N=2–Maxwell-Chern-Simons model with anomalous magnetic moment coupling via dimensional reduction

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Abstract

An N=1–supersymmetric version of the Cremmer-Scherk-Kalb-Ramond model with non-minimal coupling to matter is built up both in terms of superfields and in a component-field formalism. By adopting a dimensional reduction procedure, the N=2–D=3 counterpart of the model comes out, with two main features: a genuine (diagonal) Chern-Simons term and an anomalous magnetic moment coupling between matter and the gauge potential.

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1 Introduction

Ordinary and supersymmetric planar gauge models have been fairly well investigated over the past years, in view of several remarkable properties they exhibit. Among their most relevant features, we could quote: gauge-invariant mass [1], ultraviolet finiteness [2] and the connection between extended supersymmetry and the existence of self-dual soliton solutions [3].

A few years ago, a Maxwell-Chern-Simons gauge theory with an additional magnetic moment interaction was proposed [4], for which Bogomol'nyi-type self-dual equations can be derived and vortex-like configurations appear whenever particular relations between the parameters are obeyed [5]. An important issue that comes about is the claim of a relation between the appearance of self-duality and the N=2-supersymmetric extension of the model.

In this regard, Navrátil [6] has succeeded in writing down an N=2 Chern-Simons model with magnetic moment interaction. His paper relies on a special choice of parameters in order that the supersymmetry be extended. In our work, we also aim at an N=2 version of the Maxwell-Chern-Simons model with magnetic moment interaction. However, instead of building up our action directly in (1+2) dimensions and constraining the parameters so as to achieve an N=2 extension as in [6], we take the viewpoint of first formulating an N=1–D=4 gauge model with a BF-term with no such constraints [7]. Having in mind a magnetic moment interaction in D=3, we consider matter non-minimally coupled to a 2-form gauge potential in D=4 with completely independent coupling constants (we refer to the latter as the Cremmer-Scherk-Kalb-Ramond field). Upon a convenient dimensional reduction of the component-field action from (1+3) to (1+2) dimensions, we set out an N=2–D=3 gauge model with a Chern-Simons term and magnetic moment interaction with the matter sector.

As we shall discuss later, our dimensional reduction procedure must be supplemented by suitable field identifications that do not break the supersymmetries of the extended model. This is necessary in order to ensure that a genuine (non-mixed) Chern-Simons term drops out in 3 dimensions.

Our paper is outlined as follows. In Section 2, we propose the superfield formulation of the N=1–D=4 gauge model with a BF–term and non-minimal coupling between matter and the 2-form potential. Next, in Section 3, we present the details of the dimensional reduction scheme we adopt. The suitable field identifications, the N=2 transformations and the N=2–D=3 Maxwell-Chern-Simons action with anomalous magnetic moment interactions are the subject of Section 4. Finally, in Section 5, we draw our General Conclusions.

2 The N=1–D=4 supersymmetric action

We start off from the following superfield action:

$$\mathcal{S}_{4D} = \int d^4x d^2\theta \left\{ -\frac{1}{8}\mathcal{W}^a\mathcal{W}_a + d^2\bar{\theta} \left[-\frac{1}{2}\mathcal{G}^2 + \frac{1}{2}m\mathcal{V}\mathcal{G} + \frac{1}{16}\bar{\Phi}e^{2h\mathcal{V}}\Phi e^{4g\mathcal{G}} \right] \right\}, \qquad (1)$$

where m is a mass parameter, h and g are coupling constants, whereas $\Phi, \mathcal{W}$ and $\mathcal{V}$ are superfields defined by the θ-expansions below:

$$\Phi = e^{(-i\theta\sigma^\mu\overline{\theta}\partial_\mu)}\left(\varphi(x) + \theta^a\chi_a(x) + \theta^2 S(x)\right), \quad \overline{D}_{\dot{a}}\Phi = 0, \tag{2}$$

$$\mathcal{W}^a = -\frac{1}{4}\overline{D}^2 D^a\mathcal{V},$$

$$\mathcal{V} = C(x) + \theta^a b_a(x) + \overline{\theta}_{\dot{a}}\overline{b}^{\dot{a}} + \theta^2 H(x) + \overline{\theta}^2 H^*(x) + \theta\sigma^\mu\overline{\theta}A_\mu +$$

$$\theta^2\overline{\theta}\left(\overline{\lambda} - \frac{i}{2}\overline{\sigma}^\mu\partial_\mu b(x)\right) + \overline{\theta}^2\theta\left(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\overline{b}(x)\right) + \theta^2\overline{\theta}^2\left(\triangle(x) - \frac{1}{4}\Box C(x)\right). \tag{3}$$

