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Electron-Pair Condensation in Parity-Preserving QED_3

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Abstract

In this letter, we present a parity-preserving QED_3 with spontaneous breaking of a local $U(1)$-symmetry. The breaking is accomplished by a potential of the φ^6-type. It is shown that a net attractive interaction appears in the Møller scattering (between two electrons with opposite spin polarisations) as mediated by the gauge field and a Higgs scalar. This might favour a pair-condensation mechanism.

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Over the past years, the study of 3-dimensional field theories [1] has been well-supported in view of the possibilities they open up for the setting of a gauge-field-theoretical foundation in the description of Condensed Matter phenomena, such as High-T_c Superconductivity [2] and Quantum Hall Effect [3]. Abelian models such as QED_3 and $\tau_3\mathrm{QED}_3$ [4, 5] are some of the theoretical approaches proposed to describe more deeply some features of high-T_c materials.

The theory of superconductivity by Bardeen, Cooper and Schrieffer (BCS model) [6] succeeds in providing a microscopical description for superconducting materials: indeed, many predictions of the BCS model have been confirmed experimentally. An elegant mathematical formulation was given to it by Bogoliubov [7]. The characteristic feature of the BCS theory is that it produces an energy gap between the ground state and the excited states of a superconductor. The gap is due to the fact that the attractive phonon-mediated interaction between electrons produces correlated pairs of such particles (Cooper pairs) [8], with opposite momenta and spin; a finite amount of energy is required to break this correlation.

In a well-known paper by Nambu and Jona-Lasinio [9], it was proposed that the nucleon mass arises from a dynamical mechanism, similar to the appearance of the energy gap in the BCS model. They observed that elementary excitations in a superconductor could be described by means of a coherent mixture of electrons and holes. The framework they set up for dynamical mass generation was motivated by the observation of an analogy between the properties of Dirac particles and the quasi-particle excitations that appear in a superconductor.

The main purpose of this letter is to show that electrons scattered in D=1+2 can experience a mutual attractive interaction, depending on their spin states. This attractive scattering potential comes from processes in which the electrons are correlated in momentum space with opposite spin polarisations (s-wave state). The intermediate bosons are a massive vector meson and a Higgs scalar, both resulting from the breaking of a local $U(1)$-symmetry. The breaking-down is accomplished by a sixth-power potential. We analyse the conditions on the parameters in order to avoid metastable vacuum states. The method used here to compute the scattering potentials is based on the ideas reported in a series of papers by Sucher *et al.* [10]. The behaviour of the scattering interactions mediated by the massive vector meson and the Higgs scalar are presented for electrons scattered with opposite spin states. The issue of confinement in QED_3 [11] is also alluded to.

The action for the parity-preserving QED_3[1] with spontaneous symmetry breaking of a local $U(1)$-symmetry is given by :

$$S_{\mathrm{QED}} = \int d^3x \left\{ -\frac{1}{4}F^{mn}F_{mn} + i\overline{\psi}_+ D\!\!\!/\,\psi_+ + i\overline{\psi}_- D\!\!\!/\,\psi_- - y(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)\varphi^*\varphi + \right.$$
$$\left. + D^m\varphi^* D_m\varphi - V(\varphi^*\varphi) \right\} \qquad , \tag{1}$$

with the potential $V(\varphi^*\varphi)$ taken as

$$V(\varphi^*\varphi) = \mu^2\varphi^*\varphi + \frac{\zeta}{2}(\varphi^*\varphi)^2 + \frac{\lambda}{3}(\varphi^*\varphi)^3 \qquad , \tag{2}$$

where the mass dimensions of the parameters, μ, ζ, λ and y are respectively 1, 1, 0 and 0.

The covariant derivatives are defined as follows :

$$D\!\!\!/\,\psi_\pm \equiv (\partial\!\!\!/ + iqgA\!\!\!/)\psi_\pm \quad \text{and} \quad D_m\varphi \equiv (\partial_m + iQgA_m)\varphi \quad , \tag{3}$$

[1]The metric adopted throughout this work is $\eta_{mn} = (+,-,-)$; m, n=(0,1,2). Note that slashed objects mean contraction with γ-matrices. The latter are taken as γ^m=$(\sigma_x, i\sigma_y, -i\sigma_z)$.

where g is a coupling constant with dimension of $(\text{mass})^{\frac{1}{2}}$ and, q and Q are the $U(1)$-charges of the fermions and scalar, respectively. In the action (1), F_{mn} is the usual field strength for A_m, ψ_+ and ψ_- are two kinds of fermions (the $\pm$ subscripts refer to their spin sign [12]) and φ is a complex scalar. The $U(1)$-symmetry gauged by A_m is interpreted as the electromagnetic one, so that A_m is meant to describe the photon.

