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The Non-Abelian Aharonov-Casher Effect in the Framework of Feynman Pseudo-Classical Path Integrals

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Abstract

I analyze the non-Abelian Aharonov-Casher effect in the Feynman path integral framework.

The Aharonov-Casher effect for electromagnetism has been discussed in the literature as a basic building to theory's of high T_e-superconductivity ([1],[2]). My aim in this note is to generalize this effect for the non-abelian case by making use of Feynman pseudo-classical path integrals *as proposed by myself on refs. [3].*

Let me start our study by considering the Grassmanian path integral representation of ref. [3] for the Green function of a $U(N)$ neutral spin one half relativistic particle with a non zero coloured magnetic dipole moment $\bar{\mu}_m$ interacting with a constant $U(N)$ non abelian field strenght $F_{\alpha\beta}$:

$$
\begin{aligned}
&G^{(a,b)}_{(\mu,\eta)}[X^{(1)}_\mu, X^{(2)}_\mu] = \\
&\int_0^\infty ds \int_{\substack{X^\mu(0)=X^{(1)}_\mu \\ X^\mu(s)=X^{(2)}_\mu}} \prod_{0<\sigma<s}[dx^\mu(\sigma)] \\
&\mathbb{P}_{colour}\ \mathbb{P}_{spin} \left\{ exp\ i/4\bar{\mu} \int_0^s [\gamma^\alpha, \gamma^\beta](\sigma) F_{\alpha\beta}(X^\gamma(\sigma)) d\sigma \right\}^{(a,b)}_{(\mu,\eta)}
\end{aligned} \tag{1}
$$

where I have evaluated exactly the Grassmanians path integrals associated to the colour charge and Lorentz spin degrees of freedom yielding, thus, the $U(N)$-colour and Lorentz group valued Mandelstan phase factor written in eq. (1), namelly

$$
\begin{aligned}
W^F = &\prod_{0<\sigma<s} [d\theta(\sigma)][d\theta^*(\sigma)][d\psi_\mu(\sigma)](\theta_a(0)\theta^*_b(s))(\psi_\mu(0)\psi_\eta(s)) \\
&exp\left\{ \frac{1}{2}\ i \int_0^s (\theta\theta^*)(s) ds + \frac{1}{2}\ i \int_0^s \psi_\mu(s)\dot{\psi}_\mu(s) ds \right\} \\
&exp\left\{ \int_0^s d\sigma \frac{1}{2} [\psi_\mu(\sigma), \psi_\eta(\sigma) F^i_{\mu\eta}[X^\alpha(\sigma)(\theta_a(\lambda_i)^{ab}\theta^*_b)(\sigma)] \right\}
\end{aligned} \tag{2}
$$

Let me analyse the following non abelian Aharonov-Chaser experiment ([3]): A quantum $U(N)$ neutral particle with a $U(N)$-coloured magnetic moment travels along a closed trajectory on the region M where $F^a_{\alpha\beta}(X^\rho)\lambda_a$ is a constant field strenght. We remark now that the only Feynman bosonic trajectory that contributes to the fermionic Mandelstan phase factor in eq. (1) comes from the classical trajectory $X^{CL}_\mu(\sigma)$. This result may obtained by expanding in power of $\hbar$, the fermionic Mandelstan phase factor eq. (2):

$$\mathbb{P}_{colour}\ \mathbb{P}_{spin}\left\{exp\ i/\hbar\bar{\mu}_m\int_0^s[\gamma^\alpha,\gamma^\beta](\sigma)F_{\alpha\beta}(X_\mu^{c\ell}(\sigma)+\hbar X_\mu^q(\sigma))d\sigma\right\}-$$
$$\mathbb{P}_{colour}\ \mathbb{P}_{spin}\left\{exp\ i/\hbar\bar{\mu}_m\int_0^s d\sigma[\gamma^\alpha,\gamma^\beta](\sigma)F_{\alpha\beta}(X_\mu^{c\ell}(\sigma))\right\}=$$
$$=\sum_{n=1}^{\infty}\frac{(\hbar)^n}{n!}\int_0^s dX_{\alpha_1}^q(\sigma_1)\cdots\int_0^s dX_{\alpha_n}^q(\sigma_n)$$
$$\left[\frac{\delta^{(n)}}{\delta\eta_{\alpha_1}(\sigma_1)\cdots\delta\eta_{\alpha_n}(\sigma_n)}\mathbb{P}\left\{exp\ i/\hbar\bar{\mu}_m\int_0^s[\gamma^\alpha,\gamma^\beta](\sigma)F_{\alpha\beta}(\eta_\mu(\sigma))d\sigma\right\}\right]_{\eta_\mu(\sigma)=X_\mu^{CL}(\sigma)} \quad (3)$$

We point out that functional derivatives in eq. (3) are multiplicatively proportional to derivatives of the field strenght, which vanish as $F_{\alpha\beta}(x)$ is taken to be a constant field strenght on M. We have, thus, the validity of the above mentioned remark. We get therefore that the wave function of a $U(N)$ spin one half neutral particle with $U(N)$ non zero magnetic dipole moment in the non abelian Aharonov-Casher experiment develops a topological phase given explicitly by the Fermionic Wilson loop of ref. ([3]) for a neutral particle

$$W^F=Tr_{spin}Tr_{colour}\left[\mathbb{P}_{colour}\ \mathbb{P}_{spin}\left\{exp\ i/\hbar\ \bar{\mu}_m\int_0^s[\gamma^\alpha,\gamma^\beta](\sigma)F_{\alpha\beta}(X_\mu^{CL}(\sigma))d\sigma\right\}\right] \quad (4)$$

This result confirms the explicit appearance of Lorentz spin degrees of freedom on the phase shift of the Aharonov-Casher wave function predicted in refs. [1], [2].

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