CBPF-NF-011/06

Area density of localization-entropy II: double cone-localization and quantum origin of the Bondi-Metzner-Sachs symmetry

Bert Schroer
CBPF, Rua Dr. Xavier Sigaud 150
22290-180 Rio de Janeiro, Brazil
and Institut fuer Theoretische Physik der FU Berlin, Germany

Abstract

The holographic encoding is generalized to subalgebras of QFT localized in double cones. It is shown that as a result of this radically different spacetime encoding the modular group acts geometrically on the holographic image. As a result we obtain a formula for localization entropy which is identical to the previously derived formula for the wedge-localized subalgebra. The symmetry group in the holographic encoding turns out to be the Bondi-Metzner-Sachs group.

1 The aims of this paper

In the algebraic setting of QFT the *holographic projection* changes the spacetime indexing of subalgebras contained in a given localized algebra; the bulk localization structure becomes replaced by the spacetime indexing of subregions associated to its causal horizon. In this process the bulk algebra as a global object remains equal to the horizon algebra. In other words holography does not change the abstract algebraic substrate; not unlike enzymes acting on stem cells, holography just leads to a different organization in space and time. The simplest case is the wedge-localized algebra which is identical to its (say upper) horizon algebra $\mathcal{A}(W) = \mathcal{A}(Hor(W))$, but both spacetime interpretations (bulk and causal horizon) lead to an entirely different substructure. It turns out that each substructure is local in its own setting but non-local relative to the other[1]. In [1] the holographic method for the special case of the free scalar field was presented for the wedge algebra; in this case holography is closely related to the old "lightcone quantization". In the interacting case the naive restriction method on pointlike fields fails and instead the more structural algebraic modular localization approach takes over. Independent of the mathematics used, the main aim is always to simplify certain properties of QFT, naturally at the expense of certain others. Either way, the result is that the theory on the horizon is the restriction of a transverse-extended chiral theory on the lightfront restricted to the horizon which is half the lightfront. The main conceptual instrument for analyzing the thermal properties of the vacuum state restricted to the algebra of the horizon (which is half the lightfront) is a theorem which says that this situation is in a one-to-one relation with those of the global lightfront theory in a standard heat bath lightlike translation KMS state at temperature 2π. This theorem permits to carry the standard thermodynamic limit formalism into an approximating sequence for thermal manifestations of the vacuum state restricted to the wedge algebra $\mathcal{A}(W)$. In particular one finds for the entropy [1]

$$\begin{aligned} s_{loc}(\varepsilon) &\underset{\varepsilon\rightarrow 0}{=} A\left|ln\varepsilon\right|\frac{c\pi}{6}+o(\varepsilon) \qquad (1)\\ &= V\frac{c\pi}{6}+o(\frac{1}{R}),\ V=AR,\, \varepsilon=e^{-R} \end{aligned}$$

where $A\rightarrow\infty$ is a sequence of transverse "boxes" which exhaust the area of $Hor(W)$ (i.e. the edge of the wedge W) and ε is interpreted as a measure of distance of the (Gibbs state) approximand to $\mathcal{A}(Hor(W))$. The second line is the entropy in the heat bath interpretation of the global lightfront algebra in a $\beta = 2\pi$

[1] The well-studied conformal holography of AdS→conformal QFT is not a holography in the present sense since its spacetime encoding does not result from projecting AdS onto a null surface but rather onto a timelike boundary at infinity (the limiting case of a "brane") which just inherits the original causality [2][3]. Holography in the present sense of projection onto the null horizon is a much more radical change of spacetime encoding [1].

thermal state; here R is the "box size"[2] in lightray direction and V is the standard volume factor. The formula shows that the $|ln\varepsilon|$ divergence of the area density of the localization entropy is of a kinematical origin; the only quantum matter-dependent contribution is contained in c which measure the cardinality of holograhic matter which for simplicity has been normalized according to the c-value in the Witt-Virasoro algebra. This is precisely what one intuitively expects for an entanglement-entropy caused by vacuum fluctuations within a collar of size ε in the limit $\varepsilon \to 0$[3]. As in the classical heat bath case there is a numerical factor which depends on the quantum matter and a (diverging) size factor (the volume V) of which two transverse powers go into the area factor and the longitudinal (lightray) factor undergoes a reinterpretation (via conformal invariance) as $|ln\varepsilon|$ in terms of a lightlike short distance ε. In the *classical heat bath* case this differentiation (according to the kind of quantum matter) is absent and the classical *Bekenstein area formula* shares this area proportionality (although conceptually it has a very different origin in differential geometry without any direct relation to thermodynamics).

