leptons. This might not be a realistic case – polarized e^+e^- beams might not be available in the nearest future – but, as we shall see, the results show such a strong interesting dependence on the spin elementary dynamics, that such a possibility should not be forgotten when planning future e^+e^- colliders. Also, this work is the natural expansion and completion – with all possible cases and theoretical predictions taken into account – of the study undertaken in Ref. [3].

In the next Section we compute the value of $\rho_{+-;-+}(q\bar{q})$ with the most general spin states of e^+ and e^-; in Section 3 we obtain numerical estimates in several particular cases and in Section 4 we give some comments and conclusions.

2 - Computation of $\rho_{+-;-+}(q\bar{q})$

In case of polarized initial leptons Eq. (2) modifies into:

$$\rho^{pol}_{\lambda_q,\lambda_{\bar{q}};\lambda'_q,\lambda'_{\bar{q}}}(q\bar{q}) = \frac{1}{N^{pol}_{q\bar{q}}} \sum_{\lambda_-,\lambda_+,\lambda'_-,\lambda'_+} M_{\lambda_q,\lambda_{\bar{q}};\lambda_-,\lambda_+}\, \rho_{\lambda_-,\lambda_+;\lambda'_-,\lambda'_+}\, M^*_{\lambda'_q,\lambda'_{\bar{q}};\lambda'_-,\lambda'_+} \tag{6}$$

with

$$N^{pol}_{q\bar{q}} = \sum_{\lambda_q,\lambda_{\bar{q}};\lambda_-,\lambda_+,\lambda'_-,\lambda'_+} M_{\lambda_q,\lambda_{\bar{q}};\lambda_-,\lambda_+}\, \rho_{\lambda_-,\lambda_+;\lambda'_-,\lambda'_+}\, M^*_{\lambda_q,\lambda_{\bar{q}};\lambda'_-,\lambda'_+} \tag{7}$$

and where

$$\rho_{\lambda_-,\lambda_+;\lambda'_-,\lambda'_+}(e^-e^+) = \rho_{\lambda_-,\lambda'_-}(e^-)\, \rho_{\lambda_+,\lambda'_+}(e^+) \tag{8}$$

is the helicity density matrix of the incoming independent leptons.

The most general helicity density matrices for the incoming e^- and e^+ are given by

$$\rho(e^-) = \frac{1}{2}\begin{pmatrix} 1+\cos\alpha_- & e^{-i\beta_-}\sin\alpha_- \\ e^{i\beta_-}\sin\alpha_- & 1-\cos\alpha_- \end{pmatrix} \tag{9}$$

and

$$\rho(e^+) = \frac{1}{2}\begin{pmatrix} 1-\cos\alpha_+ & e^{i\beta_+}\sin\alpha_+ \\ e^{-i\beta_+}\sin\alpha_+ & 1+\cos\alpha_+ \end{pmatrix} \tag{10}$$

where α_- and β_- (α_+ and β_+) are respectively the polar and azimuthal angle of the e^- (e^+) spin vectors; we have chosen xz as the scattering plane with e^- (e^+) moving along the positive (negative) direction of z-axis.

Insertion of Eqs. (8)-(10) into Eqs. (6) and (7), neglecting lepton masses, yields

$$\begin{aligned}
\rho^{pol}_{\lambda_q,\lambda_{\bar{q}};\lambda'_q,\lambda'_{\bar{q}}}(q\bar{q}) &= \frac{1}{4N^{pol}_{q\bar{q}}}\Big[(1+\cos\alpha_-)(1+\cos\alpha_+)\, M_{\lambda_q,\lambda_{\bar{q}};+,-}\, M^*_{\lambda'_q,\lambda'_{\bar{q}};+,-} \\
&+ e^{-i(\beta_-+\beta_+)}(\sin\alpha_-\sin\alpha_+)\, M_{\lambda_q,\lambda_{\bar{q}};+,-}\, M^*_{\lambda'_q,\lambda'_{\bar{q}};-,+} \\
&+ e^{i(\beta_-+\beta_+)}(\sin\alpha_-\sin\alpha_+)\, M_{\lambda_q,\lambda_{\bar{q}};-,+}\, M^*_{\lambda'_q,\lambda'_{\bar{q}};+,-} \\
&+ (1-\cos\alpha_-)(1-\cos\alpha_+)\, M_{\lambda_q,\lambda_{\bar{q}};-,+}\, M^*_{\lambda'_q,\lambda'_{\bar{q}};-,+}\Big]
\end{aligned} \tag{11}$$

with

$$\begin{aligned} 4N^{pol}_{q\bar q} &= (1+\cos\alpha_-)\,(1+\cos\alpha_+)\ \left[\,|M_{+-;+-}|^2 + |M_{-+;+-}|^2\,\right] \\ &+ (1-\cos\alpha_-)\,(1-\cos\alpha_+)\ \left[\,|M_{+-;-+}|^2 + |M_{-+;-+}|^2\,\right] \\ &+ 2\,\sin\alpha_-\,\sin\alpha_+\ \mathrm{Re}\left[e^{-i(\beta_- + \beta_+)}\left(M_{+-;+-}\ M^*_{+-;-+}\right.\right. \\ &+ \left.\left. M_{-+;+-}\ M^*_{-+;-+}\right)\right]\ . \end{aligned} \tag{12}$$

In the last equation also quark masses, compared to their energies, have been neglected.

The explicit expressions of the relevant $e^+e^- \to q\bar q$ c.m. helicity amplitudes are given by [3]:

$$M_{\pm\mp;\pm\mp} = e^2(1+\cos\theta)\,[e_q - g_Z(s)(g_V \mp g_A)_l\,(g_V \mp g_A)_q] \tag{13}$$

$$M_{\pm\mp;\mp\pm} = e^2(1-\cos\theta)\,[e_q - g_Z(s)(g_V \pm g_A)_l\,(g_V \mp g_A)_q] \tag{14}$$

with the usual Standard Model coupling constants:

$$\begin{aligned} g^l_V &= -\frac{1}{2} + 2\sin^2\theta_W \qquad g^l_A = -\frac{1}{2} \\ g^{u,c,t}_V &= \frac{1}{2} - \frac{4}{3}\sin^2\theta_W \qquad g^{u,c,t}_A = \frac{1}{2} \\ g^{d,s,b}_V &= -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W \qquad g^{d,s,b}_A = -\frac{1}{2} \\ g_Z(s) &= \frac{1}{4\sin^2\theta_W\cos^2\theta_W}\ \frac{s}{(s-M_Z^2) + iM_Z\Gamma_Z}\,. \end{aligned} \tag{15}$$

By inserting Eqs. (13) and (14) into Eqs. (11) and (12) one obtains:

$$\rho^{pol}_{+-;+-}(q\bar q) = \frac{1}{4N^{pol}_{q\bar q}}\left[(1+\cos^2\theta)\ F^{pol}_{1,q} + \cos\theta\ F^{pol}_{2,q} + \sin^2\theta\ F^{pol}_{3,q}\right]\ , \tag{16}$$

$$\begin{aligned} \rho^{pol}_{+-;-+}(q\bar q) &= \frac{1}{4N^{pol}_{q\bar q}}\Bigg[(1+\cos^2\theta)\,(F^{pol}_{4,q} + iF^{pol}_{5,q}) + \cos\theta\,(F^{pol}_{6,q} + iF^{pol}_{7,q}) \\ &+ \sin^2\theta\,(F^{pol}_{8,q} + iF^{pol}_{9,q})\Bigg] \end{aligned} \tag{17}$$

with

$$N^{pol}_{q\bar q} = (1+\cos^2\theta)\,F^{pol}_{10,q} + \cos\theta\,F^{pol}_{11,q} + \sin^2\theta\,F^{pol}_{12,q}\,. \tag{18}$$

