# Cosmic Spinning String and Causal Protecting Capsules 

by

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#### Abstract

A method by which a geometry with causality violation can be taken as a part of a globally causal space-time model is presented. This procedure is applied to a pair of extensions to Gödel's space-time [1]: a "Gödel-generalized " and a stringlike solution. The latter is shown to be an intermediate region between Gödel and deformed Minkowskii geometries. Some conclusions and general ideas concerning structures with closed timelike curves are also presented.


Key-words: Closed timelike curves; Metric connection; Gödel-generalized solution; String; Causal capsules.

## 1 INTRODUCTION

In the last years a great interest has been aroused concerning the study of metrical properties associated to infinite strings. It has been shown that the exterior geometry of the string is conical, associated to a deficit angle which depends uniquely on the linear density of the string [2] [3] [4] [5]. Lately, in order to generate a more realistic situation, the case of a non-singular string (constituting a cilinder of radius $r_{0}$ and energy density $\rho>0$, with a pressure along the $z$-axis giving $\rho+p_{z}=0$ ) has been examined [6] [7].

The natural evolution of such a study led to the introduction of rotation, generating spinning strings. A new qualitative feature then arises: the question of causality violations. Recently some physicists [8] [9] [10] [11] [12] [13] dared to examine the difficulties to reconciliate standard ideas concerning causal problems in spaces admitting the ocurrence of closed timelike curves (CTC's). This kind of curve appeared for the first time in the framework of General Relativity through Gödel's cosmological solution [1]. Although there have been some comments [14] to the extent that Gödel's is an "artificial geometry", it is notwithstanding the true paradigm for spacetimes presenting CTC's and as such cannot be ignored. Nobody doubts that we do not live in a Gödel Universe, but this does not preclude the interest in this special solution. Besides, as we shall show in this paper, it occurs that Gödel's geometry can be enclosed in a compact region which possesses a well behaved exterior extension. In other words, the theory does not forbid that Gödel's geometry exists in a limited bounded region encircled by a membrane (or a capsule) which maintains the required condition of global causality.

There are two most widely used attitudes concerning the above mentioned causality problems: (i)reject all geometries presenting such features as physically undesirable; (ii)keep some form which preserves the most of traditional Physics, e. g. : Cauchy initial value problem [10] [11] [15]. Some authors [16] embarked upon a program trying to connect theorems relating CTC's to physically forbidden situations, e. g. , the violation of the weak energy condition, in a very similar way as it was done for the singularity theorems in the 60's.

This was criticized by Jensen and Soleng [7], who exhibited a solution in which the existence or not of CTC's was not related to the energy conditions. In the same framework, we shall show here that it is possible to exhibit a theoretical model, independent of the energy conditions, which may or may not present CTC's. At this point we will be able to introduce the idea of capsules of causal protection.

## 2 SYNOPSIS

In this article we exhibit a method by which any geometry admitting CTC's can generate a well behaved global causal structure. In our case, this method will be applied for a particular example, namely the Gödel geometry. In the next section we review briefly the geometry of spinning strings and the appearance of causality problems. In Section (4) we examine a deformed Gödel Universe, consisting of an extension of this geometry to a "Gödel- generalized" solution. Section (5) deals with the case of a second deformed Gödel Universe, in which the vorticity changes with the radial coordinate. This allows
us to achieve a connection between Gödel and deformed Minkowskii geometries through an intermediate string-like solution. We end with Section (6), in which some conclusions and general ideas concerning CTC's are presented.