The QED$_3$-action[2] (1) is invariant under the discrete symmetry, P, whose action is fixed below :

$$x_m \xrightarrow{P} x_m^P = (x_0, -x_1, x_2) \quad , \tag{4.a}$$

$$\psi_\pm \xrightarrow{P} \psi_\pm^P = -i\gamma^1\psi_\mp \quad , \tag{4.b}$$

$$A_m \xrightarrow{P} A_m^P = (A_0, -A_1, A_2) \quad . \tag{4.c}$$

Since we are looking for a model that preserves the parity and time-reversal in D=1+2, it should be notice that the transformation (4.c) has been imposed in such a way that the interactions respect both invariances.

The sixth-power potential, V, is the responsible for breaking the electromagnetic $U(1)$-symmetry. It is the most general renormalisable potential in $3D$.

Analysing the potential (2), and imposing that it is bounded from below and yields only stable vacua (metastability is ruled out), the following conditions on the parameters μ, ζ, λ must be set :

$$\lambda > 0 \ , \quad \zeta < 0 \quad \text{and} \quad \mu^2 \leq \frac{3}{16}\frac{\zeta^2}{\lambda} \quad . \tag{5}$$

We denote $\langle\varphi\rangle$=v and the vacuum expectation value for the $\varphi^*\varphi$-product, v^2, is chosen as

$$\langle\varphi^*\varphi\rangle = v^2 = -\frac{\zeta}{2\lambda} + \left[\left(\frac{\zeta}{2\lambda}\right)^2 - \frac{\mu^2}{\lambda}\right]^{\frac{1}{2}} \quad , \tag{6}$$

the condition for minimum being read as

$$m^2 + \zeta v^2 + \lambda v^4 = 0 \quad . \tag{7}$$

The complex scalar, φ, is parametrised by

$$\varphi = e^{i\frac{\theta}{v}}(v + H) \quad , \tag{8}$$

where θ is the would-be Goldstone boson and H is the Higgs scalar, both with vanishing vacuum expectation values.

By replacing the parametrisation (8) for the complex scalar, φ, into the action (1), the following free action comes out:

$$\begin{aligned} \hat{S}_{\text{QED}}^{\text{free}} = \int d^3x \Big\{ &-\frac{1}{4}F^{mn}F_{mn} + \frac{1}{2}M_A^2 A^m A_m + \overline{\psi}_+(i\partial\!\!\!/ - m)\psi_+ + \overline{\psi}_-(i\partial\!\!\!/ + m)\psi_- + \\ &+ \partial^m H \partial_m H - M_H^2 H^2 + \partial^m\theta\partial_m\theta + 2vQgA^m\partial_m\theta \Big\} \quad , \end{aligned} \tag{9}$$

where the parameters M_A^2, m and M_H^2 are given by

$$M_A^2 = 2v^2Q^2g^2 \ , \quad m = yv^2 \quad \text{and} \quad M_H^2 = 2v^2(\zeta + 2\lambda v^2) \quad . \tag{10}$$

[2]For more details about QED$_3$ and τ_3QED$_3$, as well as their applications and some peculiarities of parity and time-reversal in D=1+2, see refs.[1, 4, 5].

The conditions (5) and (7) imply the following lower-bound (see eq.(10)) for the Higgs mass :

$$M_H^2 \geq \frac{3}{4}\frac{\zeta^2}{\lambda} \quad . \tag{11}$$

Therefore, a *massless* Higgs is out of the model we consider here. A massless Higgs would be present in the spectrum if $\mu^2 > \frac{3}{16}\frac{\zeta^2}{\lambda}$. But, in such a situation, the minima realising the spontaneous symmetry breaking would not be absolute ones, corresponding therefore to metastable ground states, that we avoid here. One-particle states would decay with a short decay-rate if compared to an absolute minimum ground state.