The twelve functions $F^{pol}_{i,q}$ depend on the spin directions of the incoming leptons:

$$\begin{aligned} F^{pol}_{1,q} &\equiv (1+\cos\alpha_+)\,(1+\cos\alpha_-)\Big[e_q^2 + |g_Z|^2\,(g_V - g_A)^2_l\,(g_V - g_A)^2_q \\ &- e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V - g_A)_l\,(g_V - g_A)_q\Big] \\ &+ (1-\cos\alpha_+)\,(1-\cos\alpha_-)\Big[e_q^2 + |g_Z|^2\,(g_V + g_A)^2_l\,(g_V - g_A)^2_q \end{aligned}$$

$$
\begin{aligned}
&- \;\; e_q\, 2\,(\mathrm{Re}\, g_Z)\,(g_V + g_A)_l\,(g_V - g_A)_q \Big] \\
F^{pol}_{2,q} \;&\equiv\;\; (1+\cos\alpha_+)\,(1+\cos\alpha_-)\,2\,\Big[e_q^2 + |g_Z|^2\,(g_V - g_A)_l^2\,(g_V - g_A)_q^2 \\
&- \;\; e_q\, 2\,(\mathrm{Re}\, g_Z)\,(g_V - g_A)_l\,(g_V - g_A)_q \Big] \\
&- \;\; (1-\cos\alpha_+)\,(1-\cos\alpha_-)\,2\,\Big[e_q^2 + |g_Z|^2\,(g_V + g_A)_l^2\,(g_V - g_A)_q^2 \\
&- \;\; e_q\, 2\,(\mathrm{Re}\, g_Z)\,(g_V + g_A)_l\,(g_V - g_A)_q \Big] \\
F^{pol}_{3,q} \;&\equiv\;\; 2\,\sin\alpha_+\,\sin\alpha_-\,\Big[\cos(\beta_+ + \beta_-)\,\Big(e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V - g_A)_q^2 \\
&- \;\; e_q\, 2\,(\mathrm{Re}\, g_Z)\,g_V^l\,(g_V - g_A)_q\Big) + \sin(\beta_+ + \beta_-)\,e_q\,2\,(\mathrm{Im}\, g_Z)\,g_A^l\,(g_V - g_A)_q\Big] \\
F^{pol}_{4,q} \;&\equiv\;\; 2\,\sin\alpha_+\,\sin\alpha_-\,\Big[\cos(\beta_+ + \beta_-)[e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V^2 - g_A^2)_q \\
&- \;\; e_q\, 2\,(\mathrm{Re}\, g_Z)\,g_V^l\,g_V^q] + \sin(\beta_+ + \beta_-)\,e_q\,2\,(\mathrm{Im}\, g_Z)\,g_A^l\,g_V^q\Big] \\
F^{pol}_{5,q} \;&\equiv\;\; 2\,\sin\alpha_+\,\sin\alpha_-\,\Big[\cos(\beta_+ + \beta_-)\,e_q\,2(\mathrm{Im}\, g_Z)\,g_V^l\,g_A^q \\
&+ \;\; \sin(\beta_+ + \beta_-)\,e_q\,2\,(\mathrm{Re}\, g_Z)\,g_A^l\,g_A^q\Big] \\
F^{pol}_{6,q} \;&\equiv\;\; 4\,\sin\alpha_+\,\sin\alpha_-\,\Big[-\cos(\beta_+ + \beta_-)\,e_q\,2\,(\mathrm{Re}\, g_Z)\,g_A^l\,g_A^q \\
&+ \;\; \sin(\beta_+ + \beta_-)\,e_q\,2\,(\mathrm{Im}\, g_Z)\,g_V^l\,g_A^q\Big] \\
F^{pol}_{7,q} \;&\equiv\;\; 4\,\sin\alpha_+\,\sin\alpha_-\,\Big[\cos(\beta_+ + \beta_-)\,e_q\,2\,(\mathrm{Im}\, g_Z)\,g_A^l\,g_V^q - \sin(\beta_+ + \beta_-) \\
&\times \;\; [e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V^2 - g_A^2)_q - e_q\,2\,(\mathrm{Re}\, g_Z)\,g_V^l\,g_V^q]\Big] \\
F^{pol}_{8,q} \;&\equiv\;\; (1+\cos\alpha_+)\,(1+\cos\alpha_-)\Big[e_q^2 + |g_Z|^2\,(g_V - g_A)_l^2\,(g_V^2 - g_A^2)_q \\
&- \;\; e_q\, 2\,(\mathrm{Re}\, g_Z)\,(g_V - g_A)_l\,g_V^q\Big] \\
&+ \;\; (1-\cos\alpha_+)\,(1-\cos\alpha_-)\Big[e_q^2 + |g_Z|^2\,(g_V + g_A)_l^2\,(g_V^2 - g_A^2)_q \\
&- \;\; e_q\, 2\,(\mathrm{Re}\, g_Z)\,(g_V + g_A)_l\,g_V^q\Big] \\
F^{pol}_{9,q} \;&\equiv\;\; (1+\cos\alpha_+)\,(1+\cos\alpha_-)\Big[e_q\,2\,(\mathrm{Im}\, g_Z)\,(g_V - g_A)_l\,g_A^q\Big] \\
&+ \;\; (1-\cos\alpha_+)\,(1-\cos\alpha_-)\Big[e_q\,2\,(\mathrm{Im}\, g_Z)\,(g_V + g_A)_l\,g_A^q\Big] \\
F^{pol}_{10,q} \;&\equiv\;\; (1+\cos\alpha_+)\,(1+\cos\alpha_-)(1/2)\,\Big[e_q^2 + |g_Z|^2\,(g_V - g_A)_l^2\,(g_V^2 + g_A^2)_q \\
&- \;\; e_q\, 2\,(\mathrm{Re}\, g_Z)\,(g_V - g_A)_l\,g_V^q\Big] + (1-\cos\alpha_+)\,(1-\cos\alpha_-)(1/2) \\
&\times \;\; \Big[e_q^2 + |g_Z|^2\,(g_V + g_A)_l^2\,(g_V^2 + g_A^2)_q - e_q\,2\,(\mathrm{Re}\, g_Z)\,(g_V + g_A)_l\,g_V^q\Big] \\
F^{pol}_{11,q} \;&\equiv\;\; (1+\cos\alpha_+)\,(1+\cos\alpha_-)\,2\,\Big[e_q\,(\mathrm{Re}\, g_Z)\,(g_V - g_A)_l\,g_A^q
\end{aligned}
$$

$$
\begin{aligned}
&- \quad |g_Z|^2 \, (g_V - g_A)_l^2 \, (g_V g_A)_q \Big] \\
&- \quad (1 - \cos\alpha_+) \, (1 - \cos\alpha_-) \, 2 \Big[e_q \, (\mathrm{Re}\, g_Z) \, (g_V + g_A)_l \, g_A^q \\
&- \quad |g_Z|^2 \, (g_V + g_A)_l^2 \, (g_V g_A)_q \Big] \\
F_{12,q}^{pol} \;\; &\equiv \quad (\sin\alpha_+ \, \sin\alpha_-) \Big[\cos(\beta_+ + \beta_-) \, [e_q^2 - e_q \, 2 \, (\mathrm{Re}\, g_Z) \, g_V^l \, g_V^q \\
&+ \quad |g_Z|^2 \, (g_V^2 - g_A^2)_l \, (g_V^2 + g_A^2)_q] + \sin(\beta_+ + \beta_-) \, [e_q \, 2 \, (\mathrm{Im}\, g_Z) \, g_A^l \, g_V^q] \Big] .
\end{aligned}
\tag{19}
$$

Eqs. (17)-(19) give the most general expression of $\rho_{+-;-+}^{pol}(q\bar{q})$ for a $q\bar{q}$ pair obtained in the annihilation process of polarized leptons, $e^- e^+ \to q\bar{q}$, at lowest perturbative order in the Standard Model, taking into account both weak and electromagnetic interactions (γ and Z_0 exchanges).

