Can Purely Electromagnetic Interaction Produce Black Holes?

M. Novello and V. A. De Lorenci

Laboratório de Cosmologia e Física Experimental de Altas Energias.

Centro Brasileiro de Pesquisas Físicas,

Rua Dr. Xavier Sigaud, 150, Urca

CEP 22290-180 – Rio de Janeiro – RJ. Brazil.

and

E. Elbaz

Institut de Physique Nucléaire de Lyon IN2P3-CNRS

Université Claude Bernard

43 Bd du 11 Novembre 1918, F-69622 Villeurbanne Cedex, France.

Abstract

We show that non-linear electrodynamics allows the existence of a mechanism such that photons may be trapped in a compact domain. We call this region an *Electromagnetic Black Hole* (EBH).

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1 Introduction

1.1 Introductory Remarks

One of the most remarkable consequences of the attractive power of gravitational forces is probably the possibility of formation of black holes. However, from a fundamental point of view one should be tempted to ask: is this a typical and exclusive characteristic of gravity? Or, on the other hand, could it be possible that other interactions present analogous properties, e.g., to display a similar behavior like gravitational black holes (GBH), allowing the existence of a hidden part of the spacetime structure, unseen from the outside, without making appeal to gravity? In this paper we provide an affirmative answer to this question. Indeed, we will show that a model can be constructed such that purely electromagnetic forces can effectively lead to such a configuration. How is this possible?

First of all let us make some comments concerning the structure of the metric properties of spacetime in order to introduce our ideas. Since the advent of general relativity it has been widely accepted that the geometry of spacetime is driven uniquely by gravitational forces. Although this is a net consequence of the universality of such interaction, it is certainly not true that some effects of other interactions cannot be described in a similar framework, i. e., such that they can be interpreted as being nothing but a modification of the local metric properties. In order to provide an example – which will be used as the basis of the whole argumentation of the present paper – let us emphasize that it has been known for a long time that the wave propagation of nonlinear interactions could well be described in terms of an effective modification of the metric qualities of the underlying substractum. To be specific and anticipating our result, we shall see that electromagnetic disturbances generated in the framework of a nonlinear theory, do not propagate in an *a priori* Minkowskian background structure, but instead, propagate in a modified geometry that depends only on the character of the nonlinearity of the field. In the case of Born-Infeld theory, for instance, it has been show [1] that it is the energy-momentum density of the non-linear field which is the essential cause by which the characteristic surfaces are in general not null cones of the background Minkowski geometry but instead null cones of another metric. Let us emphasize that the true responsible for the associated curvature of this effective geometry has nothing to do with gravitational process: it is a pure consequence of the assumed non-linearity of the electromagnetic field. We are not interested here in pointing out the obvious differences based on the distinct property that makes the geometrisation related to gravity to be a universal one[1], in comparison with that produced by electromagnetic forces. Notwithstanding its fundamental importance, for our purposes here such a distinction is not relevant.

Since all our argumentation in the present paper rests on electrodynamics and moreover on its nonlinearity, a few comments on it seems necessary. The modification of the metric properties of the underlying geometry through which the electromagnetic waves propagate is not an universal phenomenon. This means that the effective geometry is nothing but a convenient choice of representation of the field propagation in certain

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[1]This is due to the validity of the equivalence principle.

circumstances[2] (see, ref. [2] for more details).

We are concerned here with the propagation of disturbances in a non-linear eletro-magnetic theory. This will be taken as a fundamental process. However, it seems worth to remark that such method of dealing with an equivalent geometry could well be applied to model processes occurring in the interior of a medium that contains interactions of charged particles and currents like, for instance, in a plasma. The reason for this is simple: it is the non-linear nature of the process that is the relevant condition for the application of the method of dealing with modified geometry. Thus, it should not be a surprise that some of the consequences that we present here could well be tested in a terrestrial laboratory under special circumstances. As a major consequence of the equiv-alent geometrical interpretation an important property of the theory appears which can be synthesized as:

- **The discontinuities of nonlinear electromagnetic theories propagates in an effective non-Minkowskian geometry dependent only on the field properties.**

Let us show this and examine some of its consequences.

