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Finite Size Effects in Thermal Field Theory

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Abstract

We consider a neutral self-interacting massive scalar field defined in a d-dimensional Euclidean space. Assuming that the system is in thermal equilibrium with a reservoir, we are discussing the perturbative renormalization in the one-loop level of this field theory in the presence of rigid boundary surfaces (two parallel hyperplanes), which break translation symmetry. In order to identify the singular part of the one-loop two-point and four-point Schwinger functions, we use a combination of dimensional and zeta function analytic regularization procedures. The infinities which occur in the regularized one-loop two-point and also four-point Schwinger functions fall into two distinct classes: local divergences that are renormalized by the introduction of the usual bulk counterterms, and surface divergences that demand countertems concentrated on the boundaries. We present the detailed form of the surface divergences. We discuss different strategies that we can assume to solve the problem of the surface divergences. We also briefly discuss how to overcome in the case of Neumann-Neumann boundary conditions the difficulties generated by infrared divergences.

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1 Introduction

The Casimir effect is the manifestation of the zero-point energy of the quantized electromagnetic field, by the presence of metallic plates [1]. A very simple calculation predicted that in a four-dimensional spacetime, uncharged parallel, perfectly conducting plates should attract with a force per unit area, $F(L) \propto \frac{1}{L^4}$, where L is the distance between the plates. Complete reviews of this effect can be found in Refs. [2] [3] [4] [5] [6]. As was stressed by Milloni et al [7], a brief old argument showing that the zero point-energy associated with the quantized electromagnetic field must have a physical meaning was given by Einstein and Stern [8]. These authors noted that a zero-point energy seems necessary in order to the Planck's expression for the average energy of an oscillator in equilibrium with radiation at temperature β^{-1} does not have a first-order quantum correction to β^{-1} in the classical limit $\beta >> \omega$.

Although formally divergent, the diference between the vacuum energy of different physical configurations can be finite. In the case of a free scalar field, interacting only with boundary surfaces, the Casimir approach can be summarized in the following steps: first a complete set of the mode solutions of the Klein-Gordon equation satisfying appropriate boundary conditions with the respective eigenfrequencies are found. Next, the divergent zero-point energy is regularized by the introduction of an ultraviolet cut-off. Finally, the polar part of the regularized energy is removed using a renormalization procedure. This procedure was discussed first a long time ago by Fierz [9], then followed by Boyer [10] and also by Svaiter and Svaiter [11] [12]. In these two last references, an attempt to clarify the relation between the cut-off method and analytic regularization procedures in Casimir effect has been developed. Being more specific, in these papers an analytic regularization procedure was interpreted as a cut-off method, and using a mixed cut-off in the regularized zero-point energy, it was possible to unify these two methods in a two-dimensional and also three-dimensional spacetime. Further, a general proof that in regularizing a ill-defined quantity, if the introduction of an exponential cut-off yields an analytic function with a pole at the origin, the analytic regularization using the zeta function (or a generalization for the zeta function) is equivalent to the aplication of a cut-off with the subtraction of the singular part at the origin [13] [14]. More recently, Fulling developed an interesting discussion describing the problems in the renormalization program which is carried out to find the renormalized vacuum stress-tensor in different field theories [15].

It is important to stress that these results are at one-loop level and are dealing only with free fields. It is clear that the formalism must be generalized to take into account the situation of self-interacting fields. Although higher-loop corrections to the Casimir effect seems beyond the experimental reach, at least theoretically such corrections are of interest. Nevertheless, except for very few papers, in the study of radiative corrections to the Casimir effect, only has been discussed global issues. An exception is the discussion developed by Robaschik et al [16]. With this scenario in mind, it is natural to ask the important question: how to implement using the standard weak-coupling perturbative expansion in quantum field theory, the perturbative renormalization algorithm, assuming the presence of rigid boundaries (hard-walls). Being more specific, how to implement, the one-loop perturbative renormalization of a self-interacting scalar theory assuming boundary conditions which do break translational symmetry. Our intent studying these issues is linked also with the following question: does the infrared problem have a solution in such theories where translational invariance is broken? Note that a resummation of infrared divergences, generating a thermal mass is a standard procedure in scalar theories at finite temperature.

Let us briefly discuss the infrared divergences in field theory. Although the singular ultraviolet behavior of a theory is independent of the sector (vacuum, thermal, etc.), the infrared divergences strongly depend on the sector in which a given theory is being examinated [17]. Using dimensional regularization [18] [19] it is very hard to see the physical significance of the infrared divergences. It is clear that infrared divergences should be absence in the cross section of a physically observed process. In $(QED)_4$ is refered to the Bloch-Nordsieck theorem [20]. In QCD the same mechanism is expected to work. Nevertheless the situation is quite different since in $(QED)_4$ one encounters only soft divergences and in $(QCD)_4$ appears colinear divergences. In $(QCD)_4$ soft cancelations was demonstrated at the one-loop level in some processes where also colinear divergences cancel out. There is a powerful theorem, the Kinoshita-Poggio-Quinn (KPT) theorem [21] that ensures the absence of infrared divergences in off-shell Green's functions in massless renormalizable field theories. Temperature effects also can solve the infrared problem in some models in quantum field theory [22]. For a recent treatment in non-abelian gauge theories at high temperature, and the infrared problem, see for example Ref. [23]. As we discussed, also in massless scalar $\lambda\varphi^4$ theory, if we assume thermal equilibrium with a reservoir, the infrared problem can be solved after a ressumation procedure. The standard argument is to use the Dyson-Schwinger equation to write a non-perturbative version of the self-energy gap equation, or use the composite operator formalism [24] [25] [26].

We would like to call the attention of the reader that there are some disagreement in the literature implementing the one-loop perturbative renormalization in finite size systems with break of translational invariance. For example, Albuquerque et al [27] founded in the one-loop approximation that the mass counterterm depends on the size of the compact dimension in the $\lambda\varphi^4$ theory. Also Malbouisson et al [28] assumed a self-interacting scalar field confined between two infinite parallel plates. Using the techniques developed by Ananos et al [26] these authors didn't find any surface countertem in the $\lambda\varphi^4$ theory at finite temperature. Furthermore, these authors were able to define a thermal and size dependent mass and coupling constant in these systems where translational invariance is broken.

