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On the Renormalization of Topological Yang-Mills Field Theory in N=1 Superspace

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ABSTRACT

We discuss the renormalization aspects of topological super-Yang-Mills field theory in N=1 superspace. Our approach makes use of the regularization independent BRS algebraic technique adapted to the case of a N=1 supersymmetric model. We give the expression of the most general local counterterm to the classical action to all orders of the perturbative expansion. The counterterm is shown to be a BRS-coboundary, implying that the cohomological properties of the supertopological theory are not affected by quantum effects. We also demonstrate the vanishing of the Callan-Symanzik β-function of the model by employing a recently discovered supersymmetric antighost Ward identity.

Key-words: Renormalization; Supersymmetry.

1 Introduction

One of the most fruitful theoretical achievements of the last decade has been the construction of the topological Yang-Mills field theories by Witten [1]. In their original formulation, these models were conceived to provide an operational tool for the evaluation of the Donaldson's invariants of four-manifolds [2].

The topological field theories are also interesting from a physical viewpoint: they are conjectured to describe the unbroken phase of general covariance in quantum gravity [1]. Hence, it seems justifiable to analyze its ultraviolet behaviour at one-loop level [3, 4, 5, 6, 7, 8] or even to all orders of the perturbative expansion [9, 10].

On the other hand, one is aware of the remarkable finiteness features displayed by a large class of supersymmetric quantum field models: non-renormalization theorems [11], miraculous amplitude cancellations [12, 13] and the complete vanishing of the gauge β-function of some extended theories [14]. By taking all these facts into account, we could eventually ask ourselves whether manifest supersymmetry may also entail any significant improvement to the renormalization properties of Witten's topological model. However, to investigate this issue in more detail, one should verify if the topological structure of the theory is consistent with the supersymmetric formalism, i.e. if its supersymmetric generalization is indeed feasible. Fortunately, in ref.[15], Birmingham et al clarified this last point and succeded in writting down an action functional which generates supersymmetric (anti) self-dual equations, corresponding to a topological super-Yang-Mills theory at the level of component fields. Soon after, the complete N=1 (and also N=2) superspace version of the model was built up by Ader et al in [16, 17], allowing us to employ the powerful superfield machinery which is known to keep supersymmetry manifest at all steps of the renormalization procedure.

This is the purpose of this letter: to study topological super-Yang-Mills theory in the ultraviolet regime by using the algebraic BRS technique [18, 19, 20] adapted to N=1 superspace [21, 22]. As a result, one determines the most general local counterterm to the action to all orders and in a regularization independent way. We observe then that the number of field monomials appearing in the counterterm expression is substantially reduced when compared to the already existent non-supersymmetric computations [9, 10]. This is due to supersymmetry and to an additional set of symmetry constraints which show up when a Landau gauge is enforced in superspace, suggesting that supersymmetry might play an interesting role in the perturbation theory of topological models. The present paper may be considered as the conclusion to the algebraic renormalization programme of the Witten's model which was initiated in [9].

We organize the work as follows: in section 2, one describes the super-topological model in the classical approximation and in a Landau type gauge, in section 3, we address the problem of the absence of anomaly in the Slavnov-Taylor identity and determine the counterterm underlining some of its properties. Section 4 contains some concluding comments.

2 The Classical Approximation

We start by recalling the notations and conventions adopted by Ader et al in refs.[16, 17]. In order to properly describe the supersymmetric (anti-)instantonic configurations, one is led to define the topological N=1 super-Yang-Mills theory in a rigid superspace based on a four-dimensional Euclidean space instead of a Minkowskian one. As explained in detail in [16], the geometric invariance group which is of relevance here is $SO(4,\mathbb{R}) \sim SU(2,\mathbb{C}) \otimes SU(2,\mathbb{C})$ implying that it is impossible to connect left and right handed spinors through complex conjugation. The main consequence for supersymmetry is that, in principle, the α and $\dot{\alpha}$ indices are to be understood as totally unrelated. Still, all the formal manipulations of N=1 superspace remain valid in the case at hand.

Following the standard construction of the N=1 supersymmetric gauge models, one introduces the gauge real superfield $\mathcal{V}$ (the prepotential) which takes values in the adjoint representation of an arbitrary compact gauge group $\mathcal{G}$. In the antichiral representation the two independent field-strengths read as:

$$W_\alpha = e^{\mathcal{V}} \left[-\bar{D}^2 \left(e^{-\mathcal{V}} D_\alpha e^{\mathcal{V}} \right) \right] e^{-\mathcal{V}} \tag{2.1}$$

and

$$\bar{W}_{\dot{\alpha}} = D^2 \left(e^{\mathcal{V}} \bar{D}_{\dot{\alpha}} e^{-\mathcal{V}} \right) \tag{2.2}$$

with D_α and $\bar{D}_{\dot{\alpha}}$ standing for the two supersymmetric covariant derivatives. The quantities (2.1) and (2.2) are then employed to define an action functional that generalizes the second Chern class in N=1 superspace, viz.

