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CONCORD OF COSMIC RAY COMPONENTS IN THE ATMOSPHERE*

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Qualitatively, the main features of the bulk of cosmic radiation, i.e., the nature of the particles constituting the different components and the mechanism of their transmutation and development in the atmosphere, are at present well known. Moreover, quantitatively, we know with more or less accuracy, the energy spectra of the different components including that of primary cosmic radiation (i.e., the spectrum of primary protons and of the nucleons contained in primary heavier nuclei), which has been explored by recent experiments with rockets, balloon flights, etc. So it seems a worthwhile task to give a deductive quantitative mathematical description, as

(*) This is the first of a series of papers in which the main results of a deductive theory of cosmic radiation will be represented. The work will be published in full detail in W. Heisenberg, Cosmic Radiation, 2nd Edition, Springer-Verlag, Göttingen.

(**) This work was done at a time when both the authors were at Max-Planck-Institut für Physik, Göttingen.

complete as possible, of the development of cosmic ray components in the atmosphere. This is what my friend Budini and I have tried to do.

The starting point of our theory is the spectrum of primary cosmic radiation. This spectrum, at least at not too high energies (up to about 15 BeV), shows a much slower variation than had formerly been assumed. This behavior of the primary spectrum at lower energies has been taken into account in our theory and turns out to be of great importance in the interpretation of experimental facts. At higher energies we have much support in assuming that the more rapid variation, according to a power law of an exponent of ~ 1.8 (for the integral spectrum), which formerly had been assumed to hold throughout the whole energy range, remains valid as an asymptotic law. As a representation of the integral primary spectrum (i.e., the number of primary particles whose energy exceeds a certain value E_o), we used the formula:

$$A \cdot (E_o + B)^{-1.8} \quad [\text{sterad}^{-1}\ \text{sec}^{-1}\ \text{cm}^{-2}]$$

where the constants A and B are chosen to fit with experimental data. By intervention of the constant B, which is of the order of 5 BeV, the flattening-out of the spectrum at lower energies is described. A graphic representation of our law for the primary spectrum is given in Figure 1, in double logarithmic scale, where the dotted straight line corresponds to the asymptotic 1.8 power law, while the curved part of the graph describes the slower variation of the spectrum at lower energies.

The particles (mainly protons), which constitute the primary component and which on their way from outside the earth, penetrate into the atmosphere, give rise there to the various processes of transmutation by which the different components finally are created, which we observe, for instance at sea level, as "cosmic radiation".

To give a survey of these processes let us consider a single primary proton of energy E_o, which on its way through the air after traversing a certain path length, will undergo a collision

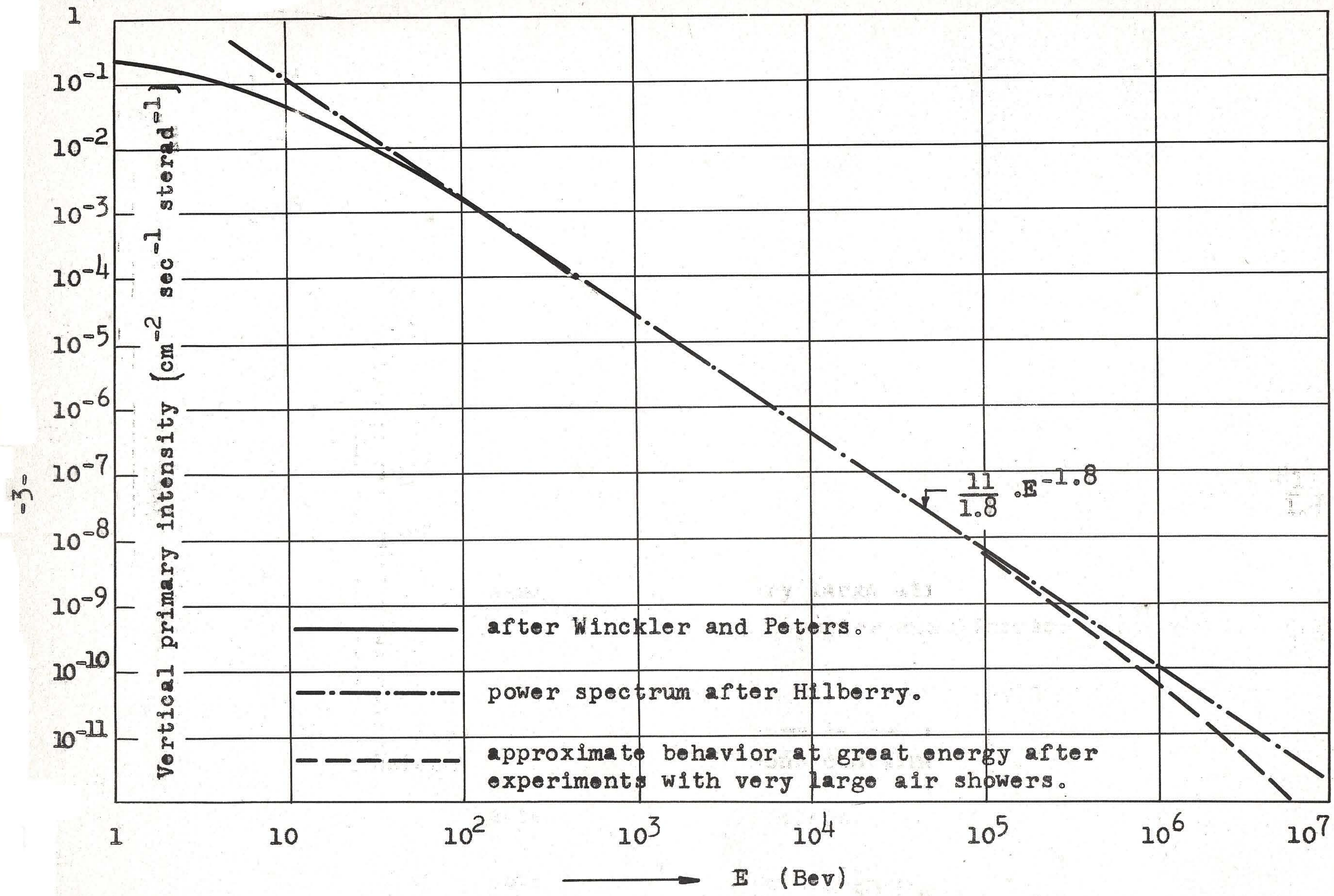

Fig. The integral spectrum of the primary nucleons (= protons + nucleons contained in nuclei).

with an air nucleus. The average value of this path length according to the geometrical cross section of the air nucleus, corresponds to a layer of about 60 gr/cm^2. In this process a shower of particles is created which are, on the one hand, nucleons pushed out of the air nucleus, and on the other, π-mesons, which in the strict sense of the word, are "created" by this collision. The average energy distribution of these two kinds of particles created by a single collision, may be described by the "creation spectra" $f_p(E_o,E)dE$ and $f_\pi(E_o,E)dE$ respectively.

The nucleons, arising from this first collision will make further collisions by which, once more, nucleons and π-mesons are created, and so forth, so that the whole process develops through the atmosphere in a kind of cascade.