The purpose of this paper is to present a detailed calculation of the renormalization at the one-loop level of the $\lambda\varphi^4$ theory at finite temperature, assuming also that one of the spatial coordinates lies in a finite interval. Since this assumption is not sufficient to explicity breaking of the translational symmetry, we are also introducing boundary surfaces where the field satisfies appropriate boundary conditions. In this situation, we are breaking the translational invariance of the theory. This paper is the natural extension of the papers of Fosco and Svaiter [29] and also Caicedo and Svaiter [30]. It may help us to understand the renormalization procedure in systems at finite temperature where there is a break of translational symmetry. We are discussing the Dirichlet-Dirichlet (DD) and also Neumann-Neumann (NN) boundary conditions. For the case of Dirichlet-Dirichlet boundary conditions the model is free of infrared divergences. In the case of Neumann-Neumann boundary conditions infrared divergences associated with the zero modes will appear in the case of bare massless fields. We are showing that there is no meaning for a thermal mass or size dependent mass in such situations. Consequently, using a resummation procedure it is not possible to solve the infrared problem in the case of in the case of Neumann-Neumann boundary conditions.

The organization of the paper is the following: in the section II we sketch the general formalism, deriving the one-loop two-point and also the four-point function of the theory. In section III we use two different analytic regularization procedures, i.e, dimensional regularization and zeta function analytic regularization to identify the polar contributions that appear in the expression of the one-loop two and also four-point Schwinger functions. In the conclusions we are discussing alternative solution for the problem of the surfaces counterterms and also how to circuvented the problem for the infrared divergences is developed in section IV. In this paper we use $\hbar = c = k_B = 1$.

2 General Formalism and the Finite Temperature Generating Functional of Schwinger Functions

The static properties of finite temperature field theory can be derived from the partition function [31]. To obtain the partition function the starting point is the Feynman, Matheus and Salam approach [32]. Thus, let us consider the generating functional of complete Green's functions for a self-interacting scalar field theory defined in a flat d-dimensional Euclidean space $Z(h)$, given by

$$Z(h) = \int [d\varphi] \, \exp\left(-S[\varphi] + \int d^dx \, h(x) \, \varphi(x)\right), \tag{1}$$

where $[d\varphi]$ is a translational invariant measure i.e., a formal product $\prod_{x\in R^d} d\varphi(x)$ of Lebesgue measures at every point in R^d and $S[\varphi]$ is the classical action associated with the scalar field. The quantity $Z(h)$ can be regarded as the functional integral representation for the imaginary time evolution operator $\langle\varphi_2|U(t_2,t_1)|\varphi_1\rangle$ with the boundary conditions $\varphi(t_1,\vec{x}) = \varphi_1(\vec{x})$ and $\varphi(t_2,\vec{x}) = \varphi_2(\vec{x})$. The quantity $Z(h)$ gives the transition amplitude from the initial state $|\varphi_1>$ to a final state $|\varphi_2>$ in the presence of some scalar source of compact support. As usual, the generating functional of the connected correlation functions shall be given by $W(h) = \ln Z(h)$. In a free scalar theory the partition function $Z(h)$ and also $W(h)$ can be calculated exactly. Regarding the Lagrangian density, we assume it to be

$$\mathcal{L}(\varphi, \partial\varphi) = \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}m_0^2\varphi^2 + \frac{1}{4!}\lambda_0\varphi^4, \tag{2}$$

where m_0 is the bare mass and λ_0 is the bare coupling constant of the model. We are assuming $m_0^2 \geq 0$ and also $\lambda_0 > 0$. The Euclidean n-point correlation functions, i.e., the n-point Schwinger functions are given by the expectation value with respect to the weight $\exp(-S(\varphi))$ defined as

$$G^{(n)}(x_1, x_2, ., x_n) = \frac{1}{Z(h)} \frac{\delta^n Z(h)}{\delta h(x_1)..\delta h(x_n)}|_{h=0}. \tag{3}$$

It is important to keep in mind that doing an appropriate analytic continuation in the time variable, the integrand of the functional integral changes from an ascillatory to a positive one, characterizing a genuine stochastic process. The n-point connected correlation functions $G_c^{(n)}(x_1,x_2,.,x_n)$ are given by

$$G_c^{(n)}(x_1,x_2,.,x_n) = \frac{1}{Z(h)}\frac{\delta^n W(h)}{\delta h(x_1)..\delta h(x_n)}|_{h=0}. \tag{4}$$

Finally, the generating functional of connected one-particle irreducible correlation functions (the effective action) is introduced by performing a Legendre transformation on $W(h)$,

$$\Gamma(\varphi_0) = -W(h) + \int d^dx\,\varphi_0(x)h(x), \tag{5}$$

and let us define the proper vertices $\Gamma^{(n)}(x_1,..,x_n)$ as:

$$\Gamma^{(n)}(x_1,..,x_n) = \frac{\delta^n\Gamma(\varphi_0)}{\delta\varphi_0(x_1),..\delta\varphi_0(x_n)}|_{\varphi_0=0}, \tag{6}$$

where the normalized vacuum expectation value of the field $\varphi_0(x)$ is given by

$$\varphi_0(x) = \frac{\delta W}{\delta h(x)}. \tag{7}$$

It is clear that for the case of a single scalar field, for a zero normalized vacuum expectation value of the field $\varphi_0(x)$, the effective action may be represented as a functional power series around the value $\varphi_0 = 0$, with the form

$$\Gamma(\varphi_0) = \sum_{n=0}^{\infty}\frac{1}{n!}\int d^dx_1..d^dx_n\,\Gamma^{(n)}(x_1,...,x_n)\varphi_0(x_1)...\varphi_0(x_n). \tag{8}$$

If the bare coupling constant vanishes, i.e., $\lambda_0 = 0$, the generating functional of all n-point Schwinger functions $Z(h)$ can be calculated exactly, since we have to evaluate only Gaussian integrals. After some manipulations we have that the Gaussian generating functional $Z_0(h)$ is given by

$$Z_0(h) = \exp\left(\frac{1}{2}\int d^dx\int d^dy\; h(x)\,G_0^{(2)}(x-y,m_0)h(y)\right), \tag{9}$$

where the two-point Schwinger function (the inverse kernel) satisfies

$$(-\Delta_x + m_0^2)G_0^{(2)}(x-y,m_0) = \delta^d(x-y). \tag{10}$$

In this situation the free Euclidean field is a gaussian random variable defined by its two-point correlation function

$$G_0^{(2)}(x-y,m_0) = \left\langle x|(-\Delta+m_0^2)^{-1}|y\right\rangle, \tag{11}$$

and the two-point Schwinger function has a well known Fourier representation given by

$$G_0^{(2)}(x-y,m_0) = \frac{1}{(2\pi)^d}\int d^dp\frac{e^{ip(x-y)}}{(p^2+m_0^2)}. \tag{12}$$