$$S_{inv.} = \frac{1}{128i} \mathrm{Tr} \left\{ \int dS \, W^\alpha W_\alpha - \int d\bar{S} \, \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \right\}, \tag{2.3}$$

where $dS = d^4x D^2$ and $d\bar{S} = d^4x \bar{D}^2$ are the chiral and antichiral integration measures respectively (see refs.[21, 22]). One can convince himself that the gauge invariant action $S_{inv.}$ is also a topologically invariant object by introducing a deformation or shift as:

$$s_1 \mathcal{V} = \Psi, \qquad s_1 \Psi = 0, \tag{2.4}$$

with Ψ representing the topological ghost, a real superfield belonging to the adjoint representation of $\mathcal{G}$ and bearing one unit of Faddeev-Popov ghost charge. Now, as it may be directly seen, the s_1 operation acts onto (2.3) to give

$$s_1 S_{inv.} = \frac{1}{64i} \mathrm{Tr} \left\{ \int dV \left(e^{\mathcal{V}} s_1 e^{-\mathcal{V}} \right) \left(\nabla^\alpha W_\alpha - \bar{\nabla}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \right) \right\}, \tag{2.5}$$

where $dV = d^4x D^2 \bar{D}^2$ is the real integration measure over the whole superspace. It turns out then that the deformation expressed in (2.5) above is zero due to the Bianchi identity:

$$\nabla^\alpha W_\alpha = \bar{\nabla}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}, \tag{2.6}$$

the gauge supercovariant derivatives ∇_α and $\bar{\nabla}_{\dot{\alpha}}$ being given in the antichiral representation by:

$$\nabla_\alpha = D_\alpha, \qquad \bar{\nabla}_{\dot{\alpha}} = e^{\mathcal{V}} \bar{D}_{\dot{\alpha}} e^{-\mathcal{V}}. \tag{2.7}$$

Table 1: Dimensions and Faddeev-Popov ghost charges of the superfields.

	$\mathcal{V}$	Ψ	c_+	$\bar{c}_+$	Φ	$\bar{\Phi}$	$\bar{\psi}_{\dot{\alpha}}$	$\bar{b}_{\dot{\alpha}}$	Λ	$\bar{\Lambda}$	β	$\bar{\beta}$	c_-	$\bar{c}_-$	B	$\bar{B}$
$dim.$	0	0	0	0	0	0	3/2	3/2	2	2	2	2	2	2	2	2
$\mathcal{N}_g$	0	1	1	1	2	2	-1	0	-2	-2	-1	-1	-1	-1	0	0

Let us also underline another noteworthy aspect about the classical action in (2.3): $S_{inv.}$ is a pure surface term in flat Euclidean space, as it is its bosonic counterpart (i.e. the second Chern class). As a consequence, this supertopological piece will not contribute to the renormalization process one is aiming to analyze. In fact, the field-strengths do obey the constraint:

$$D^2(W^\alpha W_\alpha) - \bar{D}^2(\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}) = \partial^m \Delta_m. \tag{2.8}$$

Now, to quantize the model one has to introduce additional superfields in much the same way as in [23, 24, 25]: a pair of antichiral spinor superfields $(\bar{b}_{\dot{\alpha}}, \bar{\psi}_{\dot{\alpha}})$ and two couples of chiral and antichiral ghost superfields, (c_+, Φ) and $(\bar{c}_+, \bar{\Phi})$ respectively. We need moreover the Lagrange multipliers (B, β) and the antighosts (c_-, Λ) taken as chiral superfields, as well as their antichiral partners $(\bar{B}, \bar{\beta})$ and $(\bar{c}_-, \bar{\Lambda})$. All the aforementioned variables are Lie algebra valued and possess dimensions and Faddeev-Popov ghost charges as attributed in Table 1. The nilpotent BRS transformations proposed in [16] write as below:

$$\begin{aligned}
s\mathcal{V} &= \Psi + \tfrac{1}{2}\left[\mathcal{V}, c_+ + \bar{c}_+\right] + \sum_{n=0}^{\infty} h_{2n}\, (\pounds_\mathcal{V})^{2n}\, (c_+ - \bar{c}_+) \\
&= \Psi + (c_+ - \bar{c}_+) + \tfrac{1}{2}\left[\mathcal{V}, c_+ + \bar{c}_+\right] + \tfrac{1}{12}\left[\mathcal{V}, [\mathcal{V}, c_+ - \bar{c}_+]\right] + \cdots, \\
s\Psi &= -\tfrac{1}{2}\left\{\Psi, c_+ + \bar{c}_+\right\} - \sum_{n=1}^{\infty} h_{2n} \sum_{i=1}^{2n} (\pounds_\mathcal{V})^{i-1} \pounds_\Psi\, (\pounds_\mathcal{V})^{2n-i} (c_+ - \bar{c}_+) + \\
&\quad - \tfrac{1}{2}[\mathcal{V}, \Phi + \bar{\Phi}] - \sum_{n=0}^{\infty} h_{2n}\, (\pounds_\mathcal{V})^{2n} (\Phi - \bar{\Phi}) \\
&= -\tfrac{1}{2}\left\{\Psi, c_+ + \bar{c}_+\right\} - \tfrac{1}{12}\left\{\Psi, [\mathcal{V}, c_+ - \bar{c}_+]\right\} - \tfrac{1}{12}\left[\mathcal{V}, \{\Psi, c_+ - \bar{c}_+\}\right] + \cdots\ + \\
&\quad - (\Phi + \bar{\Phi}) - \tfrac{1}{2}[\mathcal{V}, \Phi + \bar{\Phi}] - \tfrac{1}{12}[\mathcal{V}, [\mathcal{V}, \Phi + \bar{\Phi}]] + \cdots, \\
sc_+ &= -\tfrac{1}{2}\left\{c_+, c_+\right\} + \Phi, \qquad\qquad s\Phi = -\left[c_+, \Phi\right], \\
s\bar{c}_+ &= -\tfrac{1}{2}\left\{\bar{c}_+, \bar{c}_+\right\} + \bar{\Phi}, \qquad\qquad s\bar{\Phi} = -[\bar{c}_+, \bar{\Phi}], \\
s\bar{\psi}_{\dot{\alpha}} &= \bar{b}_{\dot{\alpha}}, \qquad\qquad s\bar{b}_{\dot{\alpha}} = 0, \\
s\Lambda &= \beta, \qquad\qquad s\beta = 0, \\
s\bar{\Lambda} &= \bar{\beta}, \qquad\qquad s\bar{\beta} = 0, \\
sc_- &= B, \qquad\qquad sB = 0, \\
s\bar{c}_- &= \bar{B}, \qquad\qquad s\bar{B} = 0,
\end{aligned} \tag{2.9}$$