2   The General Framework

The nonlinear electrodynamic theory[3] is described by a Lagrangian L given uniquely in terms of the invariant $F \equiv F_{\mu\nu} F^{\mu\nu}$. We set[4]

$$L = L(F). \tag{1}$$

The corresponding equation of motion is given by

$$\{L_F F^{\mu\nu}\}_{,\nu} = 0. \tag{2}$$

in which L_F represents the functional derivative of the Lagrangian $(\delta L/\delta F)$ with respect to invariant F; L_{FF} is the second derivative.

This equation can be written in another, more appealing form, by just isolating the linear Maxwell term and taking all remaining non linear parts as an additional *internal* current to be added to the external one:

$$F^{\mu\nu}{}_{,\nu} = J^\mu_{int} + J^\mu_{ext} \tag{3}$$

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[2]This is not a crucial distinction from General Relativity as it can appears in the first sight, since, as it has been shown (see, for instance refs. [2], [3]), Einstein presentation of his gravity theory is nothing but a choice of representation. Indeed, all contents of GR can be depicted without appeal to the geometrical description. However, due to its universality, such (geometric) representation acquired a status of uniqueness. It is precisely such universality that makes the most important difference between these processes. The equivalence principle guarantees that the geometry modification induced by gravity acts in all forms of existing matter and energy; on the other hand, only the *non-linear photon* experiences the modification of the geometry in the non-linear electrodynamics.

[3]We note that these remarks concern any spin-1 theory.

[4]We do not consider here the invariant constructed with the dual $F^*_{\mu\nu}$.

in which the associated *internal* current, the self-term is given by

$$J^{\mu}_{int} \equiv - \frac{L_{FF}}{L_F} F_{,\nu} F^{\mu\nu}. \tag{4}$$

Written under this form it can be thought as nothing but a modelling of the response in a self-interacting way of some special plasma medium. Indeed, let us consider the quantity χ_{ν},

$$\chi_{\nu} \equiv \frac{L_{FF}}{L_F} F_{,\nu}. \tag{5}$$

We define a normalized frame $n^{\mu} \equiv \chi^{\mu}/\chi^2$, which in the case χ^{μ} is time-like in the Minkowski background, could be identified to a real observer that co-moves with χ^{μ}. The extra term of the current assumes then the form

$$J^{int}_{\ \mu} = \sigma E_{\mu} \tag{6}$$

in which E_{μ} is the electric part of the field as seen in the frame n^{μ} and $\sigma(F)$ may depend on the field variables in a complicated way. Under the form of Eq. (6) the analogy with situations treated within Maxwell electrodynamics in material media is transparent.

From the definition of the energy-momentum tensor we obtain from the non-linear Lagrangian:

$$T_{\mu\nu} = -L\gamma_{\mu\nu} - 4L_F F_{\mu\alpha} F^{\alpha}_{\ \nu}. \tag{7}$$

Using the equation of motion and after some manipulation, one obtains the expression that contains all information of the balance of forces through the exchange of energy of the field and the currents independently of the particular form of the Lagrangian. Indeed, we obtain

$$T^{\mu\nu}_{\ \ ,\nu} = -F^{\mu\nu} J^{ext}_{\nu}. \tag{8}$$

2.1 Propagation of the Discontinuities in Non-Linear Electro-dynamics

Let Σ be a surface of discontinuity for the electromagnetic field. Following Hadamard's [4] condition let us assume that the field is continuous through Σ but its first derivative is discontinuous. We set

$$[F_{\mu\nu}]_{\Sigma} = 0, \tag{9}$$

and

$$[F_{\mu\nu,\lambda}]_{\Sigma} = f_{\mu\nu} k_{\lambda}, \tag{10}$$

in which the symbol $[J]_{\Sigma}$ represents the discontinuity of the function J through the surface Σ.