In order to preserve the manifest renormalisability of the model, the 't Hooft gauge [13] is adopted :

$$\hat{S}^{\rm gf}_{R_\xi} = \int d^3x \left\{ -\frac{1}{2\xi}\Big(\partial^m A_m - \sqrt{2}\xi M_A \theta\Big)^2 \right\} \quad , \tag{12}$$

where ξ is a dimensionless gauge parameter.

By replacing the parametrisation (8) into the action (1), and adding up the 't Hooft gauge (12), it can be directly found the following complete parity-preserving action :

$$\begin{aligned} S^{\rm SSB}_{\rm QED} = \int d^3x \Big\{ & -\frac{1}{4}F^{mn}F_{mn} + \frac{1}{2}M_A^2 A^m A_m + \overline{\psi}_+(i\partial\!\!\!/ - m)\psi_+ + \overline{\psi}_-(i\partial\!\!\!/ + m)\psi_- + \\ & + \partial^m H \partial_m H - M_H^2 H^2 + \partial^m\theta\partial_m\theta - \xi M_A^2\theta^2 - \frac{1}{2\xi}(\partial^m A_m)^2 + \\ & - qg\overline{\psi}_+ A\!\!\!/ \psi_+ - qg\overline{\psi}_- A\!\!\!/ \psi_- - y(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)(2vH + H^2) + \\ & + \Big(Q^2 g^2 A^m A_m + 2\frac{Qg}{v}A^m\partial_m\theta + \frac{1}{v^2}\partial^m\theta\partial_m\theta\Big)(2vH + H^2) + \\ & + c_3 H^3 + c_4 H^4 + c_5 H^5 + c_6 H^6 \Big\} \quad , \end{aligned} \tag{13}$$

where the constants, c_3, c_4, c_5 and c_6 are defined by

$$c_3 = 2v(\zeta + \frac{10}{3}\lambda v^2) \;\; , \quad c_4 = \frac{\zeta}{2} + 5\lambda v^2 \;\; , \quad c_5 = 2\lambda v \quad \text{and} \quad c_6 = \frac{\lambda}{3} \quad . \tag{14}$$

The Møller scattering to be contemplated will include the scatterings mediated by the gauge field and the Higgs (A_m and H). The scattered electrons exhibit opposite spin polarisations ($e^-_{(+)}$ and $e^-_{(-)}$). This study is motivated by the fact that in 4-dimensional space-time, a Cooper pair bound state (s-wave state) [8] is built up by a scattering between electrons correlated in phase-space with opposite spin states. The interactions involved in such a process are the electromagnetic and the phononic ones. The former is mediated by photons, with a repulsive behaviour, and the latter is mediated by the phonons, which is attractive. The opposite behaviour of these interactions play a central rôle for the BCS superconductivity phenomena [6] (weak-coupling superconductors), since, at temperatures below the critical one (T_c), the interaction mediated by phonons (attractive) is stronger than the electromagnetic (repulsive) interaction. For temperatures above T_c, the superconducting phase is destroyed, which means that, the net interaction becomes repulsive.

For a 3-dimensional space-time, we are now trying to understand, with the help of the model proposed here, what happens if we consider the scattering of electrons with opposite polarisations. One of the questions to be answered is whether or not there is a net attractive interaction in $e^-_{(+)}$–$e^-_{(-)}$-scattering, as mediated by the gauge field and the Higgs. Another interesting point to be analised concerns the influence of spin polarisations (+ and −) on the dynamical nature of these scattering processes.

To compute the scattering amplitudes, it will be necessary to derive the Feynman rules for propagators and interaction vertices involving the fermions, the gauge field and the Higgs. From the action (13), the following propagator and vertex Feynman rules come out :

1. fermions and Higgs propagators :

$$\langle\overline{\psi}_+\psi_+\rangle = i\frac{\not{k}+m}{k^2-m^2} \quad , \quad \langle\overline{\psi}_-\psi_-\rangle = i\frac{\not{k}-m}{k^2-m^2} \quad \text{and} \quad \langle HH\rangle = \frac{i}{2}\frac{1}{k^2-M_H^2} \quad ; \qquad (15)$$

