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HARD KAON AND PION CALCULATIONS OF THE DECAYS $K_A \rightarrow K\rho$ AND $K_A \rightarrow K^*\pi$

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INTRODUCTION

The current algebra method [1] along with soft pion hypothesis has been used with considerable success in variety of processes. However, there are cases where this technique does not work so well. Recently, new techniques of applying consistently current algebra method (with pole dominance approximation) have been suggested in these cases. The calculations on strong [2, 3, 4] A_1 and [5, 6] K_A decays, admiting that some of the form factors satisfy a subtracted dispersion relation predict, contrary to the earlier calculations [7] giving very large widths, reasonable widths consistent with experiments. Schnitzer and Weinberg [3] developed a technique of Ward-like identities for the vertex functions to calculate "hard" pion processes $A_1 \rightarrow \rho\pi$ and $\rho \rightarrow \pi\pi$ successfully. Brown and West [2], on the other hand assume dispersion relations for vertex functions with an appropriate fixed invariant, so as to include the poles in all the variables while admitting that the form factors are at most once subtracted. The results obtained by the two methods are identical. The same holds true in the case of the second order renormalization corrections to the $K_{\ell 3}$ form factors as calculated by [8] Glashow and Weinberg and [9] Srivastava. However, the unsubtracted dispersion relation technique seems to be rather straightforward and amounts to writing Feynman diagrams with form factors at the vertices and establishing appropriate current algebra identities. We will discuss in this paper the strong decays $K_A \rightarrow K\rho$ and $K_A \rightarrow K^*\pi$ following the procedure exposed in references 2 and 9. In what follows, we assume that the form factors are at most once subtracted and the non-constant part is calculated in the pole dominance approximation. Along with the expressions for the relevant decay rates we also re-derive the [10] Weinberg sum rules for SU(3) × SU(3) group and also illustrate that the hypothesis of partial conservation of axial current need not hold necessarily, even for pion, for every form factor. The calculated decay rates are in fair agreement with the experimental results considering the present uncertaintities in experimental data.

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$K_A \rightarrow K^*\pi$ and $K^* \rightarrow K\pi$ decays:

I. K_A and K^* Matrix Elements

We introduce the following matrix elements

$$\sqrt{4k_o p_o V^2}\langle \pi^o(k)|A_\mu(0)_3^1|K^{*+}(p)\rangle$$
$$= ie_\nu^{K^*}(p)\left[K_1(q^2)g^{\mu\nu} + K_2(q^2)\, k^\nu(p+k)^\mu + K_3(q^2)\, k^\nu\, q^\mu\right] \quad (1)$$

$$\sqrt{4k_0p_0V^2}\,\langle \pi^0(k)|V_\mu(0)^3_1|K_A^-(p)\rangle$$

$$= i\, e_\nu^{K_A}(p)\left[L_1'(q^2)\, g^{\mu\nu} + L_2'(q^2)\, k^\nu(p+k)^\mu + L_3'(q^2)\, k^\nu q^\mu\right] \quad (2)$$

where $q_\mu = p_\mu - k_\mu$ and the form factors $K_{1,2,3}(q^2)$ and $L_{1,2,3}(q^2)$, introduced on the considerations of Lorentz covariance, are calculated in the pole dominance approximation to be:

$$K_1(q^2) = \frac{G_{K_A}\, G_S'}{(m_{K_A}^2 - q^2)} + K_1(\infty)$$

$$K_2(q^2) = -\frac{G_{K_A}\, G_D'}{2(m_{K_A}^2 - q^2)} + K_2(\infty) \quad (3)$$

$$K_3(q^2) = \frac{G_{K_A}}{m_{K_A}^2}\, \frac{\left[G_S' + \frac{1}{2}\, G_D'(m_{K^*}^2 - m_\pi^2)\right]}{(m_{K_A}^2 - q^2)} - \frac{F_K\, G_{K^{*+}\pi^0K^+}}{(m_K^2 - q^2)} + K_3(\infty)$$

and similarly:

$$L_1'(q^2) = -\frac{G_{K^*}\, G_S''}{(m_{K^*}^2 - q^2)} + L_1'(\infty)$$

$$L_2'(q^2) = \frac{G_{K^*}\, G_D''}{2(m_{K^*}^2 - q^2)} + L_2'(\infty) \quad (4)$$

$$L_3'(q^2) = -\frac{G_{K^*}}{m_{K^*}^2}\, \frac{|G_S'' + 1/2\, G_D''(m_{K_A}^2 - m_\pi^2)|}{(m_{K^*}^2 - q^2)} - \frac{F_\varkappa\, G_{K_A^-\pi^0\varkappa^-}}{(m_\varkappa^2 - q^2)} + L_3'(\infty)$$

Here $K(\infty)$, $L(\infty)$ are subtraction constants and the various coupling constants are defined by the following invariant matrix elements:

$$\sqrt{2q_oV}\ \langle 0|A_\mu(0)^1_3|K^+(q)\rangle = i\ F_K\ q_\mu$$

$$\sqrt{2q_oV}\ \langle 0|V_\mu(0)^1_3|\varkappa^+(q)\rangle = i\ F_\varkappa\ q_\mu$$

$$\sqrt{4q_op_oV^2}\langle K^+(q)|j_{\pi^o}(0)|K^{*+}(p)\rangle = G_{K^{*+}\pi^oK^+}\ e^{K^*}(p).q$$

$$\sqrt{2q_oV}\ \langle 0|A_\mu(0)^1_3|K_A^{\ +}(q)\rangle = G_{K_A}\ e_\mu^{K_A}(q)$$

$$\sqrt{4q_op_oV^2}\langle \varkappa^-(q)|j_{\pi^o}(0)|K_A^-(p)\rangle = G_{K_A^-\pi^o\varkappa^-}\ e^{K_A}(p).q$$

$$\sqrt{2q_oV}\ \langle 0|V_\mu(0)^1_3|K^{*+}(q)\rangle = G_{K^*}\ e_\mu^{K^{*+}}(q) \tag{5}$$

$$\sqrt{4q_op_oV^2}\langle K^{*-}(q)|j_{\pi^o}(0)|K_A^-(p)\rangle = i\left[G_S''\ e^{K_A}.e^{K^{*+}} + G_D''\ e^{K_A}.q\ e^{K^{*+}}.p\right]$$

$$\sqrt{4q_op_oV^2}\langle K_A^+(q)|j_{\pi^o}(o)K^{*+}(p)\rangle = -i\left[G_S'\ e^{K_A^+}.e^{K^*} + G_D'\ e^{K^*}.q\ e^{K_A^+}.p\right]$$

