A note on Infraparticles and Unparticles

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Abstract

We remind the reader of the meaning and achievements of infraparticles which, although themselves not necessarily of zero mass, require the presence of zero mass in order get delocalized states with a singularity which dissolves the mass-shell in an inexorable way into the continuum and therefore renders the standard particle concept useless.

These objects were recently rediscovered under the name unparticles. The case of infraparticles also encompasses particle-like objects in conformal QFT when all multiparticle thresholds coalesce on top of each other and only the concept of a highly inclusive cross section survives of scattering theory. The infraparticle research has led to deep results and the recently discovered semiinfinite string-localized vectorpotentials have led to new interesting ideas of dealing with physical electrically charged states and a generalized scattering theory.

We explain why unparticles are identical to the old infraparticles. Using this relation it is shown that unparticles/infraparticles cannot lead to a natural description of darkness of dark matter. A more radical scenario for darkness comes from semiinfinite string-localized vectorpotentials in a reformulated version of nonabelian gauge theory.
1 Why the setting of the old infraparticles also covers the new “unparticles”

The post renormalization era, more precisely the interregnum between the elaboration of QED and the beginning of the standard model was despite its appearance of a relative boring period a time of significant conceptual conquests and maturing of ideas, notably about great advances in the understanding of the relation between particles and fields. The problematization of that relation started already way back in the work of Furry and Oppenheimer when these authors observed to their surprise (and that of the particle physics community in those days) that an interacting field applied to the vacuum does not just create a particle (as it would in the second quantized representation of QM), but rather leads to a state which inexorably attaches to the expected one-particle contribution an infinite vacuum (particle-antiparticle) polarization cloud. It is this interaction-caused ubiquitous polarization cloud which comes with the application of any local observable to the vacuum which is the cause of the holistic aspect of QFT; in contrast to QM (including relativistic QM) any interaction within the locality principles obeys a kind of universal benevolent particle physics version of Murphy’s law:

**Claim 1 (Murphy’s law in local particle physics)** All channels described by compactly localized states whose coupling is not forbidden by superselection rules are actually mutually coupled.

The original (weaker) version consisted in proving that if a state vector created by an interacting field $\Phi(x)$ acting on the vacuum in a theory with a mass gap does create a (Wigner) one particle state it must be a free field. This (Jost-Schroer) theorem was later generalized to zero mass situations, including the very tricky 2-dimensional case [1].

In more recent times the use of modular localization theory permitted a generalization to arbitrary compactly localized operators independent of their covariance properties [2][3]. Only operators which are localized in noncompact regions, whose causal completion is at least as large as a wedge, can generate one-particle states which are free of contamination by vacuum polarization. Murphy’s law actually implies the stronger claim

$$\langle p; p_1, p_2...p_n | A | 0 \rangle \neq 0, \quad A \subset A(O)$$

where $A(O)$ denotes the algebra of all operators localized in the compact region $O$ (for technical details see [17]) and $A$ is an $O$-localized interpolating operator associated with $p_i$ being on-shell momenta of particles. This coupling of a localized state $A|0\rangle$ to all admissible particle states is basically a property of causal localization and energy positivity; it only needs the presence of an interaction as a “catalyzer”, the kind of interaction and its strength is irrelevant.

This also explains what is behind the metaphor: “the vacuum is a broiling soup of particle/antiparticle pairs”. The idea that the energy conservation can be bypassed for short times is one of those harmless but nevertheless nonsensical metaphors. The correct statement can be red off from the previous formula and consists in recognizing that localized states in QFT not only obey Murphy’s law but also have energy content which extends to infinity large values (so there is no violation of energy conservation).

Born-factorization into a localized QM and its complement does not cost any energy since the vacuum tensor factorizes under spatial subdivisions\(^1\). The vacuum in quantum theories with a finite maximal propagation velocity on the other hand resists tensor factorization by requiring infinite energy. Only by creating a “splitting distance” between the space-time region and its causal complement can the energy and entropy content of such a situation be finite. This splitting process leads to a product vacuum, but unlike the quantum mechanical case it is accompanied by thermal manifestation (in particular by the appearance of an entropy which obeys a universal area law). The best understanding of this phenomenon is achieved by juxtaposing the Born-localization of QM with the covariant modular localization of local quantum physics [4]. As a result one also finds significant differences concerning entanglement associated with local splits in QM and QFT. Whereas the first situation leads to an information theoretical kind of entanglement, the ubiquitous vacuum polarization clouds destroy the connection with information theory and generate thermal manifestations instead.