Here D_a and $\overline{D}_{\dot{a}}$ are the supersymmetric covariant derivatives [8]

$$D_a = \partial_a - i\sigma^\mu_{a\dot{a}}\overline{\theta}^{\dot{a}}\partial_\mu$$
$$\overline{D}_{\dot{a}} = -\partial_{\dot{a}} + i\theta^a\sigma^\mu_{a\dot{a}}\partial_\mu. \tag{4}$$

and $\mathcal{G}$ is defined in terms of the chiral spinor superfield

$$\Sigma_a = \psi_a(x) + \theta^b\Omega_{ba}(x) + \theta^2\left[\xi_a(x) + i\sigma^\mu_{a\dot{a}}\partial_\mu\overline{\psi}^{\dot{a}}(x)\right] - i\theta\sigma^\mu\overline{\theta}\partial_\mu\psi_a(x)$$

$$-i\theta\sigma^\mu\overline{\theta}\theta^b\partial_\mu\Omega_{ba}(x) - \frac{1}{4}\theta^2\overline{\theta}^2\Box\psi_a(x), \quad \overline{D}_{\dot{a}}\Sigma_a = 0, \tag{5}$$

by

$$\mathcal{G} = \frac{i}{8}\left(D^a\Sigma_a - \overline{D}_{\dot{a}}\overline{\Sigma}^{\dot{a}}\right).$$

The Lorentz-group irreducible representations accommodated in $\Omega_{ba}(x)$ can be split as follows:

$$\Omega_{ba} = \epsilon_{ba}\rho(x) + (\sigma^{\mu\nu})_{ba}\mathcal{B}_{\mu\nu}(x), \tag{6}$$

with $\rho(x)$ and $\mathcal{B}_{\mu\nu}(x)$ being complex fields:

$$\rho(x) = P(x) + iM(x),$$
$$\mathcal{B}_{\mu\nu}(x) = \frac{1}{4}\left[B_{\mu\nu}(x) - i\tilde{B}_{\mu\nu}(x)\right], \tag{7}$$

with

$$\tilde{B}_{\mu\nu}(x) = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}B^{\alpha\beta}(x). \tag{8}$$

In this way, $\mathcal{B}_{\mu\nu}$ exhibits a self-dual nature:

$$\widetilde{\mathcal{B}}_{\mu\nu} = i\mathcal{B}_{\mu\nu}. \tag{9}$$

$B_{\mu\nu}$ is to be read as the 2-form field in the CSKR model which emerges when one writes the action in components; therefore $\mathcal{G}$ is referred to as the tensor multiplet[9]. The role of the remaining field components introduced above will become clearer later. Let us mention that the number of degrees of freedom is actually not as large as it seems. For instance, both Φ and Σ_a are chiral superfields, and the superfield connection $\mathcal{V}$ will be taken in the Wess-Zumino(WZ) gauge from now on,

$$\mathcal{V} = \theta\sigma^\mu\overline{\theta}A_\mu + \theta^2\overline{\theta\lambda} + \overline{\theta}^2\theta\lambda + \theta^2\overline{\theta}^2\triangle(x). \tag{10}$$

However, for the sake of clarity, we have written it in its wider form, (3), because some of the susy variations shall explicitly exhibit the compensating fields. As usual, an irreducible representation of the susy algebra, involving just the field-strength $F_{\mu\nu}$ together with the gaugino λ and the auxiliary field Δ will be found. In fact, the use of a complete expression for $\mathcal{V}$, as in eq.(3), make it easier to determine the transformation properties of the components under susy transformations, and will enable us to find the proper identifications necessary to formulate a Chern-Simons theory in 3D. Notice also that, within the action, the spinor superfield comes into play only through the field-strength superfield $\mathcal{G}$, which carries just half the degrees of freedom of Σ_a: ψ_a does not appear, ρ appears only through M, and on the same token $\mathcal{B}_{\mu\nu}$ manifests through $\tilde{G}_\mu$, making clear the resulting relevant degrees of freedom. The component-field expansion for $\mathcal{G}$ turns out to be:

$$
\begin{aligned}
\mathcal{G} \;=\; & -\frac{1}{2}M + \frac{i}{4}\theta^a \xi_a - \frac{i}{4}\bar\theta_{\dot a}\bar\xi^{\dot a} + \frac{1}{2}\theta^a \sigma^\mu_{a\dot a}\bar\theta^{\dot a}\tilde G_\mu \\
& +\frac{1}{8}\theta^a\sigma^\mu_{a\dot a}\bar\theta^2\partial_\mu\bar\xi^{\dot a} - \frac{1}{8}\theta^2\sigma^\mu_{a\dot a}\bar\theta^{\dot a}\partial_\mu\xi^a - \frac{1}{8}\theta^2\bar\theta^2\Box M,
\end{aligned}
\tag{11}
$$

where $G_{\mu\nu\kappa}$ and its dual, $\tilde G_\mu$, are given by

$$
\begin{aligned}
G_{\alpha\mu\nu} \;&=\; \partial_\alpha B_{\mu\nu} + \partial_\mu B_{\nu\alpha} + \partial_\nu B_{\alpha\mu}, \\
\tilde G_\mu \;&=\; \frac{1}{3!}\varepsilon_{\mu\nu\alpha\beta}G^{\nu\alpha\beta}.
\end{aligned}
\tag{12}
$$

Therefore, the parametrization described above for $\mathcal{G}$ exhibits a sort of WZ gauge effect for the spinor superpotential Σ_a in that the individual degrees of freedom carried by the latter can be grouped into suitable combinations that correspond to the physical fields.