where the constants h_{2n} are given by ($h_0 = 1$, $h_2 = \frac{1}{12}$, $h_4 = -\frac{1}{720}$, ...) and $\pounds_{\mathcal{V}}$ (or $\pounds_{\Psi}$) is for the Lie bracket[1], i.e.

$$\pounds_{\mathcal{V}} \cdot \ = [\mathcal{V}, \ \cdot \ ] \, . \tag{2.10}$$

The next step consists in introducing a specific gauge-fixing term in order to restrain the quantization process to the (anti-)instantonic configurations of the gauge superfield. The Landau type gauge-fixing action to be adopted here contais three BRS-exact pieces [16]:

$$\begin{aligned} S_{gf} = & \ \frac{1}{128} \, s \ \mathrm{Tr} \left\{ \int d\bar{S} \, \bar{\psi}_{\dot\alpha} \bar{W}^{\dot\alpha} + \int dV \, (\Lambda + \bar\Lambda) \Psi + \int dV \ (c_- + \bar{c}_-) \, \mathcal{V} \right\} \\ = & \ \frac{1}{128} \mathrm{Tr} \left\{ \int d\bar{S} \ \left[\bar{b}_{\dot\alpha} \bar{W}^{\dot\alpha} - \bar\psi_{\dot\alpha} D^2 \left((s_1 e^{\mathcal{V}}) \bar{D}^{\dot\alpha} e^{-\mathcal{V}} + e^{\mathcal{V}} \bar{D}^{\dot\alpha} s_1 e^{-\mathcal{V}} \right) + \bar\psi_{\dot\alpha} [\bar{c}_+, \bar{W}^{\dot\alpha}] \right] + \right. \\ & \left. + \int dV \ \left[(\beta + \bar\beta) \Psi + (\Lambda + \bar\Lambda) s \Psi + (B + \bar{B}) \mathcal{V} - (c_- + \bar{c}_-) s \mathcal{V} \right] \right\} \end{aligned} \tag{2.11}$$

where

$$\begin{aligned} s_1 e^{\mathcal{V}} = & \ e^{\mathcal{V} + \Psi} - e^{\mathcal{V}} \\ = & \ \sum_{n=1}^{\infty} \frac{1}{n!} \sum_{i=1}^{n} \ (\mathcal{V})^{i-1} \, \Psi \, (\mathcal{V})^{n-i} + \mathcal{O}(\Psi^2). \end{aligned} \tag{2.12}$$

Here, besides the supersymmetric self-duality constraint,

$$\bar{W}_{\dot\alpha} = 0, \tag{2.13}$$

two parameter free gauge-fixings [21] are imposed on the topological ghost and the prepotential respectively. In particular, the non-covariant conditions

$$D^2 \Psi = \bar{D}^2 \Psi = 0, \tag{2.14}$$

are differing from the standard covariant gauge choices of ref.[16]. We stress however that in the present work one is interested in the ultraviolet properties of the model and the equations (2.14) are most suitable to accomplish this task, and surely, they cannot modify the physical output (see also [10]). One should observe that the covariant gauge-fixing on the topological ghost does indeed play a crucial role in the computation of the topological observables of the theory [1]. However, we shall not attempt to obtain these latters presently.

To translate the BRS invariance of the model into a Slavnov-Taylor identity, we introduce external sources ($\mathcal{V}^*$, Ψ^*, c^*, $\bar{c}^*$, Φ^*, $\bar\Phi^*$) coupled to specific parts of the transformation laws in (2.9). The external coupling is then given by:

$$S_{ext.} = \mathrm{Tr} \left\{ \int dV \, [\mathcal{V}^* \mathcal{P}(\mathcal{V}, c_+, \bar{c}_+) + \Psi^* s \Psi] + \int dS \, [c^* c_+ c_+ + \Phi^* s \Phi] + \int d\bar{S} \, [\bar{c}^* \bar{c}_+ \bar{c}_+ + \bar\Phi^* s \bar\Phi] \right\} , \tag{2.15}$$

in which the polynomial

$$\mathcal{P}(\mathcal{V}, c_+, \bar{c}_+) = \frac{1}{2} [\mathcal{V}, c_+ + \bar{c}_+] + \sum_{n=0}^{\infty} h_{2n} \ (\pounds_{\mathcal{V}})^{2n} (c_+ - \bar{c}_+) \tag{2.16}$$

[1] A grading has to be understood here since Ψ is an anticommuting superfield.