Also the charged π-mesons are able to make collisions of the same kind, with production of further nucleons and π-mesons. Besides this, as a competing process, they can undergo the π-μ-decay which, at lower energies, is the predominant one, while at higher energies (beginning at about 100 BeV, because of the relativistic increase of lifetime), most of the π will collide with air nuclei and so will participate in the above described "nucleonic cascade". For the particles or component playing a rôle in it (i.e., nucleons plus those π-mesons which collide before decaying), the common name of "N-particles" or "N-component" is used.

This N-component, developing through the atmosphere by the mechanism of nucleonic cascade, may be considered the "skeleton" of cosmic radiation in the atmosphere, in so far as all other components are derived from it as secondary. So the μ component is originated by π-μ-decay of the charged π-mesons (viz. those which don't collide before decaying), which are created by collisions of the N-particles. Similarly, the electronic-photonic component in the higher atmosphere is initiated by the decay photons of neutral π-mesons, the latter being created in the same way as those charged.

So the first thing we have to do, according to the mechanism of the nucleonic cascade, is to calculate the energy spectrum of the nucleonic component in its dependence on the atmospheric depth, starting from the primary spectrum. The resultant nucleonic spectrum will serve us as the source from which the spectra of the other components have to be derived.

In performing these calculations a general procedure, based on the extensive use of a folding theorem was adopted which is equivalent to, but much simpler than, the usual method of solving diffusion equations.

Two simplifying assumptions (which are common to most of the previous work on the nucleonic cascade) are involved in our method: the firstis the neglect of all angular deviations of the secondary particles from the direction of the primaries, which is good for sufficiently high energies. The other, which is most essential, is the assumption that the above mentioned "creation spectra" are "homogeneous" (or what is sometimes called in literature, "bremsstrahlung"-type), meaning that these spectra may be written in the form $f_p(E/E_o)dE/E_o$ and $f_\pi(E/E_o)dE/E$, depending only on the fraction of the energies of the created and colliding particles. We have support for believing that for sufficiently high energies (compared with the rest energy of the particles in consideration), this assumption of homogeneous creation spectra is at least a good approximation.

For the special form of these creation spectra (about which not much is known as yet, theoretically as well as experimentally), we used simple analytical "ansatze", depending on some parameters which are determined numerically later on by comparison of the results of our calculations with experimental data. It may be noted that our result, so obtained, for the π-meson creation spectrum agrees well with Heisenberg's new theory of meson production which, particularly, leads also to a homogeneous creation spectrum.

The assumption of homogeneous creation spectra has mathematical consequences, well known from cascade theory, which are of importance for the interpretation of certain experimental facts. Suppose a primary component, the spectrum of which is of power form, produces a secondary component by some process which is described by a homogeneous creation spectrum. It follows that the secondary spectrum is of the same power form. Furthermore, if the process under consideration is a complete cascade process, in some material, it follows that the spectrum is absorbed exponentially.

Now, these two facts: reproduction of a power spectrum and exponential absorption, are just what is found experimentally, not over the whole energy range, but asymptotically for large energies and great depths in the atmosphere. Here we have the justification for two of our fundamental assumptions: the asymptotic power form of the primary spectrum and the homogeneousness of the creation spectra.

Our statement about the reproduction of a power spectrum by a homogeneous creation mechanism remains valid, with a slight modification for a spectrum which is only asymptotically of power form at high energies and varies more slowly at low energies, as our primary spectrum does. In this case the shape of the whole spectrum, including its slowly varying low energy part, is approximately reproduced with the modification that this part is shifted towards lower energies in the secondary spectrum. This behaviour can be understood from the degradation of energy by the creation process. In the case of a cascade, the shifting of the flatter part of the spectrum increases with increasing depth.

An illustration of this behaviour of a cascade spectrum is given by Fig. 2, where the results of our calculation for the spectra of the nucleonic component at different atmospheric depths are plotted in double logarithmic scale. (The figures attached to the curves refer to the depth in gr/cm^2).

In Fig. 2 are also included all available experimental data for the protonic spectrum [1], directly given by experiment, by adding a correcting factor 2, corresponding to a 1:1 ratio of neutrons: protons. It should be kept in mind, however, that this ratio, which is one at higher energies, has a larger value at lower energies ($\lesssim$ 2BeV kinetic), because of the intervention of ionization

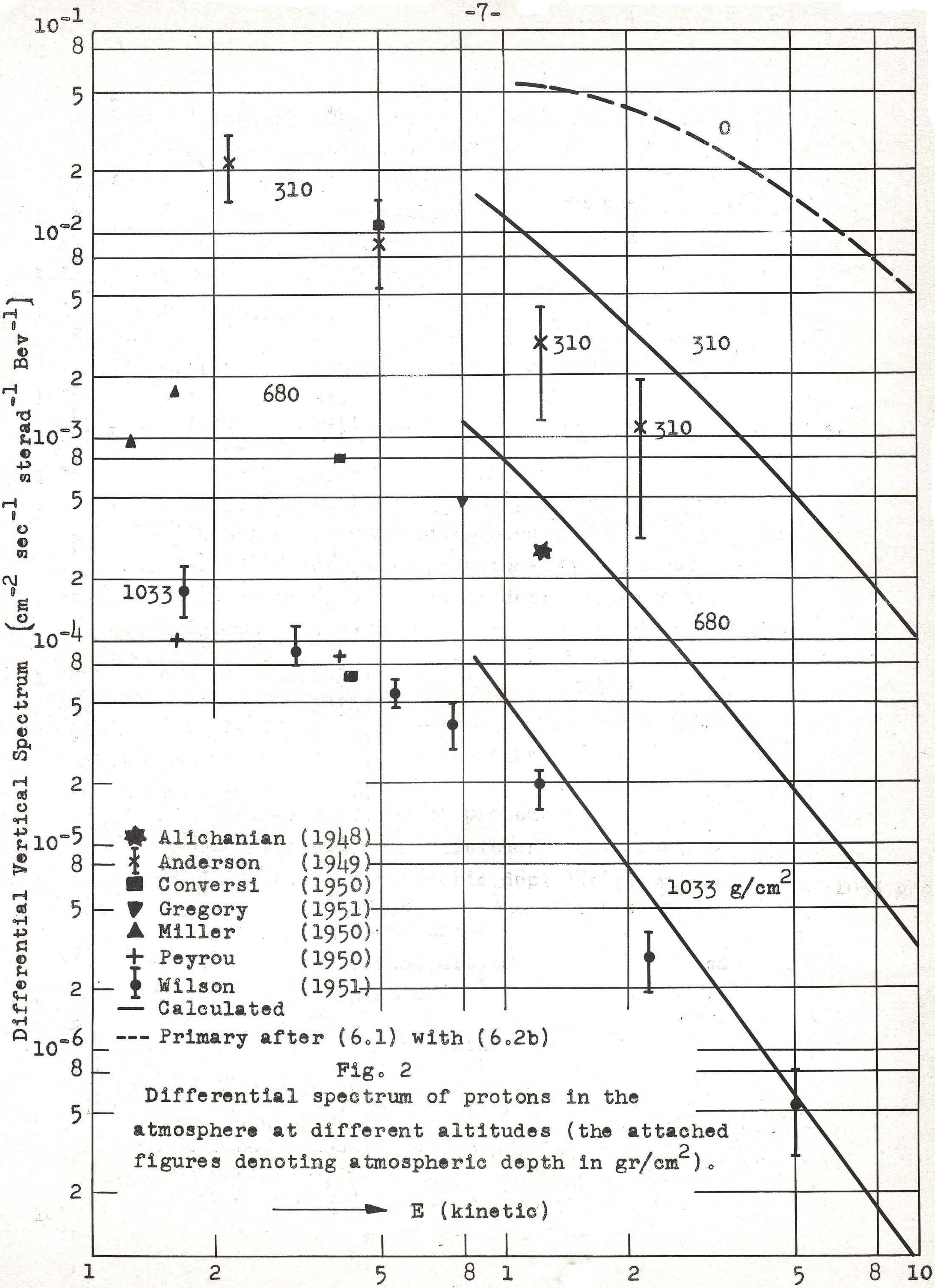

Fig. 2
Differential spectrum of protons in the atmosphere at different altitudes (the attached figures denoting atmospheric depth in gr/cm^2).

loss for protons. The experimental points in Fig. 2, therefore, are too low at low energies. On the other hand, our theory is only valid at greater energies (> 2 BeV kinetic), where ionization loss and other complications intervening at low energies, may be neglected.