Going over to components, the action S_{4D} takes the form below:

$$
\begin{aligned}
\mathcal{S}_{4D} \;=\; & \int d^4x \left\{ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{3!}G_{\mu\alpha\beta}G^{\mu\alpha\beta} + m\varepsilon^{\mu\nu\alpha\beta}A_\mu\partial_\nu B_{\alpha\beta} \right. \\
& +2\Delta^2 + \frac{i}{2}\overline\Lambda\Gamma^\mu\partial_\mu\Lambda + \partial_\mu M\partial^\mu M + \frac{i}{4}\overline\Xi\Gamma^\mu\partial_\mu\Xi + im\overline\Lambda\Gamma_5\Xi - 4mM\Delta \\
& +e^{-2gM(x)}\left[\nabla_\mu\varphi\nabla^\mu\varphi^* + \frac{i}{4}\overline X\Gamma^\mu\nabla_{\mu 5}X - \frac{g^2}{2}\partial_\mu M\left(\overline X\Gamma_L\Gamma^\mu\Xi\varphi^* + \overline\Xi\Gamma_L\Gamma^\mu X\varphi\right) \right. \\
& +\frac{g}{2}\left(\overline\Xi\Gamma^\mu\Gamma_R X\nabla_\mu\varphi + \overline X\Gamma_L\Gamma^\mu\Xi\nabla_\mu\varphi^*\right) - i\frac{g^2}{4}\varphi^*\varphi\overline\Xi\Gamma^\mu\partial_\mu\Xi - \frac{g^2}{4h}\overline\Xi\Gamma_5\Gamma^\mu\mathcal{J}_\mu\Xi \\
& +\varphi\varphi^*\left(2h\Delta + igh\overline\Lambda\Gamma_5\Xi - g^2\partial_\mu M\partial^\mu M\right) - h(\varphi\overline\Lambda\Gamma_R X + \varphi^*\overline\Lambda\Gamma_L X) \\
& \left.\left. +\left(S - \frac{ig}{2}\overline X\Gamma_L\Xi + \frac{g^2}{4}\overline\Xi\Gamma_L\Xi\varphi\right)\left(S^* + \frac{ig}{2}\overline X\Gamma_R\Xi + \frac{g^2}{4}\overline\Xi\Gamma_R\Xi\varphi^*\right) \right] \right\},
\end{aligned}
\tag{13}
$$

where we have organized the fermionic fields so as to form four-component Majorana spinors as follows:

$$
\Xi(x) \equiv \begin{pmatrix} \xi_a(x) \\ \bar\xi^{\dot a}(x) \end{pmatrix},\quad X \equiv \begin{pmatrix} \chi_a \\ \overline\chi^{\dot a} \end{pmatrix},\quad \Lambda \equiv \begin{pmatrix} \lambda_a \\ \overline\lambda^{\dot a} \end{pmatrix},
$$

and the current $\mathcal{J}_\mu$ is given by

$$\mathcal{J}_\mu = -\frac{ih}{2}\left(\varphi^*\nabla_\mu\varphi - \varphi\nabla_\mu\varphi^*\right), \tag{14}$$

with

$$\nabla_\mu\varphi = \left(\partial_\mu + ihA_\mu + ig\tilde{G}_\mu\right)\varphi. \tag{15}$$

Also, there appears a covariant derivative with Γ_5-couplings

$$\nabla_{\mu 5}X = \left(\partial_\mu - ihA_\mu\Gamma_5 - ig\tilde{G}_\mu\Gamma_5\right)X. \tag{16}$$

It is noteworthy to pay attention to the presence of the bosonic CSKR Lagrangian among the first 3 terms of eq.(13). We shall see in Section 4 how the corresponding mixing term can be manipulated so as to give rise to the usual Chern-Simons term. We can also recognize, in the first term in the square brackets, a kinetic piece which corresponds to the non-minimal coupling of scalar matter to the CSKR gauge fields. These four terms define an Abelian gauge invariant theory which we will carefully analyse as a guide to connect both gauge groups.

It is worthwhile noting that, in order to achieve the four-component spinors in the expression above, we have chosen the following representation for the Γ-matrices in (1+3) dimensions:

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{a\dot{b}} \\ \bar{\sigma}^{\mu\dot{a}b} & 0 \end{pmatrix}.$$

Of course, the action is independent of such a choice and in the next section we shall adopt a Majorana-like representation in order to perform dimensional reduction.

The susy transformations of the components fields are listed below:

$$\begin{aligned} \delta\varphi &= \varepsilon^a\chi_a, \\ \delta\chi_a &= 2\varepsilon_a S - 2i\sigma^\mu_{a\dot{a}}\bar{\varepsilon}^{\dot{a}}D_\mu\varphi, \\ \delta S &= -i\bar{\varepsilon}_{\dot{a}}\bar{\sigma}^{\mu\dot{a}a}D_\mu\chi_a + 2h\overline{\lambda}_{\dot{a}}\bar{\varepsilon}^{\dot{a}}\varphi; \end{aligned} \tag{17}$$