The indices on the currents are the usual SU(3) tensor indices; the coupling constants $G_{s,D}$ determine the decay rate of K_A; and $\varkappa$ is a scalar isospinor strangeness carrying meson. For the discussion below we also need the $K - \pi^o$ and $\varkappa - \pi^o$ form factors. The former are defined by:

$$\sqrt{4k_op_oV^2}\langle \pi^o(k)|V_\mu(0)^1_3|K^+(p)\rangle = F_+(t)(p+q)_\mu + F_-(t)(p-q)_\mu \tag{6}$$

where $t = (p-k)^2 = q^2$. In the exact SU(3) limit $F_-(q^2) = 0$ while $F_+(0) = -1/\sqrt{2}$. A similar definition is given for $\varkappa - \pi^o$ form factors $f_\pm(q^2)$, considering the matrix element of axial current.

In the pole dominance approximation we find:

$$F_+(q^2) = - \frac{G_{K^*} \; G_{K^{*+}\pi^0 K^+}}{2(m_{K^*}^2 - q^2)} + F_+(\infty) \tag{7}$$

$$F_-(q^2) = \left(\frac{m_K^2 - m_\pi^2}{2m_{K^*}^2} \right) \frac{G_{K^*} \; G_{K^{*+}\pi^0 K^+}}{(m_{K^*}^2 - q^2)} - \frac{F_\varkappa \; G_{K^+\pi^0\varkappa^+}}{(m_\varkappa^2 - q^2)}$$

where we have used the fact that the subtraction constant in $F_-(q^2)$ must be vanishing under the hypothesis of at most once subtracted dispersion relation for the matrix element of the divergence $\partial^\mu(V^1_{\mu 3})$. Here the $K\pi\varkappa$ coupling is defined by:

$$\sqrt{4q_0 p_0 V^2} \langle \varkappa^+(q) | j_{\pi^0}(0) | K^+(p) \rangle = i \; G_{K^+\pi^0\varkappa^+} \tag{8}$$

For $\varkappa^+ - \pi^0$ form factors we find:

$$f_+(q^2) = - \frac{G_{K_A} \; G_{K_A \pi^0 \varkappa^+}}{2(m_{K_A}^2 - q^2)} + f_+(\infty) \tag{9}$$

$$f_-(q^2) = \left(\frac{m_\varkappa^2 - m_\pi^2}{2 \, m_{K_A}^2} \right) \frac{G_{K_A} \; G_{K_A^+ \pi^0 \varkappa^+}}{(m_{K_A}^2 - q^2)} - \frac{F_K \; G_{\varkappa^+ \pi^0 K^+}}{(m_K^2 - q^2)}$$

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II. Sum Rules from the K_A and K^* Matrix Elements of Two Currents:

We, now, set up a set of self consistent sum rules among the various subtraction constants. The solution of these equations will lead to the Weinberg sum rules and expressions for G_S and G_D to determine the decay width of K_A. We will illustrate the procedure by discussing the case of K* matrix element of two currents introduced below.

Consider the retarded matrix element:

$$W^{K^*}_{\mu\nu} = i\sqrt{2p_0 V}\int d^4x\, e^{ik\cdot x}\theta(x_0)\langle 0|\left[A_\nu(x)^1_1 - A_\nu(x)^2_2,\ A_\mu(0)^1_3\right]|K^{*+}(p)\rangle \tag{10}$$

$$= i\sqrt{2p_0 V}\int d^4x\, e^{iq\cdot x}\,\theta(-x_0)\langle 0|\left[A_\nu(0)^1_1 - A_\nu(0)^2_2,\ A_\mu(x)^1_3\right]K^{*+}(p)\rangle$$

Then:

$$ik^\nu W^{K^*}_{\mu\nu} = -W^{K^*}_\mu + iG_{K^*}\, e^{K^*}_\mu(p) \tag{11}$$

where

$$p = k + q\ ,$$

$$W^{K^*}_\mu = i\sqrt{2p_0 V}\int d^4x\, e^{ik\cdot x}\theta(x_0)\langle 0|\left[\partial^\nu(A_\nu(x)^1_1 - A_\nu(x)^2_2),\ A_\mu(0)^1_3\right]|K^{*+}(p)\rangle \tag{12}$$

$$= i\sqrt{2p_0 V}\int d^4x\, e^{iq\cdot x}\theta(-x_0)\langle 0|\left[\partial^\nu(A_\nu(0)^1_1 - A_\nu(0)^2_2),\ A_\mu(x)^1_3\right]|K^{*+}(p)\rangle$$

and we use the current algebra equal time commutator relation

$$\delta(x^0)\left[A_0(x)^i_j,\ A_\mu(0)^k_\ell\right] = \delta^4(x)\left[\delta^i_\ell V_\mu(0)^k_j - \delta^k_j\ V_\mu(0)^i_\ell\right] \tag{13}$$

in the second term, on the right hand side, obtained on integration by parts.

