CBPF-NF-010/03

Lightfront holography and area density of entropy associated with localization on wedge-horizons

Bert Schroer
present address: CBPF, Rua Dr. Xavier Sigaud 150,
22290-180 Rio de Janeiro, Brazil
email schroer@cbpf.br
permanent address: Institut für Theoretische Physik
FU-Berlin, Arnimallee 14, 14195 Berlin, Germany
February 6, 2003

Abstract

It is shown that a suitably formulated algebraic lightfront holography (LFH), in which the lightfront is viewed as the linear extension of the upper causal horizon of a wedge region, is capable of overcoming the shortcomings of the old lightfront quantization. The absence of transverse vacuum fluctuations, which this formalism reveals, is responsible for an area (edge of the wedge) - rearrangement of degrees of freedom which in turn leads to the notion of area density of entropy for a "split localization". This area proportionality of horizon associated entropy has to be compared to the volume dependence of ordinary heat bath entropy. The desired limit, in which the split distance vanishes and the localization on the horizon becomes sharp, can at most yield a relative area density which measures the ratio of area densities for different quantum matter. In order to obtain a normalized area density one needs the unknown analog of a second fundamental law of thermodynamics for thermalization caused by vacuum fluctuation through localization on causal horizons. This is similar to the role of the classical Gibbs form of that law which relates Bekenstein's classical area formula with the Hawking quantum mechanism for thermalization from black holes.

PACS: 11.10.-z, 11.30.-j, 11.55.-m

1 Constructive Aims of Lightfront Holography

Lightfront quantum field theory and the closely related $p \to \infty$ frame method have a long history. The large number of articles on this subject (which started to appear at the beginning of the 70ies) may be separated into two groups. On the one hand there are those papers whose aim is to show that such concepts constitute a potentially useful enrichment of standard local quantum physics [1][2][3], but there are also innumerable attempts to use lightfront concepts as a starting point of more free-floating "effective" approximation ideas in high-energy phenomenology (notably for Bjorken scaling) whose relations to causal and local quantum physics remained unclear or were simply not addressed.

As will become clear in the next section, where we will recall some of the old ideas from a modern perspective, the old lightfront approach was severely limited since in d=1+3 spacetime dimensions its canonical short distance prerequisites are met only in the absence of interactions. Nevertheless algebraic lightfront holography[1] (LFH), which is the subject of this paper, may be viewed as a revitalization of the old approach with new concepts. The aim of the old approach (though never satisfactorily achieved) was the simplification of dynamics by encoding some of its aspects into more sophisticated kinematics; this is precisely what LFH aims to achieve, but this time without suffering from short distance limitations which eliminate interactions and with the full awareness of the locality issue and the problem of reconstruction of the original theory (or family of original theories) which has (have) the same holographic projection.

Whereas the old approach amounted to the lightfront restriction of pointlike fields (which also caused the mentioned limitation), the LFH reprocesses the original fields first into a net of algebras which by algebraic holography is then converted into a net of operator algebras indexed by spacetime regions in the lightfront. This net turns out to be a "generalized" chiral net (a chiral net extended by a very specific vacuum-polarization-free transverse quantum mechanics) with a 7-parametric vacuum symmetry group which corresponds to a subgroup of the 10-parametric Poincaré group of the ambient theory. It also possesses additional higher symmetries which originate from the conformal covariance of the chiral LFH theory. The latter amount to diffuse-acting automorphisms in the ambient theory ("fuzzy symmetries"). This does not only include the rigid rotation which belongs to the Moebius group with L_0 as its infinitesimal generator, but also all higher diffeomorphism of the circle $Diff(S^1)$.

It is important to realize that the absence of a direct relation between the ambient fields A and those A_{LF} generating the lightfront net i.e.

$$A_{LF}(x) \neq A(x)|_{LF} \tag{1}$$

is the prize to pay for the simplification of *interacting* quantum field theories[2] in the algebraic LFH (where the left hand side is determined by intermediate algebraic steps and not by restricting fields). The inverse holography i.e. the classification of ambient theories which belong to one LFH class (including the action of its 7-parametric symmetry group) remains as the main unsolved problem.

Although the LFH does not accomplish dynamical miracles, it shifts the separating line between kinematical substrate and dynamical actions in a helpful way by placing more structure (as e.g. compared to the canonical formalism) onto the kinematical side which is described by the holographic projection.

[1] The term "holography" was introduced by 't Hooft [4] for the description of his intuitive idea about the organization of degrees of freedom in the presence of event horizons for QFT in CST. The present setting of LFH is algebraic QFT (AQFT) in Minkowski spacetime.

[2] In fact in the presence of strictly renormalizable interactions, the right hand side is void of mathematical meaning (see (13) in section 2).

LFH also avoids having an "artistic" starting point as e.g. canonical commutation relations or functional integral representations (which the physical renormalized theory cannot maintain in the presence of interactions, apart from d=1+1 polynomial scalar interactions). Instead it fits well into the spirit of Wightman QFT or algebraic QFT (AQFT), where the result obtained at the end of a computation are in complete mathematical harmony with the requirements at the start. Among the reasonably easy structural consequences of LFH is the surface proportionality of *localization entropy* associated with a causal horizon. This will be the subject of the fourth section.

There are several papers which discuss the gain of scale symmetry through restrictions of QFT to horizons using modular theory [5][6][7] without addressing the problem of the transverse direction which is the central point of the present work.

Although the connection between the local net on the lightfront and that on the full ambient spacetime turns out to be quite nonlocal (in contradistinction to the algebraic AdS-CQFT holography, which still preserves many relative local aspects [8]), the modular localization approach succeeds to formulate this holographic procedure (including an understanding of its nonlocal relations to the original theory) in the rigorous setting of local quantum physics. With these remarks on what LFH means in this paper (as compared to many other meanings of "holography" in the recent literature), we conclude our historical and general remarks and pass to a mathematical description [3].

2 Elementary facts on pointlike fields restricted to the lightfront

For some elementary observations we now turn to the simple model of a d=1+1 massive free field

$$A(x) = \frac{1}{\sqrt{2\pi}} \int \left(e^{-ipx}a(\theta) + e^{ipx}a^*(\theta)\right) d\theta \tag{2}$$
$$p = m(ch\theta, sh\theta)$$

where for convenience we use the momentum space rapidity description. In order to get onto the light ray $x_- = t - x = 0$ in such a way that $x_+ = t + x$ remains finite, we approach the $x_+ > 0$ horizon of the right wedge $W : x > |t|$ by taking the $r \to 0$, $\chi = \hat{\chi} - ln\frac{r}{r_0} \to \hat{\chi} + \infty$ limit in the x-space rapidity parametrization

$$\begin{aligned} & x = r(sh\chi, ch\chi),\ x \to (x_- = 0, x_+ \geq 0,\ finite) \\ & A(x_+, x_- \to 0) \equiv A_{LF}(x_+) = \frac{1}{\sqrt{2\pi}} \int \left(e^{-ip_-x_+}a(\theta) + e^{ip_-x_+}a^*(\theta)\right) d\theta \\ & = \frac{1}{\sqrt{2\pi}} \int \left(e^{-ip_-x_+}a(p) + e^{ip_-x_+}a^*(p)\right) \frac{dp}{|p|} \end{aligned} \tag{3}$$

where the last formula exposes the limiting $A_{LF}(x_+)$ field as a chiral conformal (gapless P_- spectrum) field; the mass in the exponent $p_-x_+ = mr_0e^{\theta}e^{-\hat{\chi}}$ is a dimension preserving parameter which (after having taken the limit) has lost its physical significance of a mass gap (the physical mass is the gap in the $P^2 = P_-P_+$ spectrum).

Since this limit only effects the numerical factors and not the Fock space operators $a^{\#}(\theta)$, we expect that there will be no problem with the horizontal restriction i.e. that the formal method (the last line

[3] The present work combines and supersedes previous unpublished work of the author (hep-th 0106284, 0108203, 0111188) and emphasizes different aspects of the subject published in [9].

in 3) agrees with the more rigorous result using smearing functions. Up to a fine point which is related to the well-known infrared behavior of a scalar chiral $dimA = 0$ field, this is indeed the case. Using the limiting χ-parametrization we see that for the smeared field with $supp\tilde{f} \in W$, $\tilde{f}$ real, one has the identity

$$\int A(x_+, x_-)\tilde{f}(x)d^2x = \int_C a(\theta)f(\theta) = \int A_{LF}(x_+)\tilde{g}(x_+)dx_+,\ \tilde{g}\ real \tag{4}$$
$$\tilde{f}(x) = \int_C e^{ip(\theta)x} f(\theta)d\theta,\ \ \tilde{g}(x_+) = \int e^{ipx_+} g(p)\frac{dp}{|p|} = \int_C f(\theta)e^{ip_-(\theta)x_+}d\theta$$