2. gauge field propagator :

$$\langle A_m A_n\rangle = -i\left[\frac{1}{(k^2-M_A^2)}\left(\eta_{mn}-\frac{k_m k_n}{M_A^2}\right)+\frac{1}{M_A^2}\left(\frac{k_m k_n}{k^2-\xi M_A^2}\right)\right] \quad ; \qquad (16)$$

3. vertex Feynman rules :

$$\mathcal{V}_{+H+} = 2iyv \quad , \quad \mathcal{V}_{-H-} = -2iyv \quad , \quad \mathcal{V}^m_{+A+} = iqg\gamma^m \quad \text{and} \quad \mathcal{V}^m_{-A-} = iqg\gamma^m \quad . \qquad (17)$$

It should be noticed that the convention addopted, $\mathcal{V}_{+H+}$, means the vertex Feynman rule for the interaction term, $\overline{\psi}_+ H\psi_+$. This convention is addopted similarly for the other interaction vertices above.

The amplitudes for the $e^-_{(+)}$–$e^-_{(-)}$ scattering by the gauge field and Higgs are listed below :

1. scattering amplitude by A_m :

$$-i\mathcal{M}_{+A-} = \overline{u}_+(p_1)\left[iqg\gamma^m_{(+)}\right]u_+(p_1')\left\{-i\frac{\eta_{mn}}{k^2-M_A^2}\right\}\overline{u}_-(p_2)\left[iqg\gamma^n_{(-)}\right]u_-(p_2') \quad ; \qquad (18)$$

2. scattering amplitude by H :

$$-i\mathcal{M}_{+H-} = \overline{u}_+(p_1)\left[2iyv\right]u_+(p_1')\left\{\frac{i}{2}\frac{1}{k^2-M_H^2}\right\}\overline{u}_-(p_2)\left[-2iyv\right]u_-(p_2') \quad , \qquad (19)$$

where $k^2=(p_1-p_1')^2$ is the invariant squared momentum transfer. The Dirac spinors, u_+ and u_-, are the positive-energy solutions to the Dirac equations for ψ_+ and ψ_-, and they are normalised to :

$$\overline{u}_+(p)u_+(p) = 1 \quad \text{and} \quad \overline{u}_-(p)u_-(p) = -1 \quad . \qquad (20)$$

To compute the scattering potentials for the interaction between electrons with opposite spin polarisations ($e^-_{(+)}$–$e^-_{(-)}$), we refer to the works of Sucher *et al.* [10], where the concept of potential in quantum field theory and in scattering processes is discussed in great detail.

The calculation of scattering potentials will be performed in the center-of-mass frame, for in this frame the electrons scattered are correlated in momentum space.

By using the Feynman rules displayed above (eqs.(15), (16) and (17)), the following scattering potentials for the $e^-_{(+)}$–$e^-_{(-)}$-scattering processes mediated by the gauge field and the Higgs are found in the center-of-mass frame (*c.m.*):[3]

[3] In the *c.m.* frame, the squared momentum transfer is given by $k^2=-\vec{q}^{\,2}$. The notations, $\mathcal{U}_{+A-}(\vec{r})$ and $\mathcal{U}_{+H-}(\vec{r})$, with $r\equiv|\vec{r}|$, refer to the scattering potentials (in configuration space) for the process $e^-_{(+)}$–$e^-_{(-)}$ mediated by gauge field and Higgs. The product $\beta_{(+)}\beta_{(-)}$ is a spinorial factor in the space of the electrons $e^-_{(+)}$ and $e^-_{(-)}$: $\beta_{(+)}=\gamma^0_{(+)}$, $\beta_{(-)}=-\gamma^0_{(-)}$ and $\vec{\alpha}_{(\pm)}\equiv\gamma^0_{(\pm)}\vec{\gamma}_{(\pm)}$.