Applying these conditions into the equation of motion (2) we obtain

$$L_F f^{\mu\nu} k_{\nu} + 2L_{FF} \xi F^{\mu\nu} k_{\nu} = 0, \tag{11}$$

where ξ is defined by

$$\xi \equiv F^{\alpha\beta} f_{\alpha\beta}. \tag{12}$$

The cyclic identity yields

$$f_{\mu\nu} k_\lambda + f_{\nu\lambda} k_\mu + f_{\lambda\mu} k_\nu = 0. \tag{13}$$

Multiplying this equation by $k^\lambda F^{\mu\nu}$ yields

$$\xi k_\nu k_\mu \eta^{\mu\nu} + 2 F^{\mu\nu} f_{\nu\lambda} k^\lambda k_\mu = 0. \tag{14}$$

From the Eq. (11) it results:

$$f_{\mu\nu} k^\nu = -2 \frac{L_{FF}}{L_F} \xi F_{\mu\nu} k^\nu. \tag{15}$$

After some algebraic manipulations the equation of propagation of the disturbances is obtained:

$$\{\eta_{\mu\nu} + \Lambda_{\mu\nu}\} k^\mu k^\nu = 0 \tag{16}$$

in which the quantity $\Lambda_{\mu\nu}$ is

$$\Lambda_{\mu\nu} \equiv -4 \frac{L_{FF}}{L_F} F_\mu{}^\alpha F_{\alpha\nu}. \tag{17}$$

The main lesson we learn from this is that in the non-linear electrodynamics the disturbances propagate not in the Minkowskian background but in an effective geometry which depends on the energy distribution of the field. The net effect of the nonlinearity can thus be summarized in the following property.

- The disturbances of nonlinear electrodynamics are null geodesics that propagate in the modified effective geometry:

$$g_{\mu\nu} = \gamma_{\mu\nu} - 4 \frac{L_{FF}}{L_F} F_\mu{}^\alpha F_{\alpha\nu}. \tag{18}$$

In these formulas $\gamma_{\mu\nu}$ is the Minkowski metric written in an arbitrary system of coordinates. A simple inspection on this formula shows that only in the particular linear case of Maxwell electrodynamics does the discontinuity of the eletromagnetic field propagate in a Minkowski background. From equation (18), we obtain the specific form of the components of the metric tensor:

$$g_{00} = 1 - 4 \frac{L_{FF}}{L_F} E^2, \tag{19}$$

$$g_{ij} = \gamma_{ij} + 4 \frac{L_{FF}}{L_F} \left(E_i E_j + B_i B_j - \gamma_{ij} B^k B_k \right), \tag{20}$$

$$g_{oi} = -4 \frac{L_{FF}}{L_F} \epsilon_{ijk} E^j B^k, \tag{21}$$

in which we have set $E^2 = - E_\alpha E^\alpha$.

For future references we note that in the case we will be concerned here, the ratio L_{FF}/L_F is a constant. Thus, the g_{00} component of the effective geometry depends only on the electric part of the field, while the off-diagonal terms g_{0i} are directly related to the Poynting vector. We can thus exhibit the conditions in which the effective metric can be

written in a Gaussian system of coordinates: the vanishing of the Poynting vector and the constancy of the norm of the electric field.

A direct inspection on the above formulas of the associated geometry allows us to envisage the possibility of generating a surface of infinite red shift exclusively in terms of eletromagnetic processes. This situation happens to occur when the properties of the non linearity is such that it induces the equality

$$E^2 = \frac{L_F}{4L_{FF}}.$$

(22)

Before going into a specific model, let us make here a comment. Linear photons propagate in a Minkowskian underlying background. Non linear photons propagate in an effective geometry given by Eq. (18). Note, however, that this situation is not competitive to gravity processes. The reason for this is easy to understand: the above modified geometry (in case of non-linear electrodynamics) is not a universal one. Indeed, other kinds of particles and radiations behave as if the background metric is that dealt with in special relativity: the charged particles, e. g., electrons follow time-like paths with respect to Minkowski metric.

3   The Model

In order to show a specific situation that represents a configuration of an effective geometry for electromagnetic forces, let us concentrate here in a simple model. We set

$$L = b \left(1 - e^{\frac{F}{4b}} \right),$$

(23)

where the constant b has dimensionality of density of energy (we use units in which $c = 1$).