It is our aim in this paper to show that these conclusions are independent of the geometry of the localization region and its causal horizon, in particular they are not related with the noncompact extension but also remain valid in the compact case. As a typical causally closed compact region we take the double cone (i.e. the intersection of the forward lightcone with the backward lightcone whose apex has been shifted into positive timelike direction0. As a result of the non-geometric nature of the modular data for the double cone algebra with respect to the vacuum $(\mathcal{A}(D), \Omega)$, the derivation is considerably more subtle than the case of a wedge algebra.

As an interesting side result of the double cone holography we obtain a derivation of the Bondi-Metzner-Sachs symmetry group, which was discovered by the three named authors and analyzed in terms of an asymptotic classical formalism developed by Penrose [21] In the present context it arises as an intrinsic (i.e. not through quantization) property of local quantum physics. It turns out that the appearance of this huge group belongs to those properties which allow a much more natural physical understanding in the quantum context than in the classical setting. Another well-known example of conceptual simplification on the quantum level is the notion of integrable field theories. Whereas classical integrability is a very complex property whose presence has to be checked in a case by case study, quantum integrability has a simple and universal structural characterization in terms of the notion of wedge-localized vacuum-**p**olarization-**f**ree-**g**enerators (PFG) [18]. It is interesting to note that the vacuum-polarization properties are also at the root of those big symmetry groups whose ("fuzzy") action on the bulk has no classical Noether counterpart.

The framework in which we work is that of local quantum physics. This is often (especially in the older literature) called "axiomatic QFT", a term which we do not use because it has the connotation of a once and for all laid down set of postulates. Although there is a stock of universally accepted requirements (as Poincaré invariance and locality), the spirit of local quantum physics is flexible and what one adds depends a bit on the aims one wants to achieve. Whatever one adds must at least be true for free fields. A second important requirement is that it allows an intrinsic (i.e. no reference to a classical parallelism as Lagrangian quantization) and fairly precise mathematical formulation.

For the present purpose we add the quantum version of the laws of classical causal propagation. We demand the causal completion property

$$\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}'') \tag{2}$$

where the dash stands for the causal disjoint of a spacetime region $\mathcal{O}$ and the causal completion is the double causal disjoint. It is customary to also require this "causal shadow property" in the limiting case where the region degenerates into a piece of a spacelike hypersurface. The associated algebra is then defined by taking a sequence of regions $\mathcal{O}_i$ of deceasing height (time going upward) whose intersections approach the spacelike hypersurface and forming the algebra $\cap_i \mathcal{A}(\mathcal{O}_i)$ of intersection. Classically also certain characteristic data (data on null hypersurfaces) cast a causal shadow[4]. For example characteristic data on half of a lightfront (i.e. semiinfinite extension in lightray direction) casts a shadow which is identical to the wedge of which the horizon is the half of the lightfront from which we started. The shadow is a region for which all lighrays which path through it must either have passed through the

[2] The box is defined in terms of the conformal Hamiltonian $L_0^{(R)}$ [1] i.e. it is a "relativistic box" in the sense of [4].

[3] The ε-dependence in this approach can only be taken seriously in the limit $\varepsilon \to 0$. For finite ε (corresponding to finite R in the thermodynamic case) one must confront the computationally much more involved "split property" [1].

[4] The only case where this picture breaks down is that of massless 2-dim. QFTs.

characteristic null surface or (in case of an upper horizon) will still path through such a surface. On the other hand a region on the lightfront which is transversely not fully extended or which is bounded in lightray direction does not cast a causal shadow[5]. In the case of a lightcone any neighborhood of the apex on the mantle casts a causal shadow which includes a (possibly very small) double cone; but any other region on the mantle which does not contain the apex does not cast any shadow. We require

Condition 1 *If $\mathcal{C}$ is a characteristic null-surface which casts a nontrivial causal shadow $\mathcal{O}_C \equiv C''$, we require the "extended causal shadow condition"*

$$\mathcal{A}(\mathcal{C}) = \mathcal{A}(\mathcal{O}_C) \tag{3}$$

As often, the devil is in the details. It is a priori not clear how to define the left-hand side, in particular if interactions are present. An attempt to treat algebras on null-surfaces as limits of algebras on spacelike surfaces runs into difficulties. The solution is to employ holography in the sense of a radical change of spacetime-indexing of the bulk algebra. This will be done in the next section. For free fields this is achieved by explicit computations. If it turns out that there are interacting models which fulfill the spacelike but not the characteristic causal shadow property (I do not know any such example and would be somewhat surprised if there is any), the above condition and the validity of holography would be restricted to those models which fulfill this condition