We take, now, the limit $k^\nu \to 0$ so that $k^2 \to 0$ and $q^2 \to m^2_{K^*}$. Since the poles involved in $W^{K^*}_{\mu\nu}$ are due to π^0, A^0_1, K_A and K and thus there are no poles due to zero mass in k^2 or corresponding to mass $m^2_{K^*}$ in q^2, it follows that

$$\lim_{k\to 0} k^\nu W^{K^*}_{\mu\nu} = 0 \tag{14}$$

Consequently:

$$\lim_{k \to 0} W_\mu^{K^*}(k^2 = 0,\ q^2 = m_{K^*}^2) = i\, G_{K^*}\, e_\mu^{K^*}(p) \qquad (15)$$

To apply this result we first calculate the invariant form factors appearing in $W_\mu^{K^*}(k^2, q^2)$ by assuming that they satisfy an unsubtracted dispersion relation for fixed invariant μ, where

$$\mu = \alpha q^2 + (1-\alpha)k^2 \qquad (16)$$

and $\alpha\ (0<\alpha<1)$ is a fixed arbitrary constant. We evaluate them in pole dominant approximation. In this way we retain the pole contributions from both the variables k^2 and q^2. We find:

$$W_\mu^{K^*} = i\, e_\mu^{K^*}(p) \left(\frac{\sqrt{2}\, F_\pi\, m_\pi^2\, K_1(q_1^2)}{(m_\pi^2 - k^2)} - \frac{G_{K_A}\beta_1(k_2^2)}{(m_{K_A}^2 - q^2)} \right) \qquad (17)$$

+ terms (†) involving $e^{K^*} \cdot k$.

where

$$\mu = \alpha q_1^2 + (1-\alpha)m_\pi^2 = \alpha m_{K_A}^2 + (1-\alpha)k_2^2 = \alpha\ m_k^2 + (1-\alpha)k_1^2 \qquad (18)$$

and $\beta_{1,2}(k^2)$ are defined by:

$$\sqrt{4q_0 p_0 V^2}\, \langle K_A^+(q) | \partial^\mu (A_\mu(0)_1^1 - A_\mu(0)_2^2) | K^{*+}(p) \rangle$$

$$= i\left[\beta_1(k^2)\, e^{K_A^+} \cdot e^{K^*} + \beta_2(k^2)\, e^{K_A^+} \cdot p\ e^{K^*} \cdot q\right] . \qquad (19)$$

The pole dominant expressions for $\beta_{1,2}$ are

(†) See appendix.

$$\beta_1(k^2) = \frac{-\sqrt{2}\, F_\pi\, m_\pi^2\, G_S'}{(m_\pi^2 - k^2)} + \beta_1(\infty)$$

$$\beta_2(k^2) = \frac{-\sqrt{2}\, F_\pi\, m_\pi^2\, G_D'}{(m_\pi^2 - k^2)} + \beta_2(\infty) \; . \tag{20}$$

Hence

$$W_\mu^{K*}(k^2, q^2) = ie_\mu^{K*}(p)\left(\frac{\sqrt{2}\; F_\pi\, m_\pi^2\, G_{K_A} G_S'}{(m_\pi^2 - k^2)(m_{K_A}^2 - q^2)} + \right.$$

$$\left. + \frac{\sqrt{2}\, F_\pi\, m_\pi^2\, K_1(\infty)}{(m_\pi^2 - k^2)} - \frac{G_{K_A}\, \beta_1(\infty)}{(m_{K_A}^2 - q^2)}\right)$$

$$+ \text{ terms involving } (e^{K*} \cdot k) \; . \tag{21}$$

From equation (15) we then obtain the sum rule:

$$G_{K*} = \sqrt{2}\, F_\pi\, K_1(\infty) + \frac{G_{K_A}(\sqrt{2}\, F_\pi\, G_S' - \beta_1(\infty))}{(m_{K_A}{}^2 - m_{K*}^2)} \; . \tag{22}$$

A similar sum rule obtained by considering the K_A matrix element is discussed in section (V).

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III. Matrix Elements of Current Divergences

Information on $\beta_{1,2}(\infty)$ can be obtained by considering:

$$iq^\mu\, W_\mu^{K*} = -\, W^{K*} + C \tag{23}$$

where

$$W^{K^*} = i\sqrt{2p_oV}\int d^4x\, e^{iq\cdot x}\theta(-x_o)\langle 0|\left[\partial^\nu(A_\nu(0)_1^1 - A_\nu(0)_2^2), \partial^\mu A(x)_3^1\right]|K^{*+}(p)\rangle \quad (24)$$

$$= i\sqrt{2p_oV}\int d^4x\, e^{ik\cdot x}\,\theta(x_o)\langle 0|\left[\partial^\nu(A_\nu(x)_1^1 - A_\nu(x)_2^2), \partial^\mu A_\mu(0)_3^1\right]|K^{*+}(p)\rangle$$

and C is the other term obtained on integration by parts and it involves an equal time commutator. If this commutator is assumed to be a local operator, C is a constant. In the present case C is vanishing due to angular momentum considerations. W^{K^*} is expressed (Appendix) in the pole dominant approximation proceeding as in the case of $W_\mu^{K^*}$ and, finally, from equation (23) we obtain:

$$- F_\pi m_\pi^2 K_2(q_1^2) + \frac{G_{K_A}}{2}\beta_2(k_2^2)\left(\frac{\alpha}{1-\alpha}\right)$$
$$- F_\pi m_\pi^2 K_3(q_1^2)\left(\frac{1-\alpha}{\alpha}\right) - F_k m_k^2 E_1(k_1^2)$$
$$- \frac{G_{K_A}}{m_{K_A}}\left[\beta_1(k_2^2) + \frac{1}{2}(m_{K^*}^2 - k_2^2)\beta_2(k_2^2)\right] = 0 \quad (25)$$

where $E_1(k^2)$ is defined by

$$\sqrt{4q_op_oV^2}\,\langle K^+(q)|\partial^\mu(A_\mu(0)_1^1 - A_\mu(0)_2^2)|K^{*+}(p)\rangle = E_1(k^2)e^{K^*}(p)\cdot q \quad (26)$$