3 - Numerical values of $\boldsymbol{\rho_{+-;-+}(q\bar{q})}$

Let us now consider different polarization states of e^- and e^+. We choose as possible spin directions the 3 coordinate axes, $\hat{x}$, $\hat{y}$, $\hat{z}$, with spin component $\pm 1/2$ along these directions: the corresponding values of (α, β) in Eqs. (9) and (10) are as follows:

$$
\begin{aligned}
+\hat{x} &= (\pi/2, \, 0) \qquad + \hat{y} = (\pi/2, \, \pi/2) \qquad + \hat{z} = (0, \, 0) \\
-\hat{x} &= (\pi/2, \, \pi) \qquad - \hat{y} = (\pi/2, \, 3\pi/2) \qquad - \hat{z} = (\pi, \, \pi)
\end{aligned}
\tag{20}
$$

We have then a total of $6 \times 6 = 36$ possible initial spin states. Many of them will lead to the same value of $\rho_{+-;-+}^{pol}(q\bar{q})$ and it is convenient to group them into the following 9 cases (notice that Case 3 is just listed for completeness, but it gives identically null results due to helicity conservation in the $e^- e^+ Z_0$ and $e^- e^+ \gamma$ vertices):

Case 1:

$\{P(e^-, +\hat{z}) \, , \; P(e^+, +\hat{z})\}$

Case 2:

$\{P(e^-, +\hat{z}) \, , \; P(e^+, +\hat{x})\}$, $\{P(e^-, +\hat{z}) \, , \; P(e^+, -\hat{x})\}$, $\{P(e^-, +\hat{z}) \, , \; P(e^+, +\hat{y})\}$,
$\{P(e^-, +\hat{z}) \, , \; P(e^+, -\hat{y})\}$, $\{P(e^-, +\hat{x}) \, , \; P(e^+, +\hat{z})\}$, $\{P(e^-, -\hat{x}) \, , \; P(e^+, +\hat{z})\}$,
$\{P(e^-, +\hat{y}) \, , \; P(e^+, +\hat{z})\}$, $\{P(e^-, -\hat{y}) \, , \; P(e^+, +\hat{z})\}$

Case 3:

$\{P(e^-, +\hat{z}) \, , \; P(e^+, -\hat{z})\}$, $\{P(e^-, -\hat{z}) \, , \; P(e^+, +\hat{z})\}$

Case 4:

$\{P(e^-, -\hat{z}) \, , \; P(e^+, -\hat{z})\}$

Case 5:

$\{P(e^-, +\hat{x}) \, , \; P(e^+, +\hat{x})\}$, $\{P(e^-, -\hat{x}) \, , \; P(e^+, -\hat{x})\}$, $\{P(e^-, +\hat{y}) \, , \; P(e^+, -\hat{y})\}$,

$\{P(e^-,-\hat{y})\,,\,P(e^+,+\hat{y})\}$

Case 6:

$\{P(e^-,+\hat{x})\,,\,P(e^+,-\hat{x})\}$, $\{P(e^-,-\hat{x})\,,\,P(e^+,+\hat{x})\}$, $\{P(e^-,+\hat{y})\,,\,P(e^+,+\hat{y})\}$, $\{P(e^-,-\hat{y})\,,\,P(e^+,-\hat{y})\}$

Case 7:

$\{P(e^-,+\hat{x})\,,\,P(e^+,+\hat{y})\}$, $\{P(e^-,+\hat{y})\,,\,P(e^+,+\hat{x})\}$, $\{P(e^-,-\hat{x})\,,\,P(e^+,-\hat{y})\}$, $\{P(e^-,-\hat{y})\,,\,P(e^+,-\hat{x})\}$

Case 8:

$\{P(e^-,+\hat{x})\,,\,P(e^+,-\hat{y})\}$, $\{P(e^-,-\hat{y})\,,\,P(e^+,+\hat{x})\}$, $\{P(e^-,-\hat{x})\,,\,P(e^+,+\hat{y})\}$, $\{P(e^-,+\hat{y})\,,\,P(e^+,-\hat{x})\}$

Case 9:

$\{P(e^-,-\hat{z})\,,\,P(e^+,+\hat{x})\}$, $\{P(e^-,-\hat{z})\,,\,P(e^+,+\hat{y})\}$, $\{P(e^-,-\hat{z})\,,\,P(e^+,-\hat{x})\}$, $\{P(e^-,-\hat{z})\,,\,P(e^+,-\hat{y})\}$, $\{P(e^-,+\hat{x})\,,\,P(e^+,-\hat{z})\}$, $\{P(e^-,-\hat{x})\,,\,P(e^+,-\hat{z})\}$, $\{P(e^-,+\hat{y})\,,\,P(e^+,-\hat{z})\}$, $\{P(e^-,-\hat{y})\,,\,P(e^+,-\hat{z})\}$.

The corresponding expressions of the functions $F^{pol}_{i,q}$ are given by:

Case 1:

$$\begin{aligned}
F^{pol,C1}_{1,q} &= 4\Big[e_q^2 + |g_Z|^2\,(g_V - g_A)^2_l\,(g_V - g_A)^2_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V - g_A)_l\,(g_V - g_A)_q\Big] \\
F^{pol,C1}_{2,q} &= 2\,F^{pol,C1}_{1,q} \\
F^{pol,C1}_{3,q} &= F^{pol,C1}_{4,q} = F^{pol,C1}_{5,q} = F^{pol,C1}_{6,q} = F^{pol,C1}_{7,q} = F^{pol,C1}_{12,q} = 0 \\
F^{pol,C1}_{8,q} &= 4\Big[e_q^2 + |g_Z|^2\,(g_V - g_A)^2_l\,(g_V^2 - g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V - g_A)_l\,g^q_V\Big] \\
F^{pol,C1}_{9,q} &= e_q\,8\,(\mathrm{Im}\,g_Z)\,(g_V - g_A)_l\,g^q_A \\
F^{pol,C1}_{10,q} &= 2\,\Big[e_q^2 + |g_Z|^2\,(g_V - g_A)^2_l\,(g_V^2 + g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V - g_A)_l\,g^q_V\Big] \\
F^{pol,C1}_{11,q} &= 8\Big[e_q\,(\mathrm{Re}\,g_Z)\,(g_V - g_A)_l\,g^q_A - |g_Z|^2\,(g_V - g_A)^2_l\,\,(g_V\,g_A)_q\Big]
\end{aligned} \tag{21}$$

Case 2:

$$F^{pol,C2}_{i,q} = (1/2)\,F^{pol,C1}_{i,q} \quad (i = 1 - 12) \tag{22}$$

Case 3:

$$F^{pol,C3}_{i,q} = 0 \quad (i = 1 - 12) \tag{23}$$

Case 4:

$$\begin{aligned}
F^{pol,C4}_{1,q} &= 4\Big[e_q^2 + |g_Z|^2\,(g_V + g_A)^2_l\,(g_V - g_A)^2_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V + g_A)_l\,(g_V - g_A)_q\Big] \\
F^{pol,C4}_{2,q} &= -8\Big[e_q^2 + |g_Z|^2\,(g_V + g_A)^2_l\,(g_V - g_A)^2_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V + g_A)_l\,(g_V - g_A)_q\Big]
\end{aligned}$$

$$
\begin{aligned}
F_{3,q}^{pol,C4} &= F_{4,q}^{pol,C4} = F_{5,q}^{pol,C4} = F_{6,q}^{pol,C4} = F_{7,q}^{pol,C4} = F_{12,q}^{pol,C4} = 0\\
F_{8,q}^{pol,C4} &= 4\Big[e_q^2 + |g_Z|^2\,(g_V + g_A)_l^2\,(g_V^2 - g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V + g_A)_l\,g_V^q\Big]\\
F_{9,q}^{pol,C4} &= e_q\,8\,(\mathrm{Im}\,g_Z)\,(g_V + g_A)_l\,g_A^q\\
F_{10,q}^{pol,C4} &= 2\Big[e_q^2 + |g_Z|^2\,(g_V + g_A)_l^2\,(g_V^2 + g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V + g_A)_l\,g_V^q\Big]\\
F_{11,q}^{pol,C4} &= 8\Big[-e_q\,(\mathrm{Re}\,g_Z)\,(g_V + g_A)_l\,g_A^q + |g_Z|^2\,(g_V + g_A)_l^2\,(g_V g_A)_q\Big]
\end{aligned} \tag{24}
$$

Case 5:

$$
\begin{aligned}
F_{1,q}^{pol,C5} &= 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V - g_A)_q^2 - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,(g_V - g_A)_q\Big]\\
F_{2,q}^{pol,C5} &= 8\Big[-|g_Z|^2\,(g_V g_A)_l\,(g_V - g_A)_q^2 + e_q\,(\mathrm{Re}\,g_Z)\,g_A^l\,(g_V - g_A)_q\Big]\\
F_{3,q}^{pol,C5} &= 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V - g_A)_q^2 - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,(g_V - g_A)_q\Big]\\
F_{4,q}^{pol,C5} &= 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V^2 - g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q\Big]\\
F_{5,q}^{pol,C5} &= e_q\,4\,(\mathrm{Im}\,g_Z)\,g_V^l\,g_A^q\\
F_{6,q}^{pol,C5} &= -e_q\,8\,(\mathrm{Re}\,g_Z)\,g_A^l\,g_A^q\\
F_{7,q}^{pol,C5} &= e_q\,8(\mathrm{Im}\,g_Z)\,g_A^l\,g_V^q\\
F_{8,q}^{pol,C5} &= 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V^2 - g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q\Big]\\
F_{9,q}^{pol,C5} &= e_q\,4\,(\mathrm{Im}\,g_Z)\,g_V^l\,g_A^q\\
F_{10,q}^{pol,C5} &= e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V^2 + g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q\\
F_{11,q}^{pol,C5} &= 4\Big[-e_q(\mathrm{Re}\,g_Z)\,g_A^l g_A^q + |g_Z|^2\,2(g_V g_A)_l\,(g_V g_A)_q\Big]\\
F_{12,q}^{pol,C5} &= e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V^2 + g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q
\end{aligned} \tag{25}
$$