The paper is organized as follows. In the next section we present an explicit proof of our claim about double cone holography in the case of a massive free scalar field; we also present the structural arguments for the case with interactions. In section 3 it is shown that double cone holography leads to the Bondi-Metzner-Sachs symmetry group as well as to the afore-mentioned formula for the localization entropy (1).

2 Holography for double cones; from fuzzy modular bulk actions to diffeomorphisms on the horizon and back

In the setting of Minkowski space, i.e. of spacetime without curvature and with trivial topology, the wedge situation is the only non-conformal situation for which the modular group acts as a diffeomorphisms on the causally enclosed bulk matter. The KMS property of the modular group, and as will be shown here also the area density of entropy, are however equally valid if the modular flow is "fuzzy". In such a case there is no possibility to analyze the thermal structure in terms of an Unruh type Gedankenexperiment; nevertheless the thermal aspects of localization are important structural elements and it is natural to be interested in all aspects of a theory which is as stupendously successful as QFT.

Such a typical non-geometric situation arises for a double cone localized operator algebra which acts standardly on the vacuum. In case the theory describes massive free particles the modular automorphism group σ_t of the standard pair $(\mathcal{A}(D), \Omega)$ cannot be described by geometric trajectories [5][6] since it acts in a "fuzzy" manner inside D (and in its causal complement) in the following sense: a subalgebra $\mathcal{A}(\tilde{D}) \subset \mathcal{A}(D)$, with $\tilde{D} \subset D$ a smaller double cone, suffers an instantaneous "foamy explosion" inside D and its causal disjoint D', i.e. $\sigma_t(\mathcal{A}(\tilde{D}))$ for every $t > 0$ and a generic $\tilde{D}$ inside D spreads instantaneous all over D in such a way that the image does not contain any localized subalgebra[6]. Using Wightman's description in terms of smeared fields, the action of σ_t can be transferred to the space of D-supported test functions, but a further encoding into diffeomorphisms acting on the spacetime variables of the test function is not possible. It is expected [8] that for free massive fields such spreading can be described in terms of pseudo-differential generators which act on D-supported test-function spaces, leaving the support and its causal disjoint invariant; but presently this hard functional analytic problem is only partially solved [9].

Here we will avoid such a head-on brute force functional analytic attempt in favor of a radical change of spacetime indexing in terms of holography. The latter maintains the total double cone algebra (see 4) but changes the spacetime substructure from its original indexing in terms of subregions of the bulk

[5] Such a region plays no role in classical field theory, but the local quantum physics process of algebraic intersections assigns a nontrivial algebra to it. These are the algebras which are "fuzzy" in the bulk setting.

[6] A mathematically precise form of this statement (even in the presence of interactions) can be found in [7].

to that of subregions on its causal horizon. It turns out, and this is the main simplification, that the modular group acts geometric in the new indexing on the horizon. This is possible because the two kinds of spacetime-indexed subalgebras have a partially nonlocal relation relative to each other.

One could in fact substitute the net of bulk subalgebras in the AQFT approach by the family of their (lower or upper) causal horizons indexed subalgebras. In this case all the modular groups act geometrically, but the formation of intersections by using horizons without the bulk becomes awkward in terms of these "hollow" light-cone stumps. There is a kind of complementarity principle: either one uses the bulk setting and works with the geometric picture of intersections for which the modular groups act non-geometrically, or one uses the holographic projections in which case the modular groups act geometrically and instead the problem of intersections becomes awkward.

We will now translate this verbal description of the modular structure obtained after holographic projection into mathematics. In analogy to the wedge situation, we expect the holographic projection of $\mathcal{A}(D)$ to be a an extended chiral theory. The lower (upper) causal horizon of a rotational symmetric D is a characteristic boundary which, if we place classical characteristic data on it, leads to a unique classical propagation inside D. For simplicity we take the lower horizon of a rotational symmetric double cone of radius 1 which we center at the origin so that the (lower) horizon $\partial D^{(+)}$ becomes part of the mantle of the forward lightcone with the apex shifted down to $(-1,\vec{0})$.