These formulas, in which a contour C appears, require some explanation. The onshell character of free fields restricts the Fourier transformed test function to their mass shell values with the backward mass shell corresponding to the rapidity on the real line shifted downward by $-i\pi$

$$f(p)|_{p^2=m^2} = \begin{cases} f(\theta),\ p_0 > 0 \\ f(\theta - i\pi), .\ p_0 < 0 \end{cases}$$

and the wedge support property is equivalent to the analyticity of $f(z)$ in the strip -$i\pi < \mathrm{Im}\, z < 0$. The integration path C consists of the upper and lower rim of this strip and corresponds to the negative/positive frequency part of the Fourier transform. By introducing the test function $\tilde{g}(x_+)$ which is supported on the halfline $x_+ \geq 0$, it becomes manifest that the smeared field on the horizon rewritten in terms of the original Fourier transforms must vanish at $p = 0$ as required by L_1-integrability according to

$$f(p)|_{p^2=m^2,p_0>0}\frac{dp}{\sqrt{p^2+m^2}} = f(\theta)d\theta \equiv g(\theta)d\theta = g(p)|_{p^2=0,p_0>0}\frac{dp}{|p|} \tag{5}$$
$$\curvearrowright g(p=0) = 0$$

with a similar formula for negative p_0 and the corresponding θ-values at the lower rim. This infrared restriction is typical for spinless free fields with $dimA = 0$. The equality of the f-smeared $A(x)$ field with a g-smeared $A_+(x_+)$ leads to the vanishing of $g(p)$ at the origin and finally to the equality of the Weyl operators and hence of the generated operator algebras

$$\mathcal{A}(W) = \mathcal{A}(R_+) \tag{6}$$
$$\mathcal{A}(W) = alg\left\{e^{iA(f)} |\ supp\tilde{f} \subset W\right\}$$
$$\mathcal{A}(R_+) = alg\left\{e^{iA_{LF}(g)} |\ supp\tilde{g} \subset R_+, \int \tilde{g}dx_+ = 0\right\}$$

The choice of the lower causal horizon of W would have led to the same global algebra and could also serve as the starting point of LFH. The equality (6) is the quantum version of the classical causal shadow propagation property of characteristic data on the upper lightfront of a wedge. With the exception of $d = 1+1$, $m = 0$ where one needs the data on both light rays, the classical amplitudes inside the causal shadow W of R_+ are uniquely determined by either the upper or the lower data.

As in the classical analog of propagation of characteristic data, it would be incorrect to think that the global identity of the algebras persists on the local level i.e. that there exists a region in W whose associated operator algebra corresponds to a $\mathcal{A}(I)$ algebra when $I \in R_+$ is a finite interval. The spacetime net structures on W and R_+ are very different; the subalgebras localized in finite intervals $\mathcal{A}(I)$ have no local relation to localized subalgebras $\mathcal{A}(\mathcal{O})$ of $\mathcal{A}(W)$ and vice versa; rather the position of compactly localized algebras in one net is diffuse ("fuzzy") relative to the net structure of the other net. The fact that a finite interval on R_+ does not cast a 2-dimensional causal shadow does of course not come as surprise, since even in the classical setting a causal shadow is only generated by characteristic data which have at least a semi-infinite extension to lightlike infinity. Related to this is the fact that the opposite lightray translation

$$AdU_-(a)\mathcal{A}(R_+) \subset A(R_+) \tag{7}$$
$$U_-(a) = e^{-iP_-a}$$

is a totally "fuzzy" endomorphism of the $A(R_+)$ net; whereas in the setting of the spacetime indexing in the$\mathcal{A}(W)$ net, the map $AdU_-(a)$ permits a geometric interpretation.

It is very important to notice that even in the free case the horizontal limit is conceptually different from the scale invariant massless limit. The latter cannot be performed in the same Hilbert space since the $m \to 0$ limit needs a compensating $\ln m$ term in the momentum space rapidity θ in the argument of the operators $a^{\#}(\theta)$, whereas the horizon limit (3) was only taking place in the c-number factors.

There is however no problem of taking this massless limit in correlation functions if one uses the appropriate spacetime smearing functions. The limiting correlation functions define via the GNS construction of operator algebras a new Hilbert space [13] which contains two chiral copies of the conformal $dimA = 0$ field corresponding to the right/left movers. This difference between the scaling limit and the lightray holography limit for free fields is easily overlooked, since the conformal dimensions of the resulting fields are in this case the same and the only difference is that the scaling limit leads to a 2-dimensional conformal theory which decomposes into two independent chiral algebras. We will see that in the case of interacting 2-dimensional theories the appropriately defined holographic projection onto the lightray is different from the scaling limit by much more than just a doubling.

There arises the question whether the chiral theories originating from the lightray restriction are intrinsically different from those which are obtained by factorizing d=1+1 conformal theories into its two chiral components. At least in the present example this is not the case; we can always extend a free chiral field independent of its origin to a d=1+1 massive field by defining an additional action $U_-(a)$ which just creates a phase factor $e^{ip_+x_-}$ on the $a^{\#}(p)$ appearing in (3) without enlarging the Hilbert space. The situation is reminiscent of Wigner's finite helicity massless representations which allow an (in that case locally acting) extension from the Poincaré- to the conformal- symmetry without enlargement of the one-particle space.

Passing now to the higher dimensional case we notice, that by introducing an effective mass which incorporates the transverse degrees of freedom on the upper lightfront horizon of W, the previous arguments continue to hold

$$A(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left(e^{-ip_+x_- - ip_-x_+ + ip_\perp x_\perp} a(p) + h.c.\right) \frac{d^3p}{2\omega},\ x_\pm = x^0 \pm x^1 \tag{8}$$
$$p = (m_{eff}ch\theta, m_{eff}sh\theta, p_\perp),\ m_{eff} = \sqrt{m^2 + p_\perp^2},\ p_\pm = \frac{p^0 \pm p^1}{2}$$

The limiting field can again be written in terms of the same Fock space creation/annihilation operators and as before the (effective) mass looses its physical role

$$A_{LF}(x_+,x_\perp)=\frac{1}{(2\pi)^{\frac{3}{2}}}\int\left(e^{-ip_-x_++ip_\perp x_\perp}a(p)+h.c.\right)\frac{dp_-}{2\left|p_-\right|}d^2p_\perp \qquad (9)$$
$$A_{LF}(gf_\perp)=\int a^*(p_-,p_\perp)g(p_-)f_\perp(x_\perp)\frac{dp_-}{2\left|p_-\right|}d^2p_\perp+h.c.$$

In the second line the resulting operator is written in terms of a dense set of test functions which factorize in a longitudinal (along the lightray) and a transverse part. The longitudinal integration may be again brought into the rapidity space form (4) involving a path C. The dependence of the longitudinal part on the transverse momenta is concentrated in the effective mass, which in turn only enters via a numerical dimension-preserving factor in $p_-x_+=m_{eff}e^{\theta}e^{-\chi}$. Note that a description of the field in the wedge in terms of lightray coordinates $x_\pm$ would have led to the transverse dependence to be converted into the mass distribution of a generalized field which is less simple than the lightfront projection.

In fact the best way to formulate the resulting structure on the lightfront is to say that the longitudinal structure is that of a chiral QFT (with the typical vacuum polarization leading to long range correlations) whereas transversely it is quantum mechanical i.e. free of vacuum polarization and without coupling between transverse separated subsystems. The ensuing correlations are given by the inner product

$$\left\langle A_{LF}(gf_\perp)A_{LF}(g'f'_\perp)\right\rangle=\int\bar{g}(p)g'(p)\frac{dp}{2\left|p\right|}\int\bar{f}_\perp(p_\perp)f'_\perp(p_\perp)d^2p_\perp$$
$$\left[A_{LF}(x_+,x_\perp),A_{LF}(x'_+,x'_\perp)\right]=i\Delta(x_+-x'_+)_{m=0}\delta(x_\perp-x'_\perp) \qquad (10)$$

where the second line shows that the commutation structure of the transverse part is like that of Schrödinger field. In fact the analogy to QM is even stronger since the vacuum does not carry any transverse correlation at all, a fact which can be best seen in the behavior of the Weyl generators

$$\left\langle W(g,f_\perp)W(g',f'_\perp)\right\rangle=\left\langle W(g,f_\perp)\right\rangle\left\langle W(g',f'_\perp)\right\rangle \;\; if\ suppf\cap suppf'=\emptyset \qquad (11)$$
$$W(g,f_\perp)=e^{iA_{LF}(gf_\perp)}$$

i.e. the vacuum behaves like a quantum mechanical vacuum with no correlations in the transverse direction[4]. To make this relation with transverse QM complete, we will now show that the loss of vacuum correlations is accompanied by the appearance of a Galilei group acting on these transverse degrees of freedom.