The existence of rather simple well behaved *wedge-localized generators without vacuum polarization* in two-dimensional factorizing models is at the root of integrability/factorizability. After struggling for more

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\(^1\)In more popular terms, the quantum mechanical vacuum is like the Buddhist nirvana, whereas the vacuum of local quantum physics with its energy demands in entering a localized region fits the image of the Judeo-Christian-Islamic paradise (heaven).
than five decades with the problem of securing the mathematical existence of strictly renormalizable models (the renormalized perturbative series diverge and hence contain no information about the existence), one finally obtained complete mathematical control and conceptual understanding of an interesting class of strictly renormalizable QFTs\textsuperscript{2} [5][6][7].

One of the lasting achievements of the late 50s was the derivation of scattering theory from the locality principles of QFT in conjunction with positivity of the energy. Together with the assumption of a mass gap and the asymptotic completeness property, this fixes the Hilbert space in which the observables act to be of the Wigner-Fock form. With these results the old particle-field problem of Furry-Oppenheimer was brought to rest in the sense that although compactly localized vacuum polarization free generators (PFGs) of particle states do not exist in interacting QFTs at finite times (in contrast to QM), multiparticle states appear at infinite times in the sense of scattering theory and determine the Wigner-Fock multiparticle structure of the Hilbert space.

Already at the beginning of the 60s there were clouds of doubts whether this particle setting based on Wigner’s identification of particles with irreducible representations of the Poincaré group was sufficiently general to incorporate theories as QED. For such theories, despite the nonexistence of an S-matrix as a result of incurable infrared divergencies, certain probabilities in the form of \textit{inclusive cross section}, in which one sums over outgoing photons below a resolution energy, came out finite thanks to a compensation between infrared divergencies caused by virtual (inner lines in Feynman diagrams) and real photons.

From a conceptual point of view this recipe was less than satisfactory because it revealed nothing about the nature of the electrically charged particle, apart from the fact that infrared divergencies indicate problems with compact spacetime localization. What was needed was a new concept beyond the Wigner setting. The first step in this direction was taken in 1963 under the name of infraparticle [8]; the name chosen for these new particle-like states referred to the cloud of infrared excitations which prevented the formation of a mass shell.

One family of models which led to the notion of infraparticles were \textit{conformal QFT}. In fact the first models were products of massive fields with massless composites with anomalous scaling dimensions and the computations of the Kallen-Lehmann two point functions which were carried out in the very first paper were identical to those which appeared in some of the "unparticle" publications.

The more difficult problems of understanding charged particles as being infraparticles began later. At the height of the application of Kramers-Kronig dispersion relation in particle physics there was the uneasy feeling that if one imposes a Wigner zero mass representation structure, there would be no conformal interaction at all which later turned out to be justified [9]. So in order to implement interactions one must admit conformal fields with anomalous dimensions and hence one again encounters a continuous mass spectrum in the Kallen-Lehmann two-point function leading to the infraparticle situation and the problem of finding out what kind of asymptotic particle physics probabilities can be abstracted from correlation functions of anomalous dimension fields.

The Hilbert space positivity forces the scale dimension of a field to be bigger (or equal) to its canonical (free field) value which means that the singularity near the momentum space forward mass shell (in the conformal case the light cone) is weaker than a delta function [9]. There is an accumulation of singular mass spectrum on the mass shell (mantle of the forward light cone). But in contradistinction to a zero mass on-shell delta function one finds that the multi-particle continuum has been inexorably amalgamated with the delta function.

This leads to a \textit{vanishing of the large time LSZ limits}, and hence there is no LSZ time-dependent scattering theory; as a consequence one cannot compute cross sections via the S-matrix and last not least the Hilbert space does not have the structure of a Wigner-Fock multiparticle space. The physical picture about interacting conformal QFTs is that they arise as zero mass (or short distance) limits of massive theories.