Allowing $\mu \rightarrow \infty$ we find from equations (18) and (25), which holds for arbitrary $(0<\alpha<1)$, that

$$\beta_2(\infty) = 0, \qquad K_3(\infty) = 0 \quad (27)$$

and

$$- F_\pi m_\pi^2 K_2(\infty) - F_K m_K^2 E_1(\infty) - \frac{G_{K_A}}{m_{K_A}^2}\left[\beta_1(\infty) - \frac{1}{2} \lim_{k^2\to\infty} (k^2 \beta_2(k^2))\right] = 0 \tag{28}$$

Starting from

$$S_\mu^K = i\sqrt{2q_0V}\int d^4x\, e^{-ip\cdot x}\,\theta(-x_0)\langle K^+(q)|\left[\partial^\nu(A_\nu(0)_1^1 - A_\nu(0)_2^2), V_\mu(x)_1^3\right]|0\rangle \tag{29}$$

$$= i\sqrt{2q_0V}\int d^4x\, e^{ik\cdot x}\,\theta(x_0)\langle K^+(q)|\left[\partial^\nu(A_\nu(x)_1^1 - A_\nu(x)_2^2), V_\mu(0)_1^3\right]|0\rangle$$

and considering

$$- i\, p^\mu S_\mu + S = \text{constant} \tag{30}$$

where

$$S = i\sqrt{2q_0V}\int d^4x\, e^{-ip\cdot x}\theta(-x_0)\langle K^+(q)|\left[\partial^\nu(A_\nu(0)_1^1 - A_\nu(0)_2^2), \partial^\mu V_\mu(x)_1^3\right]|0\rangle \tag{31}$$

we can show, likewise, that

$$E_1(\infty) = 0 \quad . \tag{32}$$

Considering

$$S_\mu = i\sqrt{2k_0V}\int d^4x\, e^{-ip\cdot x}\,\theta(-x_0)\langle \pi^0(k)|\left[\partial^\mu A_\nu(0)_3^1, V_\mu(x)_1^3\right]|0\rangle \tag{33}$$

We are lead to:

$$E_2(\infty) = 0 \tag{34}$$

where

$$\sqrt{4k_0p_0V^2}\,\langle\pi^0(k)|\partial^\mu A_\mu(0)_3^1|K^{*+}(p)\rangle = E_2(q^2)\, e^{K^*}(p)\cdot k \tag{35}$$

that is:

$$E_2(q^2) = - K_1(q^2) + (m_{K^*}^2 - m_\pi^2)K_2(q^2) + q^2 K_3(q^2) \tag{36}$$

and, similarly, by considering

$$S_\mu = i\sqrt{2k_oV}\int d^4x\; e^{-ip\cdot x}\,\theta(-x_o)\langle\pi^o(k)\big|\big[\partial^\mu V_\mu(0)^1_3,\; A_\mu(x)^3_1\big]\big|0\rangle$$

we show that

$$E_3(\infty) = 0 \tag{38}$$

where

$$\sqrt{4k_op_oV^2}\,\langle\pi^o(k)|\partial^\mu\, V_\mu(0)^1_3|K^+_A(p)\rangle = E_3(q^2)\; e^{K_A}(p)\cdot k \tag{39}$$

that is

$$E_3(q^2) = -L_1(q^2) + (m_{K_A}^2 - m_\pi^2)\; L_2(q^2) + q^2L_3(q^2) \tag{40}$$

We note that $E_2(\infty) = 0$ leads to, again, with our assumptions, $K_3(\infty) = 0$.

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IV. Sum Rules from Pion Matrix Elements of Two Currents:

Likewise we consider:

$$S^{\pi^o}_{\mu\nu} = i\sqrt{2k_oV}\int d^4x\; e^{iq\cdot x}\theta(x_o)\langle\pi^o(k)\big|\big[A_\nu(x)^1_3,\; V_\mu(0)^3_1\big]\big|0\rangle \tag{41}$$

$$= i\sqrt{2k_oV}\int d^4x\; e^{-ip\cdot x}\theta(-x_o)\langle\pi^o(k)\big|\big[A\;(0)^1_3, V_\mu(x)^3_1\big]\big|0\rangle$$

$$= A_1\, g_{\mu\nu} + A_2\; k_\mu k_\nu + A_3\; p_\mu\; p_\nu + A_4\; k_\mu p_\nu + A_5\; k_\nu p_\mu$$

and

$$i\; q^\nu\; S^{\pi^o}_{\mu\nu} + S^{\pi^o}_\mu = \frac{F_\pi}{\sqrt{2}}\, k_\mu \tag{42}$$

where

$$S^{\pi^o}_\mu = i\sqrt{2k_oV}\int d^4x\; e^{iq\cdot x}\;\theta(x_o)\langle\pi^o(k)\big|\big[\partial^\nu A_\nu(x)^1_3,\; V_\mu(0)^3_1\big]\big|0\rangle \tag{43}$$

$$= i\sqrt{2k_oV}\int d^4x\; e^{-ip\cdot x}\;\theta(-x_o)\langle\pi^o(k)\big|\big[\partial^\nu A_\nu(0)^1_3,\; V_\mu(x)^3_1\big]\big|0\rangle$$

obtained on integration by parts and using the current algebra commutation relation. We write then unsubtracted dispersion relations for fixed for the invariants $A_i(q^2, p^2)$. After a straightforward calculation following the procedure already explained, and using the results already obtained we find:

$$L_2'(\infty) = L_3'(\infty) = K_2(\infty) = 0$$

$$K_1(\infty) = \lim_{q^2 \to \infty} \left[q^2 K_3(q^2)\right] \tag{44}$$

$$L_1'(\infty) = \lim_{q^2 \to \infty} \left[q^2 L_3'(q^2)\right]$$

In addition we derive the following sum rules:

$$\frac{F_\pi}{\sqrt{2}} = 2 F_K F_+'(\infty) - \frac{G_{K_A}}{m_{K_A}^2} L_1'(\infty) + \frac{G_{K_A}}{m_{K_A}^2} \lim_{q^2 \to \infty} \left[q^2 L_2'(q^2)\right] \tag{45}$$

and

$$F_\pi f_+(\infty) - F_K F_+'(\infty)$$

$$= - \frac{F_\pi}{\sqrt{2}} - \frac{G_{K_A}}{2m_{K_A}^2} \left[L_1'(\infty) - \lim_{q^2 \to \infty} (q^2 L_2'(q^2))\right]$$

$$+ \frac{G_{K^*}}{2m_{K^*}^2} \left[K_1(\infty) - \lim_{q^2 \to \infty} (q^2 K_2(q^2))\right] . \tag{46}$$

Here $F_\pm'(q^2)$ denotes the $K^- - \pi^0$ form factors defined by an expression similar to equation (6).

Using the pole dominant forms for the various form factors we can re-cast them as follows (*)

$$F_{\varkappa}\, f_+(0) - F_K\, F_+'(0) = -\frac{F_\pi}{\sqrt{2}} \tag{47}$$

and

$$F_K\, F_+'(0) + F_{\varkappa}\, f_+(0)$$
$$= -\frac{G_{K_A}\, G_{K^*}}{m_{K^*}^2\, m_{K_A}^2}\left[G_S' + \frac{1}{2}\, G_D'(m_{K_A}^2 + m_{K^*}^2 - m_\pi^2)\right] +$$
$$+\frac{F_{\varkappa}\, G_{K_A}\, G_{K_A^-\pi^0\varkappa^-}}{m_{K_A}^2} + \frac{F_K\, G_{K^*}\, G_{K^{*+}\pi^0 K^+}}{m_{K^*}^2}\,. \tag{48}$$

From equations (28), (32) and (44) we find:

$$\beta_1(\infty) = \frac{1}{2}\lim_{k^2\to\infty}\left[k^2\, \beta_2(k^2)\right] \tag{49}$$

and from equations (20) and (27):

$$\beta_1(\infty) = \frac{1}{2}\sqrt{2}\, F_\pi\, G_D'\,. \tag{50}$$

It is interesting to remark that according to the partial conservation of axial pion current hypothesis we should expect both $\beta_1(\infty)=\beta_2(\infty)=0$, contrary to the conclusion arrived by using our procedure.

From equations (3), (22), (44) and (50) we deduce:

(*) In arriving at this result we have been made of SU(3) symmetric coupling relations $G_{K^{*+}\pi^0K^+} = -G_{K^{*-}\pi^0K^-}$, $G_{K_A^+\pi^0\varkappa^+} = -G_{K_A^-\pi^0\varkappa^-}$, $G''_{S,D} = -G'_{S,D}$ etc.

$$\frac{G_{K^*}}{\sqrt{2}\,F_\pi} = F_K\, G_{K^{*+}\,\pi^0 K^+}$$

$$+ \frac{G_{K_A}\, m_{K^*}^2}{m_{K_A}^2\,(m_{K_A}^2 - m_{K^*}^2)} \left[G_s' + \frac{1}{2}\, G_D'(m_{K^*}^2 - m_{K_A}^2 - m_\pi^2)\right] \tag{51}$$

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V. Sum Rules From K_A, K and $\varkappa$ Matrix Elements of Two Currents.

Weinberg Sum Rule:

Considering

$$W_{\mu\nu}^{K_A} = i\sqrt{2p_oV}\int d^4x\; e^{ik\cdot x}\theta(x_0)\,\langle 0\left|\left[A_\nu(x)_1^1 - A_\nu(x)_2^2,\; V_\mu(0)_1^3\right]\right|K_A^-(p)\rangle \tag{52}$$

and proceeding as in section (II) we can obtain:

$$-\frac{G_{K_A}}{\sqrt{2}\,F_\pi} = F_\varkappa\, G_{K_A^-\,\pi^0\,\varkappa^-}$$

$$- \frac{G_{K^*}\, m_{K_A}^2}{m_{K^*}^2(m_{K_A}^2 - m_{K^*}^2)} \left[G_s' + \frac{1}{2}\, G_D'(m_{K_A}^2 - m_{K^*}^2 - m_\pi^2)\right]. \tag{53}$$

From equations (51) and (53) we show:

$$\frac{1}{\sqrt{2}\,F_\pi}\left(\frac{G_{K^*}^2}{m_{K^*}^2} - \frac{G_{K_A}^2}{m_{K_A}^2}\right)$$

$$= \frac{F_K\, G_{K^*} G_{K^{*+}\pi^0 K^+}}{m_{K^*}^2} + \frac{F_\varkappa\, G_{K_A}\, G_{K_A^-\,\pi^0\varkappa^-}}{m_{K_A}^2} -$$

$$- \frac{G_{K^*}\, G_{K_A}}{m_{K^*}^2\, m_{K_A}^2}\left[G_s' + \frac{1}{2}\, G_D'\,(m_{K_A}^2 + m_{K^*}^2 - m_\pi^2)\right] \tag{54}$$

Using equations (4), (7), (44), (47), (48) and (54) we find:

$$\left(\frac{G_{K^*}^{2}}{m_{K^*}^{2}} - \frac{G_{K_A}^{2}}{m_{K_A}^{2}}\right) = 2\sqrt{2}\, F_\pi F_K F_+'(0) - F_\pi^2 \qquad (55)$$