Case 6:

$$
\begin{aligned}
F_{1,q}^{pol,C6} &= 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V - g_A)_q^2 - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,(g_V - g_A)_q\Big]\\
F_{2,q}^{pol,C6} &= 8\Big[-|g_Z|^2\,(g_V g_A)_l\,(g_V - g_A)_q^2 + e_q\,(\mathrm{Re}\,g_Z)\,g_A^l\,(g_V - g_A)_q\Big]\\
F_{3,q}^{pol,C6} &= -2\Big[e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V - g_A)_q^2 - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,(g_V - g_A)_q\Big]\\
F_{4,q}^{pol,C6} &= -2\Big[e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V^2 - g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q\Big]\\
F_{5,q}^{pol,C6} &= -e_q\,4\,(\mathrm{Im}\,g_Z)\,g_V^l\,g_A^q\\
F_{6,q}^{pol,C6} &= e_q\,8\,(\mathrm{Re}\,g_Z)\,g_A^l\,g_A^q\\
F_{7,q}^{pol,C6} &= -e_q\,8\,(\mathrm{Im}\,g_Z)\,g_A^l\,g_V^q\\
F_{8,q}^{pol,C6} &= 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V^2 - g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q\Big]\\
F_{9,q}^{pol,C6} &= e_q\,4\,(\mathrm{Im}\,g_Z)\,g_V^l\,g_A^q
\end{aligned}
$$

$$F^{pol,C6}_{10,q} = e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V^2 + g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q$$
$$F^{pol,C6}_{11,q} = 4\Big[-e_q\,(\mathrm{Re}\,g_Z)\,g_A^l\,g_A^q + |g_Z|^2\,2(g_V g_A)_l\,(g_V g_A)_q\Big]$$
$$F^{pol,C6}_{12,q} = -e_q^2 - |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V^2 + g_A^2)_q + e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q \tag{26}$$

Case 7:

$$F^{pol,C7}_{1,q} = 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V - g_A)_q^2 - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,(g_V - g_A)_q\Big]$$
$$F^{pol,C7}_{2,q} = 4\Big[-|g_Z|^2\,2(g_V g_A)_l\,(g_V - g_A)_q^2 + e_q\,2\,(\mathrm{Re}\,g_Z)\,g_A^l\,(g_V - g_A)_q\Big]$$
$$F^{pol,C7}_{3,q} = e_q\,4\,(\mathrm{Im}\,g_Z)\,g_A^l\,(g_V - g_A)_q$$
$$F^{pol,C7}_{4,q} = e_q\,4\,(\mathrm{Im}\,g_Z)\,g_A^l\,g_V^q$$
$$F^{pol,C7}_{5,q} = e_q\,4\,(\mathrm{Re}\,g_Z)\,g_A^l\,g_A^q$$
$$F^{pol,C7}_{6,q} = e_q\,8\,(\mathrm{Im}\,g_Z)\,g_V^l\,g_A^q$$
$$F^{pol,C7}_{7,q} = -4\Big[e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V^2 - g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q\Big]$$
$$F^{pol,C7}_{8,q} = 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V^2 - g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q\Big]$$
$$F^{pol,C7}_{9,q} = e_q\,4\,(\mathrm{Im}\,g_Z)\,g_V^l\,g_A^q$$
$$F^{pol,C7}_{10,q} = e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V^2 + g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q$$
$$F^{pol,C7}_{11,q} = 4\Big[-e_q\,(\mathrm{Re}\,g_Z)\,g_A^l\,g_A^q + |g_Z|^2\,2\,(g_V g_A)_l\,(g_V g_A)_q\Big]$$
$$F^{pol,C7}_{12,q} = e_q\,2\,(\mathrm{Im}\,g_Z)\,g_A^l\,g_V^q \tag{27}$$

Case 8:

$$F^{pol,C8}_{1,q} = 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V - g_A)_q^2 - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,(g_V - g_A)_q\Big]$$
$$F^{pol,C8}_{2,q} = 8\Big[-|g_Z|^2\,(g_V g_A)_l\,(g_V - g_A)_q^2 + e_q\,(\mathrm{Re}\,g_Z)\,g_A^l\,(g_V - g_A)_q\Big]$$
$$F^{pol,C8}_{3,q} = -e_q\,4\,(\mathrm{Im}\,g_Z)\,g_A^l\,(g_V - g_A)_q$$
$$F^{pol,C8}_{4,q} = -e_q\,4\,(\mathrm{Im}\,g_Z)\,g_A^l\,g_V^q$$
$$F^{pol,C8}_{5,q} = -e_q\,4\,(\mathrm{Re}\,g_Z)\,g_A^l\,g_A^q$$
$$F^{pol,C8}_{6,q} = -e_q\,8\,(\mathrm{Im}\,g_Z)\,g_V^l\,g_A^q$$
$$F^{pol,C8}_{7,q} = 4\Big[e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V^2 - g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q\Big]$$
$$F^{pol,C8}_{8,q} = 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V^2 - g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q\Big]$$
$$F^{pol,C8}_{9,q} = e_q\,4\,(\mathrm{Im}\,g_Z)\,g_V^l\,g_A^q$$
$$F^{pol,C8}_{10,q} = e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V^2 + g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q$$
$$F^{pol,C8}_{11,q} = 4\Big[-e_q\,(\mathrm{Re}\,g_Z)\,g_A^l\,g_A^q + |g_Z|^2\,2(g_V g_A)_l\,(g_V g_A)_q\Big]$$
$$F^{pol,C8}_{12,q} = -e_q\,2\,(\mathrm{Im}\,g_Z)\,g_A^l\,g_V^q \tag{28}$$

Case 9:

$$F_{i,q}^{pol,C9} = (1/2)\, F_{i,q}^{pol,C4} \quad (i = 1 - 12) \tag{29}$$

We can now compute $\rho^{pol}_{+-;-+}(q\bar{q})$ for any initial lepton spin state, and at any energy, by using Eqs. (21)-(29), together with Eq. (15), in Eqs. (17) and (18). We do it here first at the Z_0 pole, $\sqrt{s} = M_Z$, where

$$g_Z(s = M_Z^2) = -i\, \frac{M_Z/\Gamma_Z}{4\, \sin^2\theta_W\, \cos^2\theta_W}\ . \tag{30}$$

Taking [8] $\sin^2\theta_W = 0.231$, $M_Z = 91.187$ GeV/c^2 and $\Gamma_Z = 2.490$ GeV yields for u-type quarks:

$$\begin{aligned}
\rho^{pol,C1,C2}_{+-;-+}(u\bar{u}; \sqrt{s} = M_Z) &= -0.369\,(1 + i\,0.132)\, \frac{\sin^2\theta}{1 + \cos^2\theta - 1.335\, \cos\theta} \\
\rho^{pol,C4,C9}_{+-;-+}(u\bar{u}; \sqrt{s} = M_Z) &= -0.370\,(1 - i\,0.113)\, \frac{\sin^2\theta}{1 + \cos^2\theta + 1.336\, \cos\theta} \\
\mathrm{Re}\,[\rho^{pol,C5}_{+-;-+}(u\bar{u}; \sqrt{s} = M_Z)] &= -0.371\, \frac{0.003 - \cos^2\theta}{0.008 + \cos^2\theta + 0.102\, \cos\theta} \\
\mathrm{Im}\,[\rho^{pol,C5}_{+-;-+}(u\bar{u}; \sqrt{s} = M_Z)] &= +0.371\, \frac{0.009 + 0.047\, \cos\theta}{0.008 + \cos^2\theta + 0.102\, \cos\theta} \\
\mathrm{Re}\,[\rho^{pol,C6}_{+-;-+}(u\bar{u}; \sqrt{s} = M_Z)] &= -0.371\, \frac{1 - 0.003\, \cos^2\theta}{1 + 0.008\, \cos^2\theta + 0.102\, \cos\theta} \\
\mathrm{Im}\,[\rho^{pol,C6}_{+-;-+}(u\bar{u}; \sqrt{s} = M_Z)] &= -0.371\, \frac{0.009\, \cos^2\theta + 0.047\, \cos\theta}{1 + 0.008\, \cos^2\theta + 0.102\, \cos\theta} \\
\mathrm{Re}\,[\rho^{pol,C7}_{+-;-+}(u\bar{u}; \sqrt{s} = M_Z)] &= -0.374\, \frac{0.911 - \cos^2\theta - 0.018\, \cos\theta}{1 + 0.934\, \cos^2\theta + 0.195\, \cos\theta} \\
\mathrm{Im}\,[\rho^{pol,C7}_{+-;-+}(u\bar{u}; \sqrt{s} = M_Z)] &= +0.374\, \frac{0.009\, \sin^2\theta - 1.901\, \cos\theta}{1 + 0.934\, \cos^2\theta + 0.195\, \cos\theta} \\
\mathrm{Re}\,[\rho^{pol,C8}_{+-;-+}(u\bar{u}; \sqrt{s} = M_Z)] &= -0.374\, \frac{1 - 0.911\, \cos^2\theta + 0.018\, \cos\theta}{0.934 + \cos^2\theta + 0.195\, \cos\theta} \\
\mathrm{Im}\,[\rho^{pol,C8}_{+-;-+}(u\bar{u}; \sqrt{s} = M_Z)] &= +0.374\, \frac{0.009\, \sin^2\theta + 1.901\, \cos\theta}{0.934 + \cos^2\theta + 0.195\, \cos\theta}
\end{aligned} \tag{31}$$