This raises the question whether one can at all extract from conformal theories observable particle physics informations about the real world. The standard argument why this should be possible is that in the scaling limit of a theory with mass gaps (and therefore with a well-defined scattering theory) will be a conformal QFT in which \textit{all multiparticle thresholds collapse on top of each other}. Although such a situation does not retain detailed information of the original theory since the critical limit represents a whole universality class of massive theories, one expects that one still can extract highly inclusive cross

\textsuperscript{2}On-shell quantities as in/out formfactors including the S-matrix (which is the formfactor of the identity operator) have been known already since the late 70s.
section [9] in which these differences average out. But it is unclear whether one has to average over unobserved incoming states in addition to summing over outgoing states (two-fold inclusiveness?). It should not be too difficult to resolve this problem.

Certainly the physically most important application has been to couplings between massive and massless particles, the prototype being the interaction of photons with charged particles. The masslessness of one participating component is necessary but not sufficient. A coupling of massive nucleons to massless mesons will not lead to infraparticle nucleons but rather remain within the Wigner-Fock multiparticle setting even though the mass gaps vanish. In order that the interaction amalgamates the mass-shell delta contribution together with the zero mass "cloud" to a new sub delta function (continuous) singularity, the interaction has to be sufficiently strong in the infrared regime a requirement which is fulfilled by the QED interaction but not the the zero mass pion-nucleon interaction.

Unitarity (positivity) requires the strength of this momentum space singularity to be weaker than a delta function in the Kallen-Lehmann mass distribution and this is the reason why the LSZ limits vanish and infraparticles fall outside the standard scattering theory. A lot of deep work was dedicated to the two main questions of infraparticle physics [12][13][14][15]: how to formulate an autonomous concept of "one-particle states" (which includes both Wigner particles and infraparticles) and how to use that concept in order to derive useful formulas for appropriately defined inclusive scattering cross section. Concerning the first problem the attempts carried out over almost two decades have been quite successful; but although the problem to understand scattering of infraparticles brought a lot of conceptual insight, it did not yet lead to nice compact formulas for inclusive cross section. Hence the old "dirty" methods of compensating infrared divergencies in Feynman diagrams with infrared factors from summing over real outgoing photons below a photon resolution energy are still not superseeded. There is however hope that the trading the gauge theoretic formulation in the BRST setting by semiinfinite stringlike localization (explained in the next section) will lead to further progress in the understanding of electrically charged states and scattering probabilities of infraparticles.

Recently the infraparticle issue surfaced again under the name "unparticles" [10][11]. The main purpose of this paper is to convince the reader that despite the different motivations the concepts are the same. As a side result, people who work on unparticle issues will have access to the very rich literature about infraparticles and the associated radical modification of scattering theory.

In the following the name infraparticles may also be read as unparticles but in preferring the old name infraparticles we are not only following history; the prefix un is bit unspecific i.e. void of physical connotations (but still better than capital latin letters!). At the end there will be some remarks on why we believe that such constructions do not contain a natural mechanism which could lead to non-gravitational invisibility/inertness. The proposal which I favor is more radical and aims at a structural understanding without numerical tinkering.

A scattering theory which aims directly at inclusive scattering probabilities and which contains the standard situation based on mass gaps and Wigner particles exists at least in parts [15]. Its characteristic feature is that in contrast to the standard LSZ ‘theory one is required to think much more of what constitutes a particle detector within the setting of QFT observables. Here the somewhat delicate problem comes from the omnipresence of vacuum fluctuations in locally generated states. An important role is played by the concept of particle weights on the subalgebra of counters [17] in which the locally generated vacuum clouds are fully taken care of.

Here we will not present computations involving infraparticles since on the one hand there exists an extensive literature, and last not least this would go much beyond the limited aim expressed in the title of this article. However it is interesting to note again that there is an ongoing radical revision of gauge theory [19] which in a way offers a much better understanding of the quantum delocalization of electric charges and its connection with the infraparticle structure. As a result of its shedding new light on little studied aspects of gauge theory, it also leads to a new outlook on dark matter in rather unconventional non-understood corners of the conventional standard model setting. This will be explained in the following paragraph.