To obtain information on $F_+'(0)$ we consider

$$S^K_{\mu\nu} = i\sqrt{2q_oV}\int d^4x\; e^{+ik\cdot x}\,\theta(+x_o)\langle K^+(q)\left|\left[A_\nu(x)^1_1 - A_\nu(x)^2_2,\; V_\mu(0)^3_1\right]\right|0\rangle \qquad (56)$$

and proceeding as in section (II) we find

$$\lim_{k\to 0} S^K_\mu\,(k^2=0\quad p^2 = m_K^2) = -F_K\, q_\mu \qquad (57)$$

which leads to [9]

$$-F_K = +\sqrt{2}\,F_\pi\, F_+(0) + \frac{F_\varkappa\, g(0)}{(m_\varkappa^2 - m_K^2)} \qquad (58)$$

where

$$\sqrt{4q_op_oV^2}\,\langle K^+(q)|\partial^\nu(A_\nu(0)^1_1 - A_\nu(0)^2_2)|\varkappa^+(p)\rangle = ig(k^2)\;. \qquad (59)$$

Considering, likewise,

$$S^\varkappa_{\mu\nu} = i\sqrt{2q_oV}\int d^4x\; e^{ik\cdot x}\,\theta(x_o)\langle\varkappa^+(q)\left|\left[A_\nu(x)^1_1 - A_\nu(x)^2_2,\; A_\mu(0)^3_1\right]\right|0\rangle \qquad (60)$$

we obtain

$$-F_\varkappa = \sqrt{2}\;F_\pi f_+(0) - \frac{F_K\, g(0)}{(m_K^2 - m_\varkappa^2)}\quad . \qquad (61)$$

From equations (58) and (61):

$$F_K^2 - F_\varkappa^2 = \sqrt{2}\, F_\pi\, (F_\varkappa\, f_+(0) - F_K F_+(0)) \qquad (62)$$
$$= \sqrt{2}\, F_\pi\, (F_\varkappa\, f_+(0) + F_K F_+'(0))$$

Combining with equation (47) we obtain:

$$\sqrt{2}\, F_+'(0) = \frac{F_K^2 + F_\pi^2 - F_\varkappa^2}{2\, F_\pi F_K} \qquad (63)$$

$$\sqrt{2}\, f_+(0) = \frac{F_K^2 - F_\pi^2 - F_\varkappa^2}{2\, F_\pi F_\varkappa} \qquad (64)$$

and then, from equation (58)

$$g(0) = \frac{(F_K^2 - F_\pi^2 + F_\varkappa^2)(m_K^2 - m_\varkappa^2)}{2\, F_K\, F_\varkappa} \,. \qquad (65)$$

These results were already discussed in references (8) and (9).

From equations (55) and (63) we obtain the Weinberg sum rule

$$\left(\frac{G_{K^*}^2}{m_K^2} - \frac{G_{K_A}^2}{m_{K_A}^2} \right) = (F_K^2 - F_\varkappa^2) \qquad (66)$$

as a self consistency constraint.

The results of equations (27) and (44) justify the assumptions made on the subtraction constants of the form factors in reference [5]. The equations (51) and (53) are identical to those of the reference mentioned if $F_\varkappa = 0$. The decay widths in this case were already discussed and found to be in agreement with experiments. We now discuss $K_A \rightarrow K\rho$ decay made.

* * *

$K_A \rightarrow K\rho$ Decay

I. K_A and ρ Matrix Elements

The decays such as $K_A \rightarrow K\rho$, $K_A \rightarrow K\omega$ and $\varphi \rightarrow K\bar{K}$, with the large mass of kaon, can hardly be treated by soft kaon technique. We will only sketch the calculation since the technique has been already explained above. The partial decay width calculated for $K_A' \rightarrow K\rho$ process is in agreement with the present experimental data.

We start by defining the form factors $G_{1,2,3}$ and $H_{1,2,3}$ appearing in the following invariant matrix elements

$$\sqrt{4k_o p_o V^2}\,\langle \rho^o(k)|\ A_\nu(0)^1_3|K^+(p)\rangle$$
$$= -\,i(e^\lambda_\rho(k))^+\left[g_{\lambda\nu}\bar{G}_1(q^2) - \bar{G}_2(q^2)\,q_\lambda(p+k)_\nu - \bar{G}_3(q^2)q_\lambda(k-p)_\nu\right] \tag{67}$$

$$\sqrt{4q_o p_o V^2}\,\langle K^+_A(q)|V_\mu(0)^1_1 - V_\mu(0)^2_2|K^+(p)\rangle$$
$$= -\,i\,(e^\lambda_{K_A}(q))^+\left[g_{\lambda\mu}\bar{H}_1(k^2) - \bar{H}_2(k^2)k_\lambda(q+p)_\mu - \bar{H}_3(k^2)k_\lambda(q-p)_\mu\right] \tag{68}$$

In the pole dominant approximation we calculate

$$\bar{G}_1(q^2) = - \frac{G_{K_A} \; G^S}{(m_{K_A}^2 - q^2)} + \bar{G}_1(\infty)$$

$$\bar{G}_2(q^2) = + \frac{G_{K_A} \; G^D}{2(m_{K_A}^2 - q^2)} + \bar{G}_2(\infty) \tag{69}$$

$$\bar{G}_3(q^2) = + \frac{G_{K_A}}{m_{K_A}^2} \; \frac{\left[G^S + \frac{1}{2} G^D(m_{K_A}^2 - m_\rho^2)\right]}{(m_{K_A}^2 - q^2)} -$$

$$- \frac{F_K \; G'_{K^+\rho^0K^+}}{(m_K^2 - q^2)} + \bar{G}_3(\infty)$$

and

$$\bar{H}_1(k^2) = - \frac{\sqrt{2} \; G\rho \; G^S}{(m_0^2 - k^2)} + \bar{H}_1(\infty)$$

$$\bar{H}_2(k^2) = \frac{\sqrt{2} \; G\rho \; G^D}{(m_\rho^2 - k^2)} + \bar{H}_2(\infty) \tag{70}$$

$$\bar{H}_3(k^2) = \frac{\sqrt{2} \; G\rho}{m_\rho^2} \; \frac{\left[G^S + \frac{1}{2} (m_K^2 - m_{K_A}^2)G^D\right]}{(m_\rho^2 - k^2)} + \bar{H}_3(\infty) \; .$$