If the double cone algebra $\mathcal{A}(D)$ would be a subalgebra of a conformal QFT, the modular group of $(\mathcal{A}(D),\Omega)$ would be identical the geometric conformal transformation in [10]. This situation has been recently analyzed in the spirit of a "conformal Unruh observer" [11]. Whereas the mathematics is straightforward, it seems that the authors stretch its physical aspect far beyond Unruh's Gedankenexperiment. Whereas conformal QFT is a useful idealization of certain particle physics observables in certain high energy limits for highly inclusive cross sections [12], it becomes somewhat science fiction if used on the side of the observer and his/her hardware. The fuzzy modular action on the massive bulk prevents a geometric interpretation a la Unruh; it is hard to imagine what an observer must do in order to put physical life into such a Hamiltonian with a two-sided spectrum and a fuzzy propagation. On the other hand the world of the holographic projection in which the modular Hamiltonian acts geometrically is physically unattainable for an observer and his hardware; at least for this is true for double cone horizons in Minkowski spacetime whereas in the case of black hole event horizons this is not so clear.

Here we will use the double cone situation in order to investigate structural questions of (not necessarily conformal) QFT, in particular about localization entropy. The starting point is the algebraic identity

$$\mathcal{A}(\partial D^{(+)}) = \mathcal{A}(D) \tag{4}$$

which is the local quantum physics version of the classical statement that the characteristic data on $\partial D^{(+)})$ fix uniquely the propagation inside D (the $+$ sign is used because the lower horizon of the double cone is part of the mantle of the forward cone). For the algebra of free fields this can be shown similarly to [13] i.e. the problem can be reduced to the "first quantization" level of spatial modular theory within the Wigner representation setting. Viewing this as a limiting case of the time-slice (causal shadow cast by spacelike surfaces) property, one simply takes this as the starting definition of holography for causally completed regions in the presence of interactions. If there really exist physically viable nets of local observables which obeys spacelike locality and fulfills stability (energy positivity or KMS stability [14]) but do not obey this characteristic causal completion property (and therefore are problematic from an holographic viewpoint), they simply will not be considered here.

As in the previous case of a wedge, the main achievement of holography is that the lightlike boundary has a radically different local substructure of $\partial D^{(+)}$ as compared to D. A crucial difference is that the fuzzy action of the modular group on the bulk gives way to a geometric modular action on the boundary i.e. (appealing to the continuity of the modular action) the action becomes asymptotically geometric near the horizon $\partial D^{(+)}$ where it coalesces with the restriction of the action of the conformal modular group

$$r_+ \rightarrow r_+(t) = \frac{(1+r_+) - e^{2\pi t}(1-r_+)}{(1+r_+) + e^{2\pi t}(1-r_+)}, \varphi, \psi \ invariant \tag{5}$$

where $r_+ = r + t,\ t = r - 1, -1 \leq r_+ \leq 1$ and the two angles parametrize the directions transverse to the light rays. In analogy to the wedge-localized situation, the chiral Moebius group and the associated spacetime substructure of $\partial D^{(+)}$ results from the modular inclusion $\mathcal{A}(\partial D_a^{(+)}) \subset \mathcal{A}(\partial D^{(+)})$ where $\partial D_a^{(+)}$

is a shortened part of $\partial D^{(+)}$ with $-1 \leq r_+ \leq a < 1$. Inverting the situation on the basis of property (4), the geometric boundary action would have a modular extension into the causal dependency bulk region D which for the realistic case of massive matter content would be "fuzzy". In terms of the pointlike Wightman setting, "fuzzyness" in the case of free fields is expected to mean that the space of D-supported test-function spaces suffers a transformation whose infinitesimal description is associated with a nonlocal pseudo-differential operator[7] which however preserves supports within D and within its causal complement. The holographic equation (4) implicitly fixes the action of σ_t on $\mathcal{A}(D)$ and as long as one avoids detailed questions concerning how σ_t acts on the local substructure of D (i.e. on smaller double cones inside D) it is sufficient to know the geometric action of the modular group on the holographic projection $\mathcal{A}(\partial D^{(+)})$. In fact one could encode the whole net structure of algebraic QFT into the associated holographic projections, thereby hoping to render the modular group actions more geometric[8]. Since the actions of the modular group (but of course not the anti-unitary inversion J) on algebras localized in causally closed regions are uniquely fixed in terms of the representation structure of the Poincaré group [15][16], it would be highly desirable to have a proof of consistency between the holographic definition and the one in terms of algebraic intersections starting from the geometric modular action on wedge algebras.