For this purpose it is helpful to understand the symmetry group of the lightfront restriction. It is not difficult to see that it consists of a 7-parametric subgroup of the 10-parametric Poincaré group; besides the longitudinal lightray translation and the W-preserving L-boost (which becomes a dilation in lightray direction on the lightfront) there are two transverse translation and one transverse rotation. The remaining two transformations are harder to see; they are the two "translations" in the Wigner little group (invariance group of the lightray in the lightfront). We remind the reader that the full little group is isomorphic to the double covering of the 3-parametric Euclidean $E(2)$ group in two dimensions. As a subgroup of the 6-parametric Lorentz group it consists of a rotation around the spatial projection of the lightray and two little group "translations" which turn out to be specially tuned combinations of L-boosts and rotations which tilt the edge of the wedge in such a way that it stays inside the lightfront

[4]The lightfront restriction amounts to a global change of the free field operators so that even spacelike correlations on the lightfront become modified compared with their old value in the ambient theory.

but changes its angle with the lightray. This 2-parametric abelian subgroup of "translations" corresponds in the covering $SL(2,C)$ description of the Lorentz group to the matrix

$$\begin{pmatrix} 1 & a_1 + ia_2 \\ 0 & 1 \end{pmatrix} \tag{12}$$

$$G_i = \frac{1}{\sqrt{2}}(M_{it} + M_{iz}),\ i = x, y$$

where in the second line we have written the generators in terms of the Lorentz-generators $M_{\mu\nu}$ so that the above interpretation in terms of a combination of boosts and rotations is obvious. The velocity parameter of the Galilei transformations in the $x_\perp$-x_+ variables in terms of the $a_i, i = 1, 2$ can be obtained from the $SL(2,C)$ formalism.

The important role of this Galilei group in the partial return to quantum mechanics as *the* main simplifying aspect of LFH cannot be overestimated. In the algebraic LFH see below) the lightray translation and dilation set the longitudinal net structure whereas the Galilei transformations are indispensable for re-creating the transverse localization structure which was lost by the global identification of the wedge- with the LFH-algebra (6).

Note that (as in the 2-dimensional example) the physical particle spectrum is not yet determined by these 7 generators; one rather needs to know the action of the x_- lightlike generator normal to the lightfront in order to obtain the physical mass operator of the ambient theory (which is part of inverse LFH).

For free massless Bose fields (as for certain more general chiral fields) there is no problem to cast the above pointlike formalism into the setting of bounded operator algebras by either using the spectral theory of selfadjoint unbounded operators or by Weyl-like exponentiation (see below). For free Fermi fields the test function smearing suffices to convert them into bounded operators; in this case one can elevate the above observations on smeared fields directly into properties of spacetime-indexed nets of operator algebras.

There exists however a disappointing limitation for this lightfront restriction formalism for pointlike interacting fields which forces one to adopt the operator-algebraic method. Namely this restriction method suffers from the same shortcomings which already affected the canonical equal time formalism: with the exception of some superrenormalizable interactions in low dimensional spacetime, there are no interacting theories which permit a restriction to the lightfront or to equal times. In particular fields of strictly renormalizable type (to which all interacting Lagrangian fields used in d=1+3 particle physics belong) are outside the range of the above restriction formalism. In fact a necessary condition for a restriction to exist can be abstracted from the two-point function and consists in the finiteness of the wave function renormalization constants Z which in the non-perturbative setting amounts to the convergence of the following integral over the well-known Kallen-Lehmann spectral function

$$Z \simeq \int \rho(\kappa^2) d\kappa^2 < \infty \tag{13}$$

$$\langle A(x)A(0)\rangle = \int i\Delta^{(+)}(x,\kappa^2)\rho(\kappa^2)d\kappa^2$$

The operator algebra approach overcomes this restriction by bypassing the short-distance problem of pointlike field "coordinatization". It uses the *causal shadow property* i.e. the requirement that the (weakly closed) operator algebra associated with a simply connected convex spacetime region is equal to the algebra of its causal completion[5] (causal shadow). In this way certain operator subalgebras in the net of W become identified with some subalgebras localized on the horizon. For example a 3-dim. lightlike semiinfinite strip within the horizon H_W casts a causal shadow into W which is simply the 4-dim. causal completion (the 4-dim. slab which this 3-dim. strip cuts into the ambient space). The largest such region is of course the causal horizon H_W itself, whose causal shadow is W (6). The net on the lightfront also contains regions with a finite longitudinal extension which do not cast causal shadows (independent of whether their transverse extension is compact or reaches to infinity) because they are already identical to their causal shadows. The algebras of those regions do not correspond to algebras in the ambient net; they have to be constructed by modular methods (using modular inclusions and modular intersections) which will be presented in the next section.

The fact that the canonical equal time and lightfront restriction approach suffer the same limitations for their Kallen-Lehmann spectral function does not mean that their algebraic structure is similar. In fact it is well-known that there are no pointlike generators for canonical equal time algebras for fields with infinite wave function renormalization, whereas (as will be seen in the sequel) lightfront algebras permit a description in terms of (generalized) chiral fields which allow for arbitrary noncanonical short distance behavior. But the failure of the restriction procedure prevents their *direct* construction, rather one needs the algebraic detour of LFH to get to those fields (in their role as locally coordinatizing the algebras).

Despite the different nature of the modular LFH method in the formulation of an algebraic lightfront holography of the next section, most of the structural results turn out to be analogous to those of the restriction method in this section. Considered as a QFT in its own right, the LFH has some unusual properties which are a consequence of the fact that the lightfront does not belong to the family of globally hyperbolic spacetime manifolds to which one usually restricts field theoretic considerations; but precisely this speciality makes them useful auxiliary tools in QFT. Not only do longitudinal compact regions not cast any causal shadows into the ambient Minkowski spacetime, there are even no such shadows (and a related Cauchy propagation) which extend the region *inside* the lightfront. The related transverse quantum mechanical behavior without vacuum polarization has the consequence that algebras, whose transverse localizations do not overlap admit a tensor factorization (just like the inside/outside tensor factorization in Schroedinger theory in the multiplicative second quantization formulation[6]with no vacuum entanglement). As a consequence, the application of such subalgebras with finite transverse extension to the vacuum does not lead to dense subspaces of the Fock space, rather their closures defines genuinely smaller subspaces (breakdown of the Reeh-Schlieder property). In fact this LFH is the only known mechanism by which one encounters a partial return to quantum mechanics within the setting of QFT without invoking any nonrelativistic approximation, as will be shown in the sequel.

The crucial operator-algebraic property which is responsible for this somewhat unexpected state of affairs is the existence of positive lightlike translations[7] $U_{e_+}(a)$ in (two-sided) lightlike strip (whose causal

[5]The extension by spacelike "caps" of a timelike interval is well-known to be already a consequence of the spectrum condition and spacelike commutativity.

[6]In the standard quantum mechanical formulation the statistical independence between inside/outside would corresponds to the additive decomposition of the corresponding Hilbert spaces.

[7]This argument is analogous to the proof that certain algebras have translational invariant centers [10] as in the case of the proof of the factorial property of wedge algebras [11]. The loss of correlations results from the fact that lightlike translation act in a two-sided way on strips [2].

shadows are lightlike slabs) algebra $\mathcal{A}(l\text{-}strip)$. Let $A \in \mathcal{A}(l\text{-}strip)$ and $A' \in \mathcal{A}(l\text{-}strip)'$. Since $U_{e_+}(a)$ acts on both the algebra and its commutant (associated with the complement of the strip within the lightfront) as an automorphism and leaves the vacuum invariant, we obtain for the vacuum expectation values

$$\left\langle 0 \left| AU_{e_+}(a)A' \right| 0 \right\rangle = \left\langle 0 \left| A'U_{e_+}(a)^* A \right| 0 \right\rangle \tag{14}$$

But according to the positivity of the translations this requires the function to be a boundary value of an analytic function which is holomorphic in the upper as well as the lower halfplane. Due to the boundedness the application of Liouville's theorem yields the constancy in a. The cluster property (which in the weak form as it is needed here also applies to infinite lightlike separations) then leads to

$$\langle 0 \left| AA' \right| 0 \rangle = \langle 0 \left| A \right| 0 \rangle \langle 0 \left| A' \right| 0 \rangle \tag{15}$$

which is the desired tensor factorization without entanglement of the vacuum. By successive application of this argument to a strip-subalgebra of the commutant, the lightfront algebra can be made to factorize into an arbitrary number of nonoverlapping strip algebras. In fact for those lightfront algebras which are associated with the restriction of free field, the two-sided strip algebras can easily be seen to be type I_∞ factors i.e. the full lightfront algebra (which is equal to the global ambient algebra) tensor-factorizes into full strip algebras

$$\begin{aligned} \mathcal{A}(LF) &= \mathcal{A}(M^{(3,1)}) = B(\mathcal{H}) \\ &= \bigotimes_i \mathcal{A}(LF_i) = \bigotimes_i B(H_i) \\ \mathcal{H} &= \bigotimes_i \mathcal{H}_i \end{aligned} \tag{16}$$

Algebras with smaller longitudinal localizations are imbedded as unique hyperfinite type III_1 factors in suitable two-sided strip algebras $B(H_i)$.