For our purposes, it is convenient to seek for a spherically symmetric and static solution of this theory. A direct computation shows that in the spherical coordinate system (t, r, θ, ϕ) a particular solution can be found uniquely in terms of a radial electric component given by:

$$F_{01} \;\; = \;\; f(r),$$

(24)

where the function $f(r)$ obeys the relation:

$$e^{-\frac{f(r)^2}{2b}} f(r) = \frac{4Q}{r^2}$$

(25)

and the parameter Q is related to the electric charge located at the origin.

From the previous section, we conclude that the effect of the nonlinearity on the propagation of the discontinuities is to induce the eletromagnetic waves to follow a null cone of the modified geometry given by

$$g_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{b} F_\mu{}^\beta F_{\beta\nu},$$

(26)

in which the Minkowski metric $\gamma_{\mu\nu}$ has the form

$$ds^2 = dt^2 - dr^2 - r^2\, d\,\theta^2 - r^2 \sin^2\theta\, d\phi^2. \tag{27}$$

It then follows that the non-null components of the metric tensor of the effective geometry, as seen by the photon disturbances, are:

$$g_{00} = \frac{b - f(r)^2}{b}, \tag{28}$$

$$g_{11} = -g_{00}, \tag{29}$$

$$g_{22} = -r^2, \tag{30}$$

$$g_{33} = g_{22} \sin^2\theta. \tag{31}$$

Just for completeness, let us exhibit the non-vanishing components of the energy-momentum tensor of the field:

$$T^0{}_0 = \frac{4Q}{r^2}\left[\frac{b + f(r)^2}{f(r)} - b\right], \tag{32}$$

$$T^1{}_1 = T^0{}_0, \tag{33}$$

$$T^2{}_2 = b\left(-1 + \frac{4Q}{f(r)\,r^2}\right), \tag{34}$$

$$T^3{}_3 = T^2{}_2. \tag{35}$$

In linear Maxwell theory the energy-momentum tensor is traceless. This property is no longer true in the non-linear case. A direct inspection on this expression shows that the field energy is well behaved throughout all space except at the origin.

3.1 Electromagnetic Black Holes (EBH)

Eletromagnetic disturbances in a non-linear theory follow null cones (geodesics) in an effective geometry. The best way to analyse the properties of their paths is then to examine the equations of motion of the geodesics in the effective metric $g_{\mu\nu}$ given by

$$ds^2 = \left[1 - \frac{f(r)^2}{b}\right] dt^2 - \left[1 - \frac{f(r)^2}{b}\right] dr^2 - r^2\, d\,\theta^2 - r^2 \sin^2\theta\, d\phi^2. \tag{36}$$

Solving the variational problem $\delta \int ds = 0$ the Euler-Lagrange equations that follow are:

$$\left[1 - \frac{f(r)^2}{b}\right]\dot{t} = l_0, \tag{37}$$

$$r^2\dot{\phi} = h_0, \tag{38}$$

where l_0 and h_0 are constants of motion related to the photon energy. The system was reduced to a planar orbit by the choice of the initial conditions:

$$\dot{\theta} = 0,$$
$$\theta = \frac{\pi}{2}. \tag{39}$$

The null property of the geodesics allows to obtain in a direct way the equation of the radial component. Using Eqs. (36) - (39), results:

$$\dot{r}^2 + \frac{h_0^2}{r^2 \left[1 - \frac{f(r)^2}{b}\right]} - \frac{l_0^2}{\left[1 - \frac{f(r)^2}{b}\right]^2} = 0, \tag{40}$$

We can re-write this equation in the more convenient form:

$$\dot{r}^2 + V_{eff} = l_0^2 \tag{41}$$

in which the potential V_{eff} is then defined by:

$$V_{eff} = \frac{bh_0^2}{r^2 \left[b - f(r)^2\right]} - \frac{b^2 l_0^2}{\left[b - f(r)^2\right]^2} + l_0^2. \tag{42}$$

Thus, the motion of the photon in such non-linear theory can be described as a particle dotted with energy l_0^2 immersed in a central field of forces characterized by the potential V_{eff}. Note that near the critical radius defined by $f(r_c)^2 = b$, the attractive power on photons by the field increases enormously. This is precisely the origin of the infinite barrier that forbids a photon to get out from such region.