In the following we explain how double cone holography works for a free field; since the arguments are completely analogous to the wedge case we can be brief. The restriction of the free scalar field to the mantle of the forward lightcone is obtained by parametrizing the free field in the spacelike region of the apex of the lightcone taking the limit from spacelike directions i.e.

$$A(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int e^{imr_+(sh\chi ch\theta - cos\vartheta ch\chi sh\theta)}a^*(\theta,\vec{e})\frac{d\theta}{2}d\Omega + h.c., \vartheta = \sphericalangle(\vec{x},\vec{p}) \tag{6}$$
$$t = r_+ sh\chi,\ r = r_+ ch\chi,\ p_0 = mch\theta, |\vec{p}| = msh\theta,\ \vec{p} = |\vec{p}|\,\vec{e}$$

In order to maintain t, r finite in the limit $r_+ \to 0$ it is necessary to compensate by letting $\chi \to \infty$. As for the analog case of the lightfront [1][17][18] the m looses its role of a physical mass and passes simply to a dimension-setting parameter whose contribution to the two-point function is an additive logarithmic spacetime-independent constant. The additive decomposition in the momentum space rapidity does not change the algebraic structure and the limit becomes indistinguishable from the formula for a restriction of a massless free field to the mantle of the forward light cone (where for convenience of notation we leave out the subscript LC (lightcone) replacing the LF in the lightfront restriction [1])

$$A(r(\chi), e(\vec{x})) \equiv A_{m=0}(x)|_{mantle} = lim_{r_+\to 0}A(x) \simeq \int e^{ir(\chi)p(\hat{\theta})(1-cos\vartheta)}a^*(\hat{\theta}, e(\vec{p}))\frac{d\hat{\theta}}{2}d\Omega + h.c. \tag{7}$$

$$\left[a^*(\hat{\theta},e), a(\hat{\theta}',e')\right] = \frac{1}{2}\delta(\hat{\theta}-\hat{\theta}')\delta(e-e')$$
$$[A(r,e(\vec{x})), A(r',e(\vec{x}'))] \simeq \int_{-\infty}^{\infty}\frac{dp}{2p}e^{ip(r-r')}\delta_\perp(e-e') \tag{8}$$
$$[\partial_r A(r,e(\vec{x})), \partial_{r'}A(r',e(\vec{x}'))] = \delta'(r-r')\delta_\perp(e-e')$$

where we have denoted the x-space angular behavior by the spatial unit vector $e(\vec{x})$; $r(\chi) = e^\chi$ denotes the remaining (after performing the limit) affine lightcone parameter corresponding to the x-space rapidity χ. As in the lightfront case the longitudinal part has an infrared divergence which in a proper test-function treatment (involving functions which decrease at $|\chi| \to \infty$ in the x-space rapidity variable χ) is rendered harmless (in the modular-adjusted rapidity parametrization it does not occur). The transverse factor is a quantum mechanical δ-function and expresses the absence of transverse fluctuations. From this result one concludes that for a free field the lower horizon of the double cone supports an algebraic structure which is identical to (7) except with the apex now being at (-1,0)

$$A(r(\chi), e(\vec{x})) \simeq \int e^{ip(\theta)(r-1-rcos\vartheta)}a^*(\theta, e(\vec{p}))\frac{d\theta}{2}d\Omega + h.c., -1 < r = th\chi < 1 \tag{9}$$

[7] A investigation along the line started in [5] could be helpful to clarify this issue at least for free fields and strengthen the confidence that the propagation equation associated with the "modular Hamiltonian" is a nonlinear pseudo-differential equation (assuming that the interacting field under consideration obeyed a local equation of motion).

[8] As mentione before there is however a prize to pay for working with only horizons without their bulks in that the rules about intersections would loose their geometric appeal and become messy.

If we shorten the height of the conic section we obtain an inclusion of boundary algebras.

The consistency of this modular inclusion with the ambient causal shadow property (4) requires the modular group of the larger double cone algebra σ_τ^D to act for $\tau < 0$ as a "compression" on the smaller one

$$\sigma_\tau^D(\mathcal{A}(\tilde{D})) \subset \mathcal{A}(\tilde{D}),\, \tau < 0 \tag{10}$$

It would be nice to verify these consistency results also by an explicit calculation of the relevant modular groups for the massive free field along the lines in [5] together with the conjectured pseudo-differential nature of the modular generator; as mentioned before there are partial results in this direction.