1. gauge field scattering potential :

$$\mathcal{U}^{c.m.}_{+A-}(\vec{r}) = q^2 g^2 \beta_{(+)}\beta_{(-)}\gamma^m_{(+)}\gamma^{(-)}_m \int \frac{d^2\vec{q}}{(2\pi)^2}\frac{1}{\vec{q}^{\,2}+M_A^2}e^{i\vec{q}.\vec{r}}$$
$$= -q^2 g^2 \gamma^0_{(+)}\gamma^0_{(-)}\gamma^m_{(+)}\gamma^{(-)}_m K_0(M_A r)$$
$$= -q^2 g^2 \left[1\!\!1 - \vec{\alpha}_{(+)}.\vec{\alpha}_{(-)}\right] K_0(M_A r) \quad . \tag{21}$$

The minus sign in (21) is due to the fact that $\beta_{(-)}{=}-\gamma^0_{(-)}$. This is an effect of (1+2)-dimensions. ψ_+ and ψ_- have mass terms with opposite signs (opposite spins, according to [1, 12]) and so, by looking at the Hamiltonian, one gets β-terms with opposite signs.

2. Higgs scattering potential :

$$\mathcal{U}^{c.m.}_{+H-}(\vec{r}) = 2y^2 v^2 \beta_{(+)}\beta_{(-)} \int \frac{d^2\vec{q}}{(2\pi)^2}\frac{1}{\vec{q}^{\,2}+M_H^2}e^{i\vec{q}.\vec{r}}$$
$$= -2y^2 v^2 \left[\gamma^0_{(+)}\gamma^0_{(-)}\right] K_0(M_H r) \quad , \tag{22}$$

where $K_0(Mr)$ is the zeroth-order modified Bessel function of the second kind :

$$\int \frac{d^2\vec{q}}{(2\pi)^2}\frac{1}{\vec{q}^{\,2}+M^2}e^{i\vec{q}.\vec{r}} = \frac{1}{2\pi}K_0(Mr) \quad . \tag{23}$$

This Bessel function presents the following asymptotic behaviour in terms of the Compton wave-length ($\frac{1}{M}$) :

$$K_0(Mr) \longrightarrow \begin{cases} -\ln(Mr) \ , & Mr \ll 1 \\ \sqrt{\frac{\pi}{2Mr}}e^{-Mr} \ , & Mr \gg 1 \quad . \end{cases} \tag{24}$$

Eq.(24) shows that the scattering potentials (21) and (22) are attractive and completely confining. Therefore, the interactions mediated by the gauge field and the Higgs are attractive in a scattering between electrons with opposite spin polarisations ($e^-_{(+)}{-}e^-_{(-)}$-scattering). For scatterings with a scalar exchange, the spin polarisations do not affect the behaviour of potential: it will be always attractive. This result is expected, since the Higgs particle does not *feel* the electron polarisations.

An interesting point to remark is that, in spite the scattered particles have the same electric charge, the spin polarisation is determinant for the behaviour of the scattering potential for processes where a gauge field is exchanged. In the case where the scattered electrons have opposite spin polarisations, the interaction is attractive. Nevertheless, for scatterings between electrons with the same spin state, the interaction becomes repulsive.

The coherence length of a Cooper pair, as Cooper found out for the 2-electron bound state [8], is much bigger than the electron Compton wave-length, namely, the former is of order 10^4Å and the latter of 10^{-2}Å. Therefore, for the sake of studying the possible existence of a Cooper pair condensate in the parity-preserving QED$_3$ discussed throughout this work, the electrons that are the candidates to built up a Cooper pair experience the following scattering potentials in the *c.m.* frame :

$$\mathcal{U}^{c.m.}_{+A-}(\vec{r}) = -q^2 g^2 \left[\mathbb{1} - \vec{\alpha}_{(+)}.\vec{\alpha}_{(-)} \right] \sqrt{\frac{\pi}{2M_A r}} e^{-M_A r} \quad ; \tag{25}$$

$$\mathcal{U}^{c.m.}_{+H-}(\vec{r}) = -2y^2 v^2 \left[\gamma^0_{(+)} \gamma^0_{(-)} \right] \sqrt{\frac{\pi}{2M_H r}} e^{-M_H r} \quad , \tag{26}$$

where the asymptotic approximations, $M_A r \gg 1$ and $M_H r \gg 1$, are compatible with the dimensions through which Cooper pair exists.

It should be pointed out that, in order to be sure of the existence of a bound state in such scatterings, it is more advisable to study the Bethe-Salpeter [14] equation in D=1+2 [15] for the model proposed here. Such an analysis is more reliable in view of its intrinsically non-perturbative nature. It is worthwhile to stress that our results simply suggest that, at the semiclassical level, a net attractive interaction between electrons with opposite polarisations might point out pair condensation if Bethe-Salpeter equations are taken into account [15].