Table 2: Dimensions and ghost charges of the sources.

	$\mathcal{V}^*$	Ψ^*	c^*	$\bar{c}^*$	Φ^*	$\bar{\Phi}^*$
$dim.$	2	2	3	3	3	3
$\mathcal{N}_g$	-1	-2	-2	-2	-3	-3

is the gauge transformation part in $s\mathcal{V}$. Moreover, the souces are required to transform in BRS doublets [9, 10]:

$$\begin{aligned} s\mathcal{V}^* &= 0, \qquad s\Psi^* = -\mathcal{V}^*, \\ sc^* &= 0, \qquad s\Phi^* = -c^*, \\ s\bar{c}^* &= 0, \qquad s\bar{\Phi}^* = -\bar{c}^*. \end{aligned} \tag{2.17}$$

One observes then that $S_{ext.}$ is also a pure BRS variation:

$$S_{ext.} = -s\mathrm{Tr}\left\{\int dV\,(\Psi^*\mathcal{P}(\mathcal{V},c,\bar{c})) + \int dS\;(\Phi^* c_+ c_+) + \int d\bar{S}\,(\bar{\Phi}^*\bar{c}_+\bar{c}_+)\right\} \tag{2.18}$$

We collect dimensions and Faddeev-Popov ghost charges of the sources in Table 2. The complete invariant classical action Σ is then taken to be:

$$\Sigma = S_{g.f.} + S_{ext.}, \tag{2.19}$$

which is clearly BRS exact. It should be stressed once more that, due to eq.(2.8), no contribution is to be expected from $S_{inv.}$. As a matter of fact, the constraint (2.8) being almost obvious in a Minkowskian space-time, will be taken here as a true renormalization condition specifying that, also in the Euclidean regime, $S_{inv.}$ is some topological invariant, i.e. a number.

The Slavnov-Taylor Identity:

The Slavnov-Taylor identity obeyed by the complete action Σ is:

$$\mathcal{S}(\Sigma) = 0, \tag{2.20}$$

with

$$\begin{aligned} \mathcal{S}(\Sigma) = \;& \mathrm{Tr}\left\{\int dV\left[\Psi\frac{\delta\Sigma}{\delta\mathcal{V}} + \frac{\delta\Sigma}{\delta\mathcal{V}^*}\frac{\delta\Sigma}{\delta\mathcal{V}} + \frac{\delta\Sigma}{\delta\Psi^*}\frac{\delta\Sigma}{\delta\Psi} - \mathcal{V}^*\frac{\delta\Sigma}{\delta\Psi^*}\right] + \right. \\ & +\int dS\left[\Phi\frac{\delta\Sigma}{\delta c_+} - \frac{\delta\Sigma}{\delta c^*}\frac{\delta\Sigma}{\delta c_+} + \frac{\delta\Sigma}{\delta\Phi^*}\frac{\delta\Sigma}{\delta\Phi} + \beta\frac{\delta\Sigma}{\delta\Lambda} + B\frac{\delta\Sigma}{\delta c_-} - c^*\frac{\delta\Sigma}{\delta\Phi^*}\right] + \\ & \left. +\int d\bar{S}\left[\bar{\Phi}\frac{\delta\Sigma}{\delta\bar{c}_+} - \frac{\delta\Sigma}{\delta\bar{c}^*}\frac{\delta\Sigma}{\delta\bar{c}_+} + \frac{\delta\Sigma}{\delta\bar{\Phi}^*}\frac{\delta\Sigma}{\delta\bar{\Phi}} + \bar{b}_{\dot{\alpha}}\frac{\delta\Sigma}{\delta\bar{\psi}_{\dot{\alpha}}} + \bar{\beta}\frac{\delta\Sigma}{\delta\bar{\Lambda}} + \bar{B}\frac{\delta\Sigma}{\delta\bar{c}_-} - \bar{c}^*\frac{\delta\Sigma}{\delta\bar{\Phi}^*}\right]\right\} = 0. \end{aligned} \tag{2.21}$$

From (2.20) above one reads off a nilpotent linearized operator at Σ:

$$\begin{aligned}\mathcal{B}_\Sigma = \ & \mathrm{Tr}\left\{\int dV \left[\Psi\frac{\delta}{\delta\mathcal{V}} + \frac{\delta\Sigma}{\delta\mathcal{V}^*}\frac{\delta}{\delta\mathcal{V}} + \frac{\delta\Sigma}{\delta\mathcal{V}}\frac{\delta}{\delta\mathcal{V}^*} + \frac{\delta\Sigma}{\delta\Psi^*}\frac{\delta}{\delta\Psi} + \frac{\delta\Sigma}{\delta\Psi}\frac{\delta}{\delta\Psi^*} - \mathcal{V}^*\frac{\delta\Sigma}{\delta\Psi^*}\right] + \right. \\ & + \int dS \left[\Phi\frac{\delta}{\delta c_+} - \frac{\delta\Sigma}{\delta c^*}\frac{\delta}{\delta c_+} - \frac{\delta\Sigma}{\delta c_+}\frac{\delta}{\delta c^*} + \frac{\delta\Sigma}{\delta\Phi^*}\frac{\delta}{\delta\Phi} + \right. \\ & \qquad \left. + \frac{\delta\Sigma}{\delta\Phi}\frac{\delta}{\delta\Phi^*} + \beta\frac{\delta}{\delta\Lambda} + B\frac{\delta}{\delta c_-} - c^*\frac{\delta}{\delta\Phi^*}\right] + \\ & + \int d\bar{S} \left[\bar{\Phi}\frac{\delta}{\delta\bar{c}_+} - \frac{\delta\Sigma}{\delta\bar{c}^*}\frac{\delta}{\delta\bar{c}_+} - \frac{\delta\Sigma}{\delta\bar{c}_+}\frac{\delta}{\delta\bar{c}^*} + \frac{\delta\Sigma}{\delta\bar{\Phi}^*}\frac{\delta}{\delta\bar{\Phi}} + \right. \\ & \qquad \left.\left. + \frac{\delta\Sigma}{\delta\bar{\Phi}}\frac{\delta}{\delta\bar{\Phi}^*} + \bar{b}_{\dot\alpha}\frac{\delta}{\delta\bar{\psi}_{\dot\alpha}} + \bar{\beta}\frac{\delta}{\delta\bar{\Lambda}} + \bar{B}\frac{\delta}{\delta\bar{c}_-} - \bar{c}^*\frac{\delta}{\delta\bar{\Phi}^*}\right]\right\} .\end{aligned} \tag{2.22}$$

Next, we turn to the additional classical Ward identities of the supertopological model.

The Gauge-Fixing Conditions:

The first set of identities consists of the Landau gauge-fixing conditions [20] imposed on the superfields Ψ and $\mathcal{V}$:

$$\frac{\delta\Sigma}{\delta\beta} = \tfrac{1}{128}\bar{D}^2\Psi, \qquad \frac{\delta\Sigma}{\delta\bar{\beta}} = \tfrac{1}{128}D^2\Psi, \qquad \frac{\delta\Sigma}{\delta B} = \tfrac{1}{128}\bar{D}^2\mathcal{V}, \qquad \frac{\delta\Sigma}{\delta\bar{B}} = \tfrac{1}{128}D^2\mathcal{V}. \tag{2.23}$$

The Ghost Equations:

Upon (anti)commuting the identities above with (2.20) one gets the following ghost equations:

$$\begin{aligned}&\frac{\delta\Sigma}{\delta\Lambda} + \tfrac{1}{128}\bar{D}^2\frac{\delta\Sigma}{\delta\Psi^*} = 0, \qquad\qquad \frac{\delta\Sigma}{\delta\bar{\Lambda}} + \tfrac{1}{128}D^2\frac{\delta\Sigma}{\delta\Psi^*} = 0, \\ &\frac{\delta\Sigma}{\delta c_-} + \tfrac{1}{128}\bar{D}^2\frac{\delta\Sigma}{\delta\mathcal{V}^*} = -\tfrac{1}{128}\bar{D}^2\Psi, \quad \frac{\delta\Sigma}{\delta\bar{c}_-} + \tfrac{1}{128}D^2\frac{\delta\Sigma}{\delta\mathcal{V}^*} = -\tfrac{1}{128}D^2\Psi.\end{aligned} \tag{2.24}$$

The Supersymmetric Antighost Equations:

The antighost equations[2] are global constraints obeyed by Σ and may be derived whenever a Landau gauge is imposed [26, 27]. Presently, we deal with two of them. The first equation controls the coupling of both Φ and $\bar{\Phi}$ ghosts:

$$\mathcal{G}\Sigma = \Delta^{\mathcal{G}}_{class.}, \tag{2.25}$$

[2]We thank O. Piguet for pointing out to us the existence and usefulness of these equations in N=1 superspace.

with

$$\mathcal{G} = \int dS \left(\frac{\delta}{\delta \Phi} + \left[\Lambda, \frac{\delta}{\delta B} \right] \right) + \int d\bar{S} \left(\frac{\delta}{\delta \bar{\Phi}} + \left[\bar{\Lambda}, \frac{\delta}{\delta \bar{B}} \right] \right), \tag{2.26}$$

and

$$\Delta^{\Phi}_{class.} = -\int dV \, [\Psi^*, \mathcal{V}] - \int dS \, [\Phi^*, c_+] - \int d\bar{S} \, [\bar{\Phi}^*, \bar{c}_+]. \tag{2.27}$$