$$\begin{aligned} \delta M &= \frac{i}{2}\bar{\varepsilon}_{\dot{a}}\bar{\xi}^{\dot{a}} - \frac{i}{2}\varepsilon^a\xi_a, \\ \delta\xi_a &= 2\sigma^\mu_{a\dot{a}}\bar{\varepsilon}^{\dot{a}}\left(\partial_\mu M - i\tilde{G}_\mu\right), \\ \delta\tilde{G}^\mu &= \frac{i}{2}\varepsilon^b\left(\sigma^{\mu\nu}\right)_b{}^a\partial_\nu\xi_a + \frac{i}{2}\bar{\varepsilon}_{\dot{b}}\left(\bar{\sigma}^{\mu\nu}\right)^{\dot{b}}{}_{\dot{a}}\partial_\nu\bar{\xi}^{\dot{a}}; \end{aligned} \tag{18}$$

$$\begin{aligned} \delta A^\mu &= \varepsilon^a\sigma^\mu_{a\dot{a}}\overline{\lambda}^{\dot{a}} - \bar{\varepsilon}_{\dot{a}}\bar{\sigma}^{\mu\dot{a}a}\lambda_a, \\ \delta\lambda_a &= 2\varepsilon_a\Delta + \frac{i}{2}\sigma^\nu_{a\dot{b}}\bar{\sigma}^{\mu\dot{b}b}\varepsilon_b F_{\mu\nu}, \\ \delta\Delta &= -\frac{i}{2}\varepsilon^a\sigma^\mu_{a\dot{a}}\partial_\mu\bar{\lambda}_{\dot{a}} - \frac{i}{2}\bar{\varepsilon}_{\dot{a}}\bar{\sigma}^{\mu\dot{a}a}\partial_\mu\lambda_a. \end{aligned} \tag{19}$$

Now, it is clear that the first and the second groups form respectively two irreducible representations of the susy algebra while the last three terms, together with

$$\delta F_{\mu\nu} = \varepsilon^a \sigma_{[\nu a \dot a} \partial_{\mu]} \overline{\lambda}^{\dot a} - \bar\varepsilon_{\dot a} \bar\sigma^{\dot a a}_{[\nu} \partial_{\mu]} \lambda_a, \tag{20}$$

close another one. On the other hand, A_μ transforms along with the compensating fields b and $\bar b$ but we can always fix the WZ gauge to eliminate them. In fact, it is not worth exhibiting the susy transformations of any of the compensating fields themselves as they become zero in the WZ gauge. Analogously, in the second group of variations, it can be seen that (as it occurs in the action) P, the real part of ρ, is irrelevant, ψ_a does not appear and $B_{\mu\nu}$ contributes only through $\tilde G_\mu$. We have however shown the variation of A_μ just because it enters the action not exclusively through $F_{\mu\nu}$.

The list of field variations in terms of four-component spinors is then

$$\begin{aligned}
\delta\varphi &= \overline{\mathcal{E}}\Gamma_L X \\
\delta X &= 2\left(S - i\Gamma^\mu D_\mu \varphi^*\right)\Gamma_L \mathcal{E} + 2\left(S^* - i\Gamma^\mu D_\mu \varphi\right)\Gamma_R \mathcal{E} \\
\delta S &= -i\overline{\mathcal{E}}\Gamma^\mu \Gamma_L D_\mu X + 2h\overline{\mathcal{E}}\Gamma_R \Lambda\varphi
\end{aligned} \tag{21}$$

$$\begin{aligned}
\delta M &= \frac{i}{2}\overline{\mathcal{E}}\Gamma_5 \Xi \\
\delta\Xi &= 2\Gamma^\mu \left(\Gamma_5 \partial_\mu M - i\tilde G_\mu\right)\mathcal{E} \\
\delta\tilde G^\mu &= \frac{i}{2}\overline{\mathcal{E}}\Gamma^{\mu\nu}\partial_\nu \Xi
\end{aligned} \tag{22}$$

$$\begin{aligned}
\delta F_{\mu\nu} &= \overline{\mathcal{E}}\Gamma^{[\nu}\Gamma_5 \partial^{\mu]}\Lambda \\
\delta\Lambda &= 2\Delta\mathcal{E} - \frac{i}{2}\Gamma_5 \Gamma^\mu \Gamma^\nu F_{\nu\mu}\mathcal{E} \\
\delta\Delta &= -\frac{i}{2}\overline{\mathcal{E}}\Gamma^\mu \partial_\mu \Lambda,
\end{aligned} \tag{23}$$

$\mathcal{E}$ being the (infinitesimal) Majorana-spinor parameter of the susy transformation,

$$\mathcal{E} \equiv \begin{pmatrix} \varepsilon_a \\ \bar\varepsilon^{\dot a} \end{pmatrix}.$$

3   The dimensional reduction: from D=4 to D=3

¿From now on, we shall identify four-dimensional Lorentz indices by $\hat\mu = 0, 1, 2, 3$, while in three-dimensional space-time we will keep bare greek indices, namely, $\mu = 0, 1, 2$.