Subalgebras with a semi-infinite or finite longitudinal extension which are associated with non-overlapping strip algebras inherit this factorization; in fact they fulfill a Reeh-Schlieder theorem within an appropriate factor space. The transverse correlation-free factorization for arbitrarily small strips in lightray direction suggests that the lightfront algebras are generated in terms of pointlike fields of the form (10) where in the interacting case the zero mass free fields should be replaced by generalized chiral fields with (half)integer scale dimensions (see next section). The arguments leading to the transverse factorization are not affected by interactions and therefore it is helpful to collect the result in form of a structural theorem of operator (von Neumann) algebras

Theorem 1 *A subalgebra $\mathcal{A}$ of $B(H)$ admitting a two-sided lightlike translation with positive generator is of type I, i.e. it tensor-factorizes $B(H)$ as $B(H) = \mathcal{A} \otimes \mathcal{A}'$*

The two-sidedness of the lightlike positive energy translations is important in the above argument; if they act only one-sided (i.e. as an algebraic endomorphism) as e.g. in the case of the wedge algebras, one can merely conclude that the center is translation-invariant and agrees with the center of the full algebra; in this case there is no tensor-factorization and the algebras turn out to be of the same kind as typical sharp localized algebras, namely hyperfinite type III_1 factors[8][11].

[8]A detailed understanding of such operator algebras beyond the fact that they have very different properties (no minimal projectors corresponding to optimal measurements, no pure states) from quantum mechanical algebras is not required in this paper.

With these remarks which (as shown in the next section) continue to apply in the presence of interactions, we prepared the ground in favor of area laws for transverse additive quantities (as e.g. a would be localization-entropy) associated with a horizon. Later this problem will be considered in more detail.

The restriction of free fields to a causal horizon can also be carried out for the more interesting case of the lower rotational symmetric causal horizon of a (without loss of generality) symmetric double cone. Again the analogous restriction limit (with the lower apex placed at the origin, $r_+ = t + r$ plays the role of x_+) maintains the creation and annihilation operator structure and the Hilbert space and also leads to a conformal invariant limit from which the original massive field may be reconstructed via the application of suitable symmetries which lead away from the horizon into the ambient spacetime. In this case there is however no geometric modular theory (no Killing vector) for the massive free theory; nevertheless the massless restriction to the horizon acquires a geometric modular group in form of a subgroup of a double cone-preserving conformal transformation [12]. Similar to the LF situation, the algebra on the lower horizon $H_{\mathcal{C}}$ is equal to the double cone algebra (but with different subnet structure)

$$\mathcal{A}(H_{\mathcal{C}}) = \mathcal{A}(\mathcal{C}) \tag{17}$$

This phenomenon, which leads to an enlarged symmetry in the same Hilbert space, has been termed "symmetry-enhancement" [6]. As a result of the equality of the two algebras and the shared Hilbert space, one could also say that a diffuse acting modular symmetry becomes geometric (a diffeomorphism) on the horizon. Unfortunately it is presently not known how to derive this result for double cones in the presence of interactions, when the method of restricting pointlike fields to the horizon breaks down [9].

3 Algebraic holography and modular localization

The algebraic construction for interacting theories with trans-canonical Kallen-Lehmann spectral functions starts from the position of a wedge algebra $\mathcal{A}(W)$ within the algebra of all operators in Fock space $B(\mathcal{H})$ and the action of the Poincaré group on $\mathcal{A}(W)$. It is based on the modular theory of operator algebras and adds two new concepts: *modular inclusions* and *modular intersections*. The intuitive idea of LFH consists in realizing that the causal shadow property identifies certain semiinfinite algebras on the horizon with their ambient 4-dimensional causal shadows; starting from those one may construct a full net of compactly localized subalgebras on the horizon in terms of relative commutants and intersections among those algebras. Modular theory in the form of inclusions and intersections is a mathematical tool which makes this intuitive picture precise. Since these modular concepts have already received attention in the recent literature on algebraic QFT, we will limit ourselves to remind the reader of the relevant definitions and theorems (with a formulation which suits our purpose) before commenting on them and applying them to the lightfront holography.

Definition 2 *(Wiesbrock, Borchers [11]) An inclusion of operator algebras $(\mathcal{A} \subset \mathcal{B}, \Omega)$ is "modular" if $(\mathcal{A},\Omega)$, $(\mathcal{B},\Omega)$ are standard and $\Delta_{\mathcal{B}}^{it}$ acts (for t<0 by convention) as a compression on $\mathcal{A}$*

$$Ad\Delta_{\mathcal{B}}^{it}\mathcal{A} \subset \mathcal{A} \tag{18}$$

[9] There is presently no formulation of modular inclusions (see next section) for a diffuse acting modular group which restricts to the smaller algebra as a diffuse compression of the smaller region.

A modular inclusion is standard if the relative commutant ($\mathcal{A}' \cap \mathcal{B}, \Omega$) is standard. If the sign of t for the compression is opposite it is advisable to add this sign and talk about a ±modular inclusion.

Modular inclusions are different from the better known inclusions which arise in the DHR superselection theory [13] associated with the origin of internal symmetries in quantum field theory. The latter are characterized by the fact that they possess conditional expectations [14]. The prototype of a conditional expectation in the conventional formulation of QFT in terms of charge carrying fields is the projection in terms of averaging over the compact internal symmetry group with its normalized Haar measure. If $U(g)$ denotes the representation of the internal symmetry group we have

$$\mathcal{A} = \int AdU(g)\mathcal{F}d\mu(g) \tag{19}$$
$$E : \mathcal{F} \xrightarrow{\mu} \mathcal{A}$$

i.e. the conditional expectation E projects the (charged) field algebra $\mathcal{F}$ onto the (neutral) observable algebra $\mathcal{A}$.

Modular inclusions have no conditional expectations. This is the consequence of a theorem of Takesaki [15] which states that the existence of a conditional expectation for an inclusion between two noncommutative[10] algebras (in standard position with respect to the same vector) is equivalent to the modular group of the smaller being the restriction of that of the bigger algebra. Since a genuine one-sided modular compression (endomorphism) excludes the modular group of the smaller being the restriction of that of the bigger algebra, there can be no projection E in this case. The modular inclusion situation may be considered as a *generalization* of the situation covered by the Takesaki theorem.

The main aim of modular inclusion is to generate spacetime symmetry and nets of spacetime indexed algebras which are covariant under these symmetries. From the two modular groups $\Delta_{\mathcal{B}}^{it}, \Delta_{\mathcal{A}}^{it}$ of a modular inclusion one can form the translation-dilation group with the commutation relation $\Delta_{\mathcal{B}}^{it}U(a) = U(e^{-2\pi t}a)\Delta_{\mathcal{B}}^{it}$ and a system of local algebras obtained by applying these symmetries to the relative commutant $\mathcal{A}' \cap \mathcal{B}$ which may be combined into a possibly new algebra $\mathcal{C}$

$$\mathcal{C} \equiv \overline{\bigcup_t Ad\Delta_{\mathcal{B}}^{it}(\mathcal{A}' \cap \mathcal{B})} \tag{20}$$

In general $H_{\mathcal{C}} \equiv \overline{\mathcal{C}\Omega}\mathcal{C} \subset H_{\mathcal{B}} = \mathcal{B}\otimes \equiv H$, and whereas the modular groups $\Delta_{\mathcal{B}}^{it}, \Delta_{\mathcal{A}}^{it}$ of the inclusion $\mathcal{A} \subset \mathcal{B}$ are different, the $\mathcal{C} \subset \mathcal{B}$ inclusion leads to a Takesaki situation with $\Delta_{\mathcal{C}}^{it} = \Delta_{\mathcal{B}}^{it}|_{H_{\mathcal{C}}}$ with the conditional expectation being $E : \mathcal{B} \rightarrow \mathcal{C} = P\mathcal{B}P$, $H_{\mathcal{C}} = H_{\mathcal{B}}$. If the inclusion is however standard (which means $H_{\mathcal{C}} = H$), the equality $\mathcal{C} = \mathcal{B}$ follows. In that case a modular inclusion gives rise to a chiral structure on $\mathcal{B}$ thanks to the following theorem.

Theorem 3 *(Guido,Longo and Wiesbrock [16]) Standard modular inclusions are in correspondence with strongly additive chiral AQFT*

Here chiral AQFT is any net of local algebras indexed by the intervals on a line with a Moebius-invariant vacuum vector and the terminology *strongly additive* refers to the fact that the removal of a point from an interval does not change the algebra i.e. the von Neumann algebra generated by the two

[10] In the commutative case there are no restrictions. Examples of commutative conditional expectations are the Kadanoff-Wilson renormalization-group decimation procedures in the Euclidean setting of QFT.

pieces is still the original algebra. This raises the question of whether the present use of the word chiral is the same as that in the standard literature where chiral refers to the apparently more restrictive situation of the existence of an energy-momentum tensor which generates the diffeomorphisms of a circle with the Moebius group being the maximal symmetry group of the vacuum. However there are arguments that not only the Moebius transformations (whose modular origin has been known for some time [11]) but also the diffeomorphisms beyond are of a general modular origin (for multi-local algebras with respect to states which differ from the vacuum [17][18]) . Hence the Witt-Virasoro algebra formed by the infinitesimal generators of $Diff(S^1)$ (which is the hallmark of standard chiral models and follows from the existence of an energy-momentum tensor) appears to be shared by the more general looking algebraic definition. This is helpful in connection with LFH where generalized chiral theories arise from higher dimensional QFT for which the chiral energy-momentum tensor is void of any direct physical meaning. .