Good agreement between theory and experiment, therefore, can be expected only for energies starting from about 2 BeV. Unfortunately, experimental data at these large energies are rare. But, from a look at Fig. 2, one has the impression that, with increasing energies, the experimental data (except those at the greatest heights, which are very rough and uncertain), tend asymptotically to our theoretical curves, and this is the best fit to be expected.

This fit was obtained by special choice of the above-mentioned parameters describing the creation spectrum for nucleons. Together with the requirement to give the right asymptotic absorption length these parameters could be fixed rather definitely. The information so obtained with regard to the nucleon-creation-spectrum, may be described as follows: In a single collision with an air nucleous, 80% of the energy of the colliding nucleon is transmitted to the nucleonic component. The shape of this creation spectrum is of the low multiplicity type, compatible with the picture of "plural production" for the nucleons emerging from such a collision.

It is of interest to discuss our results from the nucleonic component under the aspect of absorption lengths, since this quantity has frequently been determined by experiment.

As already pointed out, at large energies the absorption is exponential, which means that there the absorption length (i.e., the inverse of the absorption coefficient), is constant. The same is no longer true at smaller energies. As a consequence of the said shifting of the slowly varying part of the spectrum toward lower energies, which shifting increases with increasing depth, the absorption length of the nucleonic component will increase with decreasing energy as well as with decreasing depth in the atmosphere. This behaviour is in general agreement with the experimental findings, which are plotted together with our theoretical curves in Fig. 3. (Cf. for

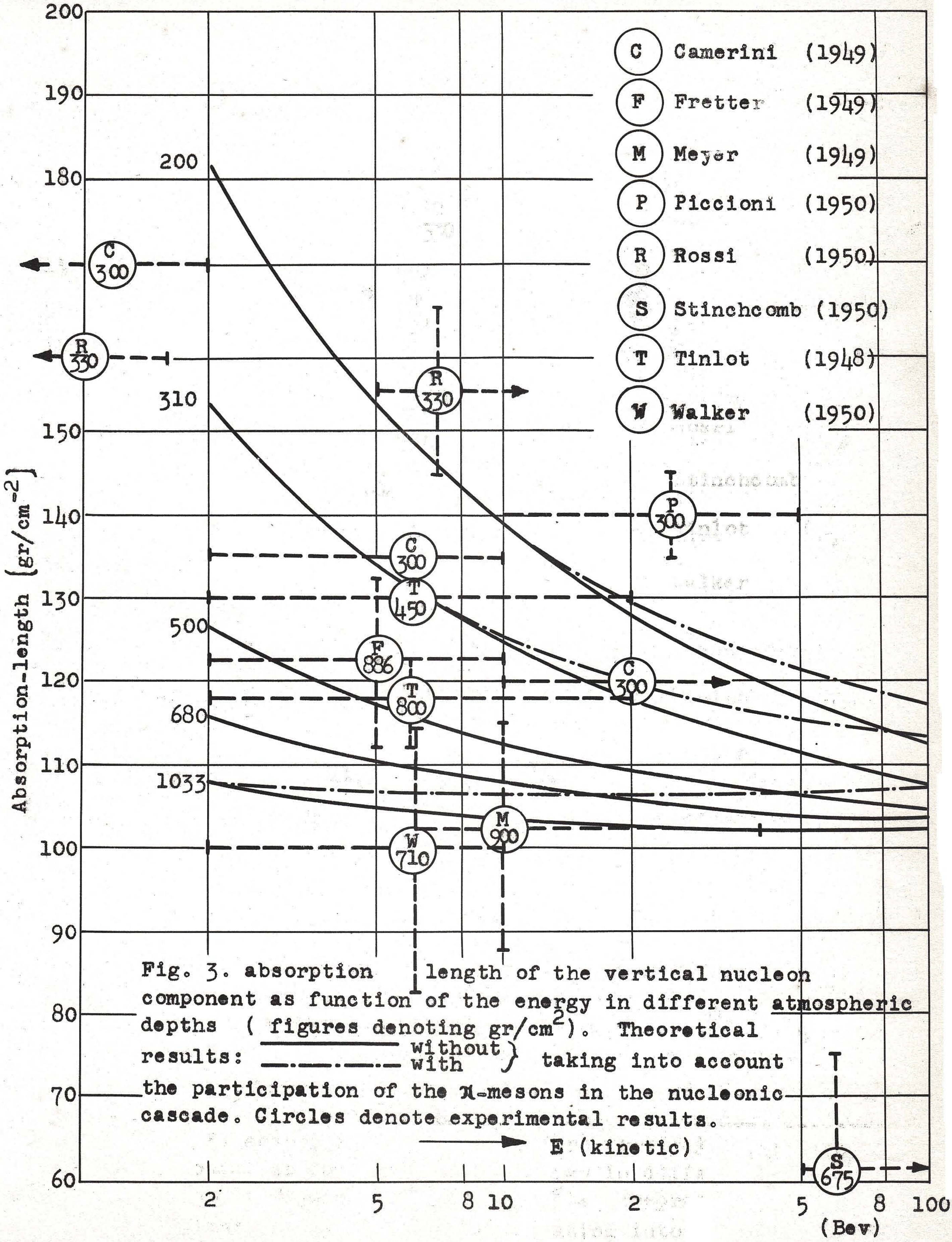

Fig. 3. absorption length of the vertical nucleon component as function of the energy in different atmospheric depths (figures denoting gr/cm^2). Theoretical results: ——— without, —·—·— with } taking into account the participation of the π-mesons in the nucleonic cascade. Circles denote experimental results.

instance the three results of Camerini and co-workers, denoted by "C" in Fig. 3, which agree well with our curve for 310 gr/cm^2).

Experimentally much better known than the nucleon spectrum is the spectrum of μ-mesons. Starting from measurements of the spectrum of the μ-component at different heights (partly performed by himself, and partly by other authors), Sands has derived the so-called source spectrum of the μ-mesons (i.e., the number of them created in a certain layer of the atmosphere). In Fig. 4, Sands' results and ours for the μ-source spectrum are compared. Theoretically, as a consequence of a homogeneous creation spectrum for the π-mesons, the μ-source spectrum is a direct reproduction (shifted toward lower energies), of the nucleonic spectrum. According to our calculations, therefore, the μ-source spectrum is of the same asymptotic power form at large energies as the primary spectrum and has, again, a more slowly varying part which is increasingly shifted toward lower energies with increasing depth. The latter feature is not correctly described by the curves of Sands, who, as a simplification in this calculations, assumed a constant absorption length, so that his spectrum appears exactly reproduced, without increasing shifting, in different depths. Nevertheless, it was possible, as Fig. 4 shows, to get an excellent fit between Sands' and our curves in the higher atmosphere where most of the μ-mesons are produced, while in the deeper atmosphere the agreement is as good as can be expected.