2 Interactions with string localized massless vector potentials and infraparticles

There are two ways to deal with interactions of zero mass finite helicity fields. One can either chose the standard gauge theoretical setting in which one circumvents the nonexistence of pointlike potential
associated to well-defined pointlike field strengths\(^3\) in a indefinite metric space containing ghosts, or one stays with the Hilbert space structure of QT and accepts covariant semiinfinite stringlike localized potentials.

Such a stringlike vectorpotential is described by a field \(A_\mu(x,e)\) with a commutation relation which makes the causal spacetime localization on the halfline \(x + \mathbb{R}_+ e\) explicit [3]. For a free stringlike vectorpotential one finds

\[
A_\mu(x,e) = \frac{1}{(2\pi)^{3/2}} \int \sum_{h=\pm 1} u_\mu(k,e) a^*(k,h) + h.c. \frac{d^3k}{2|k|}
\]

(2)

\[
U(a, \Lambda) A_\mu(x,e) U(a, \Lambda)^* = (\Lambda^{-1})_\mu^\nu A_\nu(\Lambda x + a, \Lambda e)
\]

\[
[A_\mu(x,e), A_\nu(x',e')] = 0 \text{ only if } x + \mathbb{R}_+ e < x' + \mathbb{R}_+ e
\]

Here \(u(k,e)\) is a numerical intertwiner which intertwines the unique Wigner representation with the covariant string localized covariant representation. The \(A_\mu\) is a free field which fluctuates both in \(x\) and the string direction described by a spacelike unit vector \(e\) (a point on unit 3-dim. de Sitter space). In other words \(e\) is not a (gauge) parameter but a fluctuating localization variable (a point in 3-dim de Sitter spacetime). The upshot of this two-fold fluctuation is that the short distance dimension in \(x\) which for a pointlike massive vector field is \(\text{sd}=2\) will now be reduced to 1. Perturbation theory with strings instead of pointlike objects are further removed from classical field theory (and from Lagrangian quantization and functional integrals) and presents some new problems, in particular in connection the Epstein-Glaser iteration, which are presently under investigation[19]; what is however obvious is that at least the power counting aspect fulfills the formal prerequisites of renormalizability.

Why wasn’t this seen in gauge theory? Well, in some sense it was noticed under the label “axial gauge“, but unfortunately the reading of \(e\) as a gauge parameter led to confusing infrared problems in loops involving vectorpotentials; as a consequence the axial gauge setting never led to useful perturbative calculations. The fluctuating string interpretation explains these infrared problems and suggests how to do confront these new problems in perturbation theory with are caused by objects which are distributions in \(x\) and \(e\). It also shows that \(e\), unlike a gauge parameter, participates in the Poincaré transformation of the Wigner representation theory.

Correlations for electromagnetic field strengths without external charge lines are independent of the \(e\)‘s, whereas those involving charge operators have a strong dependence on the string directions. This shows that the string localized setting replaces gauge invariance by \(e\)-independence which turns out to be the same as pointlike localizability. The main advantage of working with string localized potentials instead of potentials in the gauge setting is that, since all operators act on the physical Hilbert space, the delocalized physical charge transferring operators appear in a natural way without the necessity of complicated and highly arbitrary ”by hand”manipulations to define physical charged operators via BRST cohomological constructions. Taking care of the distribution theoretical \(e\) dependence\(^5\) with the help of a smearing function \(f\), the charge carrying operators \(\Psi(x,f)\), whose correlation function are infrared divergent only in the limit \(f \to \delta\), are ghostfree physical even though they do not belong to the local observables since the adherence to the Gauss law renders them somewhat nonlocal. This means that the relative commutator of charge-carrying physical operators with local observables falls off (the details depending on \(f\)) but do not vanish for large but finite spacelike distances [12][17].