First we adapt the abstract theorem to our concrete case of a wedge algebra in a massive interacting QFT in d=1+1 spacetime dimensions.

$$\begin{aligned} \mathcal{B} &= \mathcal{A}(W) \\ \mathcal{A} &= U(e_+)\mathcal{A}(W)U^*(e_+),\ e_+ = (1,1) \\ &\equiv \mathcal{A}(W_{e_+}) \end{aligned} \tag{21}$$

As Wiesbrock has shown [19], the modular nature of the inclusion is a consequence of the *modular covariance* for wedge algebras (the Bisognano-Wichmann property [13]) together with this positivity of the generator of the lightlike translation $U(e_+)$. Geometrically the relative commutator

$$\mathcal{A}(W_{e_+})' \cap \mathcal{A}(W) \equiv \mathcal{A}(I(0,1)) \tag{22}$$

is by causality localized in the upper horizontal interval $(0,1)$. The standardness of this inclusion then leads to a chiral conformal AQFT, i.e. a net (more precisely a pre-cosheaf [16])

$$\begin{aligned} I &\rightarrow \mathcal{A}(I),\ I \subset S^1 \\ \mathcal{A}(R_+) &= \overline{\cup_t Ad\Delta^{it}\mathcal{A}(I(0,1))} \\ \mathcal{A}(R) &= \mathcal{A}(R_+) \vee J\mathcal{A}(R_+)J \end{aligned} \tag{23}$$

on which the Moebius group (which preserves the vacuum vector) acts. With the help of the external (i.e. non-Moebius) automorphism on $\mathcal{A}(R)$ implemented by the opposite lightray translation $U_-(a)$, we are able to return from the chiral net on the right upper horizon to the original 2-dim. net on W. If we call the transition from the d=1+1 original net via the modular inclusion of wedge algebras to the $\mathcal{A}(R)$ net the *holographic projection*, then the reconstruction of the d=1+1 theory from its holographic projection together with the opposite lightray translation $U_-(a)$ (which acts as a kind of positive spectrum Hamiltonian) should be called the *holographic inversion*. Since chiral theories are simpler than massive d=1+1 models, the gain by looking first at holographic projections should be obvious. In fact the kind of chiral theory which is a candidate for such a start is restricted to models with generating fields with (half)integer scaling dimension which are closed under commutation i.e. where the delta function terms and their derivatives are multiplied with field from the generating set. Such models are "Lie-fields" in case of a finite number of generators have been called "W-algebras" (here used in the wider sense that current algebras are also included).

In analogy to the free case of the previous section the $U_{-}(a)$ translation does not enlarge the Hilbert space nor the global algebra on the horizon. But without the knowledge of its action it would not be possible to identify the physical mass via the mass operator $P_{+}P_{-}$ in that Hilbert space, nor would one be able to recover the spacetime interpretation (e. i. the physical content) of the original ambient theory. If the canonical quantization (or the equivalent functional integral representation approach) would be an ultraviolet consistent approach beyond canonical short distance behavior, then the lightfront data acted upon by P_{-} would be analogous to the canonical data acted upon by the Hamiltonian. According to the remarks in the previous section based on the Kallen-Lehmann representation, the canonical structure gets lost in the process of renormalization, whereas the LFH-structure is consistent with chiral fields of arbitrary high dimensions. This stability against trans-canonical ultraviolet behavior favors the LFH approach as compared to canonical method. Another potentially helpful aspect of the constructive use of LFH is the fact that LFH data are capable to contain more specific dynamical information since the set of chiral algebras is much richer than the set of canonical data (which for given mass and spin is essentially unique). It is generally believed that any change of the cut separating dynamics from kinematics (which by definition the latter contains the already well-understood aspects) towards the enlargement of the latter is helpful in the understanding and constructions of models of QFT.

An interesting family of models for which a study of these holographic aspects seems to be well in reach, are the d=1+1 factorizing models for which some modular localization properties are already known [20]. Another advantage of having a richer kinematical side may be that certain rather general structural properties may become more accessible. In fact in the next section we will argue that the area proportionality of localization entropy associated with the wedge horizon in d=1+3 QFT is among those properties.

For the extension of holographic projection to higher dimensional theory one needs one more mathematical definition and a related theorem about *modular intersections.*

Definition 4 *([21]) A (±) modular intersection is defined in terms of two standard pairs $(\mathcal{N},\Omega), (\mathcal{M},\Omega)$ whose intersection is also standard $(\mathcal{N}\cap\mathcal{M},\Omega)$ and which in addition fulfill*

$$\begin{aligned}
&((\mathcal{N}\cap\mathcal{M})\subset\mathcal{N},\otimes)\ \ and\ \ ((\mathcal{N}\cap\mathcal{M})\subset\mathcal{M},\otimes)\ \ is\ \ \pm\,\mathrm{mod}\,ular \qquad (24)\\
&J_{\mathcal{N}}(\lim_{t\to\mp\infty}\Delta_{\mathcal{N}}^{it}\Delta_{\mathcal{M}}^{-it})J_{\mathcal{N}}=\lim_{t\to\mp\infty}\Delta_{\mathcal{M}}^{it}\Delta_{\mathcal{N}}^{-it}=J_{\mathcal{M}}\lim_{t\to\mp\infty}\Delta_{\mathcal{N}}^{it}\Delta_{\mathcal{M}}^{-it})J_{\mathcal{M}}
\end{aligned}$$

All limits in the modular setting are to be understood in the sense of strong convergence on Hilbert space vectors.

We will now explain how this definition is used in LFH in order to achieve a transverse localization on the lightfront. Setting Ω =vacuum, $\mathcal{M}=\mathcal{A}(W)$ and $\mathcal{N}=AdU(\Lambda_{e_{+}}(1))\mathcal{M}$ where $\Lambda_{e_{+}}(a)$ denotes a "translation" in the Wigner little group which fixes the lightray vector e_{+} (a transverse Galilei transformation in the lightfront according to the previous section), we first check the prerequisites of the definition. The modularity of the inclusion follows again from the geometric properties of the action of the modular group of the bigger algebra as a compression on the smaller, and since the limit in the second line is (24) is nothing else but $U(\Lambda_{e_{+}}(1))$ and the commutation relation with $J_{\mathcal{N},\mathcal{M}}$ with $U(\Lambda_{e_{+}}(1))$ in the last line are geometric relation in the extended Lorentz group, the intersection property holds in our case. In d=3+1 there are two independent "translations" which maintain the given lightray and implement a

Galilei transformation inside the lightfront. As in the free field reduction formalism of the previous section, there is a seven-parametric subgroup of the Poincaré group which is a symmetry of the LFH. Since the Galilei transformations change the transverse directions inside the lightfront, they tilt the transverse strips in such a way that the intersection of the original with a transformed strip is a compact 3-dim. region on the lightfront. These intersections define a net structure on the lightfront containing arbitrarily small regions in the longitudinal as well as transverse sense. Whereas it appears relatively straightforward to extend a given bosonic/fermionic chiral theory by transverse quantum mechanical degrees of freedom in a covariant way with respect to the seven-parametric symmetry group, the remaining three symmetries, notably the Hamiltonian-like lightray translation away from the lightfront (which are important to re-install the particle physics interpretation of the ambient theory), pose the more challenging task of holographic inversion. It is presently not even clear whether all generalized chiral theories appear as images in LFH. From the experience with the canonical Hamiltonian formulation one would expect that this step is highly non-unique, or to phrase it the other way around that the holographic projection is a many-to-one universality class relation.

The notion of a universality class projections is well-known in connection with the scale and conformally covariant short distance limit which is the basis of critical phenomena. The holographic projection classes constitute a quite different universality class relation between (massive) theories in any spacetime dimensions $d \geq 1+1$ and chiral theories in which only (half)integer dimensions appear. Even if the higher dimensional interacting models do possess conventional pointlike field generators, there will be in general no direct relation between these fields and those which generate the chiral algebras. The holographic relation rather involves a radical spacetime reprocessing which presently can only be formulated in terms of changing the spacetime net structure; there seems to be no way to express such a radical change in terms of a (necessarily) nonlocal formula which relates pointlike fields.

Since the theorem about the transverse tensor factorization of lightlike strips and their causal shadows (lightlike slabs) of the previous section follows from a general theorem about strip algebras with a two-sided action of a positive lightlike translation which made no reference to free fields, it remains valid in the interacting situation.