## 3 THE DEFORMED VACUUM

The exterior metric of a spinning string can be written in the cylindrical coordinate system $(t, r, \varphi, z)$ in the form:

$$
\begin{equation*}
d s^{2}=a^{2}\left(d t^{2}-d r^{2}+2 h(r) d \varphi d t+g(r) d \varphi^{2}-d z^{2}\right) \tag{1}
\end{equation*}
$$

In the absence of any energy content the functions $h$ and $g$ are given by:

$$
\begin{align*}
& h(r)=4 G J=\text { constant } \\
& g(r)=(4 G J)^{2}-(1-4 G M)^{2} r^{2} \tag{2}
\end{align*}
$$

This geometry is locally flat but has global gravitational effects which have been examined by many authors ( see, for instance, [2] [3] [6] [7]). We will concentrate here in its causal properties. In order to clarify this let us perform a change of coordinates to a Gaussian system. A very particular choice made by Deser et al [17] [18] yields a new time $T=t+4 G J \varphi$. This generates causal difficulties due to the cyclic character of $\varphi$.

One should wonder if this unusual choice, by imposing a cyclic condition on time, should not be responsible for such a strange causal behaviour. That this is not the case can be shown by a simple inspection of all possible Gaussian coordinate systems for this geometry [19]. To provide for such class of special coordinates one has to solve Jacobi's equation

$$
g^{\mu \nu} \frac{\partial T}{\partial x^{\mu}} \frac{\partial T}{\partial x^{\nu}}=1
$$

for the geometry (1). It then follows that the Gaussian time $T$ is given by the Ansatz:

$$
\begin{equation*}
T=\lambda_{1} t+\lambda_{2} \varphi+\lambda_{3} z+F(r) \tag{3}
\end{equation*}
$$

in which

$$
F(r)=\sqrt{\mu^{2} r^{2}-\nu^{2}}+\nu \arcsin \left(\frac{\nu}{\mu r}\right)
$$

and the constants $\mu$ and $\nu$ are given in terms of the parameters $\lambda_{i}$ by the expressions:

$$
\begin{aligned}
& \mu^{2}=\lambda_{1}^{2}-1 \\
& \nu=\frac{1}{\alpha}\left(4 G J \lambda_{1}-\lambda_{2}\right)
\end{aligned}
$$

Just for completeness, let us exhibit the remaining spacial coordinates $\xi, \eta, z^{\prime}$ :

$$
\begin{aligned}
& \xi=t+\frac{\lambda_{1}}{\mu^{2}} \sqrt{\mu^{2} r^{2}-\nu^{2}}+\frac{4 G J J}{\alpha} \arcsin \left(\frac{\nu}{\mu r}\right) \\
& \eta=\varphi-\frac{1}{\alpha} \arcsin \left(\frac{\nu}{\mu r}\right) \\
& z^{\prime}=z
\end{aligned}
$$

The geometry takes then the Gaussian form

$$
\begin{align*}
d s^{2} & =d T^{2}-\mu^{2} d \xi^{2}-\left(16 G^{2} J^{2} \mu^{2}+(1-4 G M)^{2} r^{2}\right) d \eta^{2} \\
& -8 G J \mu^{2} d \xi d \eta-d z^{\prime 2} \tag{4}
\end{align*}
$$

in which $r=-\mu T+\mu \lambda \xi+4 G J \mu \lambda \eta$. We note that the determinant of $g_{\mu \nu}$, given by

$$
\operatorname{detg}_{\mu \nu}=-\mu^{4}(1-4 G M)^{2}(-T+\lambda \xi+4 G J \lambda \eta)^{2}
$$

becomes singular at the hypersurface $T=\lambda(\xi+4 G J \eta)$.
It is worth to examine two particular values of the parameter $\nu$.
Case i: $\nu=0$
A global Cauchy surface is provided by the equation $T=$ constant. The new time is defined by means of a family of geodesics which intersects a global space-like hypersurface $\Sigma$. In this case the parameter $\lambda_{2}$ cannot be made zero, which then results that $T$ is cyclic: $T=\lambda(t+4 G J \varphi)$. For a matter of continuity the values $T_{0}$ and $T_{0}+8 G J \pi \lambda \varphi$ must be identified.