and for d-type quarks

$$\begin{aligned}
\rho^{pol,C1,C2}_{+-;-+}(d\bar{d}; \sqrt{s} = M_Z) &= -0.176\,(1 + i\,0.108)\, \frac{\sin^2\theta}{1 + \cos^2\theta - 1.871\, \cos\theta} \\
\rho^{pol,C4,C9}_{+-;-+}(d\bar{d}; \sqrt{s} = M_Z) &= -0.176\,(1 - i\,0.092)\, \frac{\sin^2\theta}{1 + \cos^2\theta + 1.871\, \cos\theta}
\end{aligned}$$

$$
\begin{aligned}
\mathrm{Re}\,[\rho^{pol,C5}_{+-;-+}(d\bar{d};\sqrt{s}=M_Z)] &= -0.176\,\frac{0.004-\cos^2\theta}{0.006+\cos^2\theta+0.142\,\cos\theta} \\
\mathrm{Im}\,[\rho^{pol,C5}_{+-;-+}(d\bar{d};\sqrt{s}=M_Z)] &= +0.176\,\frac{0.008+0.069\,\cos\theta}{0.006+\cos^2\theta+0.142\,\cos\theta} \\
\mathrm{Re}\,[\rho^{pol,C6}_{+-;-+}(d\bar{d};\sqrt{s}=M_Z)] &= -0.176\,\frac{1-0.004\,\cos^2\theta}{1+0.006\,\cos^2\theta+0.142\,\cos\theta} \\
\mathrm{Im}\,[\rho^{pol,C6}_{+-;-+}(d\bar{d};\sqrt{s}=M_Z)] &= -0.176\,\frac{0.008\,\cos^2\theta+0.069\,\cos\theta}{1+0.006\,\cos^2\theta+0.142\,\cos\theta} \\
\mathrm{Re}\,[\rho^{pol,C7}_{+-;-+}(d\bar{d};\sqrt{s}=M_Z)] &= -0.184\,\frac{0.872-\cos^2\theta-0.014\,\cos\theta}{1+0.953\,\cos^2\theta+0.276\,\cos\theta} \\
\mathrm{Im}\,[\rho^{pol,C7}_{+-;-+}(d\bar{d};\sqrt{s}=M_Z)] &= +0.184\,\frac{0.007\,\sin^2\theta-1.855\,\cos\theta}{1+0.953\,\cos^2\theta+0.276\,\cos\theta} \\
\mathrm{Re}\,[\rho^{pol,C8}_{+-;-+}(d\bar{d};\sqrt{s}=M_Z)] &= -0.184\,\frac{1-0.872\,\cos^2\theta+0.014\,\cos\theta}{0.953+\cos^2\theta+0.276\,\cos\theta} \\
\mathrm{Im}\,[\rho^{pol,C8}_{+-;-+}(d\bar{d};\sqrt{s}=M_Z)] &= +0.184\,\frac{0.007\,\sin^2\theta+1.855\,\cos\theta}{0.953+\cos^2\theta+0.276\,\cos\theta} \qquad (32)
\end{aligned}
$$

At lower energies, instead, where one can neglect all weak interactions [that is, setting $g_Z = 0$ in Eqs. (21)-(29) and taking into account quark masses] one obtains for any flavour:

$$
\begin{aligned}
\rho^{pol,C1,C2,C4,C9}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_Z) &= \frac{1}{2}\,\frac{\sin^2\theta}{1+\cos^2\theta+\epsilon^2\sin^2\theta} \\
\rho^{pol,C5}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_Z) &= \frac{1}{2} \\
\rho^{pol,C6}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_Z) &= -\frac{1}{2}\,\frac{\cos^2\theta}{\cos^2\theta+\epsilon^2\sin^2\theta} \\
\mathrm{Re}\,[\rho^{pol,C7,C8}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_Z)] &= \frac{1}{2}\,\frac{\sin^2\theta}{1+\cos^2\theta+\epsilon^2\sin^2\theta} \\
\mathrm{Im}\,[\rho^{pol,C7}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_Z)] &= \frac{-\cos\theta}{1+\cos^2\theta+\epsilon^2\sin^2\theta} \\
\mathrm{Im}\,[\rho^{pol,C8}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_Z)] &= \frac{\cos\theta}{1+\cos^2\theta+\epsilon^2\sin^2\theta}\,, \qquad (33)
\end{aligned}
$$

where $\epsilon = 2m_q/\sqrt{s}$, which, for heavy flavours, might not be negligible at $\sqrt{s} \ll M_Z$.

Insertion of Eqs. (31) and (32) or (33) into Eq. (1) allows to give predictions for the relation between $\rho_{1,-1}(V)$ and $\rho_{0,0}(V)$, both of them measurable quantities. Eq. (1) holds for vector mesons with a large energy fraction x_E and collinear with the parent jet; q is the quark flavour which contributes dominantly to the final vector meson production (*e.g.* c in D^*); an average should be taken if more than one flavour contributes [3].

Notice that we expect [3] $\rho_{0,0}(V)$ to be independent of the production angle θ, so that the sign of $\rho_{1,-1}(V)$ and its θ dependence are entirely given by the elementary dynamics, via $\rho_{+-;-+}(q\bar{q})$; for unpolarized e^+ and e^- such dynamics is given by Eq. (4) or (5), and for polarized ones by Eqs. (31) and (32) or (33). We turn now to a discussion of these equations and a comparison with the unpolarized case.

4 - Comments and conclusions

We show our numerical results for $\rho^{pol}_{+-;-+}(q\bar{q})$ in Figs. 1-6. We give results only for those cases which strongly differ from the unpolarized case and have such peculiar features which would make a measurement of $\rho_{1,-1}(V)$ in agreement with them an unquestionable test of our approach. In Figs. 1-4 we consider the LEP high energy case, $\sqrt{s} = M_Z$, and in Figs. 5-6 the lower energy case, $\sqrt{s} \ll M_Z$.

In Fig. 1 we plot as functions of θ (the V production angle in the e^-e^+ c.m. frame) the real part of $\rho^{pol}_{+-;-+}(u\bar{u})$ at LEP energy for cases: $C5$, $C6$, $C1,2$ and $C4,9$. Also the value of $\rho_{+-;-+}(u\bar{u})$ for unpolarized leptons is reported for comparison. In Fig. 2 we do the same for d-type quarks.

In Fig. 3 we plot the imaginary part of $\rho^{pol}_{+-;-+}(u\bar{u})$ at LEP energy for cases: $C5$, $C7$ and $C8$. In all other cases, *including the unpolarized one*, such imaginary part is much smaller and should lead to a measurement of Im $\rho_{1,-1}(V) \simeq 0$. The same is done in Fig. 4 for d, s and b quarks.

In Fig. 5 we plot the real part of $\rho^{pol}_{+-;-+}(q\bar{q}; \sqrt{s} \ll M_Z)$ taking into account only electromagnetic interactions for cases $C5$ and $C6$. All other cases give the same result as unpolarized leptons, result which is reported for comparison. Quark masses have been taken into account, setting $\epsilon = 2m_q/\sqrt{s} = 0.1$.