The various coupling constants are defined by:

$$\sqrt{2p_oV} \langle 0|V_\mu(0)_1^1 - V_\mu(0)_2^2|\rho^0(p)\rangle = \sqrt{2} \; G\rho \; e_\mu^\rho(p)$$

$$\sqrt{4q_op_oV^2} \langle K^+(q)|j_\rho^\mu(0)|K^+(p)\rangle = - \frac{G'_{K^+\rho^0K^+}}{2} \; (p+q)^\mu$$

$$\sqrt{4q_o p_o V^2}\,\langle K_A^+(q)|j_\rho^\mu(0)|K^+(p)\rangle = i\left(e^{K_A}(q)_\lambda\right)^\dagger \left[G^S g^{\mu\lambda} + G^D k^\lambda q^\mu\right] \quad (71)$$

where k = (p-q).

We also introduce the form factor $e(k^2)$ defined by

$$\langle K^+(q)|V_\mu(0)_1^1 - V_\mu(0)_2^2|K^+(p)\rangle = (p+q)_\mu \; e(k^2) \quad (72)$$

and find easily:

$$e(k^2) = -\frac{\sqrt{2}\,G\rho\,G'_{K^+\rho^o K^+}}{2(m_\rho^2 - k^2)} + e(\infty)\;. \quad (73)$$

There is only one form factor in this case if we assume that the SU(2) currents $V_{\mu j}^i$ (i, j = 1,2) are conserved. This assumption also leads to

$$\bar{H}_3(\infty) = 0\;. \quad (74)$$

* * *

II. Sum Rules from K Matrix Element of Two Currents

Following arguments similar to those in sections (IV) and (V) we can show:

$$\bar{H}_2(\infty) = \bar{G}_2(\infty) = \bar{G}_3(\infty) = 0$$
$$\bar{H}_1(\infty) = -\lim_{k^2\to\infty}\left[k^2\,\bar{H}_3(k^2)\right] \quad (75)$$
$$\bar{G}_1(\infty) = -\lim_{k^2\to\infty}\left[k^2\,\bar{G}_3(k^2)\right]$$

Consider the retarded matrix element:

$$S_{\mu\nu}^K = i\sqrt{2p_oV}\int d^4x\, e^{ik\cdot x}\,\theta(-x_o)\,\langle 0|\left[A_\nu(0)_3^1,\; V_\mu(x)_1^1 - V_\mu(x)_2^2\right]|K^+(p)\rangle$$

$$= A_1\, g_{\mu\nu} + A_2 p_\mu p_\nu + A_3 k_\mu k_\nu + A_4 p_\mu k_\nu + A_5\, p_\nu\, k_\mu \quad (76)$$

We obtain after integrating by parts and using current algebra commutation relation

$$ik^{\mu} S^{K}_{\mu\nu} = - F_K \, p_{\nu} \tag{77}$$

where we have also used the conserved vector current hypothesis for $V_{\mu}{}^{1,2}_{1,2}$. This relation leads to two sum rules:

$$e(0) = 1 \tag{78}$$

and

$$F_K = - \frac{\sqrt{2}\, G\rho \; G'_{K^+\rho^o K^+}}{m_{\rho}^{2}}$$

$$+ \frac{\sqrt{2}\, G\rho \; G_{K_A}}{m_{K_A}^{2}\, m_{\rho}^{2}} \left[G^{s} + \frac{1}{2} \left(m_K^2 - m_{K_A}^2 - m_{\rho}^2 \right) G^{D} \right] . \tag{79}$$

The result in equation (78) is already expected from the C.V.C. hypothesis used above.

* * *

III. K_A and ρ Matrix Elements of Two Currents. Weinberg Sum Rule:

Considering the matrix elements

$$W^{\rho}_{\mu\nu} = i \sqrt{2k_o V} \int d^4x \; e^{-ip.x} \; \theta(-x_o) \langle \rho^{o}(k) | \left[A_{\nu}(0)^{1}_{3}, \; A_{\mu}(x)^{3}_{1} \right] | 0 \rangle \tag{80}$$

and

$$W_{\mu\nu}^{K_A} = i \sqrt{2q_o V} \int d^4x \; e^{-ik.x} \; \theta(-x_o) \langle K_A^{+}(q) | \left[V_{\nu}(0)^{1}_{1} - V^{1}_{1}(0)^{2}_{2}, \; A_{\mu}(x)^{3}_{1} \right] | 0 \rangle \tag{81}$$

and the corresponding relations

$$\lim_{p_{\nu} \to 0} W^{\rho}_{\nu} \left(p^2 = 0, \; q^2 = m_{\rho}^2 \right) = - \frac{iG\rho}{\sqrt{2}} \, e^{\rho}_{\nu}(k) \tag{82}$$

and

$$\lim_{p_\nu \to 0} W_\nu^{K_A}(p^2 = 0,\ k^2 = m_{K_A}^2) = i\ G_{K_A}\ e_\nu^{K_A}(q) \tag{83}$$

where

$$W_\nu^\rho = i\sqrt{2k_oV}\int d^4x\ e^{-ip\cdot x}\ \theta(-x_o)\langle \rho^o(k)\Big|\Big[A_\nu(0)_3^1, \partial^\mu A_\mu(x)_1^3\Big]\Big|0\rangle \tag{84}$$