Case ii: $\nu \neq 0$ with $\lambda_{2}=0$
In this case the Gaussian coordinate system has a hole. The parameters $\mu$ and $\nu$ do not vanish but the Gaussian system is defined only for

$$
r>r_{g} \equiv \frac{\nu}{\mu} .
$$

The region where this system is not defined contains closed timelike curves (CTC's), once $g(r)<0$ for $r<r_{c} \equiv \frac{4 G J}{(1-4 M)}$. We remark that $g(r)<0$ constitutes the condition that the curve with constant $t, r$ and $z$ is closed.

In both cases presented above we face a causality problem. This is probably a consequence of the idealized configuration of the structureless string that one deals with in these cases. Jensen et al papers [6] [7] support this argument. They exhibit an internal solution concerning the spinning string adapted to an exterior Minkowskian geometry in such a way that the radius of the string stands beyond the previous acausal domain thus inhibiting any causal deficiencies to appear. The source of this internal solution is a fluid endowed with anisotropic pressure $\pi_{\mu \nu}$ and a non-vanishing heat flux $q_{\mu}$ in the $\varphi$-direction. The geometry has the form presented in (1) in which functions $h(r)$ and $\Delta(r) \equiv \sqrt{h^{2}-g}$ are given by:

$$
\begin{align*}
& h_{I}(r)=\left(r-r_{S}\right) \cos \lambda r-\frac{1}{\lambda} \sin \lambda r+r_{S}  \tag{5}\\
& \Delta_{I}(r)=b \sin \lambda r
\end{align*}
$$

where the index $I$ stands for the internal geometry and $r_{S}$ is the string radius;likewise the index $I I$ will stand for the external geometry. Note that $\lambda$ is related to the matter content of the solution.

This solution must be connected to (2) obeying the Darmois-Lichnerowicz conditions [20]. Choosing the hypersurface $\Sigma$ - the boundary between the two regions - to be given by the radius $r=r_{S}$, the DL conditions reduce to the continuity of the functions $h$ and $\Delta$ and their respective derivatives through $\Sigma$.

Denoting $[h] \equiv h_{I}-h_{I I}$ as the discontinuity through the hypersurface $\Sigma$, the DL conditions are:

$$
\begin{equation*}
[h]=[\Delta]=\left[h^{\prime}\right]=\left[\Delta^{\prime}\right]=0 \tag{6}
\end{equation*}
$$

in which a prime ( ${ }^{\prime}$ ) indicates derivatives with respect to the radial coordinate. Applying condition (6) in the above solution it follows that its free parameters ( $J, M, \lambda, r_{S}$ and $b$ ) must satisfy:

$$
\begin{align*}
4 G J & =r_{S}-\frac{1}{\lambda} \sin \lambda r_{S}  \tag{7}\\
(1-4 G M) & =b \lambda \cos \lambda r_{S}  \tag{8}\\
b \sin \lambda r_{S} & =(1-4 G M) r_{S}-4 G J \tag{9}
\end{align*}
$$

We note that only two parameters ( $M$ and $J$ ) are completely independent physical quantities. That is, by a convenient choice of $M$ and $J$ one can control the appearance or not of CTC's in the different models. Then, by filling in the interior region with a convenient matter configuration - respecting the previous symmetry - one can modify drastically the causality properties of the geometry. This procedure will be generalized further on and, as we shall see in a subsequent section, it can support a conjecture regarding the absence of causality violations in our Universe.

## 4 THE DEFORMED GÖDEL'S UNIVERSE $I$

Let us come back to the form (1) of the geometry. Choose a tetrad frame $e^{(\alpha)}{ }_{A}$ (in which $A$ is a tetrad index) given by

$$
\begin{align*}
& e^{(0)}{ }_{0}=e^{(1)}{ }_{1}=e^{(3)}{ }_{3}=a \\
& e^{(0)}{ }_{2}=-\frac{h}{a \Delta}  \tag{10}\\
& e^{(2)}=\frac{1}{a \Delta}
\end{align*}
$$