It can be commuted with (2.20) to give a further Ward identity:

$$\mathcal{F}\Sigma = \Delta^{\mathcal{F}}_{class.}, \tag{2.28}$$

with

$$\begin{aligned} \mathcal{F} = & \int dS \left(\frac{\delta}{\delta c_+} - \left[\beta, \frac{\delta}{\delta B} \right] + \left[\Lambda, \frac{\delta}{\delta c_-} \right] + \left[c_+, \frac{\delta}{\delta \Phi} \right] - \left[\Phi^*, \frac{\delta}{\delta c^*} \right] \right) + \\ + & \int d\bar{S} \left(\frac{\delta}{\delta \bar{c}_+} - \left[\bar{\beta}, \frac{\delta}{\delta \bar{B}} \right] + \left[\bar{\Lambda}, \frac{\delta}{\delta \bar{c}_-} \right] + \left[\bar{c}_+, \frac{\delta}{\delta \bar{\Phi}} \right] - \left[\bar{\Phi}^*, \frac{\delta}{\delta \bar{c}^*} \right] \right) + \\ + & \int dV \left(\left[\mathcal{V}, \frac{\delta}{\delta \Psi} \right] - \left[\Psi^*, \frac{\delta}{\delta \mathcal{V}^*} \right] \right), \end{aligned} \tag{2.29}$$

and

$$\Delta^{\mathcal{F}}_{class.} = \int dV \, (-[\mathcal{V}^*, \mathcal{V}] + [\Psi^*, \Psi]) - \int dS \, ([c^*, c_+] + [\Phi^*, \Phi]) - \int d\bar{S} \, ([\bar{c}^*, \bar{c}_+] + [\bar{\Phi}^*, \bar{\Phi}]). \tag{2.30}$$

The second antighost equation is related to the ghosts c_+ and $\bar{c}_+$:

$$\mathcal{G}_-\Sigma = \Delta^{\mathcal{G}_-}_{class.}, \tag{2.31}$$

with

$$\begin{aligned} \mathcal{G}_- = & \int dS \left(\frac{\delta}{\delta c_+} - \left[\Lambda, \frac{\delta}{\delta \beta} \right] - \left[c_-, \frac{\delta}{\delta B} \right] \right) + \\ + & \int d\bar{S} \left(\frac{\delta}{\delta \bar{c}_+} - \left[\bar{\Lambda}, \frac{\delta}{\delta \bar{\beta}} \right] - \left[\bar{c}_-, \frac{\delta}{\delta \bar{B}} \right] + \left\{ \bar{\psi}_{\dot{\alpha}}, \frac{\delta}{\delta \bar{b}_{\dot{\alpha}}} \right\} \right), \end{aligned} \tag{2.32}$$

and

$$\Delta^{\mathcal{G}_-}_{class.} = \int dV \, (-[\mathcal{V}^*, \mathcal{V}] + [\Psi^*, \Psi]) - \int dS \, ([c^*, c_+] + [\Phi^*, \Phi]) - \int d\bar{S} \, ([\bar{c}^*, \bar{c}_+] + [\bar{\Phi}^*, \bar{\Phi}]). \tag{2.33}$$

It is important to remark that the two supersymmetric antighost equations obtained here, (2.25) and (2.31), are both classically broken, the breaking pieces being integrated monomials which are linear in the quantum fields and do not demand specific renormalizations.

Rigid Invariance:

Finally, the anticommutation of the antighost equation (2.31) with (2.20) leads us to the exact rigid gauge invariance of the model. Indeed, one has:

$$W_{rig.}\Sigma = 0, \tag{2.34}$$

where

$$
\begin{aligned}
W_{rig.} = & \int dV \left(\left[\mathcal{V}, \frac{\delta}{\delta \mathcal{V}} \right] + \left\{ \Psi, \frac{\delta}{\delta \Psi} \right\} + \left\{ \mathcal{V}^*, \frac{\delta}{\delta \mathcal{V}^*} \right\} + \left[\Psi^*, \frac{\delta}{\delta \Psi^*} \right] \right) + \\
+ & \int dS \left(\left\{ c_+, \frac{\delta}{\delta c_+} \right\} + \left[\Phi, \frac{\delta}{\delta \Phi} \right] + \left[c^*, \frac{\delta}{\delta c^*} \right] + \left\{ \Phi^*, \frac{\delta}{\delta \Phi^*} \right\} + \right. \\
& \left. + \left[\Lambda, \frac{\delta}{\delta \Lambda} \right] + \left\{ c_-, \frac{\delta}{\delta c_-} \right\} + \left\{ \beta, \frac{\delta}{\delta \beta} \right\} + \left[B, \frac{\delta}{\delta B} \right] \right) + \\
+ & \int d\bar{S} \left(\left\{ \bar{c}_+, \frac{\delta}{\delta \bar{c}_+} \right\} + \left[\bar{\Phi}, \frac{\delta}{\delta \bar{\Phi}} \right] + \left[\bar{c}^*, \frac{\delta}{\delta \bar{c}^*} \right] + \left\{ \bar{\Phi}^*, \frac{\delta}{\delta \bar{\Phi}^*} \right\} + \left\{ \bar{\psi}_{\dot{\alpha}}, \frac{\delta}{\delta \bar{\psi}_{\dot{\alpha}}} \right\} + \right. \\
& \left. + \left[\bar{\Lambda}, \frac{\delta}{\delta \bar{\Lambda}} \right] + \left\{ \bar{c}_-, \frac{\delta}{\delta \bar{c}_-} \right\} + \left[\bar{b}_{\dot{\alpha}}, \frac{\delta}{\delta \bar{b}_{\dot{\alpha}}} \right] + \left\{ \bar{\beta}, \frac{\delta}{\delta \bar{\beta}} \right\} + \left[\bar{B}, \frac{\delta}{\delta \bar{B}} \right] \right) .
\end{aligned}
\tag{2.35}
$$

In the next section one will be concerned with the construction of the most general local counterterm to the complete action given in (2.19). To this aim we shall discuss the quantum extension of the above derived Ward identities as well as the Slavnov-Taylor identity (2.20).