In Fig. 6 we plot the imaginary part of $\rho^{pol}_{+-;-+}(q\bar{q}; \sqrt{s} \ll M_Z)$ taking into account only electromagnetic interactions (and quark masses, $\epsilon = 0.1$) for cases $C7$ and $C8$. In all other cases, *including the unpolarized one*, the imaginary part is zero.

Figs. 1-6 show beyond any possible doubt how the elementary dynamics might lead to very different values of $\rho_{1,-1}(V)$, according to the different spin states of the initial e^+ and e^-. A measurement in agreement with our predictions would confirm in a definite way the necessity of coherent effects in the quark fragmentation and prove all subtleties of the Standard Model dynamics.

Let us further comment on the most typical cases. The possible spin configurations and the definitions of the various cases are listed at the beginning of Section 3. Concerning the real parts at LEP energy – Figs. 1 and 2 – case $C5$ presents the most striking features, both in sign and θ dependence and shows a drastic difference from the unpolarized case; also $C6$ has a peculiar, almost constant, θ dependence which should be easily detectable. These two cases correspond to e^+ and e^- transversely polarized in the same direction, with either parallel or opposite spins. Cases $C1,2$ and $C4,9$ also deviate largely from the unpolarized case, in particular for charge -1/3 quarks: $C1$ and $C4$ correspond to initial leptons with opposite helicities and $C2$, $C9$ to spin configurations in which one of the lepton is longitudinally polarized and the other is transversely polarized.

Cases $C7$ and $C8$, leptons transversely polarized in different directions, lead to results similar to unpolarized leptons for the real part of $\rho^{pol}_{+-;-+}(q\bar{q})$; however, contrary to the unpolarized case, they give large values, strongly varying with θ – Figs. 3 and 4 – for Im $\rho^{pol}_{+-;-+}(q\bar{q})$, which makes them very interesting. Also $C5$ exhibits a peculiar θ dependence in Im $\rho^{pol}_{+-;-+}(q\bar{q})$

At lower energy, when only electromagnetic interactions contribute, cases $C5$ and $C6$ are simple and very interesting – see Fig. 5 – for the real parts of $\rho^{pol}_{+-;-+}(q\bar{q})$; cases $C7$ and $C8$ are unique providers of sizeable imaginary parts of $\rho^{pol}_{+-;-+}(q\bar{q})$, Fig. 6.

We have thus completed the study of the off-diagonal helicity density matrix element $\rho_{1,-1}(V)$ of vector mesons produced in e^+e^- annihilations into two jets, selecting vector mesons with a large energy fraction and small transverse momentum inside one of the jets. The idea was suggested in Refs. [1] and [2], and the first numerical predictions, given in Ref. [3], have been confirmed by some experimental data [4, 5]. We have considered here the most general case of polarized e^+ and e^-; we have given numerical results both at LEP energy, $\sqrt{s} = M_Z$, and for $\sqrt{s} \ll M_Z$, but our formulae, Eqs. (17)-(19) and (21)-(29), are valid at any energy and take into account both electromagnetic and weak interactions.

At the moment, there is no operating e^+e^- collider with polarized beams; however, future generations of linear colliders are being planned and our study might indicate very good reasons to seriously consider polarization options.

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Figure Captions

Fig. 1 - Plot of $\mathrm{Re}[\rho^{pol}_{+-;-+}(u\bar{u};\sqrt{s}=M_Z)]$ as a function of θ (the production angle of the vector meson in the e^-e^+ c.m. frame) for cases: $C5$, $C6$ (both leptons transversely polarized with spins either parallel or anti-parallel); $C1$, $C4$ (leptons with opposite helicities); $C2$, $C9$ (one lepton longitudinally polarized, the other transversely polarized). Also the value of $\rho_{+-;-+}(u\bar{u};\sqrt{s}=M_Z)$ for unpolarized leptons is shown for comparison. In all other cases one obtains results similar to the unpolarized case.

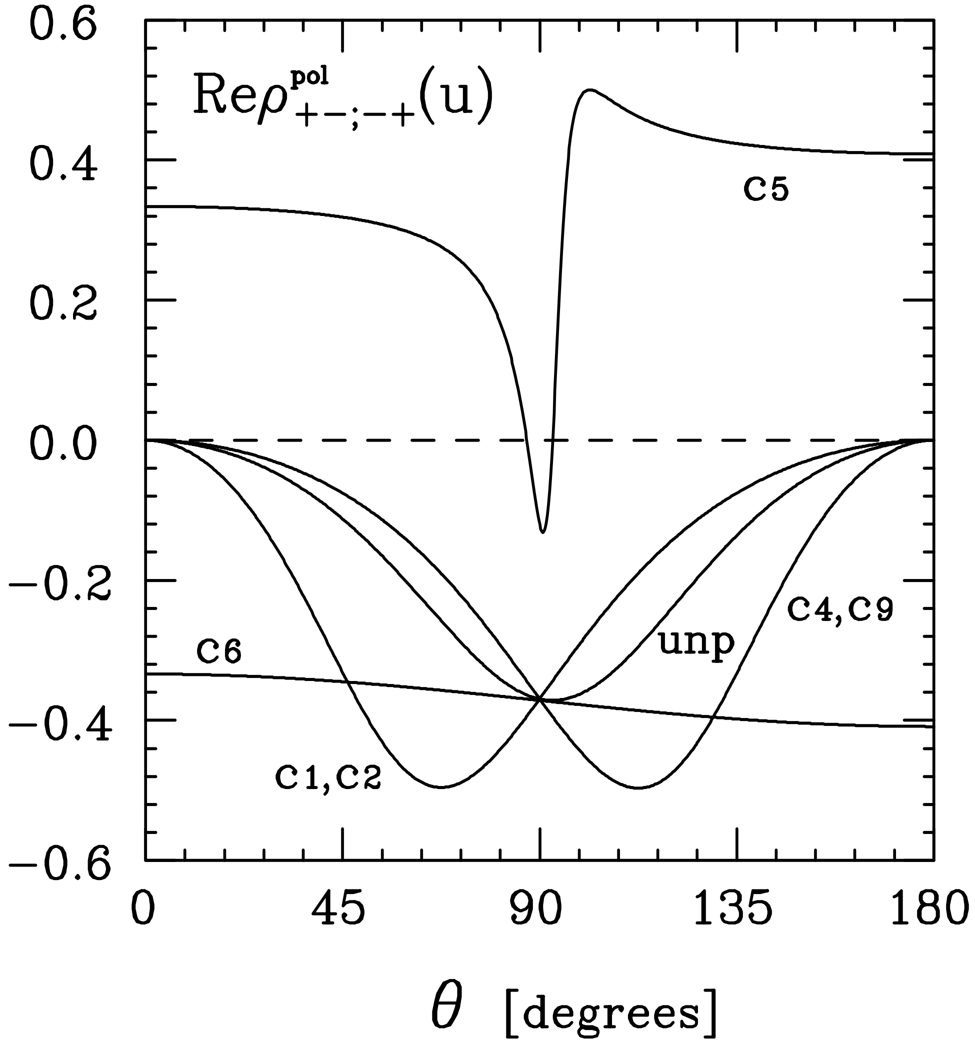
Fig. 2 - The same as in Fig. 1, for d-type quarks.

Fig. 3 - Plot of $\mathrm{Im}[\rho^{pol}_{+-;-+}(u\bar{u};\sqrt{s}=M_Z)]$ as a function of θ (the production angle of the vector meson in the e^-e^+ c.m. frame) for cases: $C5$ (both leptons transversely polarized with spins either parallel or anti-parallel); $C7$, $C8$ (both leptons transversely polarized, in different directions). In all other cases, *including the unpolarized one*, $\mathrm{Im}[\rho^{pol}_{+-;-+}(u\bar{u};\sqrt{s}=M_Z)]\simeq 0$.

Fig. 4 - The same as in Fig. 3, for d-type quarks.

Fig. 5 - Plot of $\mathrm{Re}[\rho^{pol}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_Z)]$ as a function of θ (the production angle of the vector meson in the e^-e^+ c.m. frame) for cases $C5$ and $C6$ (both leptons transversely polarized with spins either parallel or anti-parallel). All other cases give the same result given by unpolarized leptons, which is shown for comparison. Quark masses have been taken into account, with $\epsilon = 2m_q/\sqrt{s} = 0.1$.