In this frame the only non-identically zero Ricci components are:

$$
\begin{equation*}
R_{00}=-\frac{1}{2 a^{2}}\left(\frac{h^{\prime}}{\Delta}\right)^{2} \tag{11}
\end{equation*}
$$

$$
\begin{align*}
& R_{11}=\frac{1}{a^{2}}\left[\frac{\Delta^{\prime \prime}}{\Delta}-\frac{1}{2}\left(\frac{h^{\prime}}{\Delta}\right)^{2}\right]  \tag{12}\\
& R_{22}=R_{11}  \tag{13}\\
& R_{02}=-\frac{1}{2 a^{2}}\left(\frac{h^{\prime}}{\Delta}\right) \tag{14}
\end{align*}
$$

The geodesic congruence generated by a set of comoving observers $V^{A}=\delta_{0}^{A}$ has no shear or expansion but has a non-null vorticity vector $\omega^{A}=\left(0,0,0,-\frac{1}{2 a} \frac{h^{\prime}}{\Delta}\right)$.

We define the quantity $\omega(r)=-\frac{1}{2 a} \frac{h^{\prime}}{\Delta}$ as the vorticity. In the Minkowskii geometry we have $\omega(r)=0$. We now consider the case in which $\omega(r)$ is a constant $\omega_{0}$ different from zero. This was precisely the case studied by K. Gödel in his 1949 remarkable paper [1]. From this condition and using (14) it follows that the source of this geometry has no heat flux. If the pressure vanishes, the energy density $\rho$, the vorticity $\omega_{0}$ and the cosmological constant $\Lambda$ are related, through Einstein's equations, by:

$$
\begin{equation*}
\rho=2 \omega_{0}^{2}=-2 \Lambda \tag{15}
\end{equation*}
$$

yielding Gödel's solution

$$
\begin{align*}
& h(r)=\sqrt{2} \sinh ^{2} r  \tag{16}\\
& \Delta(r)=\sinh r \cosh r \tag{17}
\end{align*}
$$

The analysis of the whole set of timelike and null geodesics in this geometry shows that the vorticity provides for a strong attractive power that produces a confinement of all free particles within a critical radius $r_{C}=\operatorname{arcsinh} 1$. This confinement is responsible for the impossibility of constructing a unique Gaussian system of coordinates which allows the existence of CTC's beyond $r_{C}$ (see for instance [19]).

In fact, Gödel's geometry was the first cosmological model known to admit CTC's and became a paradigm for causality violations in Gravitational theory.

As a general method of attack to stablish a framework to inhibit the occurrence of CTC's, one could try to make use of the experience gained in the previous section and deform this geometry, by destroying its homogeneity, notwithstanding preserving its axial symmetry.

Our problem can thus be defined as follows: to find a new solution of Einstein's equations for a geometry with the form given by (1), which can be joined to an interior Gödel's geometry through a hypersurface $\Sigma$ defined by the equation $r=r_{\Sigma}=$ constant. If this can be done for an arbitrary value of $r_{\Sigma}$, then we could choose it as less than the confining region ( $r_{\Sigma}<r_{C}$ ) to inhibit the appearance of closed timelike curves, in the same way as it was done in section (3).

We thus set for the stress-energy tensor the form

$$
\begin{equation*}
T_{\mu \nu}=\rho V_{\mu} V_{\nu}-p h_{\mu \nu}+\pi_{\mu \nu}+q_{(\mu} V_{\nu)} \tag{18}
\end{equation*}
$$

It then follows that the quantity $\omega(r)$, defined earlier in this section, is a constant (i. e. no heat flux).