In analogy to the upper lightfront horizon for the wedge we take the mantel of the upper backward lightcone as the causal horizon of a double cone. Haag duality then localizes the commutant in the backward causal shadow cast by the extension of the lightcone to infinity in positive time direction. In this way the horizon $\partial D^{(+)}$ of the double cone becomes encoded into the interval $-1 \leq r_+ \leq 1$ of a transverse extended chiral theory. The situation is similar to the wedge situation except that the action of the modular group on the bulk is fuzzy and that the transverse part is a compact angular region instead of a infinite Cartesian extension. The same theorem which in the wedge case [1] assures the absence of transverse vacuum fluctuations (Takesaki's theorem on the structure of subalgebras and their relative commutants which are left invariant by the modular group of the larger algebra). In the wedge case these were the transverse finitely extended subalgebras whose boundaries consist of parallel lightlike intervals, whereas for double cones these are regions of finite φ, ψ angular extension which are bounded by focussing lightrays[9]. In the latter case the manifest subsymmetries coming from the Poincaré group is much smaller than in the case of the wedge horizon (the group of subsymmetries include the transverse φ, ψ rotations) but again there are new chiral conformal subsymmetries which as a result of (4) act in a fuzzy manner on the ambient algebra. We collect the results for the characteristic holographic boundary algebras of double cones in case of massive free fields into the following theorem

Theorem 2 *The inclusion of double cone algebras in geometric situations where the lower causal horizon of the smaller algebra $\mathcal{A}(\hat{D})$ is contained in that of the bigger one $\mathcal{A}(D)$ is a half-sided modular inclusion [28] i.e.*

$$\sigma_\tau^D(\mathcal{A}(\tilde{D})) \subset \mathcal{A}(\tilde{D}),\, \tau < 0 \tag{11}$$

We view this as a geometric limitation on the fuzziness of modular propagation imposed by the geometric modular inclusion of the holographic projections to the (lower) horizon together with the causal shadow identity (4). It is the only geometric relic of the geometric conformal modular flow of the bulk which survives its delocalization caused by the mass.

The proof consists in noting that this compressive (endomorphism) one-sided modular inclusion property holds for the holographic spacetime indexing. Hence by the characteristic causal shadow conditions formulated in the introduction this one-sided inclusion property is inherited by the corresponding bulk algebras. Although the modular action on the bulk is not geometric, it is still "partially geometric" in the sense of this theorem.

The partial extension of the ambient Poincaré symmetry to certain fuzzy acting conformal symmetries[10] is a new interesting result of the modular setting. It seems to be part of a wider story of modular symmetries which act as local diffeomorphisms on subalgebras [18] where the case of subalgebras indexed by subregions on horizons is certainly the most interesting and radical illustration of this point.

3 The symmetry group and the localization entropy of the holographic projection

Without the transverse angular part the commutation relation for the holographic lightcone restriction of the free field would be that of a potential associated with an abelian chiral current. It is well-known

[9]In both cases the algebras associated with the region between two light rays are totally "fuzzy" in the sense of the ambient spacetime indexing. They are born in the process of intersections and their possible pointlike field coordinatization cannot be done (apart from free fields) in terms of the ambient Borchersclass.

[10]Those symmetries beyond the Poincaré and Moebius symmetries have no globally invariant state. As well-known from chiral theories, the higher diffeomorphisms are unitarily implemented but they possess at most partially invariant states.

that for chiral currents the $Diff(S^1)$ automorphism group of the commutator can be lifted to a unitary (ray) representation [19]. By the same argument this also can be verified for its transverse angular extension. But the symmetry group of the extended chiral theory is much larger than that of a pure chiral theory. Since the tensor-factorizing transverse quantum mechanical behavior manifests itself in the presence of a rotational invariant angular delta function, the full symmetry group includes now in addition to transverse rotations and circular (through lightray compactification) e-independent diffeomorphisms of a chiral theory e-dependent diffeomorphisms. The generated big group is known under the name Bondi-Metzner-Sachs (BMS group) groupwhich generalizes the chiral diffeomorphisms [20]. It contains "supertranslations" (angular dependent lightray translations) as a normal subgroup. To be more precise, the symmetry group of the holographic image on the mantle of a double cone is a subgroup of the BMS group; the full group is only achieved by taking the limit of an infinitely extended double cone. In the classical theory this group arose from the asymptotic $t \pm r$ lightlike behavior of classical zero mass finite helicity equations [21]; in the application to the Einstein-Hilbert classical gravity, which involves nonlinear helicity two fields, one also needs the assumption of asymptotic flatness [20].