It is reasonable to ask whether these factorizing chiral strip algebras really do permit the introduction of pointlike generating fields. The results in [22][23] on standard chiral theories suggest strongly that group-representation methods may also be sufficient for their construction in the present case. In case of a finite number of generating fields these fields should be of the form of generalized W-algebras (Lie-fields) i.e. their exists a finite collection of generating fields $A_{LF}^{(i)}(x_+, x_\perp), i = 1, 2, ...$with the following (anti)commutation relation

$$\left[A_{LF}^{(i)}(x_+, x_\perp), A_{LF}^{(j)}(x'_+, x'_\perp)\right] = \left\{\sum_{k=0}^{n(i,j)} B_{LF,k}^{(i,j)}(x_+, x_\perp)\delta^{(k)}(x_+ - x'_+)\right\}\delta(x_\perp - x'_\perp) \tag{25}$$

where the sum extends over chiral δ-functions and their derivatives multiplied with fields B which consist of linear combinations of generating fields A with the same scale dimension which together with the dimension carried by the (derivatives of) δ-functions match the balance of scale dimension of the left hand side. For readers who are familiar with chiral conformal QFT, these formula are straightforward transverse extensions of W-algebra commutation with covariance properties under the seven-dimensional symmetry group of lightfront. The longitudinal compactification adds the rigid rotation symmetry of the circle. As in the case of standard W-algebras this structure is easily extended to include Fermion fields.

The quantum mechanical behavior in the transverse direction is described by the common transverse δ-function. Besides the covariance under the seven-parametric subgroup of the Poincaré group, the expectation values fulfill the strong factorization

$$\langle CD \rangle = \langle C \rangle \langle D \rangle \tag{26}$$

where C, D are products of generating fields so that the transverse coordinates in the C-cluster are disjoint from those of the D-cluster. Another way to describe lightfront fields which makes the quantum mechanical transverse structure more explicit would be to use the notion of wave functions on transverse coordinates which are chiral field-valued.

It should be mentioned that modular intersections which we used to establish the Galilei invariance of the transverse quantum mechanics associated with LFH also played an important role in the construction of 3- and higher- dimensional AQFT starting from a finite set of wedge algebras [35].

4 Area density of horizon-associated entropy

The use of the LFH for a coarse classification of higher dimensional theories with the aim of their construction via holographic inversion is an ambitious new program. A more modest goal would consists in trying to pinpoint general properties of the LFH projection which pertain to the LFH universality classes. In the sequel we will argue that the area proportionality of *localization entropy* associated with the horizon of a wedge is such a property.

It has been known for a long time that the restriction of the vacuum state to the wedge algebra leads to a thermal state with a fixed Hawking temperature. To place this formal mathematical observation on a physical footing, Unruh [31] has interpreted this effect as a physical phenomenon seen by a uniformly accelerated observer with a fixed acceleration (creating the causal wedge horizon). The Hawking-Unruh temperature arises from the fact that an Unruh observer is immersed in a KMS thermal state for which the wedge-preserving Lorentz-boost defines the dynamics (the "Hamiltonian"). There are however obstacles against directly associating a notion of entropy with this thermal situation, the most obvious being the fact that the Lorentz boost is not a trace-class operator. In this section we will argue that the LFH method combined with the so-called split property applied to the chiral theories on the lightfront allows to extract a (relative or unnormalized) area density of localization entropy. Our arguments depend on the assumption that the (logarithmic) increase of vacuum polarization at the boundary of localization in chiral theories in the limit of sharp boundaries (vanishing splitting distance) is universal, so that different chiral models only lead to different numerical factors which determine finite ratios. Only in this case an assignment of an additive measure of the cardinality of degrees of freedom associated with the vacuum state restricted to a sharp localizes spacetime region as a wedge or its horizon is conceivable. The determination of a *normalized* area densities would then hinge on the validity of fundamental thermodynamic laws involving localization entropy, an issue which is outside the scope of this paper.

Whereas our conclusions are still preliminary, the ideas by which we try to relate transverse area density of localization entropy on the lightfront with vacuum polarization at the boundaries of localization are interesting in their own right and may turn out to be useful in more profound future studies of this problem.

It is interesting to recall that Heisenberg discovered vacuum fluctuations through surface effects. Phrased in a more modern setting of partial charges defined by smeared Noether currents

$$Q(f_{R,\varepsilon}f_T) = \int j_0(x) f_{R,\varepsilon}(\vec{x}) f_T(x_0) d^4x \tag{27}$$

where the spatial test function equals one inside a sphere of radius R and vanishes outside $R+\varepsilon$ and the time smearing is a positive test function symmetric around $x_0 = 0$ and with total integral equal to one. This partial charge has the same commutation relation with other operators as the total charge as long as these operators are localized inside a (origin-centered) double cone of size R. By use of the Kallen-Lehmann representation one can calculate the charge fluctuation

$$\left\langle \Omega \left| Q(f_{R,\varepsilon}f_T)^2 \right| \Omega \right\rangle = \left\| Q(f_{R,\varepsilon}f_T)\Omega \right\|^2 \sim R^2 c(\varepsilon) \tag{28}$$

they are (for small ε) proportional to the surface of the sphere with a numerical coefficient which diverges as a inverse power in ε. This is not in contradiction with the expected zero value of the total charge in the vacuum state because the convergence for $R \to \infty$ is in the sense of weak convergence[11].

In the algebraic setting the consequences of vacuum polarization have more dramatic structural manifestations. Their omnipresence even in the absence of interactions is behind the loss of such quantum mechanical properties as the existence of minimal projectors and even that of pure states for the sharply localized algebras which are the basic objects of the algebraic approach. As already mentioned before in terms of von Neumann types they are isomorphic to the unique hyperfinite type III_1 von Neumann factor instead of the unique type I_∞ factor isomorphic to the algebra of all operators $B(H)$ in an infinite dimensional Hilbert space H. In the multiplicative second quantization formulation of Schroedinger quantum mechanics the spatial division into an algebra inside and outside a spatial box would lead to a tensor factorization of the two algebras. Interactions would correlate (entangle) the inside with the outside of the box but not affect the tensor product factorization. The nonrelativistic vacuum would remain a (nonentangled) tensor product of the in and outside vacuum even in the presence of interactions. Hence the restriction of the global vacuum to the inside of the box would never create an impure state with thermal aspects. The situation changes radically in the presence of field theoretic vacuum fluctuations. In that case not only does the vacuum become an entangled state in a tensor product description, but the very tensor product structure, which is the basis of the entanglement, gets lost. Namely the global algebra is not representable as a tensor product of the local algebra $\mathcal{A}(\mathcal{C})$ (with say a double cone $\mathcal{C}$ being the analog of a nonrelativistic box) and its commutant $\mathcal{A}(\mathcal{C})'$ (equal to the causally disjoint algebra $\mathcal{A}(\mathcal{C}')$ according to Haag duality), although it is generated by both commuting factors: $B(H) = \mathcal{A}(\mathcal{C}) \vee \mathcal{A}(\mathcal{C}')$. This radical change is consistent with the global vacuum turning into a KMS state at the Hawking temperature upon restriction from the global to a local algebra; however the von Neumann entropy requires type I and cannot be defined in such a situation. In order to obtain a tensor factorization one invokes the "split property" (for a rather complete bibliography see [13]) which is a kind of algebraic analog of the above test function transition region of size ε and intuitively speaking corresponds to a "fuzzy" surface of a spacetime localization region. Algebraically one takes instead of sharply localized algebra $\mathcal{A}(\mathcal{C})$ a fuzzy-localized type I algebra whose localization is between $\mathcal{C}$ and the by ε bigger double cone $\mathcal{C}_\varepsilon$. Let us first recall the general algebraic definition without reference to our geometric setting.

[11] One can also have strong convergence at the expense of a time smearing which depends on the spatial smearing [24].

Definition 5 *An inclusion $\mathcal{A} \subset \mathcal{B}$ is called split if there exists an intermediate type I factor $\mathcal{N}$: $\mathcal{A} \subset \mathcal{N} \subset \mathcal{B}$. This yields the following tensor factorization:* $B(H) = \mathcal{N} \otimes \mathcal{N}'$, $\mathcal{A} \to \mathcal{A} \otimes 1$, $\mathcal{B}' \to 1 \otimes \mathcal{B}'$

We remind the reader that in theories with reasonable phase space properties there exists a canonical way of constructing for a standard inclusion (i.e. an inclusion for which $(\mathcal{A}' \cap \mathcal{B}, \Omega)$ is also standard) a type I factor [13] which is explicitly given by the formula

$$\mathcal{N} = \mathcal{A}J\mathcal{A}J = \mathcal{B}J\mathcal{B}J \tag{29}$$
$$with\ J \equiv J_{(\mathcal{A}' \cap \mathcal{B}, \Omega)}$$

The verification of these structural properties is not difficult [26], but the explicit computation of a modular conjugation J which belongs to not simply connected or disconnected localization region has not been possible up to now [12]. The fact that $\mathcal{N}$ is type I implies that the modular group of $(\mathcal{N}, \Omega)$ is implemented by the unitary group $h^{it} = e^{it\mathbf{H}_{\mathcal{N}}}$ with a generator $\mathbf{H}_{\mathcal{N}}$ which is affiliated with the algebra $\mathcal{N}$

$$Ad\Delta^{it}_{(\mathcal{N},\Omega)}\mathcal{N} = Adh^{it}\mathcal{N} \tag{30}$$
$$\Delta^{it}_{(\mathcal{N},\Omega)}H = h^{it}H_{\mathcal{N}} \otimes h^{-it}H_{\mathcal{N}'},\ H = H_{\mathcal{N}} \otimes H_{\mathcal{N}'}$$

where the positive operator $h = h^{it}|_{t=-i}$ represents the Connes-Radon-Nikodym derivative of the vacuum state $\omega_{\Omega}(\cdot)$ restricted to $\mathcal{N}$ with respect to the trace (tracial weight) on $\mathcal{N}$. By construction this operator has discrete spectrum and finite trace $tr\, h < \infty$ (which by normalization $\omega_{\Omega}(\cdot) = tr(h\cdot)$ can be set equal to one). The finiteness of the entropy

$$E_{(\mathcal{N},\Omega)} \equiv -tr|_{H_{\mathcal{N}}} hlnh \tag{31}$$

requires in addition the absence of an infrared accumulation of too many eigenvalues near zero.