3 The BRS Cohomology: the Local Counterterm

In this section we follow the same reasoning of refs.[9, 10] to obtain the general counterterm to (2.19). As a first step, one has to prove the renormalizability of the Slavnov-Taylor and the Ward identities of the previous section. The gauge conditions (2.23), the ghost equations (2.24) and the antighost equations (2.25) and (2.31) may be directly shown to hold at the quantum level just by repeating the arguments of [21, 26]. The Ward identity for rigid transformations (2.34) is known to be renormalizable thanks to the Whitehead lemma. Thus, we concentrate our attention on the Slavnov-Taylor identity (2.20).

To study the cohomology of the linearized operator $\mathcal{B}_\Sigma$ one introduces a filtering:

$$
\begin{aligned}
\mathcal{N} = & \int dV \left(\mathcal{V}\frac{\delta}{\delta \mathcal{V}} + \Psi\frac{\delta}{\delta \Psi} + \mathcal{V}^*\frac{\delta}{\delta \mathcal{V}^*} + \Psi^*\frac{\delta}{\delta \Psi^*} \right) + \\
+ & \int dS \left(2c_+\frac{\delta}{\delta c_+} + 2\Phi\frac{\delta}{\delta \Phi} + 2B\frac{\delta}{\delta B} + 2c_-\frac{\delta}{\delta c_-} + 2\beta\frac{\delta}{\delta \beta} + 2\Lambda\frac{\delta}{\delta \Lambda} \right) + \\
+ & \int d\bar{S} \left(2\bar{c}_+\frac{\delta}{\delta \bar{c}_+} + 2\bar{\Phi}\frac{\delta}{\delta \bar{\Phi}} + 2\bar{b}_{\dot{\alpha}}\frac{\delta}{\delta \bar{b}_{\dot{\alpha}}} + 2\bar{\psi}_{\dot{\alpha}}\frac{\delta}{\delta \bar{\psi}_{\dot{\alpha}}} + 2\bar{B}\frac{\delta}{\delta \bar{B}} + 2\bar{c}_-\frac{\delta}{\delta \bar{c}_-} + 2\bar{\beta}\frac{\delta}{\delta \bar{\beta}} + 2\bar{\Lambda}\frac{\delta}{\delta \bar{\Lambda}} \right) ,
\end{aligned}
\tag{3.1}
$$

which induces a decomposition such that

$$
\mathcal{B}_\Sigma = \sum_{n=0}^{\infty} \mathcal{B}_\Sigma^{(n)}, \tag{3.2}
$$

and

$$
\left[\mathcal{N}, \mathcal{B}_\Sigma^{(n)} \right] = n\mathcal{B}_\Sigma^{(n)}. \tag{3.3}
$$

We give the expressions of $\mathcal{B}_\Sigma^{(0)}$ and $\mathcal{B}_\Sigma^{(1)}$:

$$\begin{aligned}\mathcal{B}_\Sigma^{(0)} = & \int dV \left(\Psi\frac{\delta}{\delta\mathcal{V}} - \mathcal{V}^*\frac{\delta}{\delta\Psi^*}\right) + \int dS\left(\Phi\frac{\delta}{\delta c_+} + \beta\frac{\delta}{\delta\Lambda} + B\frac{\delta}{\delta c_-} - c^*\frac{\delta}{\delta\Phi^*}\right) + \\ & + \int d\bar{S}\left(\bar{\Phi}\frac{\delta}{\delta\bar{c}_+} + \bar{b}_{\dot\alpha}\frac{\delta}{\delta\bar{\psi}_{\dot\alpha}} + \bar{\beta}\frac{\delta}{\delta\bar{\Lambda}} + \bar{B}\frac{\delta}{\delta\bar{c}_-} - \bar{c}^*\frac{\delta}{\delta\bar{\Phi}^*}\right),\end{aligned} \tag{3.4}$$

$$\begin{aligned}\mathcal{B}_\Sigma^{(1)} = & \int dV \Bigg((c_+ - \bar{c}_+)\frac{\delta}{\delta\mathcal{V}} - (\Phi - \bar{\Phi})\frac{\delta}{\delta\Psi} - \tfrac{1}{128}(\bar{D}_{\dot\alpha}\bar{b}^{\dot\alpha} + B + \bar{B})\frac{\delta}{\delta\mathcal{V}^*} + \\ & \qquad + \tfrac{1}{128}(\beta + \bar{\beta} + c_- + \bar{c}_-)\frac{\delta}{\delta\Psi^*}\Bigg) + \\ & + \int dS\left((\bar{D}^2\mathcal{V}^*)\frac{\delta}{\delta c^*} + (\bar{D}^2\Psi^*)\frac{\delta}{\delta\Phi^*}\right) + \int d\bar{S}\left((D^2\mathcal{V}^*)\frac{\delta}{\delta\bar{c}^*} + (D^2\Psi^*)\frac{\delta}{\delta\bar{\Phi}^*}\right).\end{aligned} \tag{3.5}$$

Now, by means of a simple inspection, one notices that the nilpotent operator $\mathcal{B}_\Sigma^{(0)}$ transforms fields and sources according to a BRS-doublet pattern. Hence the cohomology of $\mathcal{B}_\Sigma^{(0)}$ is empty [28]. As a consequence, the cohomology of $\mathcal{B}_\Sigma$ is empty too [29]:

$$\mathcal{H}^*(\mathcal{B}_\Sigma) = \emptyset. \tag{3.6}$$

The main conclusion here is that the Slavnov-Taylor identity is non-anomalous.