As a direct consequence of the axial symmetry of the metric (1), the only non-zero components of the anisotropic pressure tensor satisfy the following relation:

$$
\begin{equation*}
\pi_{11}=\pi_{22}=-\frac{1}{2} \pi_{33} \equiv \pi_{0} \tag{19}
\end{equation*}
$$

We find the solution of Einstein's equations as:

$$
\begin{align*}
\Delta(r) & =\frac{1}{\lambda} \cosh 2 r_{\Sigma} \sinh u+\frac{1}{2} \sinh 2 r_{\Sigma} \cosh u  \tag{20}\\
h(r) & =\frac{2 \sqrt{2}}{\lambda^{2}} \cosh 2 r_{\Sigma}(\cosh u-1)+\frac{\sqrt{2}}{\lambda} \sinh 2 r_{\Sigma} \sinh u \\
& +\sqrt{2} \sinh ^{2} r_{\Sigma} \tag{21}
\end{align*}
$$

with $u \equiv \lambda\left(r-r_{\Sigma}\right)$, in which $\lambda$ is a constant related to the matter terms by:

$$
\begin{equation*}
\lambda^{2}=a^{2}\left[2 p-\pi_{0}-2 \Lambda\right] \tag{22}
\end{equation*}
$$

and $r_{\Sigma}$ is a positive constant.
The isotropic ( $p$ ) and anisotropic ( $\pi_{0}$ ) pressure are given by:

$$
\begin{align*}
\pi_{0} & =\frac{1}{2}\left(\rho+p-2 \omega^{2}\right)  \tag{23}\\
p & =\frac{2}{3}\left(\Lambda+2 \omega^{2}-\frac{1}{2} \rho\right) \tag{24}
\end{align*}
$$

Let us emphasize that the weak and the strong energy conditions can be satisfied for convenient choices of the free parameters. We can now join this solution to Gödel's geometry through the hypersurface $r=r_{\Sigma}$, satisfying DL conditions (equation (6)).

The final geometrical configuration is thus the following:for $0<r<r_{\Sigma}$ the geometry is provided by Gödel;for $r>r_{\Sigma}$ the geometry has the same cylindrical symmetry but the metrical coeficients $h(r)$ and $\Delta(r)$ are given by (20) and (21). This exterior solution can be considered as a "Gödel- generalized ", since it is proved that it reduces to Gödel for the special case $\lambda=2$.

Now we can consider the causality properties of this structure. Choosing the connection point to be given by $r_{\Sigma}=\delta \operatorname{arcsinh} 1$, for $\frac{1}{2}<\delta<1$, it follows that CTC's are completely forbidden if the anisotropic pressure and the cosmological constant are bounded, i. e.

$$
\begin{align*}
& -\frac{4}{3 a^{2}(\sqrt{2}-1)}<\pi_{0}<-\frac{4}{3 a^{2}}  \tag{25}\\
& \Lambda<\frac{2}{a^{2}}+3 \pi_{0}
\end{align*}
$$

There are also two other matter configurations which provide valid solutions, but we will not consider them here.

## 5 THE DEFORMED GÖDEL'S UNIVERSE $I I: T H E$ NON-CONSTANT VORTICITY

In this section we will combine the results of what we have learned above in order to generate a more complex structure than the previous one. We have studied a model which joined two different solutions of Einstein's equations:the interior (Gödel) and a new exterior solution with the same constant vorticity for all values of the radial coordinate $r$.

As we remarked before, our goal is to join Gödel's solution to a deformed Minkowskii geometry (i. e. , one with a topological defect - an angular deficit). Both solutions possess constant vorticity, but their values are not the same. To achieve a valid model, we must look for an intermediate solution, with vorticity $\omega(r)$, which varies from $\omega_{0}$ (Gödel's value) to zero (deformed Minkowskii's value).

This intermediate solution is found to be a general case of the spinning string, as presented by Jensen and Soleng [6] [7]. The solution presented by these authors differs from ours in the sense that it tends to Minkowskii geometry for $r=0$, whereas our solution must tend necessarily to Gödel's geometry for the interior region (defined by a given width).

In order to make our calculations more systematic, we will thus consider a model containing three regions:

- Region $I$ - Gödel's solution.
- Region $I I$ - Spinning String solution.
- Region $I I I$ - deformed Minkowskii solution.

