ON THE SPREAD OF THE SOFT COMPONENT OF COSMIC RADIATION*

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Recently, in a paper under the same title [1] the authors H. S. Green and H. Messel criticized "all previous work" on the subject in question. A reply to their criticism seems worth while in order to avoid confusion and to clear up the situation.

1) Green's and Messel's criticism, at first, concerns the higher angular and radial moments, due to multiple Coulomb scattering, of the cascade electrons of a given energy. The numerical values of these higher moments depend essentially on the manner in which the effect of single scattering is taken into account. With respect to this, the following cases can be distinguished:

A. Pure multiple scattering: This is the approximation of Landau's equations. Single scattering is entirely neglected and the angular and radial distribution functions $f(E,\theta)$ and $f(E,r)$, therefore, fall off too rapidly at great arguments θ and r. The higher moments are closely connected with the fall-off at great arguments and are therefore, strictly speaking, without physical significance in this approximation. Nevertheless, their computation can be useful. These are the higher moments which have been exactly

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calculated by Eyges and Fernbach [2]. In the present author's theory [3] these higher moments were duly taken into account up to a certain stage of the calculation (cf. below).

B. Single scattering is taken into account with due consideration of the effect of the finite size of the nuclei: This is, in principle, the most exact procedure and it is this which was proposed by Green and Messel in their paper. The limitation of the great single scattering angles as a consequence of the finite size of the nucleus, thereby, may be taken into account (according to E. J. Williams) by introducing a cut-off angle θ_{max}. The higher moments in this case, of course, are greater than in case A ($\langle\theta^4\rangle_{av}$ by 18% and $\langle\theta^6\rangle_{av}$ by 45%, according to Green and Messel). - But it is obvious that the higher moments in this case depend very sensibly upon the choice of the cut-off θ_{max}. On the other hand, E. J. Williams' expression for θ_{max} merely represents an estimation of the order of magnitude and the cut-off procedure as a whole, of course, is somewhat rough. (By a more rigorous treatment the greatest scattering angles would not be entirely suppressed but only reduced in frequency by a factor Z^{-1} since in this region the protons of the nucleus scatter individually.) The exact numerical values of the higher moments in this case also, are therefore physically meaningless. Moreover, the influence of the finite size of the nucleus is practically negligible in shower theory (cf. below). Finally, it may be stated that the way of calculating the distribution functions by means of the moments, as proposed by Green and Messel, seems not very suitable because a very large number of moments (strictly speaking, all of them), would be needed if one wished to obtain some accuracy in the interesting region of small and medium arguments.

C. Single scattering is taken into account neglecting the influence of the finite size of the nuclei: In shower theory single scattering altogether is of little importance. This has been pointed out by Nishimura and Kamata [4] who have shown that the main contribution to strongly deflected electrons (i.e., those found at great θ and r), is due to multiple scattering of electrons slowed down by ionization loss rather than to single scattering. Great single scattering angles, in general, are rare events. The modification of the frequency of very great single scattering angles by the finite size of the nucleus, therefore, is of still less importance. (As an illustration, it may be estimated that the influence of the finite size of the nucleus on the distribution function f(E,θ) is not appreciable up to angles of about seven times the root-mean-square angle of multiple scattering.) - It may be noted, further, that in case C and also in the above mentioned rigorous treatment of case B, all the moments turn out to be infinite in the usual small-angle-approximation. This illustrates the irrelevance of the moments proposed by G'een and Messel which entirely depend on the cut-off.