In the next chapter we will show that the two-point function $G_0^{(2)}(x-y,m_0)$ can be expressed in terms of the modified Bessel function of the third kind, or the Macdonald's function $K_\mu(x)$. At the moment we are not interested to evaluate the two-point Schwinger function, but only analyse the behavior of $G_0^{(2)}(x-y,m_0)$ in an ϵ neighborhood. Let us assume that $m|x-y| << 1$. In this case for $d \geq 3$ we can use that $G_0^{(2)}(x-y,m_0^2) \approx G_0^{(2)}(x-y,m_0^2=0) = |x-y|^{-(d-2)}$. For $d=2$, the mass parameter can not be eliminated from the denominator and we have the following short distance behavior: $G_0^{(2)}(x-y,m_0^2) \propto \ln(m|x-y|)$. It is well known that a massless two-dimensional scalar field theory is not a consistent theory, since the model has severe infrared divergences. There are different proposes that try to circumvent the problem. We only enumerate some of them. First one may violate the positivity of the state vector space. Another choice is to restrict the test function of the theory, and finally we can introduce a cut-off in the definition of the positive and negative Wightman functions. It is clear that this is equivalent to introduce a box to regulate the theory in the infrared. Latter we will discusss other strategies to solve the problem of the infrared divergences in scalar theoies.

Going back to the generating functional of all Schwinger functions, for $\lambda_0 \neq 0$ it is not possible to find a closed exact expression for the partition function, and a perturbative expansion is mandatory. Let us assume the weak-coupling perturbative expansion of the theory. It is important to point out that the partition function can be defined in arbitrary geometries and classical boundary conditions must be implemented in the two-point Schwinger function, restricting the space of functions that appear in the functional integrals. If we want to include thermal effects, and assuming thermal equilibrium, from the Feynman, Matheus and Salam formula we have:

$$\left\langle\varphi_b|e^{-iH(t_f-t_i)}|\varphi_a\right\rangle = \int_{\varphi(t_i)=\varphi_a}^{\varphi(t_f)=\varphi_b} \exp\left(i\int_{t_i}^{t_f} dt \int d^{d-1}x\,\mathcal{L}(\varphi,\,\partial\varphi)\right), \tag{13}$$

where we have to assume that $t_f - t_i = -i\beta$ and set also $\varphi_a = \varphi_b$ and the sum over all φ_a must be performed in order to produce the trace. The partition function $Tr\left[e^{-\beta\,H}\right]$ is given by

$$Tr\left[e^{-\beta\,H}\right] = \int_{periodic} [d\varphi] \exp\left(i\int_{ti}^{t_i-i\beta} dt \int d^{d-1}x\,\mathcal{L}(\varphi,\,\partial\varphi)\right), \tag{14}$$

where the integration over the fields satisfying $\varphi(t_i - i\beta, \vec{x}) = \varphi(t, \vec{x})$. Since the time integration must go from some t_i to $t_i - i\beta$, let $t_i = 0$ and set the contour sum along the negative imaginary axis from 0 to $-i\beta$. Thus $t = -i\tau$ where $0 \leq \tau \leq \beta$, and we have

$$Z(h)|_{h=0} = \int_{periodic} [d\varphi] \exp\left(\int_0^\beta d\tau \int d^{d-1}x\,\mathcal{L}(\varphi, \partial\varphi)\right). \tag{15}$$

To generate the n-point Schwinger functions we need to coupled the field with an external source. We will assume that the system is confined between two paralel hyperplates, that we call the Casimir configuration, localized at $z = 0$ and $z = L$, and we are using cartesian coordinates $x^\mu = (\vec{r}, z)$, where $\vec{r}$ is a $(d-1)$ dimensional vector perpendicular to the $\vec{z}$ vector. Note that since we assume thermal equilibrium with a reservoir, we have periodicity in the first coordinate and $0 \leq r_1 \leq \beta$. See for example Ref. [33], or for a complete review of quantum field theory at thermal equilibrium, see for example Ref. [34]. The choice of Dirichlet-Dirichlet boundary conditions means that the scalar field satisfies

$$\varphi(\vec{r}, z)|_{z=0} = \varphi(\vec{r}, z)|_{z=L}, \tag{16}$$

and Neumann-Neumann boundary conditions means that

$$\frac{\partial}{\partial z}\varphi(\vec{r}, z)|_{z=0} = \frac{\partial}{\partial z}\varphi(\vec{r}, z)|_{z=L}. \tag{17}$$

In the next section we discuss the perturbative renormalization at the one-loop level of the field theory in the presence of rigid boundaries. The great interest of this question is that when systems contain macroscopic structures, how it is possible to implement the renormalization program. We examinate how does the weak-coupling perturbative expansion and the renormalization program can be implemented. In order to identify the singular part of the one-loop two-point Schwinger function, we use a combination of dimensional and zeta function analytic regularization procedures. We also present the detailed form of the surface divergences. Note that due to our choice (two-parallel hyperplates), the region outside the boundaries is the union of two-simple connected domains. The renormalization of the field theory in such exterior regions must be carried out along the same lines as for the interior region. For simplicity we are considering only the interior region.

3 The regularized one-loop two and four-point Schwinger functions

The aim of this section is to rederive a well known result, adding finite temperature effects in the problem. In order to implement the renormalization program in a scalar field theory where we assume Dirichlet-Dirichlet or Neumann-Neumann boundary conditions on rigid surfaces introduce surface divergences. To write the full renormalized action for the theory with rigid boundaries we need two regulators: the first one is the usual ϵ that is introduced in the dimensional regularization procedure and the second one, that we call η, represents the distance to a boundary. According to this we will show that the full renormalized action must be given by:

$$\begin{aligned} S(\varphi) \quad &= \int_0^L dz \int d^{d-1}r \left(\frac{A(\epsilon)}{2} (\partial_\mu \varphi)^2 + \frac{B(\epsilon)}{2} \varphi^2 + \frac{C(\epsilon)}{4!} \varphi^4 \right) \\ &+ \int d^{d-1}r \left(c_1(\eta) \varphi^2(\vec{r}, 0) + c_2(\eta) \varphi^2(\vec{r}, L) \right) \end{aligned}$$

$$+\int d^{d-1}r\left(c_3(\eta)\varphi^4(\vec{r},0)+c_4(\eta)\varphi^4(\vec{r},L)\right), \tag{18}$$

where $A(\epsilon)$, $B(\epsilon)$ and $C(\epsilon)$ are the usual coefficients for the bulk counterterms and the coefficients $c_i(\eta)$ $i=1,..4$, which depend on the boundary conditions for the field, are the coefficients for the surface counterterms. As usual all of these coefficients must be calculated order by order in perturbation theory. Note that the system that we are interested in is invariant under translation along the directions parallel to the plates. This implies that what is conserved is not the full momentum. For such conditions, to implement the perturbative renormalization a more convenient representation for the n-point Schwinger functions is a mixed $(\vec{p},z)$ representation. Careless one-loop perturbation theory generates ultraviolet counterterms that depends on the distance between the plates or also absence of surface counterterms [27] [28].