From Eq. (37) it follows that the time of an observer at the critical radius runs infinitely slow compared to a clock at infinity. Thus, we can make the statement:

- There exists an infinite red-shift at the critical radius.

4   Conclusion

From what we have learned in this paper, we conclude that configurations of structures containing hidden regions, like black holes, are not restricted to gravitational forces. Non linear eletromagnetic interactions can also produce similar objects. In this paper we presented a simple purely electromagnetic model in which we have neglected all gravitational effects. In principle this is not a wrong procedure, since gravitational forces, in comparison with electromagnetic ones, is by far a weaker process. Nervertheless from a fundamental point of view, one should examine the complete situation by combining both interactions. We postpone this analysis for a forthcoming paper.

5   Acknowledgements

Appendix A

In this appendix we will present a compilation of results of the behavior of non-linear photons in another particular solution in our model. It represents the static field generated by an electric and a magnetic monopoles at the origin of the coordinate system. The solution is given by:

$$
\begin{aligned}
F_{01} &= \frac{Q}{r^2}, \\
F_{23} &= Q \sin \theta.
\end{aligned}
\tag{43}
$$

This solution has the property that the invariant F vanishes, consequently it is also a solution of the linear Maxwell's equations. The effective geometry is provided by:

$$
g_{00} = 1 - \frac{Q^2}{br^4},
\tag{44}
$$

$$
g_{11} = -g_{00},
\tag{45}
$$

$$
g_{22} = -r^2 \left(1 + \frac{Q^2}{br^4} \right),
\tag{46}
$$

$$
g_{33} = g_{22} \sin^2 \theta,
\tag{47}
$$

and the corresponding energy distribution in the field, results:

$$
T^0{}_0 = \frac{Q^2}{r^4},
\tag{48}
$$

$$
T^1{}_1 = T^0{}_0,
\tag{49}
$$

$$
T^2{}_2 = -T^0{}_0,
\tag{50}
$$

$$
T^3{}_3 = T^2{}_2.
\tag{51}
$$

We note that it is singular only in the origin. Since, in this case the invariant F vanishes, the energy momentum tensor becomes traceless.

The wave propagation follows null geodesics in the above effective geometry. The properties of this curves are similar to the previous one, but the potential in which the photon propagates takes the form:

$$
V_{eff} = \frac{h_0^2}{r^2 \left(1 - \frac{Q^4}{b^2 r^8} \right)} - \frac{l_0^2}{\left(1 - \frac{Q^2}{br^4} \right)^2} + l_0^2.
\tag{52}
$$

In this case the constants of motion l_0 and h_0 are given by:

$$
\begin{aligned}
\left(1 - \frac{Q^2}{br^4} \right) \dot{t} &= l_0 \\
r^2 \left(1 + \frac{Q^2}{br^4} \right) \dot{\phi} &= h_0.
\end{aligned}
\tag{53}
$$

Appendix B

The result that we have shown here, concerning the existence of EBH depends not only on the nonlinearity but also on the specific form of theory. For instance, one of the most popular non-linear model, the Born-Infeld theory, does not admit such structure (EBH) in its corresponding static configuration. Indeed, the simplest way to show this is by a direct inspection on the properties of the corresponding static solution for this case. The Born-Infeld Lagrangian is

$$L = -\frac{b^2}{4} \left\{ \sqrt{1 + \frac{2\,F}{b^2}} - 1 \right\}. \tag{54}$$

From the previous formuli, the g_{00} component of the effective geometry takes the form

$$g_{00} = \frac{b^2}{4} \sqrt{1 + \frac{2\,F}{b^2}} + \frac{E^2}{\sqrt{1 + \frac{2\,F}{b^2}}} \tag{55}$$

that is

$$g_{00} = \frac{b}{4} \frac{b^2 + 2\,E^2}{\sqrt{b^2 - 2\,E^2}} \tag{56}$$

which never vanishes for real values of the eletric field.

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