This fit, for the asymptotic power part at high energies as well as for the more slowly varying low energy part of the μ-source spectrum, was obtained by determining numerical values for the two parameters describing the above mentioned creation spectrum for π-mesons, the result for the latter being as follows:

(i) The shape of the π-creation spectrum, as already mentioned, is in good agreement with that given by Heisenberg's new theory.

(ii) The average energy fraction, transmitted by a single collision of a nucleon with an air nucleous to the created π-mesons, is about 20 % .

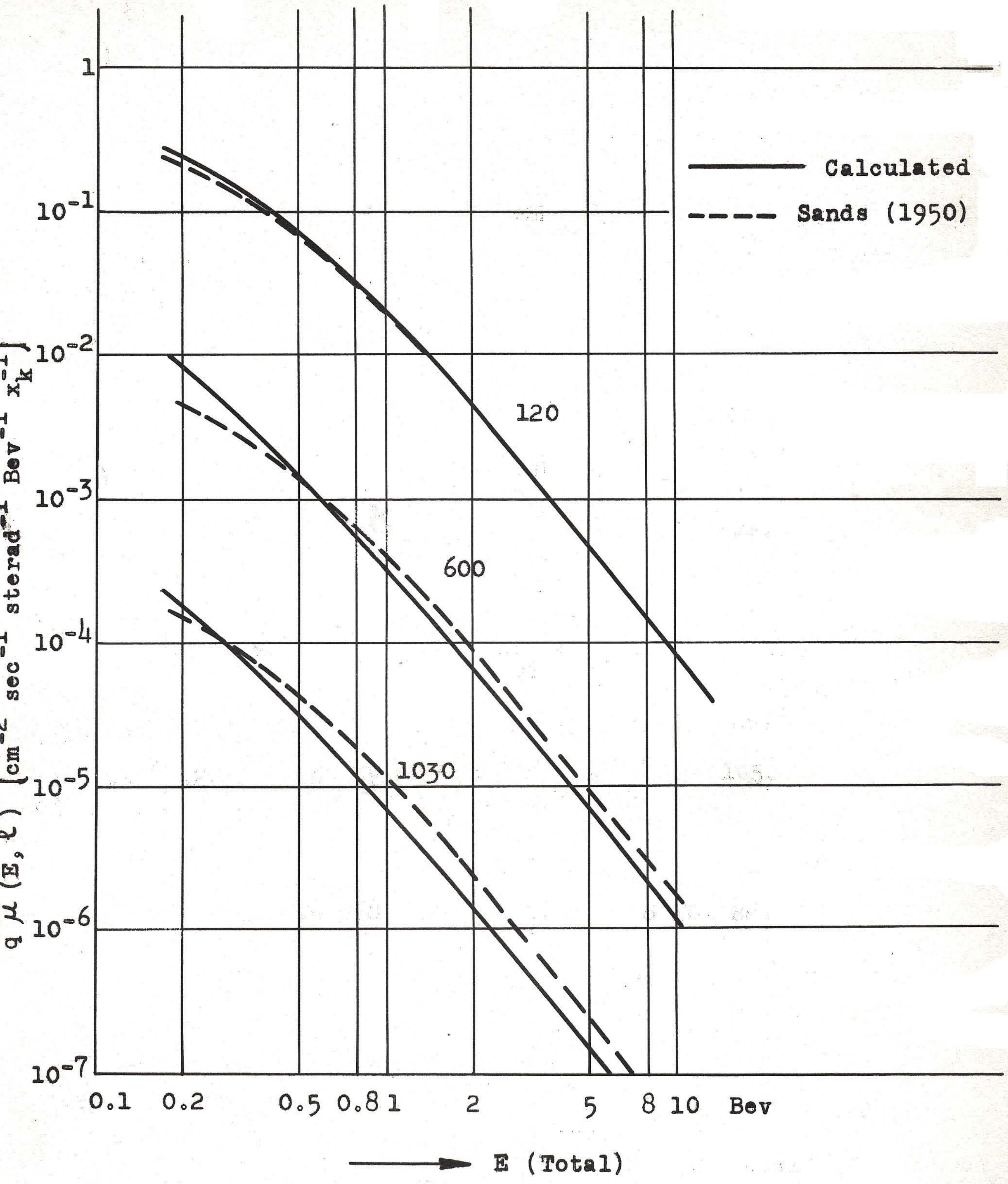

Fig. 4

Source spectrum of μ-mesons at different depths (the attached figures denoting gr/cm^2).

The latter result is of interest with respect to the question of energy balance in cosmic radiation. Together with the corresponding result for the fraction of energy of a colliding nucleon transferred to nucleons, viz. 80%, it shows that our picture, according to which only two kinds of particles (viz. nucleons and π-mesons), are created by nucleonic collision, duly accounts for the conservation of energy.

This conclusion, of course, is restricted to the range of energy (extending for nucleons from about 2 to 100 BeV), where it is valid. If there exists a lack of energy balance, as found by investigations of Rossi, it can only occur at low energies where our theory is no longer valid. --On the other hand, at very high energies we have to expect that other particles, such as k-mesons and (perhaps), nucleon-pairs, will come into play. The question of k-mesons will be discussed later on in this paper.

Sands' curves for the μ-source spectrum, as represented in Fig. 4, are somewhat smoothed. Really, Sands found an anomaly consisting of a sudden change in slope at a certain energy as shown by the curve "S" in Fig. 5. Our explanation for this anomaly is based on the fact that the spectrum of primary protons is cut-off at a certain energy by the geo-magnetic field, while the following generations of the nucleon cascade extend continuously toward smaller energies. As a consequence of these facts, the one part of the source spectrum of μ-mesons which is produced by collisions of primary protons will fall off rapidly with decreasing energy, while the other part of it, which is due to the following cascade generations of the nucleonic component will, on the contrary, increase with decreasing energy. At small atmospheric depths, where most of the μ-mesons are produced, the contribution of the first part will surpass that of the other at higher energies, while at lower energies the latter part will preponderate. The anomaly in form of a sudden change in slope will occur approximately at that energy where the contributions of both parts are equal.