Briefly speaking, in the Matsubara formalism all the Feynman rules are the same as in the zero temperature case, except that the momentum-space integrals over the zeroth component is replaced by a sum over discrete frequencies. For the case of bosons fields we have to perform the replacement

$$\int\frac{d^dp}{(2\pi)^d}f(p)\to\frac{1}{\beta}\sum_n\int\frac{d^{d-1}p}{(2\pi)^{d-1}}f(\frac{2n\pi}{\beta},\vec{p}), \tag{19}$$

and note that we are using the following notation: $(\int d^{d-1}r=\int_0^\beta dr_1\int d^{d-2}r)$.

We begin the study of the interacting theory by building the one-loop correction $\left(G_1^{(2)}(\lambda_0,x,x')\right)$ to the bare two-point Schwinger function $G_0^{(2)}(x,x')$, for both the DD and NN boundary conditions. Using the Feynman rules we have that $G_1^{(2)}(\lambda_0,\vec{r}_1,z_1,\vec{r}_2,z_2)$ can be writtem as

$$G_1^{(2)}(\lambda_0,\vec{r}_1,z_1,\vec{r}_2,z_2)=\frac{\lambda_0}{2}\int d^{d-1}r\int_0^L dz\;G_0^{(2)}(\vec{r}_1-\vec{r},z_1,z)\,G_0^{(2)}(\vec{0},z)\,G_0^{(2)}(\vec{r}-\vec{r}_2,z,z_2). \tag{20}$$

Here we point out that even though the functions $G_0^{(2)}(\vec{r}_1-\vec{r}_2,z_1,z_2)$ and $G_0^{(2)}(\vec{r}_2-\vec{r}_3,z_2,z_3)$ are singular at coincident points $(\vec{r}_1=\vec{r}_2,\ z_1=z_2)$ and $(\vec{r}_2=\vec{r}_3,\ z_2=z_3)$, the singularities are integrable for points outside the plates. It is worth mentioning that the most simple way to take into account the boundary is the following: both boundary conditions can be implemented through the explicit form of the free two-point Schwinger function $G_0^{(2)}(x-y,m_0)$. A straightforward substitution yields the order λ_0 correction to the bare two-point Schwinger function in the one-loop approximation for the case of Dirichlet-Dirichlet boundary conditions. Using the Feynman rules, $G_2^{(4)}(\lambda_0,x_1,x_2,x_3,x_4)$, i.e., the $O(\lambda_0^2)$ correction to the bare one-loop four-point Schwinger function, is given by

$$\begin{aligned} G_2^{(4)}(\lambda_0,\vec{r}_1,z_1,\vec{r}_2,z_2,\vec{r}_3,z_3,\vec{r}_4,z_4) &= \frac{\lambda_0^2}{2}\int d^{d-1}r\int d^{d-1}r'\int_0^L dz\int_0^L dz'\; G_0^{(2)}(\vec{r}_1-\vec{r},z_1,z) \\ & G_0^{(2)}(\vec{r}_2-\vec{r},z_2,z)\left(G_0^{(2)}(\vec{r}-\vec{r'},z,z')\right)^2 \\ & G_0^{(2)}(\vec{r'}-\vec{r}_3,z',z_3)G_0^{(2)}(\vec{r'}-\vec{r}_4,z',z_4). \end{aligned} \tag{21}$$

Note that we supress the m_0 term in each expression. Again, all G_0's are singular at coincident points, but the singularities are integrable for points outside the plates, except for $G_0^{(2)}(\vec{r}-\vec{r'},z,z')$. Having in mind the discussion above, in this section we are interested to study the following expressions:

$$\frac{\lambda_0}{2}\int d^{d-1}r\int_0^L dz\;\left(G_0^{(2)}(\vec{0},z)\right), \tag{22}$$

and

$$\frac{\lambda_0^2}{2}\int d^{d-1}r\int d^{d-1}r'\int_0^L dz\int_0^L dz'\left(G_0^{(2)}(\vec{r}-\vec{r'},z,z')\right)^2. \tag{23}$$

Consequently, let us study $\frac{1}{2}G_0^{(2)}(\vec{0},z)\equiv I(z,m_0,L,\beta,d)$ and define the following quantities: $\frac{1}{b}=\frac{2}{\beta}$, $L=a$ and finally the dimensionless coupling constant $g=\mu^{4-d}\lambda_0$. Therefore, we have that the argument in the integral defined in Eq.(22), $I(z,m_0,a,b,d)$ can be written as

$$I(z,m_0,a,b,d)=\frac{g}{2(2\pi)^{d-2}ab}\sum_{n=-\infty}^{\infty}\sum_{n'=1}^{\infty}\sin^2(\frac{n'\pi z}{a})\int d^{d-2}p\frac{1}{\left(\vec{p}^2+(\frac{n'\pi}{a})^2+(\frac{n\pi}{b})^2++m_0^2\right)}. \tag{24}$$

There are two points that we would like to stress. First is the fact that to perform analytic regularizations, we have to introduce a parameter μ with dimension of mass in order to have dimensionless quantities raised to a complex power. Second is the fact that the generalization for the case of Neumann boundary conditions is straightforward, although in this case infrared divergences associated with the $n = 0$ mode will appear in the case of massless scalar field. To circumvent this situation we must have a finite Euclidean volume to regularize the model in the infrared or trying to implement a resummation to generate a thermal mass. We will return to this point latter.

Using trigonometric identities, it is convenient to write the amputated one-loop two-point Schwinger in two parts. The first is the contributions that do not depend on the distance to the boundary and the second part is the contributions that depend on the distance to the boundary. Therefore we have that the quantity $I(z, m_0, a, b, d)$ can be split in two parts $T_1(m_0, a, b, d)$ and $T_2(z, m_0, a, b, d)$, i.e.:

$$I(z, m_0, a, b, d) = T_1(m_0, a, b, d) + T_2(z, m_0, a, b, d). \tag{25}$$

The first quantity $T_1(m_0, a, b, d)$ that does not depend on the distance to the boundaries can be expressed in the following way:

$$T_1(m_0, a, b, d) = I_0(m_0, a, b, d) + I_1(m_0, a, b, d) + I_2(m_0, a, b, d), \tag{26}$$

where each of the terms are given respectivelly by:

$$I_0(m_0, a, b, d) = -\frac{g}{16(2\pi)^{d-2}ab}\int d^{d-2}p\frac{1}{(\vec{p}^2 + m_0^2)}, \tag{27}$$

$$I_1(m_0, a, b, d) = \frac{g}{8\,(2\pi)^{d-2}\,ab}\sum_{n=1}^{\infty}\int d^{d-2}p\frac{1}{\left(\vec{p}^2 + m_0^2 + (\frac{n\pi}{a})^2\right)}, \tag{28}$$

and finally

$$I_2(m_0, a, b, d) = \frac{g}{4(2\pi)^{d-2}ab}\sum_{n,n'=1}^{\infty}\int d^{d-2}p\frac{1}{\left(\vec{p}^2 + (\frac{n\pi}{a})^2 + (\frac{n'\pi}{b})^2 + m_0^2\right)}. \tag{29}$$