Fig. 6 - Plot of $\mathrm{Im}[\rho^{pol}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_Z)]$ for cases $C7$ and $C8$ (both leptons transversely polarized, in different directions). In all other cases, *including the unpolarized one*, $\mathrm{Im}[\rho^{pol}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_Z)]=0$. Again, $\epsilon=0.1$.



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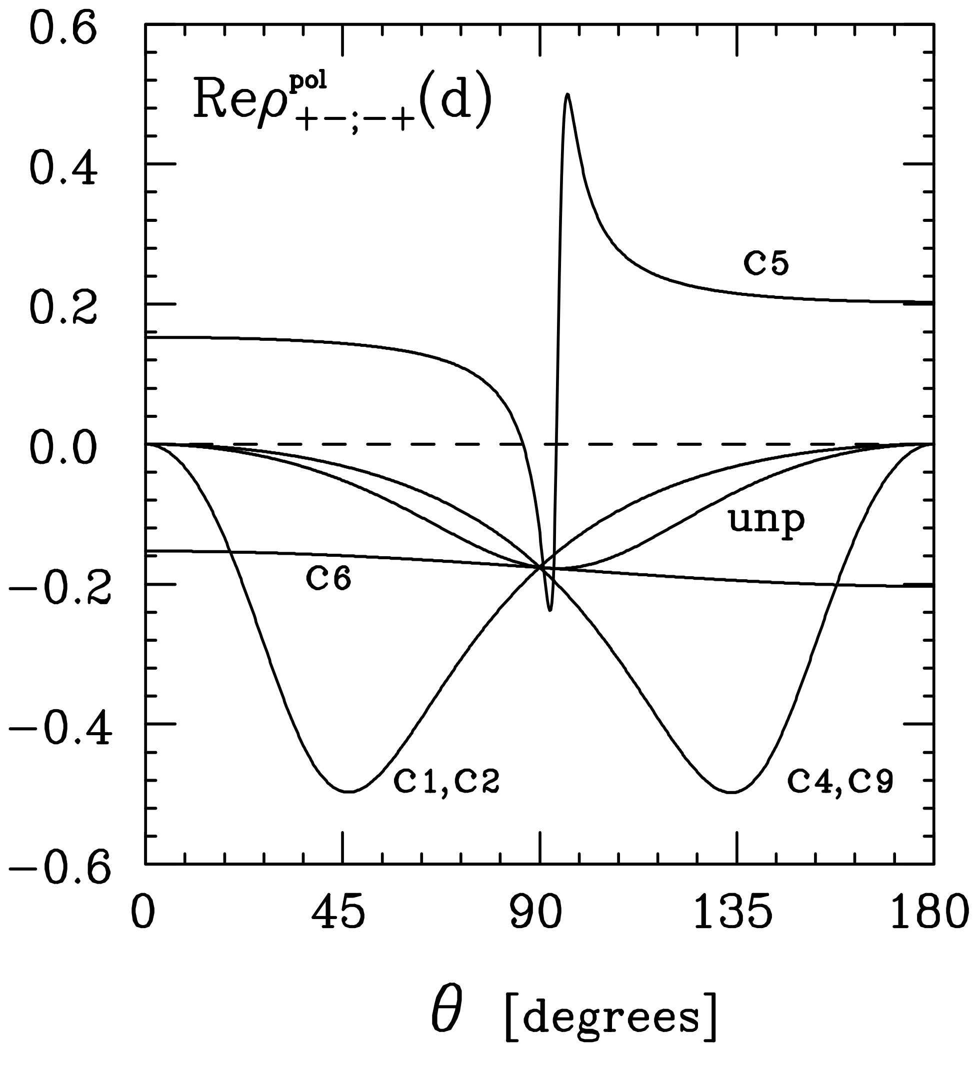

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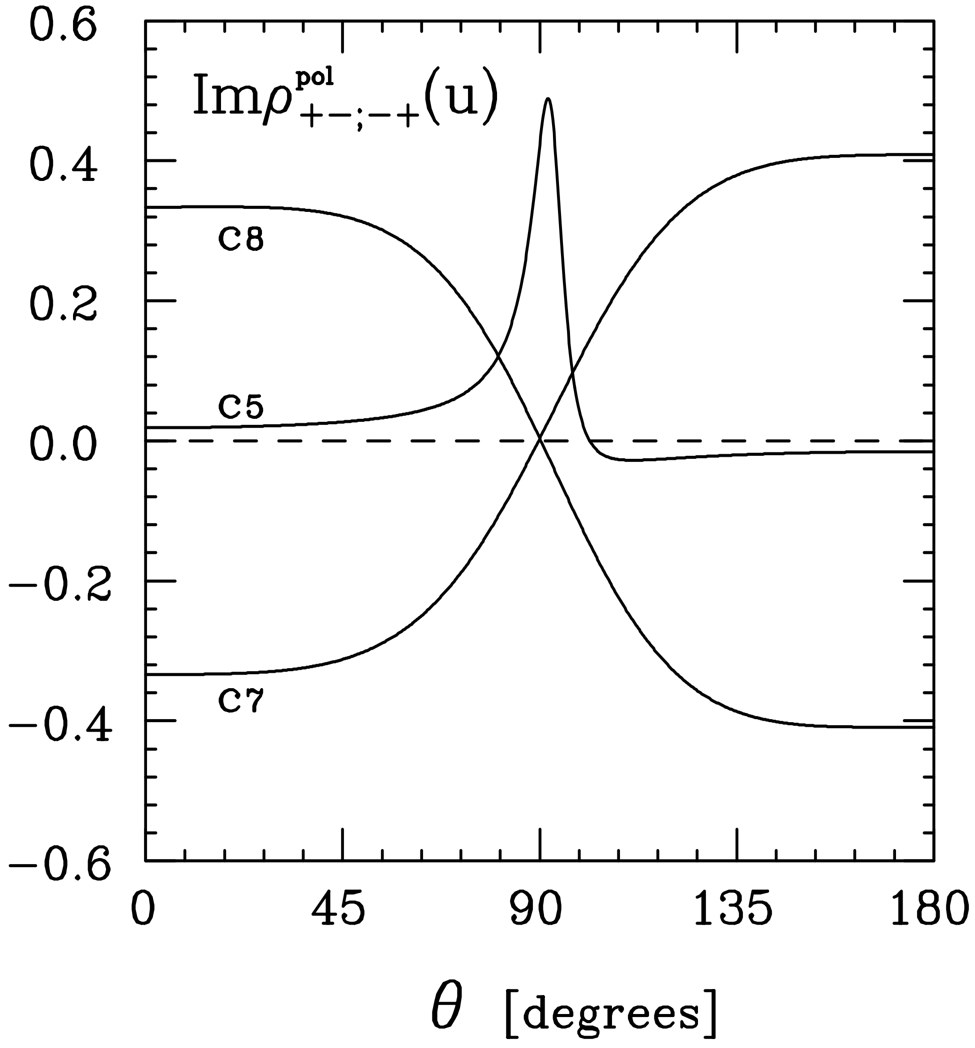