The nonlocal infarparticle properties of charge-carrying operators and states, as well as the spontaneous broken Lorentz covariance caused by their infrared photon clouds depend on the smeared string direction \(e\). Strictly speaking only the neutral field strength correlation are \(L\)-invariant and strictly local in the usual Wightman sense.

In his new semiinfinite string setting the compactly localizable (pointlike generated) observables are exactly equal to the gauge invariant local polynomials in the standard approach. But whereas the famous question why can there be no gauge choice-dependent physical variable may provoke different answers from different individuals, the semiinfinite string setting leads to an unambiguous answer by replacing

\(^3\) For \(h=1\) these are the ordinary positivity violating electromagnetic vectorpotentials and their associated positivity obeying field strengths. Stringlike “potential” with helicity-independent \(\text{sd}=1\) and their associated “field strength” with increasing \(\text{sd}\) depending on the helicity exist for each helicity [3].

\(^4\) In the string description of the vectorpotentials the infrared divergencies in charged states result from the ultraviolet divergencies in the directional 3-dim. de Sitter space.

\(^5\) The directional de Sitter short distance behavior corresponds to the infrared singularity in Minkowski space.
the classical gauge principle by the quantum locality principle. The removal of the classical gauge formalism and its substitution by the pointlike locality principle within a wider context of semiinfinite stringlike localized fields is a step away from classical metaphors towards an autonomous quantum realm. All properties of QFT including inner symmetries and their spontaneous breaking, the Schwinger-Higgs screening, the TCP and the spin-statistics theorems and the occurrence of plektons in $d=1+2$ have their origin in the richness of manifestations of causal quantum localization. The gauge principle is on the way to be added to this list and (see the remarks below) there is a good chance that also dark matter is an origin in the richness of possibilities of ordering matter in spacetime.

For abelian couplings (i.e. no coupling among vectorpotentials) the use of semiinfinite string localized potentials (instead of the indefinite metric gauge setting for abelian vectorpotentials) is basically a conceptual improvement in the formulation. It does not lead to new results which in principle cannot also be obtained (less naturally) in the gauge setting. However for gauge invariant operators which are not of the pointlike polynomial kind but rather require nonlocal expressions in the vectorpotential as gauge invariant charged fields as functions of the formal (gauge dependent) charge fields and exponential line integrals in the vectorpotentials extending to infinity, the string-localized description is much simpler because the difficult part of the stringlike approach has been encoded into the Epstein-Glaser iteration of renormalization theory for semiinfinite stringlike fields [19].

One for a long time quite mysterious family of positive energy representations is Wigner’s third kind representation, the so-called infinite spin representations. Together with the massless finite helicity representation and the massive finite spin representation (referred to as the second and first kind) these three representation classes exhaust all positive energy possibilities. The third kind is described by fields $\Phi(x, e)$ as in (2) but with more complicated intertwiners. These free fields generate an algebra which, unlike the QED case, contains no compactly localizable subalgebra. In other words not only is their no differential operator which transforms the $\Phi$ into a pointlike field as the $A_{\mu} \rightarrow F_{\mu \nu}$ map, but there is not even a composite which has an $e$-independent pointlike localization.

This matter would be invisible in the sense that it cannot be produced by ordinary matter. It is also hard to image that it interacts at all with normal matter since in contrast to the previous abelian gauge model the $\Phi(x, e)$ does not generate pointlike composites. Since the third kind of Wigner matter does not have a pointlike energy-momentum tensor and hence a gravitational coupling in the Einstein-Hilbert setting is not possible. As all finite energy matter one expects it to couple in some vague sense to classical gravity. In such a situation physicists like to lump two non-understood properties together. For the case at hand this amounts to the conjecture that the origin of this kind of dark matter is related to primordial role of quantum gravity.