As we are interested in solutions with non-constant vorticity, equation (18) implies that the source of the geometry must have a non zero heat flux. This can be seen combining Einstein's equation for $R_{02}$ with (18), which gives:

$$
\begin{equation*}
\left(\frac{h^{\prime}}{2 a^{2} \Delta}\right)^{\prime} \equiv \omega^{\prime}(r)=q_{\varphi} \tag{26}
\end{equation*}
$$

Let us limit ourselves here to the special case in which $q_{\varphi}=$ constant. We will see that this condition is sufficient for our purposes. Now we are able to perform a simple integration in (26), obtaining

$$
\begin{equation*}
\frac{h^{\prime}}{2 a^{2} \Delta}=q_{\varphi}(r+\xi) \tag{27}
\end{equation*}
$$

where $\xi$ is an arbitrary integration constant, to be soon determined. We will also redefine the parameter $\lambda$ as:

$$
\begin{equation*}
\lambda^{2} \equiv a^{2}\left(-2 p+\pi_{0}+2 \Lambda\right) \tag{28}
\end{equation*}
$$

mantaining it positive, as in the previous section. In this case, equation (14) gives:

$$
\begin{equation*}
\Delta^{\prime \prime}+\lambda^{2} \Delta=0 \tag{29}
\end{equation*}
$$

which is easily solved for $\lambda \equiv$ constant.

The constant $\xi$ can be obtained if we impose that the solution for region $I I$ is joined exteriorly (for a radius $r_{S}$ ) to deformed Minkowskii geometry, given by [6] [7]:

$$
\begin{align*}
h_{I I I}(r) & =4 G J  \tag{30}\\
\Delta_{I I I}(r) & =(1-4 G M)\left(r+r_{0}\right) \tag{31}
\end{align*}
$$

where $J$ is the string's angular momentum per lenght, $M$ is the string's mass per lenght and $r_{0}$ is an arbitrary constant. Imposing DL condictions (continuity for $r=r_{S}$ ) we obtain:

$$
\begin{equation*}
\omega\left(r_{S}\right)=0 \Longrightarrow \xi=-r_{S} \tag{32}
\end{equation*}
$$

Substituting (32) in (27) and solving Einstein's equations we find the desired spinning string solution for region $I I$ as:

$$
\begin{align*}
\Delta_{I I}(r) & =A \sin \lambda r+B \cos \lambda r  \tag{33}\\
h_{I I}(r) & =\frac{2 a^{2} q_{\varphi} A}{\lambda}\left(\frac{1}{\lambda} \sin \lambda r+\left(r_{S}-r\right) \cos \lambda r\right) \\
& +\frac{2 a^{2} q_{\varphi} B}{\lambda}\left(\frac{1}{\lambda} \cos \lambda r-\left(r_{S}-r\right) \sin \lambda r\right) \\
& +\alpha \tag{34}
\end{align*}
$$

Einstein's equations and (28) give also the following result for the energy density:

$$
\begin{align*}
\rho & =3 \pi_{0}-\Lambda+a^{2} q_{\varphi}^{2}\left(r_{S}-r\right)^{2}  \tag{35}\\
& =\left(\frac{\lambda}{a}\right)^{2}-\Lambda+3 a^{2} q_{\varphi}^{2}\left(r_{S}-r\right)^{2} \tag{36}
\end{align*}
$$

Now we apply DL junction conditions for the complete model, given by:

- Region $I\left(0 \leq r \leq r_{\Sigma}\right)$, Gödel's solution, equations (16) and (17);
- Region $I I$ ( $r_{\Sigma} \leq r \leq r_{S}$ ), the spinning string generalized solution, equations (33) and (34);
- Region $I I I\left(r \geq r_{S}\right)$, deformed Minkowskii solution, equation (31).

Note that, in order to obtain an analytically joined model, these conditions must be satisfied for two hypersurfaces: $r=r_{\Sigma}$ and $r=r_{S}$. Consequently, this gives us two sets of equations of the kind (6), which must be simultaneously valid. Therefore these equations provide us with the means to obtain the integration constants $A, B, \alpha$ and $r_{0}$ and $J, M$ and $q_{\varphi}$ - related to the matter which generates the solution for region $I I$ - in terms of the parameters $\lambda, r_{\Sigma}$ and $r_{S}$.