Let us first perform the dimensional reduction of the bosonic sector of the susy action, eq.(13). For this, we will adopt the following procedure: we eliminate the third spatial coordinate so as to make the D=3-fields x_3-independent [10],

$$\partial_3(\text{fields}) = 0. \tag{24}$$

On the other hand, we will assume that the $\hat{\mu} = 3$ component of the D=4-fields are taken as scalars in (1+2) dimensions. In this scheme, the Poincaré invariance in D=1+3 has been broken down to the direct product between Poincaré invariance in D=1+2 and a U(1) factor. Thus, A_μ is in the vector representation of the Lorentz group and is a singlet of such a U(1) while A_3 is an independent scalar field. The relevant (off-shell) degrees of freedom of $B_{\hat{\mu}\hat{\nu}}$ are $B_{\mu\nu}$ and $B_{\mu 3}$ as it is an antisymmetric tensor. Accordingly, in three-dimensional space, we shall make the following identifications:

$$N \equiv A^3, \quad B^\mu \equiv B^{3\mu}, \quad B^{\mu\nu} \equiv \varepsilon^{\mu\nu\rho} Z_\rho,$$
$$\partial_\mu Z^\mu = -\tilde{G}^3, \quad \partial^\mu B^\nu - \partial^\nu B^\mu = G^{\mu\nu}, \tag{25}$$

with $\varepsilon^{\mu\nu\rho} \equiv \varepsilon^{\mu\nu\rho 3}$. Hence, after decomposition, the bosonic terms reduce as given below:

$$-\frac{1}{6}G_{\hat{\mu}\hat{\nu}\hat{\rho}}G^{\hat{\mu}\hat{\nu}\hat{\rho}} \quad \longrightarrow \quad -\frac{1}{2}G_{\mu\nu}G^{\mu\nu} + \partial_\mu Z^\mu \partial_\nu Z^\nu,$$
$$-\frac{1}{4}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}} \quad \longrightarrow \quad -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu N \partial^\mu N,$$
$$m\varepsilon_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\lambda}}A^{\hat{\mu}}\partial^{\hat{\nu}}B^{\hat{\rho}\hat{\lambda}} \quad \longrightarrow \quad 2m\varepsilon_{\mu\nu\rho}A^\mu\partial^\nu B^\rho + 2mN\partial_\mu Z^\mu,$$
$$\nabla_{\hat{\mu}}\phi^*\nabla^{\hat{\mu}}\phi \quad \longrightarrow \quad \nabla_\mu\phi^*\nabla^\mu\phi - (hN - g\partial_\mu Z^\mu)^2 |\varphi|^2.$$

In order to proceed with the dimensional reduction in the fermionic sector, let us mention that one can always construct a representation of the Clifford algebra in the form of a tensor product of lower dimensional matrices. We use capital $\Gamma^{\hat{\mu}}$ for Dirac matrices in the higher dimension and lower case γ^μ in the lower dimension. A suitable set of the 4D Γ-matrices is the following

$$\Gamma^\mu = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & -\gamma^\mu \end{pmatrix}, \qquad \Gamma^3 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

Taking $\gamma^0 \equiv \sigma_y$, $\gamma^1 \equiv i\sigma_z$, and $\gamma^2 \equiv i\sigma_x$, we have a Majorana representation both in D=4 and D=3. So, in this way, a Majorana spinor in D=4 is real and splits into a doublet of real Majorana spinors in D=3.

It is worth noting, before dimensionally reducing the fermionic sector, that the relevant degrees of freedom reorganize themselves as follows. The field content of four-component spinors:

$$X \quad \rightarrow \quad \chi, \, \omega$$
$$\Xi \quad \rightarrow \quad \xi, \, \zeta$$
$$\Lambda \quad \rightarrow \quad \lambda, \, \eta$$

gives rise to the following Dirac spinors

$$X_\pm = \chi \pm i\omega$$
$$\Xi_\pm = \xi \pm i\zeta \tag{26}$$
$$\Lambda_\pm = \lambda \pm i\eta$$

Namely, the two-component Majorana fermions corresponding to each 4D spinor become completely independent in 3D and, further, in the reduced action they appear as Dirac spinors in the particular way indicated in eq.(26). On the same footing, the infinitesimal susy parameter will break down into two dissociated spinorial species

$$\mathcal{E} \longrightarrow \varepsilon, \delta \;\; \rightarrow \varepsilon_{\pm} = \varepsilon \pm i\delta, \tag{27}$$

revealing the existence of two supersymmetries in the reduced theory.

4   The N=2–D=3 Model

In terms of the D=3 bosonic fields and Dirac fermions defined above, the three-dimensional action reads