The contributions that depends on the distance to the boundaries given by $T_2(z, m_0, a, b, d)$ can be split in the following way:

$$T_2(z, m_0, a, b, d) = I_3(z, m_0, b, d) + I_4(z, m_0, a, b, d) + I_5(z, m_0, b, d) + I_6(z, m_0, a, b, d). \tag{30}$$

Each of the terms that contributes to $T_2(z,m_0,a,b,d)$ are given by:

$$I_3(z,m_0,b,d)=\frac{g}{2b}h(d)\int_{m_0}^{\infty}dv(v^2-m_0^2)^{\frac{d-4}{2}}\exp(-2vz), \tag{31}$$

$$I_4(z,m_0,a,b,d)=\frac{g}{2b}h(d)\int_{m_0}^{\infty}dv\,(v^2-m_0^2)^{\frac{d-4}{2}}(\coth av-1)\cosh 2vz, \tag{32}$$

$$I_5(z,m_0,b,d)=\frac{g}{b}h(d)\sum_{n=1}^{\infty}\int_{m_0}^{\infty}dv\,\left(v^2-m_0^2-(\frac{n\pi}{b})^2\right)^{\frac{d-4}{2}}\exp(-2vz), \tag{33}$$

and finally

$$I_6(z,m_0,a,b,d)=\frac{g}{b}h(d)\sum_{n=1}^{\infty}\int_{\alpha}^{\infty}dv\,\left(v^2-m_0^2-(\frac{n\pi}{b})^2\right)^{\frac{d-4}{2}}(\coth av-1)\cosh 2vz. \tag{34}$$

In the expression above we have that the quantity α is given by

$$\alpha=\left(m_0^2+(\frac{n\pi}{b})^2\right)^{\frac{1}{2}}, \tag{35}$$

and $h(d)$ that appears in Eqs.(31), (32), (33) and (34) is an entire function given by

$$h(d)=\frac{1}{4(4\pi)^{\frac{d-2}{2}}}\frac{1}{\Gamma(\frac{d-2}{2})}. \tag{36}$$

Let us investigate each contribution in details. Using dimensional regularization we obtain for $I_0(m_0,d)$ the following expression:

$$I_0(m_0,a,b,d)=-\frac{g}{16\,ab(2\sqrt{\pi})^{d-2}}\Gamma(2-\frac{d}{2})(m_0^2)^{\frac{d}{2}-2}. \tag{37}$$

An analytic expression for the Gamma function $\Gamma(z)$, defining in the whole complex plane can be found and in the neighborhood of a pole $z=-n$, $(n=0,1,2..)$ the Gamma function has the representation

$$\Gamma(z)=\frac{(-1)^n}{n!}\frac{1}{(z+n)}+\Omega(z+n), \tag{38}$$

with regular part $\Omega(z+n)$. Using that $4-d=\epsilon$ an the duplication formula for the Gamma function $\Gamma(z)$ we have

$$I_0(m_0,a,b,d)|_{d=4}=-\frac{g}{16\pi\,ab}\frac{1}{m_0^{\epsilon}}\left(\frac{1}{\epsilon}+\Omega(\epsilon)\right). \tag{39}$$

At this moment we have different renormalization schemes to choose. We can choose the minimal subtraction (MS) scheme. In the minimal subtraction scheme we eliminate only the pole term $\frac{1}{\epsilon}$ in the dimensionaly regularized expression for the Schwinger functions. Another choice is the modified minimal subtraction ($\overline{\rm MS}$) scheme. In the modified minimal subtraction scheme we eliminate not only the pole term $\frac{1}{\epsilon}$ but also the regular part around the pole. Note that in the minimal subtraction scheme the counterterms acquire the simplest expression while the renormalized Schwinger functions have more complicated expressions. Let us analyse the second expression given by $I_1(m_0,a,b,d)$. Using dimensional regularization it is possible to show that $I_1(m_0,a,b,d)$ is given by

$$I_1(m_0,a,b,d) = \frac{g}{8(2\sqrt{\pi})^{d-2}ab}\Gamma(2-\frac{d}{2})\sum_{n=1}^{\infty}\frac{1}{\left(m_0^2+(\frac{n\pi}{a})^2\right)^{2-\frac{d}{2}}}. \tag{40}$$

We note that to extract a finite result from $I_1(m_0,a,b,d)$ we still have to use the analytic extension of the Epstein-Hurwitz zeta function. A direct calculation gives

$$\begin{aligned} I_1(m_0,a,b,d) \;=\; & -\frac{g}{8ab}m_0^{d-4}\frac{\sqrt{\pi}}{(2\sqrt{\pi})^{d-1}}\Gamma(2-\frac{d}{2}) + \\ & \frac{g\,m_0^{d-3}}{8b}\frac{1}{(2\pi)^{d-1}}\left(\Gamma\left(\frac{3-d}{2}\right)+4\sum_{n=1}^{\infty}(am_0n)^{\frac{3-d}{2}}K_{\frac{3-d}{2}}(2m_0na)\right). \end{aligned} \tag{41}$$

The first term in the above equation is a polar part and the second one is finite. Assuming then minimal subtraction scheme $I_1(m_0,a,b,d)$ becomes finite. The next term that we have to analyse is $I_2(m_0,a,b,d)$ defined by:

$$I_2(m_0,a,b,d) = \frac{g}{4ab}\frac{1}{(2\pi)^{d-2}}\sum_{n,n'=1}^{\infty}\int d^{d-2}p\frac{1}{\left(\vec{p}^2+(\frac{n\pi}{a})^2+(\frac{n'\pi}{b})^2+m_0^2\right)}. \tag{42}$$

The contribution given by the above equation is one part of the amputated one-loop two-point Schwinger function which does not depend from the distance to the boundaries and in the renormalization procedure will require only a usual bulk counterterm. The form of the counterterm is given by the principal part of the Laurent expansion of Eq.(42) around some d, which must be given by the analytic extension of the Epstein zeta function in the complex d plane. The structure of the divergences of the Epstein zeta function is well know in the literature [35] [36] [37] [38]. Since the polar structure of the above equation can be found in the literature, we will concentrate only on the position dependent divergent part given by $T_2(z, m_0, a, b, d)$. We are now in position to discuss the behavior of $I_3(z, m_0, b, d), I_4(z, m_0, a, b, d), I_5(z, m_0, b, d)$ and finally $I_6(z, m_0, a, b, d)$.