On the basis of this idea we calculated the μ-source spec-

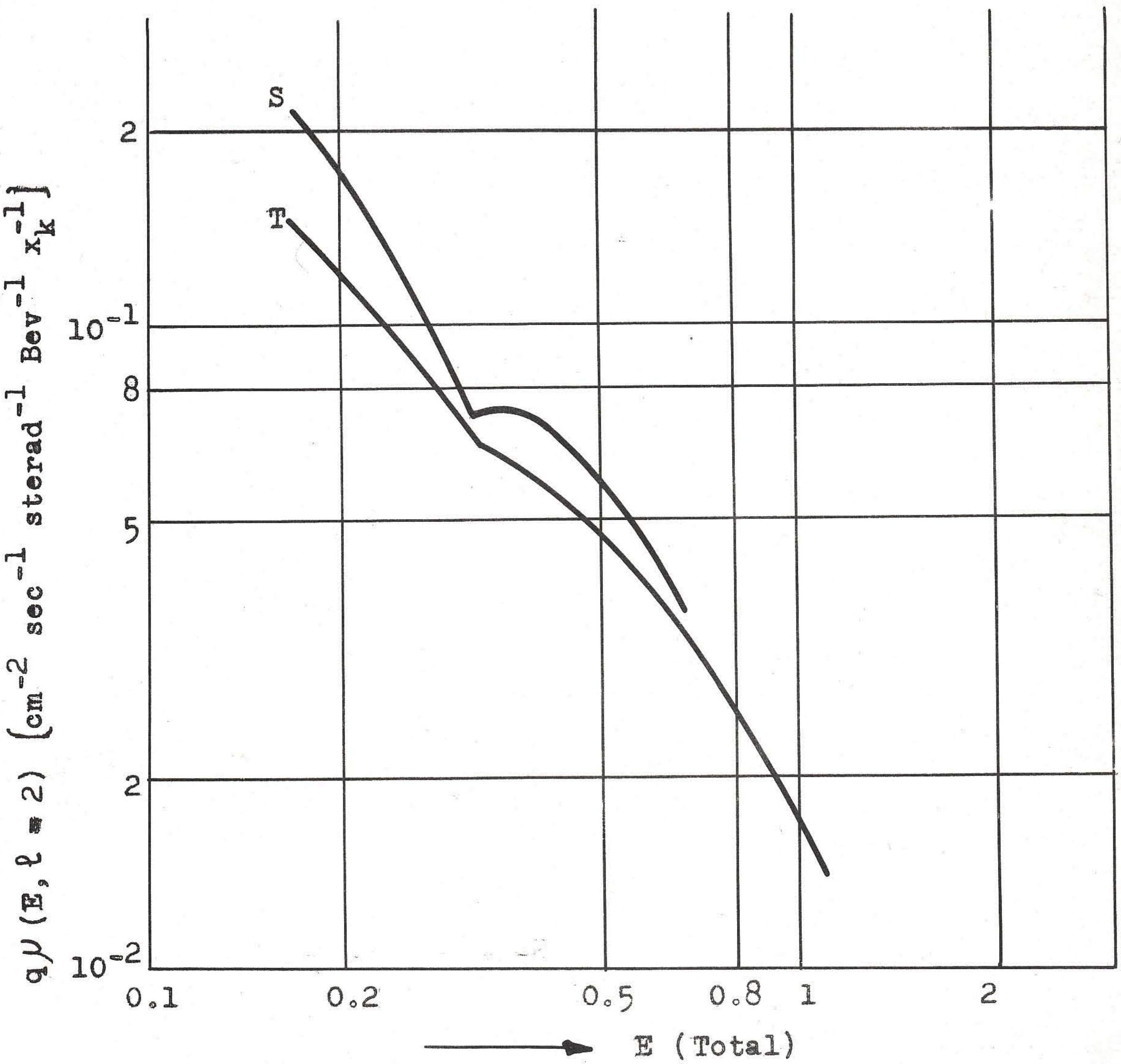

Fig. 5

Sands' anomaly of the source spectrum of μ-mesons.

T : theoretical for a depth of 120 gr/cm^2 ;

S : after Sands.

trum for an atmospheric depth of 120 gr/cm^2, the result being represented by curve T in Fig. 5, which, in fact, shows a behaviour similar to Sands' curve. The similarity would have been still more pronounced at a lesser depth of about 60 gr/cm^2, where the spectrum of primary protons is still more predominant.

It should be noted that a similar anomaly has been found by Camerini and co-workers, for the π-source spectrum.

The correctness of our explanation of Sands' anomaly could be checked by measurements at different geo-magnetic latitudes.

Our above described results concerning the components of nucleons and of μ-mesons are much confirmed by the application of our theory to the electronic component in the atmosphere. P. Budini calculated the total electronic intensity as a function of the atmospheric depth, the results being given by Fig. 6. The experimental results are represented in Fig. 6 by the full curve with the two branches "E_R" and "E_p", denoting the data of Rossi and Pomerantz respectively, at great heights. The two dotted curves "π_1" and "π_2" give the theoretical result for that part of the electronic intensity which is due to the decay of neutral π-mesons, the difference between them consisting of two different assumptions, viz. 1 BeV and 2 BeV, for the threshold energy for π-creation. The dotted curves "Z" and "S" respectively, denote the contributions of decay and knock-on electrons of μ-mesons. The sum of the different contributions is given by the dash-dot curves "T_1" and "T_2", differing by the two assumptions about the threshold. As Fig. 6 shows, there is excellent agreement between the theoretical curva T_2 and the experimental results, and at great heights, especially those of Pomerantz.

It should be emphasized that the calculations of that part of the electronic intensity which is due to the decay of neutral π-mesons, assuming that the numbers of neutral to charged π-mesons are 1:2, was based on the same creation spectra as used before with parameters determined by comparison with experimental data on nucleons and μ-mesons only.