A milder and more realistic form of invisibility and inertness arises from the string reformulation of nonabelian gauge theory. Unlike a string localized interacting abelian vectorpotential which can be written as a semiinfinite line integral over a field strength, vectorpotential arising from interactions among themselves cannot be approximated by approximated by local observables since there is no linear relation between such stringlike potentials and the smaller subalgebra generated by pointlike composites [23]. In this case the energy-momentum tensor is a pointlike composite and there is no problem with the back reaction in a curved spacetime environment. In contradistinction to the infraparticle case the Hilbert space is bigger than the space generated by the application of only pointlike composites. In this case the stringlike formulation contains more information than the gauge setting; in addition to the gauge invariant pointlike fields which agree with pointlike composites of the stringlike setting there are genuine stringlike localized fields which have no counterpart in the classical gauge theory setting. This is the arena for invisible matter which cannot be produced from visible matter. Since the energy-momentum tensor is part of normal matter, there is no problem with quasi-classical gravity coupling.

This stringlocalized matter may be even inert with respect to Araki-Haag infraparticle counters. Even if one starts with a There are also problems to register such strings in Araki-Haag counters. So these semiinfinite strings are the ideal objects for interpreting dark matter as a natural property of a yet unexplored extension of normal matter in which $s \geq 1$ mutually interacting massless objects play a crucial role. The difficulty of investigating such a situation is not different from the difficulty in studying nonabelian gauge theories: the perturbative approach is beset by off-shell infrared problems of a much more serious kind than the on-shell infrared problems of infraparticles.

The delocalization of infraparticles/unparticles is not strong enough to make them inert in fact, charged particles like electrons are extremely visible. Similar to WHIMPS which do not belong to the

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6 The pair production of such semiinfinite spacelike strings would contradict causal locality.
standard model particle set but are certainly standard Wigner particles, they can at best be somewhat "gray" but not dark. Darkness in the sense of the present work means that the cross section for pair production of dark matter by pair production is exactly zero. If dark matter would be the result of metaphoric unnatural linking as in the unparticle proposals it would be quite boring. But of course boredom is not a physical argument; on the other hand naturalness seems to be a widespread accepted concept (which is however difficult to define in rigorous terms).

I have explained elsewhere that the string with respect to localization have nothing in common with the objects of string theory [18]. The latter are (in their free Nambu-Goto form) pointlike localized fields which, in distinction to ordinary fields, have a larger cardinality of mass and spin values (an infinite mass and spin “tower”) and are described by special generalized free fields with a spin spectrum. The meaning of string in string theory is metaphoric and helps to formulate interactions in terms of Euclidean “tubism” (splitting and recombining tubes) but, as will be explained in somewhat more detail below, it has nothing to do with those little strings in Minkowski spacetime which constitute the opening mantra of talks on string theory.

Some of the recent ideas in connection with unparticles are not only ill-fitted for natural autonomous concept of darkness but they are also not consistent with a more intrinsic understanding of QFT. It is not possible to couple an interacting anomalous dimension (conformal) field to a massive field because there is no perturbative systematics which such an Ansatz could lead to; one can only couple composites of free fields. There are however indications that the systematics of perturbation theory may be extended to conformal generalized free fields which can have any arbitrary anomalous dimensions above the minimal value. Such fields result from the standard free field on AdS via the AdS-CFT correspondence [20]. But to place such an Ansatz on solid ground would require additional conceptual investments along the lines of the cited work.

For those readers who are not familiar with generalized free fields we mention that these are pointlike localized fields with c-number commutators which contain either continuous mass distributions (especially if they are conformal) or discrete (possibly infinite) mass- and spin-towers. Generalized free fields have some remarkable properties which distinguishes them from ordinary free fields. For example the set of fields which are relative local with respect to a generalized free field is not exhausted by the classical rule of Wick-ordering the classical local polynomial expressions and their thermal states either come with a Hagedorn temperature or in the worst case such states do not exist at all. A famous illustration is provided by the Nambu-Goto ”string field” [21][22]. In contradistinction to its totally metaphoric name, the field is not string-localized [23] in any intrinsic quantum sense as explained above.

Some readers may find it disenchanting that with increasing frequency part of pre-electronic work is overlooked as the infraparticle work by the rather large unparticle community. There are other examples and perhaps this is the prize we have to pay for the exponentially increasing publication volume.

What is however really worrying is that the conceptual level of re-discovered knowledge is more metaphoric and less intrinsic than its original version. This problem is very serious since it points to a loss of objective standards on the level of refereeing. Fore more illustrations and critical remarks see [24].

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