The appearance of the BMS group in the holography of local quantum physics permits to remove the somewhat mysterious aspect of the classical setting which resulted from noticing that the asymptotic group is much bigger than the global symmetry (in our case the Poincaré group). The local quantum physical setting removes this mystery because any symmetry of the holographic image is also a symmetry of the bulk matter albeit not necessarily one which enters through quantization i.e. one which of the Noetherian type. Modular localization puts into evidence those nongeometric "fuzzy" symmetry actions which contains important informations about the dynamical structure.

In case of interacting QFTs the holography becomes part of a structural analysis based on the use of modular localization theory. This was explained in some detail in a previous paper [1] for the case of the noncompact wedge region where a 7-parametric subgroup of the Poincaré group which leaves the lightfront invariant was identified. We will defer the corresponding details for the compact double cone and the ensuing infinitely extended mantle of the lightcone for a separate paper [23].

Again one expects that the holographic lightcone fields have pointlike generators, although apart from the free field one cannot hope for a simple linear relation to the generators of the original ambient QFT (assuming that the ambient massive net has pointlike generators). The generalization of the commutation structure of pointlike lightfront generators to the interacting case suggests to expect generalized (e-dependent) chiral fields which, as a result of the lightlike conformal invariance and the transverse factorization must be of the following form

$$[A(r,e), B(r',e')] = \delta_{\perp}(e-e') \sum_{n} \delta^{(n)}(x_+ - x_+) C_n(r,e) \qquad (12)$$

where the sum goes over a finite number of derivatives of delta functions and C_n are (composite) operators of the model. The presence of the quantum mechanical $\delta_{\perp}$-function and the absence of transverse derivatives expresses the transverse tensor-factorization of the vacuum i.e. all the field theoretic vacuum polarization has been compressed into the r lightray direction. In view of the fact that in chiral theories the existence of pointlike field generators follows from the net and covariance structure of the algebraic setting [24], there seems to be no problem to construct pointlike fields in this transverse extended chiral theory; the local structure of the commutation relations is then a consequence of locality and Moebius covariance [1].

For the same reasons as in the above special case the BMS group is the symmetry group of this algebraic structure.

Finally we should return to the problem of localization entropy which according to the title and the presentation of the final result in the introduction was the main purpose of this work. Since the transverse structure has no vacuum polarization and the vacuum polarization due to longitudinal localization are described by those of a chiral theory, the localization entropy law is identical to the area law for the wedge localization (or localization on the lightfront) since the different transverse part does not generate entropy. A full proof which requires to relate the localization entropy on a lightcone to a global KMS state at temperature 2π on the lightcone will be deferred to [23]

4 Concluding remarks

For reasons of clarity we have limited the presentation of holography and its consequences for localization entropy and the BMS symmetry to QFTs on Minkowski spacetime. The most interesting case arises however when gravitationally caused curvature converts the present structural relations into direct observational physics as will be the case from the appearance of black holes in observational astrophysics. The Hawking temperature state after the formation of a black hole is a KMS state[11] for an observer outside the black hole with the KMS automorphism being that of the Killing observer time. Whatever localization entropy is in detail, it should be the other side of the coin corresponding to the same localization-caused thermal behavior as Hawking radiation. All entropy calculations for black holes, including those done within string theory must take place in this KMS thermal setting since the problem is to understand the entropy associated to the thermal Hawking effect and not to lend support to the classical Bekenstein area law by some quantum mechanical kind of degree of freedom counting[12]. The present derivation via holography fulfils this requirement, in fact it relates the Hawking effect, localization entropy and the quantum BMS group to the same basic physical principle.

Holography is a radical change of the spacetime-indexing of bulk matter in such a way the new horizon-based indexing is partially nonlocal with respect to the original natural bulk indexing. It is precisely this partial nonlocality and the ensuing associated change of symmetry (in contrast to the symmetry-preserving AdS-CFT holography) which is at the root of the extraordinary analytic strength of the holographic method in the present context. In the light of this, discussions as in [26] about whether it is the bulk or the horizon which is reponsible for thermal manifestations, are somewhat academic.