To investigate the rôle played by k-mesons in the produc-

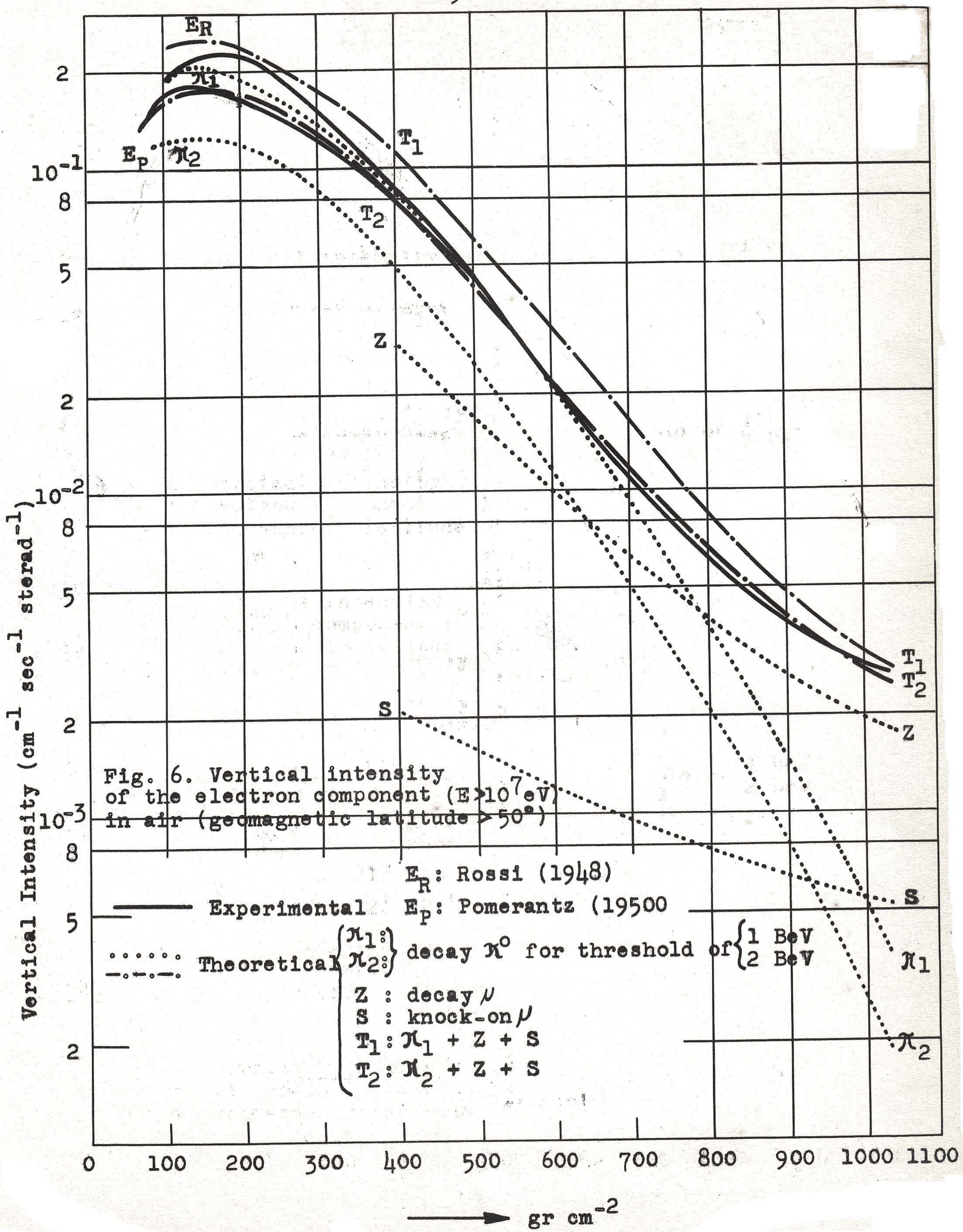

Fig. 6. Vertical intensity of the electron component ($E > 10^7$ eV) in air (geomagnetic latitude $> 50°$)

tion of high energy μ-mesons, we performed some calculations on the μ-component up to energies as high as $\sim 10^4$ BeV, which shall be reported here.

The first of these calculations concerns the spectrum of the vertical μ-component at sea level, the results being shown in Fig. 7. The two curves (dotted and dash-dotted), represent our theoretical results, obtained under alternative assumptions: the dotted curve corresponds to the assumption that all μ-mesons are created by decay of π-mesons - i.e., by the same mechanism responsible for μ-production at lower energies. This curve falls off rapidly at high energies because of the fact that high energy π-mesons(as first pointed out by Greisen), collide with air nuclei and therefore contribute less to μ-production. The other curve, dot-dash, was obtained on the basis of the alternative assumption that μ-mesons are exclusively created by decay of k-mesons. As a consequence of their shorter lifetime, the latter curve falls off more slowly.

By the comparison of these two results, obtained under extreme assumptions, with experimental data, we hope to get information about the mechanism of μ-production at high energies.

Since direct experimental results on the μ-spectrum at these extremely high energies are not available, we calculated the spectrum of μ-mesons at sea-level from underground measurements of the μ intensity by means of the range-energy-relation, as given by George[2]. The result is represented by the shaded band in Fig. 7.

The comparison of this "experimental band" with our theoretical curves seems to indicate that up to about 500 BeV, most of the μ-mesons are created by decay of π-mesons, while at higher energies almost all of them are due to k-decay.

It should be emphasized, however, that this conclusion is not free of uncertainties, one of which, e.g. being introduced by uncertainties in the range-energy-relation at high energies. Direct measurements of the μ-spectrum at high energies, therefore, would be of great importance.

Another possible source of information about the rôle of

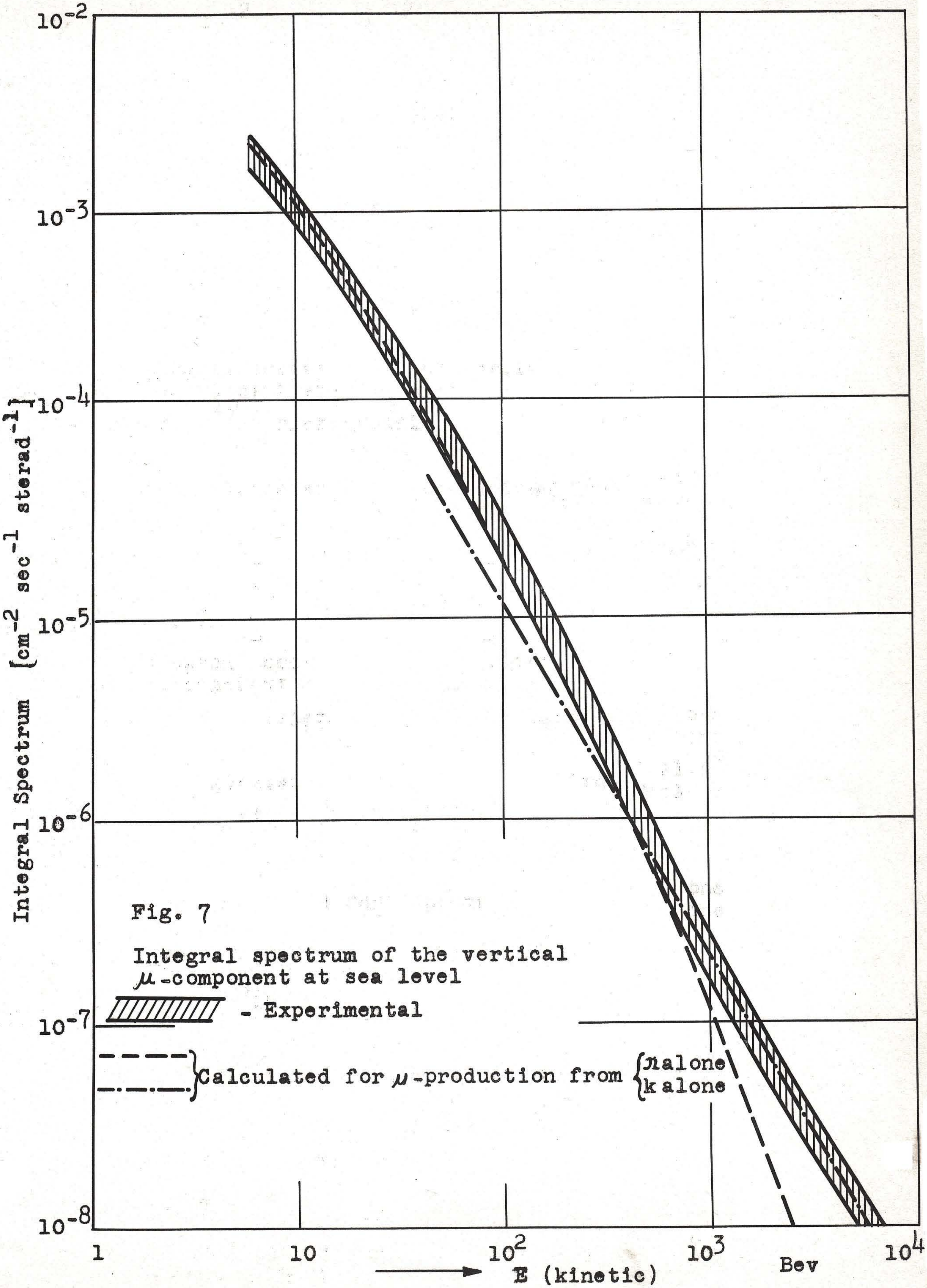

Fig. 7

Integral spectrum of the vertical μ-component at sea level k-mesons is given by the angular distribution of high energy μ-mesons.