Returning to the above geometric case, $\mathcal{A}$ and $\mathcal{B}$ should be identified as $\mathcal{A} = \mathcal{A}(\mathcal{C}), \mathcal{B} = \mathcal{A}(\mathcal{C}_{\varepsilon})$. The main interest would be the study of the limiting $\varepsilon \to 0$ behavior of the localization entropy $E_{(\mathcal{N},\Omega)}$. It is only in this limit that one would expect the localization entropy to become independent of the choice of the splitting type I subfactor $\mathcal{N}$ (similar to the dependence of the partial charge on the $f_{R,\varepsilon}(\vec{x})$ test function dependence in the transition region). In the algebraic setting one can actually avoid this problem of arbitrariness in the splitting by realizing that all different choices of $\mathcal{N}$ yield the same product state

$$\omega_p(AB') = \omega(A)\omega(B'),\ A \in \mathcal{A}, B' \in \mathcal{B}' \tag{32}$$

This together with Kosaki's elegant variational formula [25][28] for Araki's relative entropy $E(\omega_p, \omega)$ of the original vacuum state ω with respect to the product vacuum would get rid of the dependence of the special kind of tensor product implementation even before going to the limit. In either formulation there have been attempts to compute such relative localization entropies, but even in the case of algebras associated with free fields the problem has turned out to be quite intractable [27][28]

In the following we will argue that the issue of localization entropy for the wedge can be significantly simplified by LFH. Although the wedge algebra is identical to that of its horizon, the latter offers a better

[12] There has been some recent progress in the understanding of modular objects associated with double interval chiral algebras which may turn out to be helpful in future attempts. [17][18].

spacetime arrangement of degrees of freedom for the computation of localization entropy. The reason is obvious since the transverse symmetry together with strong factorization property of the vacuum in transverse direction permit the reduction of the entropy problem to that of an area density (entropy per unit volume of the edge of the wedge) which is determined by the chiral theory on the lightray. One then hopes to profit from the simplicity of chiral theories.

It is well-known that the split property of chiral theories is a consequence of the nuclearity property which in turn follows from the convergence of the partition function [29]

$$tre^{-\beta L_0} < \infty \tag{33}$$

where L_0 is the rigid rotation operator on the compactified light ray. Since the transverse symmetry (which would lead to an infinite degeneracy of these eigenvalues) has been taken out, this condition does not seem to be unduly restrictive. After the transverse reduction the horizon now corresponds to the right halfline or in the compactified circular description to the upper semi-circle. The chiral split inclusion to be studied is now $\mathcal{A} \subset \mathcal{B}$ with $\mathcal{A} = \mathcal{A}(I(0+\varepsilon, \pi-\varepsilon))$, $\mathcal{B} = \mathcal{A}(I(0,\pi))$. The practical problem is then to find an implementation of the split isomorphism Φ

$$\mathcal{A} \vee \mathcal{B}' \overset{\Phi}{\simeq} \mathcal{A} \otimes \mathcal{B}' \tag{34}$$

which is computationally manageable. It turns out that implementation of Φ with the help of the “flip trick” used in [30] is useful. It consists in implementing Φ in the duplicated tensor representation space $H \otimes H$ with the duplicated vacuum $\Omega \otimes \Omega$. Assume for simplicity that the algebra is generated by one pointlike field ψ and with $\psi_1 \equiv \psi \otimes \mathbf{1}$, $\psi_2 \equiv \mathbf{1} \otimes \psi$ the implementation of Φ reads

$$\Phi(\psi(x)) = \begin{cases} \psi_1(x), \ x \in I(0+\varepsilon, \pi-\varepsilon) \\ \psi_2(x), \ x \in I(\pi, 2\pi) \end{cases}$$

If we now view $\psi_i, i = 1.2$ formally as a $SO(2) \simeq U(1)$ charge doublet (each living in a separate tensor factor of $H \otimes H$), then we may implement Φ in the spirit of the well-known Noether current formalism

$$\Phi(\psi_1(x)) = U(f)\psi_1(x)U(f)^*, \ U(f) = e^{ij(f)} \tag{36}$$

$$j(f) \equiv \int f(x)j(x)dx, \ f = \begin{cases} 0, \ x \in I(0+\varepsilon, \pi-\varepsilon) \\ 1, \ x \in I(\pi, 2\pi) \end{cases}$$

The bilinear expression for the current in terms of the two-component field has a precise mathematical meaning in the case when the generating field is a free chiral Boson or Fermion field[13]. This implementation of Φ by a unitary operator depends again on the choice of the smearing function f in the transition regions. Before looking at the issue of localization entropy it is instructive to consider the overlap between the tensor duplicated representation of the original vacuum $\Omega_{vac} = \Omega\bar{\otimes}\Omega$ and the choice of implementing vector $\Omega_p \equiv U(f)(\Omega\bar{\otimes}\Omega)$ of the product vacuum

[13] Iin the latter case the duplicated Fermion formalism is related to the tensor product description by a Klein transformation.

$$\langle\Omega_{vac}|\Omega_p\rangle = \langle\Omega_{vac}|U(f)|\Omega_{vac}\rangle = e^{-\frac{1}{2}\langle j(f),j(f)\rangle_0} \tag{37}$$
$$\langle\eta'|AB'|\eta'\rangle = \langle\Omega_{vac}|A|\Omega_{vac}\rangle\langle\Omega_{vac}|B'|\Omega_{vac}\rangle$$
$$A \in \mathcal{A}(I(0+\varepsilon,\pi-\varepsilon)),\ B' \in \mathcal{A}(I(\pi,2\pi))$$

The fluctuation of a smeared free current is easily shown to have a logarithmic dependence on the ε in (36) which leads to a linear vanishing of the overlap in ε. The vanishing of the overlap is of course related to the expected unitary inequivalence of the product representation in the limit $\varepsilon = 0$. In fact the vanishing of the overlap can be shown to hold for all basis vectors in $H \otimes H$ which are generated by polynomials in the creation operators

$$\langle\Omega_{vac}|U(f)|a_1^*(p_1)...a_n^*(p_n) \otimes a_2^*(k_1)...a_2^*(k_m)\Omega_{vac}\rangle \to 0 \tag{38}$$

By tracing the square of these matrix elements over the second tensor factor basis, one obtains a density matrix ρ_ε which represents the vacuum Ω_{vac} as a mixed state on the factor space $H = \overline{\mathcal{A}(I(0+\varepsilon,\pi-\varepsilon))\otimes_p}$. The associated von Neumann entropy

$$s_\varepsilon = -\rho_\varepsilon ln\rho_\varepsilon = -ln\varepsilon \cdot s \tag{39}$$

represents the desired localization entropy for this particular implementation of the split localization. The implementation independent description which is intrinsically determined in terms of product states is obtained by minimizing over all implementations of the product state. The associated entropy is the minimum of all split entropies and can be explicitly written in terms of the aforementioned Kosaki variational formula.

The only presently computational accessible entropy is the one in the above two fold replica tensor-factorized implementation for free chiral theories. We hope to be able to present an explicit calculation along this scenario in a future paper.

It is important to stress that the success of the LFH formalism to the problem of area density of localization entropy depends on whether chiral theories lead to a universal divergence for this density as the splitting distance approaches zero. Only if this turns out to be true can one assign a split independent additive area density to the horizon which is fixed by LFH up to a common normalization constant (independent of the specific chiral model). Under these circumstances the LFH analysis of Rindler wedges would lead to well defined ratios between area densities of entropy. This (relative or unnormalized) area density would be an intrinsic aspect of the LFH algebra rather than (as in standard field theoretic attempts) a consequence of the accidental short distance behavior of special field coordinatization.