Let us first analyse $I_3(z, m_0, b, d)$. Using the following integral representation of the modified Bessel functions of third kind, or the Macdonald's functions $K_\nu(x)$ [39]:

$$\int_u^\infty (x^2 - u^2)^{\nu-1} e^{-\mu x}\, dx = \frac{1}{\sqrt{\pi}} (\frac{2u}{\mu})^{\nu-\frac{1}{2}} \Gamma(\nu) K_{\nu-\frac{1}{2}}(u\mu), \tag{43}$$

which is valid for for $u > 0$, $Re\,(\mu) > 0$ and $Re\,(\nu) > 0$, $I_3(z, m_0, a, b, d)$ can be written in terms of these functions. A simple substitution gives

$$I_3(z, m_0, a, b, d) = \frac{2}{b} \frac{h(d)}{(2\sqrt{\pi})^{d-1}} (\frac{m_0}{z})^{\frac{d-3}{2}} K_{\frac{d-3}{2}}(2m_0 z). \tag{44}$$

Using a asymptoptic formula for the Bessel function we have that $I_3(z, m_0, a, b, d)$ is given by

$$I_3(z, m_0, a, b, d) = \frac{2}{b} \frac{h(d)}{(2\sqrt{\pi})^{d-1}} \frac{\Gamma(\frac{d-3}{2})}{z^{d-3}}. \tag{45}$$

We can see that we have a divergent behavior as $z \to 0$, that demands a surface counterterm. Let us show that the other terms also contains surface divergences. Consequently, let us study the $I_4(z, m_0, a, b, d)$. To proceed in the calculations we have to extend the binomial series for positive or negative integral exponents that is written in the form

$$(1 + x)^k = \sum_{n=0}^{\infty} C_n^k\, x^n. \tag{46}$$

It is possible to show that the binomial expansion holds first for any real exponent α, $|x| < 1$ and $\alpha\, \epsilon\, R$, i.e.,

$$(1 + x)^\alpha = \sum_{n=0}^{\infty} C_\alpha^n\, x^n, \tag{47}$$

where C_α^n are the generalization of the binomial coefficients. Since we are using dimensional regularization, it is possible to extend the binomial expansion when both the exponent α as well the variable x assume complex values. For this purpose we have to use the following theorem:

For any complex exponent α and any complex z in $|z| < 1$, the binomial series

$$\sum_{n=0}^{\infty} C_{\alpha}^{n} z^{n} = 1 + C_{\alpha}^{1} z + .. + C_{\alpha}^{n} z^{n} + .. \tag{48}$$

converges and has the the sum of the principal value of the power $(1+z)^{\alpha}$, where the principal value of the power b^{a} where a and b denotes any complex numbers, with $b \neq 0$ as the only condition is given by the number uniquely defined by the formula $b^{a} = \exp(a \ln b)$, where $\ln b$ is given its principal value. Going back to $I_4(z, m_0, a, b, d)$ using the generalization of the binomial theorem, let us define $C^{(1)}(d,k) = \frac{1}{2} h(d)(-1)^{k} C^{k}_{\frac{d-4}{2}}$ to obtain

$$I_4(z, m_0, a, b, d) = \frac{g}{a^{d-3} b} \sum_{k=0}^{\infty} C^{(1)}(d,k)(m_0 a)^{2k} \int_{m_0 a}^{\infty} u^{d-4-2k} (\coth u - 1) \cosh(\frac{2uz}{a}). \tag{49}$$

Let us use the following integral representation of the Gamma function,

$$\int_{0}^{\infty} dt\, t^{\mu-1} e^{-\nu t} = \frac{1}{\nu^{\mu}} \Gamma(\mu), \quad Re(\mu) > 0, \;\; Re(\nu) > 0, \tag{50}$$

and also the following integral representation of the product of the Gamma function times the Hurwitz zeta function

$$\int_{0}^{\infty} dt\, t^{\mu-1} e^{-\alpha t} (\coth t - 1) = 2^{1-\mu} \Gamma(\mu) \zeta(\mu, \frac{\alpha}{2} + 1) \quad Re(\alpha) > 0, \;\; Re(\mu) > 1, \tag{51}$$

where $\zeta(s, u)$ is the Hurwitz zeta function defined by [39]

$$\zeta(s, u) = \sum_{n=0}^{\infty} \frac{1}{(n+u)^{s}}, \quad Re(s) > 1 \quad u \neq 0, -1, -2... \tag{52}$$

It is not difficult to show that $I_4(z, m_0, a, b, d)$ contains surface divergences at $z = 0$ and also $z = a$. For more details, see for example Ref. [40]. The other expression that we have to study is $I_5(z, m_0, a, b, d)$. Using an integral representation of the Bessel function of third kind we have:

$$I_5(z, m_0, a, b, d) = \frac{1}{b} \frac{1}{(2\sqrt{\pi})^{d-1}} \sum_{n=1}^{\infty} (\frac{\alpha}{z})^{\frac{d-3}{2}} K_{\frac{d-3}{2}}(2\alpha z). \tag{53}$$

Using an asymptotic representation of the Bessel function it is posssible to present also the singular behavior near $z = 0$. Let us finally investigate $I_6(z, m_0, a, b, d)$. A simple calculation for the massless case gives

$$I_6(z, m_0, a, b, d)|_{m=0} = \frac{1}{a^{d-3}b} \sum_{k=0}^{\infty} C^{(2)}(d, k)(\frac{a}{b})^{2k} \sum_{n=1}^{\infty} n^{2k} \int_{\frac{n\pi a}{b}}^{\infty} du\, u^{d-4-2k}(\coth u - 1) \cosh(\frac{2uz}{a}), \tag{54}$$

where $C^{(2)}(d, k) = h(d)(-1)^k C^k_{\frac{d-4}{2}} \pi^{2k}$ is an entire function in the complex d plane. The integral that appear in Eq.(54) cannot be evaluated explicity in terms of well known functions. Nevertheless it is possible to write Eq.(54) in a convenient way where the structure of the divergences near the plate when $y \to b$ appear. For details see Ref. [40]. In the next section we will investigate the singularities of the four-point Schwinger function.