$$
\begin{aligned}
\mathcal{S}_{3D} \;=\; & \int d^3x \left\{ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}G_{\mu\nu}G^{\mu\nu} + 2m\varepsilon^{\mu\nu\alpha}A_\mu\partial_\nu B_\alpha + 2\Delta^2 + (\partial_\mu Z^\mu)^2 \right. \\
& + \frac{i}{2}\overline{\Lambda}_-\,\slashed{\partial}\,\Lambda_- + \frac{i}{4}\overline{\Xi}_-\,\slashed{\partial}\,\Xi_- + \frac{i}{2}m(\overline{\Lambda}_+\Xi_- - \overline{\Lambda}_-\Xi_+) + \frac{1}{2}\partial_\mu N\partial^\mu N \\
& - 4mM\Delta + 2mN\partial_\mu Z^\mu + \partial_\mu M\partial^\mu M + e^{-2gM}\left[\nabla_\mu\varphi\nabla^\mu\varphi^* - (hN - g\partial_\mu Z^\mu)^2\,\varphi\varphi^* \right. \\
& + \frac{1}{4}(hN - g\partial_\mu Z^\mu)\,X_+X_+ + \frac{i}{8}(\overline{X}_-\,\slashed{\nabla}_-X_- + \overline{X}_+\,\slashed{\nabla}X_+) \\
& + \frac{g}{4}\left[(\overline{\Xi}_-\gamma^\mu X_- + \overline{X}_+\gamma^\mu\Xi_+)\nabla_\mu\varphi - i(\overline{\Xi}_-X_-\varphi + \overline{X}_-\Xi_-\varphi^*)(hN - g\partial_\mu Z^\mu)\right] \\
& - \frac{g^2}{4}\partial_\mu M\left(\overline{X}_-\gamma^\mu\Xi_-\varphi^* + \overline{\Xi}_-\gamma^\mu X_-\varphi\right) - i\frac{g^2}{8}\varphi^*\varphi\left(\overline{\Xi}_-\,\slashed{\partial}\,\Xi_- + \overline{\Xi}_+\slashed{\partial}\,\Xi_+\right) \\
& + \frac{g^2}{4h}\left(\frac{1}{2}(\overline{\Xi}_-\gamma^\mu\mathcal{J}_\mu\Xi_- - \overline{\Xi}_+\gamma^\mu\mathcal{J}_\mu\Xi_+) + \overline{\Xi}_-\Xi_-h(hN - g\partial_\mu Z^\mu)\varphi\varphi^*\right) \\
& \left. + \varphi\varphi^*\left(2h\Delta + \frac{igh}{2}(\overline{\Lambda}_+\Xi_- - \overline{\Xi}_-\Lambda_+) - g^2\partial_\mu M\partial^\mu M\right) - \frac{h}{2}(\varphi\overline{\Lambda}_+X_- + \varphi^*\overline{X}_-\Lambda_+)\right. \\
& \left.\left. + \left| S - \frac{ig}{4}\overline{X}_-\Xi_+ + \frac{g^2}{8}\varphi\overline{\Xi}_-\Xi_+ \right|^2 \right]\right\}. \tag{28}
\end{aligned}
$$

Although it looks so large an expression, it has been written in a rather compact notation and, again, it can be recognized that it contains a mixed CS theory. Of course, we would like to know more about its physical meaning, which is still fairly obscure. Actually, the bosonic sector characterizes a parity-preserving statistical gauge theory which can be related to superconductivity at finite temperature [11]. At this point, we should draw the attention to the Z_μ–field, dual of the two-form B in three dimensions. Its kinetic term is not built up from the usual field strenght, as it is the case for ordinary gauge vector fields. An inspection of its Abelian transformation shows that its transverse part can be gauged away. This is why only its longitudinal part propagates off-shell. Such a peculiar gauge field does not correspond to any phyisical excitation: a two-form gauge field presents no on-shell degree of freedom in $D = 3$. So, the kinetic term for Z_μ is harmless, for no ghost excitation is present in the spectrum.

In the next section, we shall find a remarkable result coming from a detailed inspection of the Lagrangian (28) and the susy transformations, suggesting a simple identification between some of the several fields appearing at the present stage. Indeed, as we shall propose later, the 3-divergence of Z_μ will be identified as the auxiliary component of the gauge superfield. Now, let us evaluate the two susy transformations acting on the 3D fields. The scalar multiplet transforms as

$$
\begin{aligned}
\delta\varphi &= \frac{1}{2}\bar\varepsilon_- X_+,\\
\delta S &= -\frac{i}{2}\bar\varepsilon_+ \left(\slashed{D} - ihN\right)X_+ + h\bar\varepsilon_+\Lambda_-\varphi,\\
\delta X_+ &= 2S\varepsilon_+ + 2\left(hN\varphi - i\slashed{D}\varphi\right)\varepsilon_-,\\
\delta X_- &= 2S^*\varepsilon_- + 2\left(hN\varphi^* - i\slashed{D}\varphi^*\right)\varepsilon_+;
\end{aligned}
\tag{29}
$$

the vector multiplet transformations read as follows

$$
\begin{aligned}
\delta N &= -\frac{1}{2}(\bar\varepsilon_+\Lambda_+ + \bar\varepsilon_-\Lambda_-),\\
\delta\Lambda_\pm &= (2\Delta + i\slashed{\partial} N)\varepsilon_\pm \pm \gamma_\mu\tilde{F}^\mu\varepsilon_\pm,\\
\delta\Delta &= -\frac{i}{2}(\bar\varepsilon_+\slashed{\partial}\Lambda_+ + \bar\varepsilon_-\slashed{\partial}\Lambda_-),\\
\delta\tilde{F}^\mu &= -\frac{1}{2}(\bar\varepsilon_+\varepsilon^{\mu\nu\lambda}\gamma_\lambda\partial_\nu\Lambda_+ - \bar\varepsilon_-\varepsilon^{\mu\nu\lambda}\gamma_\lambda\partial_\nu\Lambda_-),
\end{aligned}
\tag{30}
$$

