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On the Baryon Asymmetry of the Universe

by

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Abstract

Based on symmetry transformations relating the Hamiltonian constraint of the radiation-dominated phase, and the Hamiltonian constraint of the matter-dominated phase of Robertson-Walker-Friedmann (RWF) universes, we derive an expression for the baryon asymmetry n_B/n_γ as a function of known thermodynamic constants, the gravitational constant, the radius of the present universe and the present temperature of the cosmic background radiation. The only parameter in the formula is the total number of (massless) degrees of freedom of relativistic particles composing the radiation density. We obtain $n_B/n_\gamma \sim 5.49 \times 10^{-10}$. Physical implications of this result are commented.

Key-words: Baryon asymmetry; Baryogenesis.

The observed baryon asymmetry of the universe [1, 2], $n_B/n_\gamma \sim 10^{-10}$, poses some fundamental questions for the models of formation and evolution of the universe. In the last thirty years the focus on this problem has shifted from specifying this number simply as an initial condition, to considering that the baryon asymmetry evolved from an initially symmetric configuration as consequence of processes which violate baryon number conservation in an expanding universe [3]. Of course the resulting baryon asymmetry depends on a detailed model of baryon non-conservation, and the observed value should therefore place constraints on the baryon-number violating interactions occuring in the early universe.

In the present letter we derive an expression for the baryon asymmetry as a function of known thermodynamic constants, the gravitational constant and other constants related to the present universe. The formula results from a constraint imposed by symmetry transformations relating the Hamiltonians of the radiation-dominated phase and matter-dominated phase, of RWF models.

Let us consider a closed RWF model with expansion scale factor B (as discussed later, analogous results arise for open models). For the matter-dominated phase (case of pressureless matter) the Hamiltonain constraint is given by [4]

$$H_m = \frac{1}{24B}\, P_B^2 + \frac{3}{2}\, B - 2k\, C_m = 0 \tag{1}$$

where P_B is the canonical momentum conjugated to B, k is Einstein's constant, and the constant of motion C_m is related to the matter density by

$$\rho_m = C_m/B^3 \tag{2}$$

Analogously for the radiation-dominated phase the Hamiltonian constraint is given by

$$H_r = \frac{1}{24B}\, P_B^2 + \frac{3}{2}\, B - \frac{2k}{B} C_r = 0 \tag{3}$$

where the constant of motion C_r is related to the radiation energy density by

$$\rho_r = C_r/B^4 \tag{4}$$

Now the following fact is essential: the Hamiltonian constraints (1) and (3) are related by the transformation [5]

$$B \to \overline{B} = B - \frac{2}{3}\, k\, C_m \tag{5}$$

namely [6], $\overline{B}\, H_r(\overline{B}) = B\, H_m(B)$, provided that

$$C_m^2 = \frac{3C_r}{k} \tag{6}$$

The symmetry transformation [7] (5) thus imposes a constraint between the radiation content of the universe in the radiation-dominated phase, and the matter content (baryon matter) in the matter-dominated phase. With the use of thermodynamics relations in RWF cosmologies we derive from (6), (2) and (4) that

$$\frac{n_\gamma}{n_B} = \sqrt{\frac{16\pi G a}{3g_\star}}\left(\frac{\mu_B}{2.7\beta}\right) B_0 T_0 \tag{7}$$

Here n_γ/n_B is the ratio photon to baryon number density, G is the gravitational constant, a the blackbody constant, β the Boltzmann constant, μ_B the baryon mass (for which we take the value of the proton mass), B_0 the present radius of the universe and T_0 the present temperature of the cosmic background radiation. The numerical factor 2.7 comes from the expression of the average energy per photon in the radiation-dominated phase. The parameter $g_\star$ is the total number of (massless) degrees of freedom of relativistic particles composing the radiation density; it appears in the relation $\rho_r = \frac{g_\star}{2}\rho_\gamma$, where ρ_γ is the energy density of photons. For $g_\star$ we take the value $g_\star \approx 106$ (cf. Ref. [2], chapter 3) which corresponds to the radiation component ρ_r at temperatures $T \gtrsim 300$ GeV. This is the early radiation-dominated era where baryogenesis is expected to have occurred. We assume that all particle species contributing to ρ_r are in equilibrium and have a common temperature, which is the photon temperature. Also since the product BT is a constant along the evolution of RWF universes, it follows that the ratio n_B/n_γ is constant in time. Expression (7) can be recast in the simpler form

$$\frac{n_\gamma}{n_B} = \sqrt{\frac{16\pi^3}{45g_\star}}\left(\frac{\mu_B}{\mu_P}\right)\left(\frac{\beta}{2.7\hbar c}\right) B_0 T_0 \tag{8}$$

where μ_P is Planck's mass. Using

$$\begin{aligned} B_0 &= 1.7 \times 10^{28} cm \\ T_0 &= 2.7\ ^0K \end{aligned}$$

we obtain from (8)

$$\frac{n_B}{n_\gamma} \sim 5.49 \times 10^{-10} \tag{9}$$

which is consistent with the observevations. On the other hand, since $g_\star$ is a temperature-dependent parameter appearing in (8) the consistency with observations limits the domain of temperature for which the symmetry should apply, and actually corresponds to the early radiation-dominated epoch.

Several questions arise related to the physical meaning of (8): (i) Is the consistency with the observed value an implication that the transformation (5) is a true symmetry of the universe? (ii) Is there an implication of (5) for theories with baryon non-conservation in an expanding universe? (theories in which a net baryon asymmetry is produced should ultimately reproduce (8) or, at least, have their parameters constrained by (8)). Affirmative answers to these questions would give a more substantial physical basis to (8). We intend to address these issues in a future publication.

Finally we note that in open RWF models, transformations of type (5) relate the radiation Hamiltonian (3) to the Hamiltonian of uncoupled matter and radiation, provided that a constraint is imposed between the matter content and the radiation content of the latter Hamiltonian. This analogously implies that the ratio baryon to photon number density is also given by (9) up to terms of the order 10^{-46}.

References

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[4] S. Weinberg, Gravitational and Cosmology, Wiley & Sons, New York, 1972, Chapter 15.

[5] To see the geometrical meaning of (5), related to the homotopy type of the domain manifolds of (1) and (3), Cf. M. Morse, The Calculus of Variations in the Large, AMS Colloquium Publications XVIII, 1934.

[6] We note that under (5) the momentum P_B transforms as $P_B \to P_{\overline{B}} = P_B$, where $P_{\overline{B}}$ is the canonical momentum conjugated to $\overline{B}$. This results from the invariance of the action $\int P_B \dot{B} dt$. Cf. also E.C.G. Sudarshan and N. Mukunda, Classical Mechanics, Wiley & Sons, New York (1974).

[7] We refer the transformaion (5) as a symmetry because it takes the Hamiltonian constraint of the radiation-dominated phase of the universe into the Hamiltonian constraint of the matter-dominated phase of the same universe.