4 The four-point Schwinger function in the one-loop approximation

We now turn our attention back to the four-point Schwinger function in the one-loop approximation. For simplicity we are studying only the zero temperature case. In this chapter we are following the discussion developed in Ref. [30]. Introducing new variables as $u_\pm \equiv z \pm z'$, and also $(\vec{\rho} = \vec{r} - \vec{r'})$, the zero-temperature two-point Schwinger function in the tree-level $G_0^{(2)}(\vec{\rho}, z, z')$ can be split into

$$G_0^{(2)}(\vec{\rho}, z, z') = G_+^{(2)}(\vec{\rho}, u_+) + G_-^{(2)}(\vec{\rho}, u_-), \tag{55}$$

where we are defining $A_n(a, m_0, d, \vec{\rho})$ by

$$A_n(a, m_0, d, \vec{\rho}) = \frac{1}{(2\pi)^{d-1}} \int d^{d-1}p \frac{e^{i\vec{p}.\vec{\rho}}}{(\vec{p}^2 + (\frac{n\pi}{a})^2 + m_0^2)}, \tag{56}$$

and we have that $G^{(2)}_{\pm}(\vec{\rho},u_{\pm})$ can be expressed as

$$G^{(2)}_{\pm}(\vec{\rho},u_{\pm}) = \mp\frac{1}{a}\sum_{n=1}^{\infty}\cos(\frac{n\pi u_{\pm}}{a})A_n(a,m_0,d,\vec{\rho}). \tag{57}$$

Before continuing, let us present a explicit formula of the free two-point Schwinger function $G^{(2)}_{\pm}(\rho,u_{\pm})$ in terms of Bessel functions. Let us define an analytic function $f(d)$ by

$$f(d) = \frac{1}{\sqrt{\pi}(2\pi)^{\frac{d-1}{2}}}\frac{\Gamma(\frac{d-2}{2})}{\Gamma(\frac{d-3}{2})}. \tag{58}$$

Strictly speaking, it is possible to show that we can write $G^{(2)}_{\pm}(\rho,u_{\pm})$ in terms of the Bessel function of third kind. To this end we use the standard formula

$$\frac{1}{(2\pi)^d}\int d^d r F(r)e^{i\vec{k}.\vec{r}} = \frac{1}{\sqrt{\pi}(2\pi)^{\frac{d}{2}}}\frac{\Gamma(\frac{d-1}{2})}{\Gamma(\frac{d-2}{2})}\int_0^{\infty}F(r)r^{\frac{d}{2}}J_{\frac{d-3}{2}}(kr)dr, \tag{59}$$

which takes us to:

$$G^{(2)}_{\pm}(\rho,u_{\pm}) = \mp\frac{f(d)}{\rho^{\frac{d-3}{2}}a}\sum_{n=1}^{\infty}\cos(\frac{n\pi u_{\pm}}{a})\int_0^{\infty}dp\frac{p^{\frac{d-1}{2}}}{(p^2+(\frac{n\pi}{L})^2+m_0^2)}J_{\frac{d-3}{2}}(p\rho), \tag{60}$$

where $J_{\nu}(x)$ is the Bessel function of the first kind of order ν. The integral in Eq.(60) can be calculated by using the result [39]

$$\int_0^{\infty}dx\frac{x^{\nu+1}J_{\nu}(ax)}{(x^2+b^2)} = b^{\nu}K_{\nu}(ab), \tag{61}$$

implying that it is possible to write $G^{(2)}_{\pm}(\rho,u_{\pm})$ as

$$G^{(2)}_{\pm}(\rho,u_{\pm}) = \mp\frac{f(d)}{\rho^{\frac{d-3}{2}}a}\sum_{n=1}^{\infty}\cos(\frac{n\pi u_{\pm}}{a})\left((\frac{n\pi}{a})^2+m_0^2\right)^{\frac{d-3}{4}}K_{\frac{d-3}{2}}\left(\rho\sqrt{m_0^2+(\frac{n\pi}{a})^2}\right). \tag{62}$$

Using Eq.(55) and the above formula gives us the explicit expression for the two-point Schwinger function in a generic d-dimensional Euclidean space confined between two flat paralel hyperplanes where we assume Dirichlet-Dirichlet boundary conditions. It is hard to use the above expressions for $G^{(2)}_{\pm}(\rho,u_{\pm})$ to investigate the analytic structure of the four-point function for both the bulk and near the boundaries. Nevertheless it is clear that the divergences of the four-point function in the one-loop approximation appear at coincident points and therefore the singular behavior is encoded in the polar part of $M(\lambda_0, a, m, d)$ given by

$$M(\lambda_0, a, m_0, d) = g^2 \int d^{d-1}r \int d^{d-1}r' \int_0^a dz \int_0^a dz' F(\vec{r}, \vec{r}', z, z') \left(G_0^{(2)}(\vec{r} - \vec{r}', z, z')\right)^2 . \quad (63)$$

It is easy to show that $G_2^{(4)}(\lambda_0, a, m_0, d)_{amp}$ is given by

$$G_2^{(4)}(\lambda_0, a, m_0, d)_{amp} =$$
$$\frac{g^2}{2(2\pi)^{2d-2}} \int d^{d-1}r \int d^{d-1}r' \int d^{d-1}k \int d^{d-1}q \sum_{n=1}^{\infty} \frac{e^{i\vec{\rho}.(\vec{q}-\vec{k})}}{(\vec{q}^2 + (\frac{n\pi}{a})^2 + m_0^2)(\vec{k}^2 + (\frac{n\pi}{a})^2 + m_0^2)}, \quad (64)$$

where $F(\vec{r}, \vec{r}', z, z')$ is a regular function. As with the one-loop two-point function, it is not difficult to realize that the above equation has two kinds of singularities, those coming from the bulk and those arising from the behavior near the surface. As before, the behavior in the bulk is as that found in thermal field theory and consequently we will only discuss the singularities that arise from the boundaries. This can be done by studying the polar part of $\tilde{M}(\lambda_0, a, m_0, d)$ given by

$$\tilde{M}(\lambda, a, m_0, d) = \frac{g^2}{2} \int_0^a dz \int_0^a dz' \mathcal{F}(z, z') \left(G_0^{(2)}(\vec{0}, z, z')\right)^2 , \quad (65)$$

where $\mathcal{F}(z, z')$ is a regular function. Now, we recall that the form of $G_{\pm}^{(2)}(\rho, u_{\pm})|_{\rho=0}$ is given by,

$$G_{\pm}^{(2)}(\rho, u_{\pm})|_{\rho=0} = \mp \frac{1}{(2\pi)^{d-1}a} \sum_{n=1}^{\infty} \cos(\frac{n\pi u_{\pm}}{a}) \int d^{d-1}p \frac{1}{\left(\vec{p}^2 + m_0^2 + (\frac{n\pi}{a})^2\right)}, \quad (66)$$

where it is not difficult to show that

$$G_{\pm}^{(2)}(\rho, u_{\pm})|_{\rho=0} = \mp \left(-\frac{1}{2a} A_0(\rho, L, m_0)|_{\rho=0} + f_2(a, m_0, d, \frac{u_{\pm}}{2})\right) . \quad (67)$$