If an attraction is felt at the level of tree amplitudes, we would not expect that loop corrections, that bring about powers of $\hbar$, might work against pair condensation. In any case, to our mind, it would be more reasonable to pursue an investigation of the Bethe-Salpeter equations (rather than computing higher-loop corrections) in order to infer about electron-pair condensation in the model discussed throughout this paper.

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References

[1] W. Siegel, *Nucl. Phys.* **B156** (1979) 135 ; J. Schonfeld, *Nucl. Phys.* **B185** (1981) 157 ; R. Jackiw and S. Templeton, *Phys. Rev.* **D23** (1981) 2291 ; S. Deser and R. Jackiw, *Phys. Lett.* **B139** (1984) 371 ; S. Deser, R. Jackiw and S. Templeton, *Phys. Rev. Lett.* **48** (1982) 975 and *Ann. Phys. (N.Y.)* **140** (1982) 372.

[2] J.G. Bednorz and A.K. Müller, *Z. Phys.* **B64** (1986) 189.

[3] K. Von Klitzing, G. Dorda and M. Peper, *Phys. Rev. Lett.* **45** (1980) 494 ; D. Tsui, H. Störmer and A. Gossard, *Phys. Rev. Lett.* **48** (1982) 1559.

[4] N. Dorey and N.E. Mavromatos, *Phys. Lett.* **B266** (1991) 163 and *Nucl. Phys.* **B386** (1992) 614.

[5] R.D. Pisarski, *Phys. Rev.* **D29** (1984) 2423 ; T.W. Appelquist, M. Bowick, D. Karabali and L.C.R. Wijewardhana, *Phys. Rev.* **D33** (1986) 3704 ; R. Mackenzie and F. Wilczek, *Int. J. Mod. Phys.* **A3** (1988) 2827 ; G.W. Semenoff and P. Sodano, *Nucl. Phys.* **B328** (1989) 753 ; G.W. Semenoff and L.C.R. Wijewardhana, *Phys. Rev. Lett.* **63** (1989) 2633 ; A. Kovner and B. Rosenstein, *Mod. Phys. Lett.* **A5** (1990) 2661 ; M.I. Dobroliubov, D. Eliezer, I.I. Kogan, G.W. Semenoff and R.J. Szabo, *Mod. Phys. Lett.* **A8** (1993) 2177.

[6] J. Bardeen, L.N. Cooper and J.R. Schrieffer, *Phys. Rev.* **108** (1957) 1175.

[7] N.N. Bogoliubov, *Soviet Phys.-JETP* **34** (1958) 41 and *Soviet Phys.-JETP* **34** (1958) 51.

[8] L.N. Cooper, *Phys. Rev.* **104** (1956) 1189.

[9] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122** (1961) 345 and *Phys. Rev.* **124** (1961) 246.

[10] W. Johnson, P. Mohr and J. Sucher, *Relativistic, Quantum Electrodynamics, and Weak Interaction Effects in Atoms*, AIP Conference Proceedings (1989) 513p, New York (USA) ; J. Sucher, *Potentials from field theory: non-uniqueness, gauge dependence and all that*, University of California preprint, NSF-ITP-89-63 ; J. Sucher, *Phys. Rev.* **D49** (1994) 4284 ; J. Sucher, *Phys. Rev.* **D51** (1995) 5965 ; J. Sucher, *The concept of potential in quantum field theory*, hep-ph/9412388, Talk given at Quantum Systems: New Trends and Methods (QS 94), Minsk, Belarus, Russia.

[11] P. Maris, *Phys. Rev.* **D52** (1995) 6087.

[12] B. Binegar, *J. Math. Phys.* **23** (1982) 1511.

[13] G. 't Hooft, *Nucl. Phys.* **B33** (1971) 173 and *Nucl. Phys.* **B35** (1971) 167.

[14] H.A. Bethe and E.E. Salpeter, *Phys. Rev.* **82** (1951) 309 and *Phys. Rev.* **84** (1951) 1232.

[15] C.J. Burden, *Nucl. Phys.* **B387** (1992) 419 ; T.W. Allen and C.J. Burden, *Positronium States in* QED_3, hep-th/9512168.