This angular distribution is usually described by a $\cos^{\rho}\vartheta$-law, where ϑ is the angle of inclination with respect to the vertical, and ρ is an exponent which slowly varies with energy. This exponent ρ, as a function of energy, is represented by Fig. 8, according to our theory. The available experimental values involved in the figure agree well with the theory within the range of its validity. (The lower values of ρ at low energies are due to the angular spread occuring at low energy creation processes, neglected in the theory.)

The two branches of our curve in Fig. 8 correspond to those in Fig. 7: the branch "Π" belongs to the assumption of μ-mesons created exclusively by Π-mesons, while the branch "exp" corresponds to the "experimental band" in Fig. 7 or to μ-mesons of highest energies created by k-decay.

The latter branch tends with increasing energy to the limiting value of $\rho = 0$, corresponding to an isotopic angular distribution of μ-mesons. The branch "Π", instead, tends to the limiting value of -1, which means that the number of highest energy μ-mesons, provided they are created by Π-decay, will increase with increasing inclination. This strange behaviour, as it seems at the first sight, is readily understood by the realization that Π-mesons, created in the high atmosphere, travelling in a more inclined direction, have a longer path through low density air and therefore, a greater chance to decay instead of colliding.

Measurements of the angular distribution of high energy μ-mesons, therefore, would be of great importance with regard to the question of k-mesons.

Our theory was also applied to the phenomenon of "extensive air showers" which are considered the nucleon-meson-electron-photon cascades initiated by single primary protons of high energy ($> 10^{12}$ eVsay).

According to this model, the structure of a single air

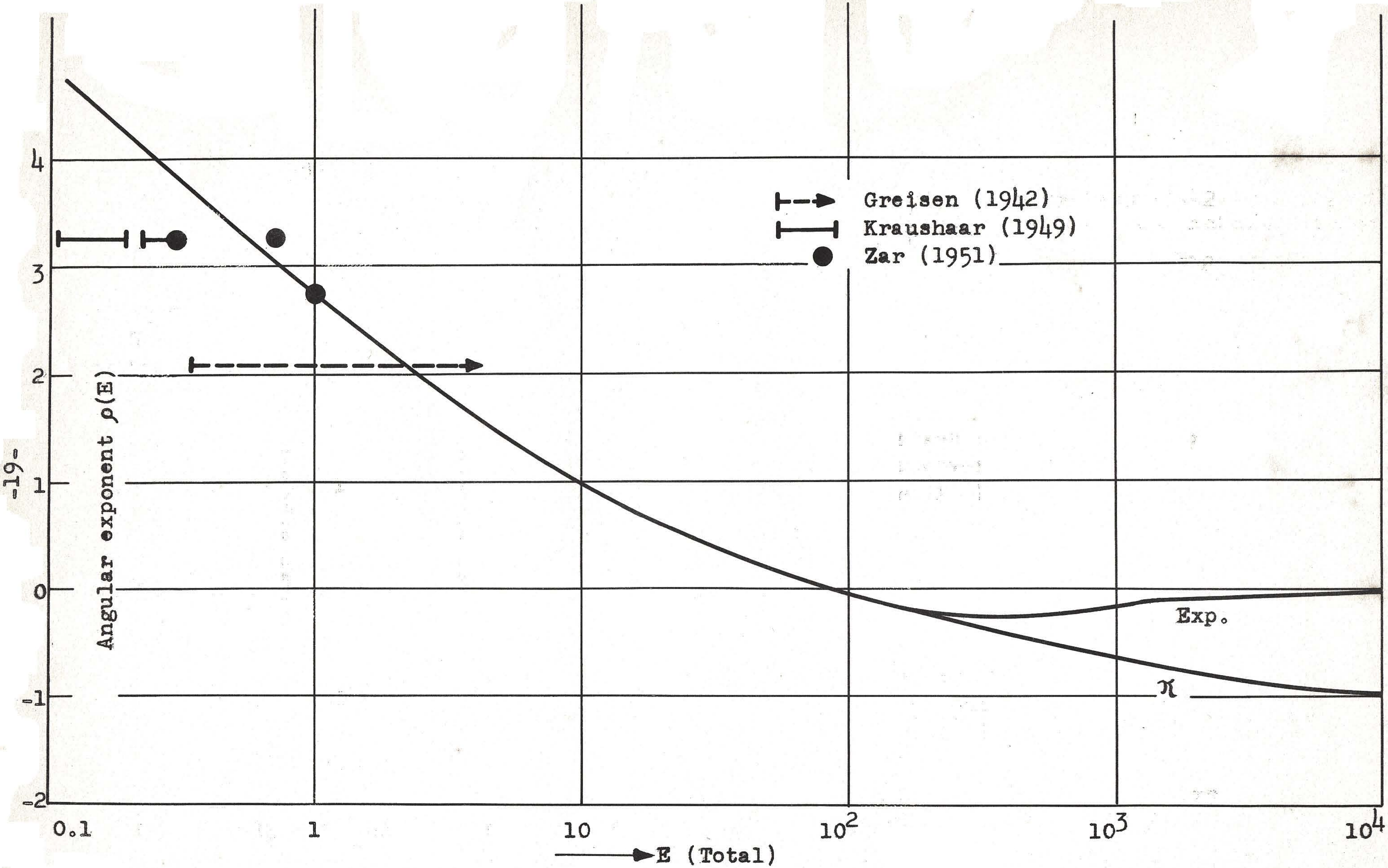

Fig. 8. Exponent ρ of the cosine-angular-law of the μ-mesons at sea level as function of the energy. "Exp." corresponds to the "experimental band" and "π" to the π-curve in Fig. 7.

shower is very similar to that of the bulk of cosmic radiation at lower energies, the difference consisting only of the different primary spectrum which, in the case of the air shower, is given by a peak function corresponding to a single proton of definite energy. In our calculations, the same creation spectra with the same parameters were used as at lower energies for the bulk of cosmic radiation.

In Fig. 9, the resulting intensities (i.e., total number of particles) of the different components of an air shower are plotted as functions of atmospheric depth (in gr/cm^2) for the three cases of primary proton-energy 10^{12}, 10^{14} and 10^{16} eV. The electronic component is represented by full lines. For comparison, the electronic intensity of a shower initiated by an electron of the same energy is also plotted (dotted lines). The electronic intensity in the primary-proton-case is in general lower by a factor four to seven and reaches its maximum earlier than in the primary electron case. The nucleonic intensity is represented in Fig. 9 by dash-dot lines, and that of the μ-component by broken lines.

Table I gives our results for some characteristic quantities of air showers of three differente primary energies and two different altitudes, viz. columns 2 and 7, the "average density" (defined below); columns 3 and 8, the ratio of numbers of electrons in the primary electron and primary proton case; columns 4 and 9, the ratio of numbers of nucleons to electrons; 5 and 10, μ-mesons to electrons and 6 and 11, penetrating particles to electrons.

As is seen, the relative number of penetrating particles in high energy air showers is a few percent, in agreement with experiments. Furthermore, it is shown by Table I that for air-showers of low primary energy, especially at sea level, an appreciable contribution of μ-mesons should be expected. This is in agreement with recent experimental findings of Watagin and Schwachheim (University of São Paulo, Brasil).