In the above definition we are making use of the auxiliary function $f_2(a, d, m_0, z)$ defined by

$$f_2(a, m_0, d, z) = \frac{1}{2(2\pi)^{d-1}} \int d^{d-1}p \frac{1}{\sqrt{\vec{p}^2 + m_0^2}} \frac{\cosh((a - 2z)\sqrt{\vec{p}^2 + m_0^2})}{\sinh(a\sqrt{\vec{p}^2 + m_0^2})}. \quad (68)$$

Before continue, note that the amputated one-loop two-point Schwinger function can be decomposed in a translational invariant part and another that break the translational invariance, given exactly by $f_2(a, m_0, d, z)$. When we add to find the free propagator we end up with the following expression

$$G_0^{(2)}(\rho, z, z')|_{\rho=0} = f_2(a, m_0, d, \frac{u_-}{2}) - f_2(a, m_0, d, \frac{u_+}{2}). \tag{69}$$

For the sake of simplicity we will discuss only the massless case since the singularities of the massive case have the same structure as the massless one. The function $f_2(a, m_0, d, \frac{u_+}{2})$ is non singular in the bulk, i.e., in the interior of the interval $[0, a]$, while $f_2(a, m_0, d, \frac{u_-}{2})$ has a singularity along the line $z = z'$. Indeed, closer inspection shows that for $0 \leq z, z' \leq a$ the only singularities are those at $u_+ = 0$, $u_+ = 2a$ and also $u_- = 0$. The former two are genuinely boundary singularities (the two conditions imply $z, z' \to 0$ or $z, z' \to a$) while the other coming from $z = z'$ in the whole domain is just the standard bulk singularity. In fact, using the structure of the two point function and showing just those terms from which singularities might arise, one finds that the counterterms for $\tilde{M}$ are given by

$$-\text{pole} \int_0^a dz \int_0^a dz' [\frac{C_1}{(z+z')^{d-2}} + \frac{C_2}{(2a - z - z')^{d-2}} + \frac{C_3}{(z - z')^{d-2}} + ...]^2, \tag{70}$$

where $C_i, i = 1, ..3$ are regular functions that do not depend on z or z'. From this discussion it is clear that in order to render the field theory finite, we must introduce surface terms in the action. This is a general statement. For any fields that satisfy boundary condition that breaks the translational invariance, in addition to the usual bulk counterterms, it is sufficient to introduce surface counterterms in the action to render the theory finite in the ultraviolet [41] [42] [43]. Now we are able to discuss if in the Casimir configuration we are able to solve the infrared problems for the case of Neumann boundary conditions. For the case of massless $(\lambda\varphi^4)_d$ theory at finite temperature, the infrared problem can be solved after a resummation procedure [22] [23] [24] [25] [44]. The key point for the solution of the infrared problem is to use the Dyson-Schwinger equation to rewrite the self-energy gap equation. Simple inspection in Eq.(24) show us that it is not possible to implement such scheme in a situation where there is a break of translational invariance.

A different possibility to solve the infrared problem is to separate the zero mode component of the field, treating the non-zero modes perturbativelly and treating the zero mode exactly. This is a standard procedure in high-temperature field theory, where using the dimensional reduction idea, we relate the thermal Schwinger functions in a d-dimensional Euclidean space to zero temperature Schwinger functions in a $(d-1)$ dimensional Euclidean space [45] [46] [47]. In this situation we have a dimensionally reduced effective theory. The key point in this construction is the fact that the leading infrared behavior of any field theory at high temperature in a d dimensional Euclidean space is governed by the zero frequency Matsubara mode.

5 Discussions and conclusions

In this paper we were interested to analyse the important question of the perturbative expansion and the renormalization program in quantum field theory with boundary conditions that breaks translation symmetry, assuming also that the system is in equilibrium with a reservoir at temperature β^{-1}. To be more specific, the purpose of this paper is to study the renormalization procedure up to one-loop level in the $(\lambda\varphi^4)_d$ theory at finite temperature assuming that the scalar field satisfies Dirichlet-Dirichlet or also Neumann-Neumann boundary conditions on two parallel hyperplates.

We first obtained the regularized one-loop diagrams associated with scalar field defined in the Casimir configuration in a d-dimensional Euclidean space. We first rederive a well-know result that surface divergences appear in the one-loop two-point and four-point Schwinger functions as consequence of the uncertaintly principle. There are at least three different possible solutions that can eliminate these divergences. The first is to take into account that real materials have imperfect conductivity at high frequencies. As was stressed by many authors, the infinities that appear in renormalized values of local observables for the ideal conductor (or perfect mirror) represent a breakdown of the perfect-conductor approximation. A wavelength cutoff corresponding to the finite plasma frequency must be included. The second is is to substitute classical boundary conditions by classical potentials. For previous papers using this idea see for example [48] [49] [50]. A localized boundary with some cut-of can also be use to substitute the potential. As we discussed, it is necessary to renormalize the potential [30]. The third would be given by a quantum mechanical treatment of the boundary conditions . A fruitful approach to avoid surface divergences, discussed by Kennedy et al [51] is to treat the boundary as a quantum mechanical object. This approach was developed by Ford and Svaiter [52] to produce finite values for the renormalized $< \varphi^2 >$ and other quantities that diverge as one approach the classical boundary.

Consequently, we have two main distincts directions in future investigations. The first is related to the infrared divergences of our model. Being more specific, the infrared divergences of massless thermal field theory arise from the zero frequency Matsubara modes. Thus we construct an effective $(d-1)$ dimensional theory by integrating out the nonstatic modes and the zero frequency Matsubara modes which are responsible for infrared divergences can be treated separately. The second direction is related to the surface divergences. In the Euclidean formalism for field theory, one may imagine that, our simplified model of rigid boundaries, is a good approximation only for points in the bulk. However, one might imagine that for points close to the surfaces our approximation is no longer acurate and a model taking into account at least thermal fluctuations of the boundaries must be developed [53]. In other words, a fundamental understanding of the perturbative renormalization algorith in the standard weak-coupling perturbative expansion of an Euclidean field in the presence of fluctuating boundaries is desired. This interesting situation of thermal fluctuating boundaries is under the investigation by the authors.

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