and, finally, the tensor multiplet components transform according to

$$
\begin{aligned}
\delta M &= \frac{i}{4}(\bar\varepsilon_+\Xi_- - \bar\varepsilon_-\Xi_+),\\
\delta\Xi_\pm &= \pm2(\slashed{\partial} M + i\partial_\mu Z^\mu)\varepsilon_\mp - 2i\gamma_\mu\tilde{G}^\mu\varepsilon_\mp,\\
\delta\tilde{(\partial_\mu Z^\mu)} &= \frac{1}{4}(\bar\varepsilon_+\slashed{\partial}\Xi_- - \bar\varepsilon_-\slashed{\partial}\Xi_+),\\
\delta\tilde{G}^\mu &= -\frac{i}{4}(\bar\varepsilon_-\varepsilon^{\mu\nu\lambda}\gamma_\lambda\partial_\nu\Xi_+ + \bar\varepsilon_+\varepsilon^{\mu\nu\lambda}\gamma_\lambda\partial_\nu\Xi_-).
\end{aligned}
\tag{31}
$$

These susy variations show on the one hand that S_{3D} is indeed N=2–supersymmetric, and, on the other hand, they exhibit the key to get one's hands on the underlying Chern-Simons Lagrangian by making manifest the relevant degrees of freedom to realize the supersymmetry algebra.

Once the N=2 transformation laws have been cleared up, we shall demonstrate that the previous susy action (28) may be suitably manipulated so as to give rise to a more familiar system, namely, the supersymmetric extension of a non-minimal Maxwell-Chern-Simons theory.

The first natural attempt is to associate the two vector gauge fields and then look for the corresponding fermionic connection in order to keep both supersymmetries. Doing so, one gets to the conclusion that, by means of simply identifying A_μ and B_μ $(A_\mu \equiv B_\mu)$, we can complete the remaining identifications:

$$
N \equiv -M, \quad \Delta \equiv -\frac{1}{2}\tilde{G}^3, \quad \Lambda_\pm \equiv \pm\frac{i}{2}\Xi_\mp
\tag{32}
$$

so as to achieve the connection between the two gauge groups of the outset and obtain a proper MCS N=2-supersymmetric theory with non-minimal coupling. Of course, as long as the two sets of fields transform identically under a symmetry transformation, we may identify them without breaking supersymmetry.

We also found it convenient to redefine the superfields and coupling constants of eq.(1) as below:

$$\mathcal{V} \to \mathcal{V}/\sqrt{3} \quad \mathcal{G} \to \mathcal{G}/\sqrt{3}$$

$$h \to \sqrt{3}h \quad g \to \sqrt{3}g \quad m \to \frac{3}{4}m$$

the reason being that we wish to get the D=3-action with the canonical kinetic terms. In so doing, the action of eq.(1) becomes

$$\mathcal{S}_{4D} = \int d^4x d^2\theta \left\{ -\frac{1}{24}\mathcal{W}^a\mathcal{W}_a + d^2\overline{\theta} \left[-\frac{1}{6}\mathcal{G}^2 + \frac{1}{8}m\mathcal{V}\mathcal{G} + \frac{1}{16}\overline{\Phi}e^{2h\mathcal{V}}e^{4g\mathcal{G}}\Phi \right] \right\} \tag{33}$$

bringing about the usual MCS terms in the N=2-susy action:

$$
\begin{aligned}
\mathcal{S}_{MCS}^{N=2} = & \int d^3x \left\{ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m}{2}\varepsilon^{\mu\nu\alpha}A_\mu\partial_\nu A_\alpha + 2\Delta^2 + \frac{1}{2}\partial_\mu N\partial^\mu N + 2mN\Delta \right. \\
& + \frac{1}{2}\overline{\Lambda}_-(i\slashed{\partial} + m)\Lambda_- + e^{2gN}\left[\nabla_\mu\varphi\nabla^\mu\varphi^* - (hN - 2g\Delta)^2\,\varphi\varphi^* \right. \\
& + \frac{1}{4}(hN - 2g\Delta)\,X_+X_+ + \frac{i}{8}(\overline{X}_-\slashed{\nabla}_-X_- + \overline{X}_+\slashed{\nabla}X_+) \\
& + \frac{ig}{2}\left(\overline{\Lambda}_+\slashed{\nabla}\varphi X_- - h.c.\right) - \frac{g}{2}(hN - 2g\Delta)\left(\overline{\Lambda}_+X_-\varphi + h.c.\right) \\
& - \frac{ig^2}{2}\partial_\mu N\left(\overline{X}_-\gamma^\mu\Lambda_+\varphi^* - \overline{\Lambda}_+\gamma^\mu X_-\varphi\right) - i\frac{g^2}{2}\varphi^*\varphi\left(\overline{\Lambda}_+\slashed{\partial}\Lambda_+ + \overline{\Lambda}_-\slashed{\partial}\Lambda_-\right) \\
& + \frac{g^2}{h}\left(\frac{1}{2}(\overline{\Lambda}_+\gamma^\mu\mathcal{J}_\mu\Lambda_+ - \overline{\Lambda}_-\gamma^\mu\mathcal{J}_\mu\Lambda_-) + \overline{\Lambda}_+\Lambda_+h(hN - 2g\Delta)\varphi\varphi^* \right) \\
& + \varphi\varphi^*\left(2h\Delta + 2gh\overline{\Lambda}_+\Lambda_+ - g^2\partial_\mu N\partial^\mu N \right) \\
& \left. \left. - \frac{h}{2}(\varphi\overline{\Lambda}_+X_- + \varphi^*\overline{X}_-\Lambda_+) + \left| S + \frac{g}{2}\overline{X}_-\Lambda_- - \frac{g^2}{2}\varphi\overline{\Lambda}_+\Lambda_- \right|^2 \right] \right\},
\end{aligned}
\tag{34}
$$

where now

$$\nabla_\mu\varphi = \left(\partial_\mu + ihA_\mu + ig\tilde{F}_\mu \right)\varphi. \tag{35}$$

Note that the last transformation changes only the kinetic and topological-mass terms, but not the interaction terms.