To write down the general counterterm, $\tilde{\Sigma}$, we notice from the BRS invariance and from (3.6) that:

$$\tilde{\Sigma} = \mathcal{B}_\Sigma\Delta, \tag{3.7}$$

here Δ is an integrated local functional of dimension four and ghost charge minus one. Moreover, $\tilde{\Sigma}$ has to obey the following stability constraints:

$$\frac{\delta\tilde{\Sigma}}{\delta\beta} = 0, \quad \frac{\delta\tilde{\Sigma}}{\delta\bar{\beta}} = 0, \quad \frac{\delta\tilde{\Sigma}}{\delta B} = 0, \quad \frac{\delta\tilde{\Sigma}}{\delta\bar{B}} = 0, \tag{3.8}$$

together with

$$\frac{\delta\tilde{\Sigma}}{\delta\Lambda} + \tfrac{1}{128}D^2\frac{\delta\tilde{\Sigma}}{\delta\Psi^*} = 0, \quad \frac{\delta\tilde{\Sigma}}{\delta\bar{\Lambda}} + \tfrac{1}{128}\bar{D}^2\frac{\delta\tilde{\Sigma}}{\delta\Psi^*} = 0, \quad \frac{\delta\tilde{\Sigma}}{\delta c_-} + \tfrac{1}{128}D^2\frac{\delta\tilde{\Sigma}}{\delta\mathcal{V}^*} = 0, \quad \frac{\delta\tilde{\Sigma}}{\delta\bar{c}_-} + \tfrac{1}{128}\bar{D}^2\frac{\delta\tilde{\Sigma}}{\delta\mathcal{V}^*} = 0, \tag{3.9}$$

and

$$\mathcal{G}\tilde{\Sigma} = 0, \quad \mathcal{F}\tilde{\Sigma} = 0, \quad \mathcal{G}_-\tilde{\Sigma} = 0, \quad W_{rig.}\tilde{\Sigma} = 0. \tag{3.10}$$

Hence, up to $\mathcal{V}$ superfield redefinitions, the local counterterm expressing the potential divergences of the model turns out to be:

$$\tilde{\Sigma} = \mathcal{B}_\Sigma\mathrm{Tr}\left\{\int dV\left(a_1\hat{\mathcal{V}}^*\mathcal{V} + a_2\hat{\Psi}^*\Psi + a_3\hat{\Psi}^*\Psi\mathcal{V}\right)\right\} \tag{3.11}$$

where,

$$\hat{\mathcal{V}}^* = \mathcal{V}^* - \tfrac{1}{128}(c_- + \bar{c}_-), \quad \hat{\Psi}^* = \Psi^* - \tfrac{1}{128}(\Lambda + \bar{\Lambda}), \tag{3.12}$$

and a_1, a_2 and a_3 stand for arbitrary coefficients. At this point, one observes that the quantum effects do not modify the cohomological character of the theory, as is also the case of the purely bosonic topological Yang-Mills theory.

As another interesting conclusion, we notice that, analogously to the bosonic case [6, 7, 9, 10], the Callan-Symanzik β-function does indeed vanish here. This last fact is entirely due to the supersymmetric antighost equation (2.31) which rules out the counterterm

$$\mathcal{B}_{\Sigma}\mathrm{Tr}\int d\bar{S}\,(\bar{b}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}) = \mathrm{Tr}\int d\bar{S}\,(\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}) = \mathrm{Tr}\int dS\,(W^{\alpha}W_{\alpha}), \tag{3.13}$$

where the $\bar{b}_{\dot{\alpha}}$-superfield equation of motion and eq.(2.8) were used.

Let us close our investigation by mentioning once more that the above presented results are regularization scheme independent and hold to all orders of the perturbative expansion.

4 Concluding Remarks

In this work we studied some of the renormalization aspects of the topological super-Yang-Mills field theory defined in N=1 superspace. By using the algebraic BRS technique in superspace, i.e. renormalized supersymmetry [21], one was able to show that many of the well-known properties of the bosonic topological Yang-Mills theory were manifest also in its supersymmetric extension. In fact, the most general local counterterm of the supersymmetric model is a BRS-exact integrated polynomial in the fields and sources, indicating that the cohomological nature of the theory is not affected by quantum fluctuations. Moreover, the Callan-Symanzik β-function was shown to vanish due to a specific stability constraint, namely, the antighost equation eq.(2.31), which is seen to hold when the non-covariant Landau gauge-fixing is imposed onto the topological ghost superfield eqs.(2.14). We believe that the present study may be of interest in connection with some supertopological gauge theories which have been proposed in the recent literature [30].

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