In experiments on extensive air showers the so-called "density spectrum" has frequently been determined. We calculated this density spectrum under the following two assumptions: (i) that the

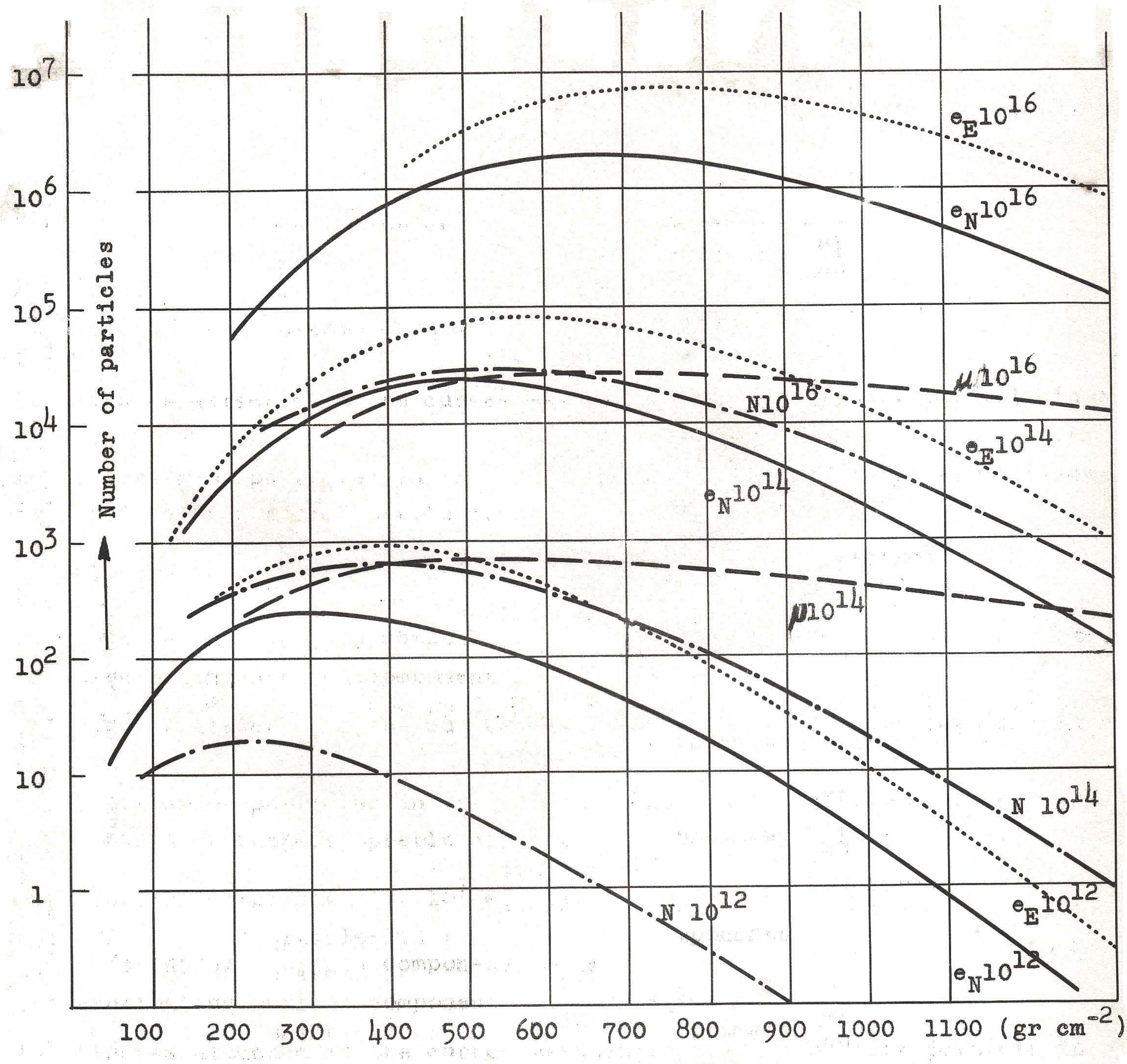

Fig. 9.

Total number of particles in the different components of large air showers as function of the atmospheric depths.

e_E, e_N: electron component ($E > 10^7$ eV) caused by primary electron / nucleon

μ: penetrating μ-meson component ($E_{tot} > 400$ MeV).

N : penetrating nucleon-component ($E_{kin} > 3$ BeV).

The figures attached to the curves mean energy of the primary particle in eV.

TABLE 1

	Sea level (1030 gr/cm^2)				
E_0 (eV)	$\bar{\Delta}$ (m^{-2})	$\frac{\text{N.el.pr.el.}}{\text{N.el.pr.nucl.}}$	$\frac{\text{N.nucl.}}{\text{N. el.}}$ x100	$\frac{\text{N.}\mu\text{mes.}}{\text{N. el.}}$ x 100	$\frac{\text{N.penetr.}}{\text{N. el.}}$ x 100
(1)	(2)	(3)	(4)	(5)	(6)
10^{12}		4.1	1.2		
10^{14}	0.4	6.9	0.98	25	25.5
10^{16}	400	7.1	0.56	2.8	3.08

	3500 m (670 gr/cm^2)				
E_0(eV)	$\bar{\Delta}$ (m^{-2})	$\frac{\text{N.el.pr.el.}}{\text{N.el.pr.nucl.}}$	$\frac{\text{N.nucl.}}{\text{N. el.}}$ x100	$\frac{\text{N. }\mu\text{mes.}}{\text{N. el.}}$ x100	$\frac{\text{N.penetr.}}{\text{N. el.}}$ x 100
(1)	(7)	(8)	(9)	(10)	(11)
10^{12}		4.7	1.4		
10^{14}	0.86	4.5	1.25	4.2	4.8
10^{16}	750	3.5	1	1.4	1.9

primary spectrum is of power form with an exponent γ and (ii), that the density distribution in a single air shower is given by the function which has formerly been determined by one of us for the maximum of large air showers and which has recently been confirmed by Nishimura and Kamata[5].

A certain density measured by a suitable apparatus may be due to showers of different primary energy. But the probability distribution of primary energies which have caused a certain density in our apparatus has a sharp maximum, so that a most probable primary energy can be attributed to each density or vice-versa. In this sense, the columns 2 and 7 in Table 1 and column 2 in Table 2 are to be understood.

TABLE 2

$\bar{E}_0$(eV)	Δ (m^{-2}) Sea level	n (Δ) (Co)	(Br 50)	(Wi 48)	γ (E_0) (Co)	(Br 50)	(Wi 48)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
10^{14}	0,4	1,27			1,81		
10^{15}	8,5	1,38	1,425		1,84	1,89	
10^{16}	400	1,54	1,425	1,5	1,96	1,81	1,78
10^{17}	1.2×10^4	(1,67)		~1,9	(2,04)		~2,1

The resulting density spectrum may be described by a power law with exponent n. The latter turns out to be very sensitively dependent on the exponent γ of the primary spectrum. By comparison with experimental results for n, as found by Cocconi, Broadbent and Williams, the exponent γ was determined. The results are given in Table 2, which shows that the value of $\gamma \approx 1.8$ remains valid up to relatively high energies, while at extremely high energies a more rapid fall-off of the primary spectrum is indicated by our results.

1 We used the data reported by Puppi and Dallaporta in "Progress in Cosmic Ray Physics", 1952.

2 Cf. George in "Progress in Cosmic Ray Physics", 1952.

3 J. Nishimura and K. Kamata, "Progress of Theoretical Physics", Vol.VI, p.628, 1952.