Thus applying DL conditions results in the following relations:

$$
\begin{align*}
& A=\frac{1}{\lambda}\left\{\lambda\left(\sinh r_{\Sigma} \cosh r_{\Sigma}\right)+\left(2 \sinh ^{2} r_{\Sigma}+1\right) \cos \lambda r_{\Sigma}\right\}  \tag{37}\\
& B=\frac{1}{\lambda}\left\{\lambda\left(\sinh r_{\Sigma} \cosh r_{\Sigma}\right)-\left(2 \sinh ^{2} r_{\Sigma}+1\right) \sin \lambda r_{\Sigma}\right\}  \tag{38}\\
& \alpha=\frac{\sqrt{2}}{\lambda^{2}\left(r_{S}-r_{\Sigma}\right)}\left\{2 \sinh r_{\Sigma} \cosh r_{\Sigma}+2\left(r_{S}-r_{\Sigma}\right)\left(2 \sinh ^{2} r_{\Sigma}+1\right)\right\} \\
& +\sqrt{2} \sinh ^{2} r_{\Sigma}  \tag{39}\\
& r_{0}=\frac{\lambda\left(\sinh r_{\Sigma} \cosh r_{\Sigma}\right) \cos u+\left(2 \sinh ^{2} r_{\Sigma}+1\right) \sin u}{\left(2 \sinh ^{2} r_{\Sigma}+1\right) \cos u-\lambda\left(\sinh r_{\Sigma} \cosh r_{\Sigma}\right) \sin u} \\
& -r_{S}  \tag{40}\\
& q_{\varphi}=-\frac{\sqrt{2}}{a^{2}\left(r_{S}-r_{\Sigma}\right)}  \tag{41}\\
& M=\frac{1}{4 G}\left\{1-\left(2 \sinh ^{2} r_{\Sigma}+1\right) \cos u\right\} \\
& +\frac{\lambda}{4 G} \sinh r_{\Sigma} \cosh r_{\Sigma} \sin u  \tag{42}\\
& J=\frac{\sqrt{2}}{2 G \lambda^{2}\left(r_{S}-r_{\Sigma}\right)}\left(\sinh r_{\Sigma} \cosh r_{\Sigma}\right)[1-\cos u] \\
& +\frac{\sqrt{2}}{2 G \lambda^{3}\left(r_{S}-r_{\Sigma}\right)}\left(2 \sinh ^{2} r_{\Sigma}+1\right)\left[\lambda\left(r_{S}-r_{\Sigma}\right)-\sin u\right] \\
& +\frac{\sqrt{2}}{4 G} \lambda^{3}\left(r_{S}-r_{\Sigma}\right) \sinh ^{2} r_{\Sigma} \tag{43}
\end{align*}
$$

where $u$ stands again for $\lambda\left(r_{S}-r_{\Sigma}\right)$.
By a convenient choice of the three parameters $\lambda, r_{\Sigma}$ and $r_{S}$ one can obtain models that do not violate causality for any value of the coordinate $r$. All that is required is that the function $g(r)$ satisfy the inequality

$$
g(r) \equiv h^{2}-\Delta^{2}<0
$$

for each region simultaneously. We also note that this model joins successfully a region with constant vorticity (Gödel) to another with zero vorticity (deformed Minkowskii) through an intermediate region (generalized spinning string) which presents variable vorticity (in terms of the coordinate $r$ ). The quantity $\omega$ is given, for each region, by:

- Region $I: \omega \equiv \omega_{0}=\frac{2}{a^{2}}$;
- Region $I I: \omega \equiv \omega(r)=\frac{2\left(r_{S}-r\right)^{2}}{a^{2}\left(r_{S}-r_{\Sigma}\right)^{2}}$;
- Region $I I I: \omega=0$.