and

$$W_\nu^{K_A} = i\sqrt{2q_oV}\int d^4x\ e^{-ip\cdot x}\ \theta(-x_o)\langle K_A^+(q)\Big|\Big[V_\nu(0)_1^1 - V_\nu(0)_2^2, \partial^\mu A_\mu(x)_1^3\Big]\Big|0\rangle \tag{85}$$

we obtain the following sum rules:

$$\frac{G\rho}{\sqrt{2}} = -F_K^2\ G'_{K^+\rho^oK^+} - \frac{F_K G_{K_A}\ m_\rho^2}{m_{K_A}^2(m_{K_A}^2 - m_\rho^2)}\Big[G^S - \tfrac{1}{2}G^D(m_\rho^2 - m_{K_A}^2 - m_K^2)\Big] \tag{86}$$

and

$$-G_{K_A} = \frac{\sqrt{2}\ G\rho\ F_K\ m_{K_A}^2}{m_\rho^2(m_{K_A}^2 - m_\rho^2)}\Big[G^S + \tfrac{1}{2}G^D(m_\rho^2 - m_{K_A}^2 + m_K^2)\Big] \tag{87}$$

From equations (78), (86) and (87) we derive the following Weinberg sum rule:

$$\left(\frac{G_\rho^2}{m_\rho^2} - \frac{G_{K_A}^2}{m_{K_A}^2}\right) = F_K^2 \tag{88}$$

Equations (66) and (88) are derived here from entirely different point of view than was used in their original derivation in ref. (10).

We also remark that for the form factors $D_{1,2}$ defined by

$$\sqrt{4k_op_oV^2}\langle \rho^o(k)|\partial^\mu A_\mu(0)_3^1|K_A^+(p)\rangle$$
$$= i\ e_\mu^{K_A}(p)\ e_\nu^{\rho^+}(k)\Big[D_1(q^2)\ g^{\mu\nu} + D_2(q^2)p^\nu k^\mu\Big]\ , \tag{89}$$

as in the case of $\beta_{1,2}$, only D_2 is unsubtracted while $D_1(\infty) \neq 0$.

PCAC hypothesis would require both of them to be unsubtracted.

We can use equations (86), (87) and the Weinberg sum rules to determine the couplings G^S and G^D and calculate the partial width for the decay $K_A \to K\rho$, which is given by:

$$\Gamma_{K_A^+ \to K\rho} = \frac{1}{8\pi}\left(\frac{k}{m_{K_A}^2}\right)\left[G_S^2\left(3 + \frac{k^2}{m_\rho^2}\right) + G^{D^2}\frac{m_{K_A}^2}{m_\rho^2}k^4 - 2G^S G^D \frac{m_{K_A}}{m_\rho}k^2\sqrt{1 + \frac{k^2}{m_\rho^2}}\right] \qquad (90)$$

We determine F_π from decay of pion, F_K from the relation [8, 9] $(F_K/F_\pi)^2 \simeq 1.17$ and $G\rho$ and G_{K_A} from Weinberg sum rules assuming $G\rho = G_{A_1}$. If we assume that ρ couples universally so we can make use of $G_{K^+\rho^0K^+} = \frac{1}{2} G_{\pi^+\rho^0\pi^+}$ and the experimental decay width of ρ^0 to obtain $G_{K^+\rho^0K^+}$. The partial width [11] thus calculated for the decay $K_A^+ \to K\rho$ comes out to be $\simeq$ 8 MeV.

* * *

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APPENDIX

The complete expressions of W_{μ}^{K*} and W^{K*} are given by:

$$W_{\mu}^{K*} = i \left[e_{\mu}^{K*}(p) \left(\frac{\sqrt{2}\, F_{\pi}\, m_{\pi}^{2}\, K_{1}(q_{1}^{2})}{(m_{\pi}^{2} - k^{2})} - \frac{G_{K_A}\, \beta_{1}\, (k_{2}^{2})}{(m_{K_A}^{2} - q^{2})} \right) \right.$$

$$+ e^{K*}\!\cdot k\, (p+k)_{\mu} \left(\frac{\sqrt{2}\, F_{\pi}\, m_{\pi}^{2}\, K_{2}(q_{1}^{2})}{(m_{\pi}^{2} - k^{2})} + \frac{G_{K_A}\, \beta_{2}(k_{2}^{2})}{2(m_{K_A}^{2} - q^{2})} \right)$$

$$+ e^{K*}\!\cdot k\, q_{\mu} \left(\frac{\sqrt{2}\, F_{\pi}\, m_{\pi}^{2}\, K_{3}(q_{1}^{2})}{(m_{\pi}^{2} - k^{2})} - \frac{F_{K}\, E_{1}(k_{1}^{2})}{(m_{K}^{2} - q^{2})} \right.$$

$$\left.\left. - \frac{G_{K_A}}{m_{K_A}^{2}(m_{K_A}^{2} - q^{2})} \left\{ \beta_{1}(k_{2}^{2}) + \frac{1}{2}\, (m_{K*}^{2} - k_{2}^{2})\, \beta_{2}(k_{2}^{2}) \right\} \right) \right]$$

$$W^{K*} = (e^{K*}\!\cdot k) \left[- \frac{F_{K}\, m_{K}^{2}\, E_{1}(k_{1}^{2})}{(m_{k}^{2} - q^{2})} + \right.$$

$$\left. + \frac{F_{\pi} m_{\pi}^{2}}{(m_{\pi}^{2} - k^{2})} \left(- K_{1}(q_{1}^{2}) + (m_{K*}^{2} - m_{\pi}^{2}) K_{2}(q_{1}^{2}) + q_{1}^{2} K_{3}(q_{1}^{2}) \right) \right]$$

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11. In view of the lack of precise information about the two K_A resonances, we ignore mixing and assume in our discussions that K_A is a pure resonance with mass $\simeq$ 1320 MeV.