It is interesting to note that this idea of a relative area density of localization entropy would be in complete harmony with the already well understood temperature aspect of localization on wedges or on their horizons since the modular operator for the split situation converges against the Lorentz-boost (which corresponds to the dilation in the LFH on the horizon) representing the dynamics of the thermal KMS state. This suggests that the LFH is capable to extract a physically meaningful measure of entropy area density from the Lorentz boost by extracting an infinite area factor together with the splitting dependence. Throughout the splitting process the restricted vacuum remains a thermal KMS state (it is even a Gibbs state as long as ε remains larger than zero) with the Hawking temperature An ad hoc cutoff (in order to change the Lorentz boost into a trace class operator) in the short distance regime of fields would not be physically reasonable since it is a brute force method which wrecks the (local aspects of the) theory in an uncontrollable manner unrelated to the Hawking temperature aspects of localization. It is important to realize that despite its appearance the splitting method (unlike cutoffs) does not destroy degrees of freedom, it just redistributes them inside the boundary of thickness ε.

5 Conclusions and outlook

By re-investigating the structure of lightfront QFT with the help of recent concepts from local quantum physics we succeeded to give a concise meaning to the idea of (algebraic) LFH. The most useful new result of the present LFH formalism is the compression of vacuum fluctuations into the direction of the lightray and a very strong form of quantum mechanical statistical independence in transverse directions within the lightfront which manifests itself in the absence of any transverse vacuum polarization. This appearance of quantum mechanics within relativistic QFT (without having performed any nonrelativistic approximation) is accompanied by the appearance of a genuine transverse Galilei covariance which results from projecting the Wigner little group "translations" into the lightfront.

The statistical independence of transverse separated subsystems has immediate consequences for quantities which behave additively for uncoupled subsystems (as entropy). Namely the problem of assigning an entropy to the vacuum state restricted to the horizon of a wedge is transferred to the question whether a transverse area density of entropy can be introduced by restricting the vacuum state of a global chiral theory to a local chiral subalgebra (for the case at hand associated with half of the lightray).

It is well-known that the global vacuum becomes a thermal KMS state at the Hawking temperature, but since the relevant generator of the KMS dynamics is not given by a traceclass operator, the problem of finding a corresponding entropy as a measure of the impurity of the vacuum caused by the restriction, poses somewhat of a dilemma. Namely the same mechanism which causes the impurity aspects (which are the prerequisites for a nontrivial entropy) prevents the relevant dynamical operator to be of trace class. To overcome this problem we inferred the split property of AQFT which replaces the KMS state by an ε-dependent family of Gibbs states which for $\varepsilon \to 0$ approximates the KMS state. Different from a cutoff implemented on the short distance fluctuation of distinguished pointlike fields, this split procedure only re-shuffles the degrees of freedom within a fuzzy interval of thickness ε around the endpoints within the same local chiral theory. Therefore contrary to the cutoff method it serves as an excellent starting point for investigating a universal behavior in the limit of $\varepsilon \to 0$. If further investigations of the split limit in chiral theories confirm our conjecture of a universal (logarithmic) divergence independent of the field content of the local algebras, then the LFH universality classes (which were shown to be classified in terms of chiral models) would only show up in numerical coefficients which are determined up to a common model independent factor. In that case the distinction between heat bath thermality and localization thermality, which presently manifests itself in the fact that the heat bath temperature is variable whereas the localization-caused temperature is determined by geometrical aspects, could be enriched by a characteristic difference in their entropy behavior. Whereas the heat bath situation leads to a volume density, the relevant concept for causal localization is an area density. To link this to observable physics, the process of localization which creates causal horizons should be taken out of its abstract "Gedanken" setting in this paper and supplied with a concrete physical blueprint in the spirit of Unruh [31].

Note that the arguments in the present work hold for all causal quantum theories with a maximal speed of causal propagation leading to a notation of causal disjointness and local commutativity. This means in particular that all (acoustical, optical, hydrodynamic) black hole analogs [32] have a LFH. The difference of gravitational interactions to their analogs lies in the scale of couplings between the local quantum physics with geometric properties (the analogs of "surface gravity").

The reader may ask why (despite obvious similarities) we refrained up to now from relating our LFH based quantum results to the classical Bekenstein area law. Of course we do not believe that the area law behavior is the result of an accidental coincidence, we rather think that this problem of comparison requires more detailed and profound future studies, in particular a proof of the conjectured universal behavior for vanishing split $\varepsilon \to 0$. Local quantum theories have a much richer structure than their classical counterparts. Whereas we find it entirely conceivable that the study of fundamental dynamical laws for localization entropy in the context of black holes and their analogs may lead to a normalized entropies (with the normalization depending on the kind of analog), it is less plausible to expect that all the quantum differences between LFH universality classes will be lost[14] in favor of a totally universal classical Bekenstein law.

The results presented in this paper do not support the optimistic view that the area density of entropy associated with the thermal aspect of restriction to a horizon reveals more about the nature of quantum gravity than the Hawking temperature and radiation. But while de-emphasizing the relation to a yet unknown quantum theory of gravity they strengthen the (still somewhat mysterious [34]) links between thermal behavior and geometry.

Acknowledgments: I am indebted to Jacob Yngvason for a invitation for a visit of the ESI and I thank the organizers of the ESI workshop "Quantum field theory on curved space time" for the opportunity to present the main results of this paper.

References

[1] H. Leutwyler, Acta Phys. Austr., Suppl.V. H. Leutwyler in: Magic without Magic (J. R. Klauder ed.). Freeman, 1972. F. Jegerlehner, Helv. Phys. Acta **46**, (1974) 824

[2] W. Driessler, Acta Phys. Austr. **46**, (1977) 63

[3] W. Driessler, Acta Phys. Austr. **46**, (1977) 163

[4] G. 't Hooft, in Salam-Festschrift, A. Ali et al. eds., World Scientific 1993, 284

[5] G. L. Sewell, Ann. Phys. **141**, (1982) 291

[6] S. J. Summers and R. Verch, Lett. Math. Phys. **37**, (1996) 145

[7] D. Guido, R. Longo, J. Roberts and R. Verch, *Charged sectors, spin and statistics in quantum field theory on curved spacetimes*, math-ph/9906019

[14] Apart from the classical "surface gravity" constant through which the gravitational coupling enters into the Bekenstein formula. Derivations of thermodynamic fundamental laws from classical field models can be found in [33].

[8] K.-H. Rehren, Ann. Henry Poincaré **1**, (2000) 607, see alsoM. Duetsch and K.-H. Rehren hep-th/0204123 for remarks about the relation with Maldacena's original duality idea.

[9] B. Schroer, JPA **35**, (2002) 9165

[10] H. J. Borchers, Commun. Math. Phys. **2**, (1966) 49

[11] H. J. Borchers, J. Math. Phys. **41**, (2000) 3604

[12] P. D. Hislop and R. Longo, Commun. Math. Phys. **84**, (1982) 71

[13] R. Haag, *Local Quantum Physics*, Springer Verlag (1992)

[14] R. Longo and K.-H. Rehren, Rev. Math. Phys. **7**, (1995) 567

[15] M. Takesaki, *Lecture Notes in Mathematics Vol. 128, Springer* Berlin-Heidelberg-New York 1970

[16] D. Guido, R. Longo and H.-W. Wiesbrock, Commun. Math. Phys. **192**, (1998) 217

[17] Lucio Fassarella and B. Schroer, Phys. Lett. **B 538**, (2002) 415

[18] K. Ebrahimi-Fard, *Comment on Modular Theory and Geometry*, math-ph/001149, to be published in JPA

[19] H.-W. Wiesbrock, Commun. Math. Physics **157**, (1993) 83, Erratum in Commun. Math. Phys. **184**, (1998) 683

[20] B. Schroer, J. Math. Phys. **41**, (2000) 3801 and ealier papers of the same author quoted therein

[21] H.-W. Wiesbrock, Commun. Math. Phys. 193, (1998) 269

[22] K. Fredenhagen and M. Joerss, Commun. Math. Phys. **176**, (1996) 541

[23] M. Joerss, Lett. Math.Phys. **38**, (1996) 257

[24] M. Rehquardt, Commun. Math. Phys. **50**, (1976) 259

[25] H. Kosaki, J. Operator Theory **16**, (1966) 335

[26] S. Doplicher and R. Longo, Invent. Math. **75**, (1984) 493

[27] Private communication from D. Buchholz

[28] H. Narnhofer, in *The State of Matter*, ed. by M. Aizenman and H. Araki (Wold-Scientific, Singapore) 1994

[29] C. D'Antoni, R. Longo and F. Radulescu, J. Operat. Theor. **45**, (2001) 195

[30] C. D'Antoni, S. Doplicher, K. Fredenhagen and R. Longo, Commun.Math. Phys. **110**, (1987) 325

[31] W. G. Unruh, Phys. Rev. D **14**, (1976) 870

[32] R. Schuetzhold and W. G. Unruh, *Gravity wave analogs of black holes*, gr-qc/0205099

[33] R. M. Wald, *Quantum Field Theory in Curved Spacetime and Black-Hole Thermodynamics*, University of Chicago Press 1994

[34] B. Schroer and H. W. Wiesbrock, Rev. Math. Phys. **12**, (2000) 461

[35] R. Kähler and H.W. Wiesbrock, JMP **42**, (2001) 74.