It is easy to verify that $\omega(r)$ is continuous both for $r=r_{\Sigma}$ and for $r=r_{S}$, as it should.
The above structure provides a way to suspend the ocurrence of causality violations in Gödel's geometry, by the presence of a protecting capsule.

## 6 CONCLUSIONS

Nowadays causality is still an open question. Concerning this problem there are three typical attitudes: (i)the laws of Physics forbid the appearance of closed timelike curves (Hawking) [16]; (ii)the laws of Physics allow for CTC's and Nature exhibits them (Thorne) [15] [21]; (iii)the laws of Physics allow the appearance of CTC's, but Nature organizes itself in such a way as to hide them. In this paper we have opted for attitude (iii), by exhibiting a specific case in which Nature can make CTC's inaccessible. We argue that the best way to achieve this result is to examine Gödel's geometry, once it is the true paradigm of solutions which admit CTC's, although this method can be generalized for other kinds of metric. A complex structure is produced, consisting of solutions of Einstein's equations for different continuous regions, in such a way that only part of Gödel's geometry, bounded by its critical radius (which delimits the separation between a well behaved region and the one in which CTC's can occur) is considered. This region possess an analytical continuation (i. e. , which satisfy the standard Darmois-Lichnerowicz junction conditions) to a topologically deformed Minkowskii solution, with a generalized spinning string solution in between them. This structure leads us to a further conjecture: all would be CTC's regions of a given geometry are ocuppied by other geometries which do not allow closed timelike curves. These causal protecting capsules would thus prevent causality violation from ocurring by enclosing the causal region of the basic geometry inside another geometry, without causality violation.

## References

[1] Gödel, K. Rev. Mod. Phys. 21, 3 (1949).
[2] Gott III, R. J. Phys. Rev. Letters 66, 9 (1991), 1126-1129.
[3] Gott III, R. J. Astrophys. J., 288 (1985), 422-427.
[4] Hiscock, J. A. Phys. Rev. D, 31, 12 (1985), 3288-3290.
[5] Vilenkin, A. Phys. Rep. 121, 5 (1985), 263.
[6] Jensen, B. Class. Quantum Grav. 9 (1992).
[7] Jensen, B. and Soleng, H. H. Phys. Rev. D 45, 8 (1992).
[8] Novello, M., Soares, I. D. and Tiomno, J. Phys. Rev. D, 27(4), 779 (1983).
[9] Tipler, F. J. Ann. of Phys. 108, 1 (1977), 36.
[10] Gibbons, G. W. and Russel, R. A. Phys. Rev. Lett. 30 (1973), 398.
[11] Novikov, I. D. Sov. Phys. JETP 68, 3 (1989), 439.
[12] Malament, D. B. J. Math. Phys. 26, 4 (1985), 774.
[13] Chandrasekhar, S. and Wright, J. P. Proc. Nat. Acad. Sci. USA 47, 341 (1961).
[14] Held, A., private communication (1992).
[15] Morris, M. S., Thorne, K. S. and Yurtsever, U. Phys. Rev. Lett. 61 (1988), 148.
[16] Hawking, S. W. "The Chronology Protection Conjecture". University of Cambridge (1991), preprint.
[17] Deser, S., Jackiw, R. and 't Hooft, G. Ann. Phys. 152, 220 (1984).
[18] Mazur, P. O. Phys. Rev. Lett. 57, 8 (1986), 929.
[19] Novello, M., Svaiter, N. F. and Guimarães, M. E. X. Mod. Phys. Lett. A 7 (1992), 38.
[20] Lake, K. in $V^{t h}$ Brazilian School of Cosmology and Gravitation, Editor M. Novello, Rio de Janeiro, 1987. World Scientific.
[21] Friedmann, J., Morris, M. S., Novikov, I. D., Echeverria, F., Klinkhammer, G., Thorne, K. S. and Yurtsever, U. Phys. Rev. D 42, 6 (1990), 1915-1930.