Perhaps the strongest indication that there is yet another reality to QFT which we are only beginning to comprehend is the way in which modular theory realizes the famous "monade setting" of Leibniz [27] within QFT [18][1]. Here the monade stands for the ubiquitous localized operator algebra of QFT, an algebra which is distinctively different from all other classified factor algebras and which in Murray von Neumann classification terminology is called the *hyperfinite type* III_1 *factor algebra* [28]. A finite set of copies of this monade (2 in chiral theories, 6 in 4-dim. QFT), carefully positioned according to modular theory within a common Hilbert space, captures the full physical and mathematical richness of a QFT [29].

. The presence of such rigorous observations together with the holographic projection setting of the present work, shows in a most dramatic way that there is a lot of virgin territory out there in QFT to be discovered. In view of these new facts it is hard to believe that the problem of quantum gravity can be solved without a more profound understanding of these modular encodings of geometry coming from these recently observed fundamental properties of QFT beyond Lagrangian quantization.

References

[1] .B. Schroer, *Area density of localization-entropy I: the case of wedge-localization*, hep-th/0507038 and previous work cited therein

[2] K.-H. Rehren, *QFT Lectures on AdS-CFT*, hep-th/0411086

[3] M. Duetsch and K-H. Rehren, Lett.Math.Phys. 62 (2002) 171-184

[4] N. M. Nicolov and I. T. Todorov, *Lectures on Elliptic Functions and Modular Forms in Conformal Field Theory*, math-ph/0412039

[5] F. Figliolini and D. Guido, Ann. Inst. Henri Poincaré **51**, (1989) 419

[6] B. Schroer and H. W. Wiesbrock, Rev. Math. Phys. **12**, (2000) 461

[7] S. Trebels, University of Goettingen thesis 1997

[11] The dynamical derivation of the Hawking radiation as a result of a collapsing star [22] is however not presentable in terms of an equilibrium KMS state.

[12] This quantum mechanical inspired kind of energy level counting has been criticised recently in [25].

[8] B. Schroer and H.-W. Wiesbrock, Rev. Math. Phys. **12**, (2000) 139

[9] T. Saffary, *On the generator of massive modular groups*, math-ph/0601036 and University of Hamburg thesis 2005

[10] P. D. Hislop and R. Longo, Commun. Math. Phys. **84**, (1982) 71

[11] P. Martinetti and C. Rovelli, Class. Quant. Grav. **29**, (2003) 4919

[12] B. Schroer, Phys.Lett. **B494** (2000) 124

[13] D. Guido, R. Longo, J. E. Roberts and R. Verch, Rev. Math. Phys. **13**, (2001) 125

[14] R. Haag, *Local Quantum Physics*, Springer 1996

[15] R. Brunetti, D. Guido and R. Longo, Rev.Math.Phys. **14**, (2002) 759

[16] String-localized quantum fields and modular localization, math-ph/0511042, in print in CMP

[17] B. Schroer, Ann. of Phys. **319**, (2005) 48

[18] B. Schroer, Ann. of Phys. **321**, (2006) 435

[19] A. Pressley and G. Segal, *Loop groups*, Clarendon Press, Oxford 1986

[20] R. M. Wald, *General Relativity*, the University of Chicago Press

[21] R. Penrose, Proc. Roy. Soc. **A284**, (1965) 159

[22] K. Fredenhagen and R. Haag, Commun. Math. Phys. **127**, (1990) 1

[23] J. Barata, P. Lauridsen and B. Schroer, *Holography and its quantum Bondi-Metzner-Sachs symmetry*, in preparation

[24] M. Joerss, Lett. Math. Phys. **38**, (1996) 257

[25] Hollands and R. E. Wald, *Quantum Field Theory Is Not Merely Quantum Mechanics Applied to Low Energy Effective Degrees of Freedom,* Gen.Rel.Grav. **36** (2004) 2595

[26] T. Jakobson, D. Marolf and C. Rovelli, *Black hole entropy: inside or out*?, hep-th/0501103

[27] http://en.wikipedia.org/wiki/Gottfried_Leibnizhttp://en.wikipedia.org/wiki/Gottfried_Leibniz

[28] S. J. Summers, *Tomita-Takesaki Modular Theory*, math-ph/0511034

[29] R. Kaehler and H.-W. Wiesbrock, JMP **42**, (2000) 74