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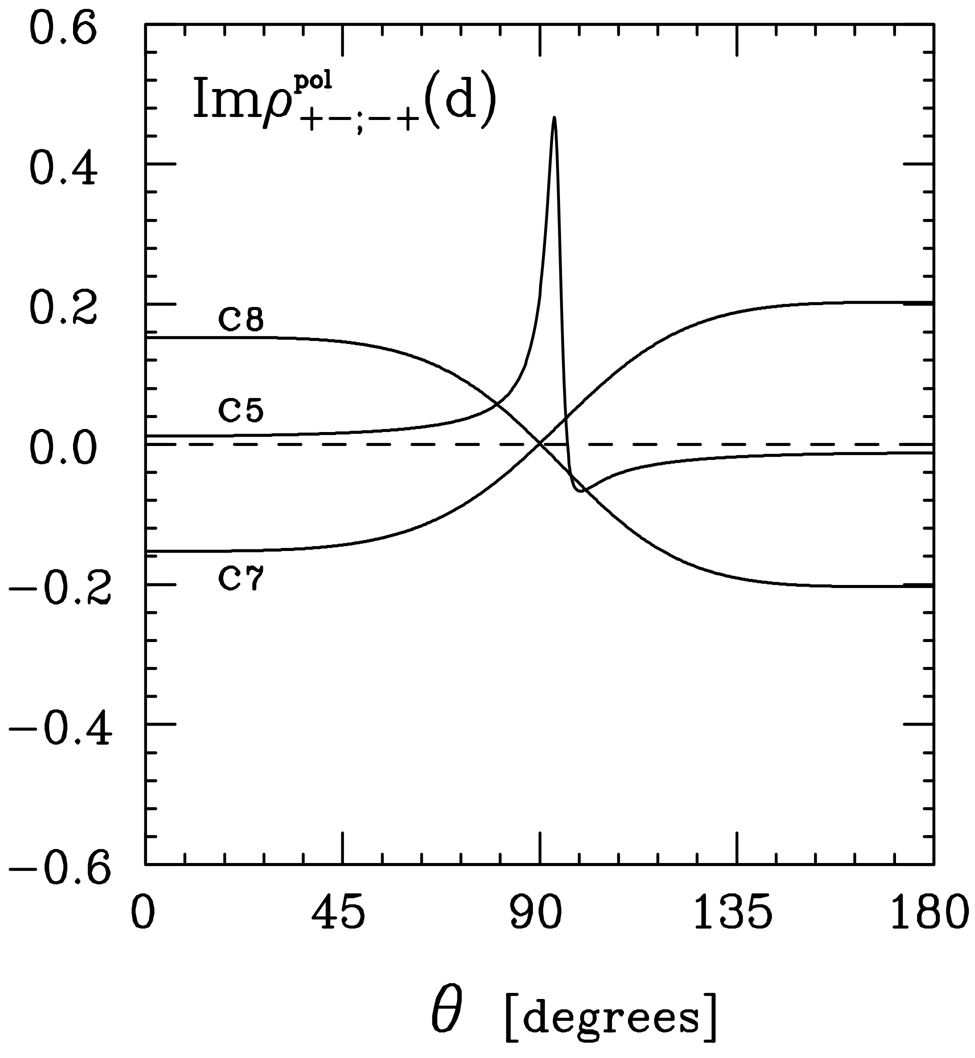

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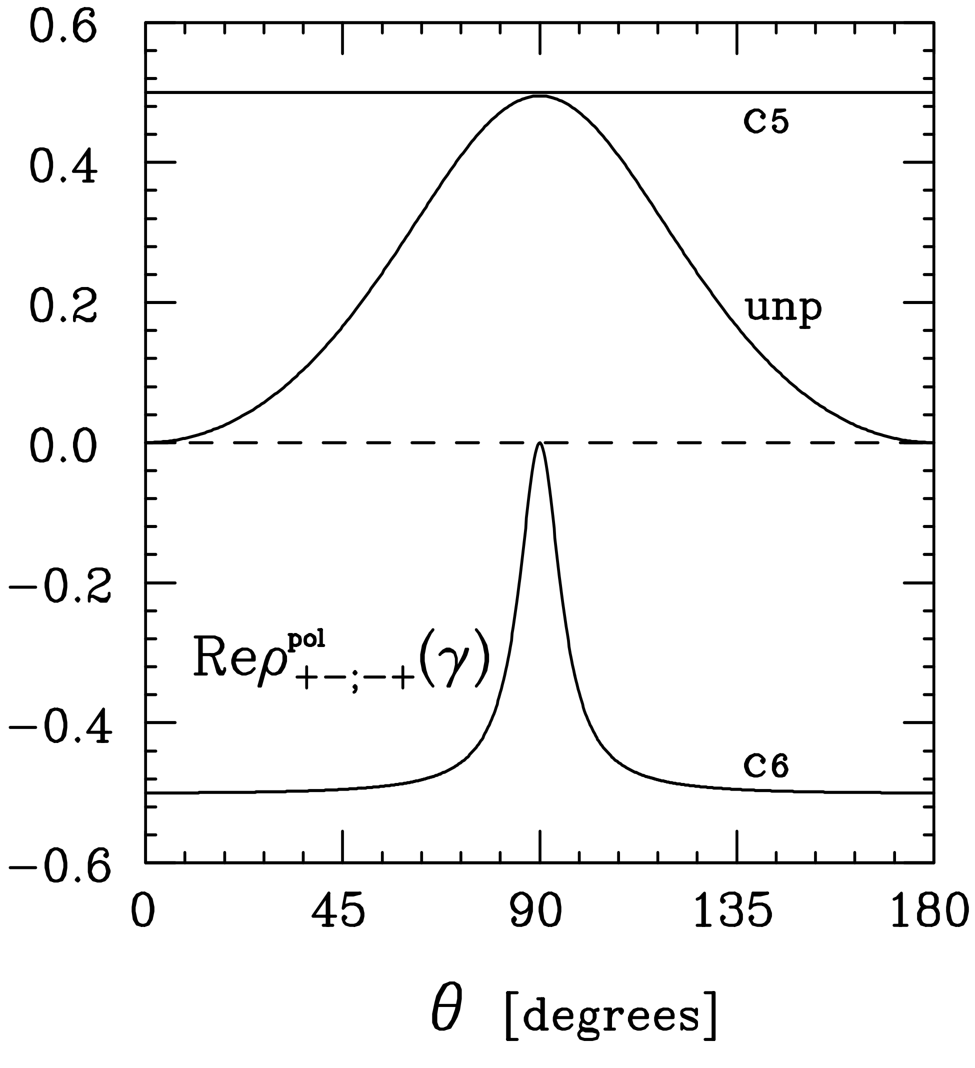

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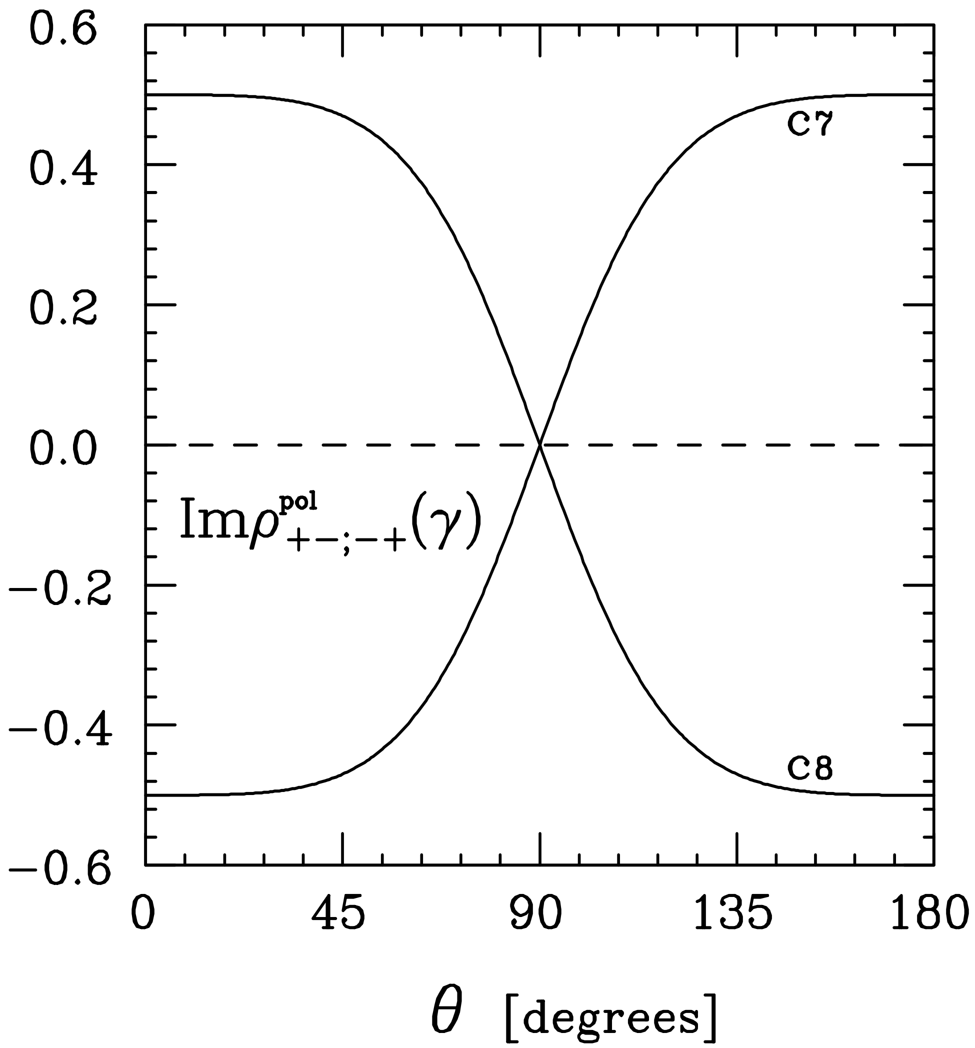

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