5   General conclusions

We have here obtained the N=2–supersymmetric extension of a Maxwell-Chern-Simons system with (non-minimal) magnetic moment interaction as the by-product of an N=1–D=4 Maxwell-BF model with non-minimal coupling between matter and the 2-form Abelian gauge potential.

Concerning the results of ref.[6], where the author extends supersymmetry in the absence of a neutral scalar superfield, we remark that our procedure provides an N=2–model containing such an "extra" scalar, N, as a natural consequence of the dimensional reduction. Thus, its presence is well-justified: it appears as a three-dimensional descent of the D=4 gauge sector.

Also in contrast to the results of ref.[6], we do not need to impose that all fields have the same mass. The reason is that, in our case, we build up the N=2–D=3 action with independent matter and gauge multiplets. The mass degeneracy then takes place only inside each multiplet. In ref. [6], in turn, the N=2–D=3 model is formulated in terms of a single gauge multiplet that encompasses the physical scalar among its components. We claim that we place ourselves in the appropriate context for generating *topological* vortices [12], an issue which has not still been thoroughly investigated.

Our construction is performed in terms of D=3–component fields. It would also be interesting to carry out the dimensional reduction while working in superspace, namely, to dimensionally reduce the N=1–D=4 superspace action without passing through components. The D=3 superspace action must be manifestly N=2 supersymmetric and its component-field projection is to be compared with the action as written in eq. (34). The three dimensional version of the model may also be written in terms of N=1–superfields. The results of our efforts in this direction will be soon reported elsewhere [13].

Finally, by choosing a suitable N=2–matter self-interaction potential one can find out self-dual vortex configurations and derive the central charge of the N=2–susy algebra. These results will be presented in a forthcoming communication [14].

Acknowledgements

References

[1] R. Jackiw and S. Templeton, Phys. Rev. **D23** (1981) 2291; J. Schonfeld, Nucl. Phus. **B185** (1981) 157; S. Deser, R. Jackiw and S. Templeton, Phys. Rev. Lett. 48 (1982) 975;

[2] A. Blasi and R. Collina, Nucl.Phys. **B345** (1990) 472; F. Delduc, C. Lucchesi, O. Piguet and S.P. Sorella, Nucl. Phys. **B346** (1990) 313; A. Blasi, O. Piguet and S.P. Sorella, Nucl. Phys. **B356** (1991) 154; C. Lucchesi and O. Piguet, Nucl. Phys. **B381** (1992) 281.

[3] R. Jackiw, K. Lee and E.J. Weinberg, Phys. Rev. **D42** (1990) 3488; R. Jackiw and E.J. Weinberg, Phys. Rev. Lett. **64** (1990) 2234.

[4] Jan Stern, Phys. Lett. **B265** (1991) 119.

[5] M. Torres, Phys. Rev. **D46** (1992) R2295; T. Lee and H. Min, Phys. Rev. **D50** (1994) 7738; M. Torres, Phys. Rev. **D51** (1995) 4533.

[6] P. Navrátil, Phys. Lett. **B365** (1996) 119.

[7] G.T. Horowitz, Commun. Math. Phys. **125** (1989) 417; M. Blau and G. Thompson, Ann. Phys. **205** (1991) 130; D. Birmingham, M. Blau, M. Rakowski and G. Thompson, Phys. Rep. **C209** (1991) 129.

[8] O. Piguet and S.P. Sorella, *Algebraic Renormalization,* Monographs Series, m 28, Springer Verlag, Berlin, 1995.

[9] V.E.R. Lemes, A.L.M.A. Nogueira and J.A. Helayël-Neto, *to appear* in Int. J. Mod. Phys. **A**, (1998).

[10] J. Scherk, *Extended Supersymmetry and Extended Supergravity Theories,* in *Recent Developments in Gravitation,* Cargèse 1978, Edited by M. Lévy and S. Deser, Plenum Press; L. Brink, J.H. Schwarz and J. Scherk, Nucl. Phys. **B121** (1977) 77.

[11] N. Dorey and N.E. Mavromatos, Phys. Lett. **B 266** (1991) 163; *idem* Nucl. Phys. **B386** (1992) 614.

[12] M.S. Cunha, H.R. Christiansen, C.A. Almeida, `hep-th/9706206`.

[13] H.R. Christiansen, M.S. Cunha, J.A. Helayël-Neto, L.R.U. Manssur, A.L.M.A. Nogueira, forthcoming communication.

[14] H.R. Christiansen, M.S. Cunha, J.A. Helayël-Neto, L.R.U. Manssur, A.L.M.A. Nogueira, forthcoming communication.