Case C was used in the present author's theory [3], the procedure being the following: Starting with case A the Fourier-transforms of the distribution functions were calculated numerically with great accuracy. The moments valid in case A play the rôle of the coefficients of the power series of the Fourier-transforms and were duly used in this calculation. As a next step, for convenience in performing the Fourier-transformation, the exact Fourier-transforms were approximated by analytical expression in such a way that stress was laid upon a good representation at great and medium arguments of the Fourier-transforms. In this way great accuracy was reached at small and medium arguments θ and r of the distribution functions resulting from the Fourier-transformation, whereas errors were admitted in the region of great θ and r where the distribution functions due to multiple scattering are practically fallen off to zero, these errors also influencing the higher moments. The criticism expressed in literature in this connection (Blatt [5], Eyges [6]) is insignificant because in this domain of great arguments the distribution function is solely due to single scattering. The influence of the latter was determined in a final step of the theory, as follows: The resulting distribution functions of case A were decomposed into Gauss-functions ("Gauss-transformation"), each Gauss-function belonging to electrons of a certain "energetic history", characterized by a certain parameter. These Gauss-functions, then, were replaced by the more exact functions which contain the "single-scattering-tail". Finally, by the inverse Gauss-transformation, the distribution functions of shower theory duly containing the effect of single scattering were obtained.

2) Another point of Green's and Messel's criticism concerns the neglect of the variation of the atmospheric density in all previous theories. The influence of this variation of density is closely connected with the path length which is needed for equilibrium in the distribution in θ and r. As to this question, Green and Messel have given an example: They have calculated $\langle r^2 \rangle_{av}$ as a function of depth in a homogeneous layer of matter in the case of a primary (integral) power spectrum of exponent 1.5. From the figures given by them the conclusion may be drawn that in the case of a power 1.5 a path length of about 12 to 14 radiation lengths is needed for equilibrium in the radial distribution. Roughly, this means that the radial deviations of the electrons in the depth of observation have their origin mainly at a depth smaller by 6 to 7 radiation lengths. For showers observed at sea level, therefore, the inhomogeneousness of the atmosphere can be roughly accounted for by using a value of the radiation length which is greater by 25 to 30 %. For a power law with exponent 1, - i.e., for air showers near the maximum, the situation is still better: It may be estimated that in this case 6 to 8 radiation lengths are sufficient to reach equilibrium which means that a layer higher by 3 to 4 radiation lengths is responsible for the radial distribution. The radiation length, therefore, has to be increased by 15 to 20 % for observation at sea level and by 30 to 40 % for 5000 m.

A more exact computation of this correction, viz., an improved theory taking into account the variation of density, would be useful. But the assertion of Green and Messel that the neglect of the effect of density variation would introduce errors as large as 5000 % (!) can not be understood. Presumably this large error would concern the higher moments and so is of no interest.

3) In a further point in their criticism Green and Messel lay blame that previous authors either consider only the maximum of showers

or integrate over all depths. However, both the maximum and the integration over all depths are suitable starting points. Besides, Green and Messel seem to overlook the fact that in the meantime, Nishimura and Kamata, l.c., extended the theory to a shower age of 1.5 .

The points 4 and 5 enumerated by the authors Green and Messel need no comentary. - The further criticism expressed by Green and Messel in the text of their paper, claiming that ionization loss has not been duly treated by previous authors, is also made out of date by the work of Nishimura and Kamata, which seems to be unknown to them.

Finally it may be noted that the lateral spread of the electronic component of large air showers is only slightly modified by the contribution of the nucleon-meson component. This contribution is practically restricted to small distances from the center and consists in the formation of plural cores at separations of the order of some tens of centimeters from each other.[7].

1 H. S. Green and H. Messel, Phys. Rev., in press. We are thankful to Prof. W. Heisenberg for sending us a prepublication print of the paper.

2 L. Eyges and S. Fernbach, Phys. Rev. 82, 23, 1951.

3 G. Molière in "Cosmic Radiation" edited by W, Heisenberg. The theory of the spread of large air showers will be presented in detail in the 2nd edition of this book (Springer-Verlag, Göttingen, in press).

4 J. Nishimura and K. Kamata, Progress of Theoretical Physics, 6, 262, 1951; 6, 628, 1951 and 7, 185, 1952.

5 J. M. Blatt, Phys. Rev. 75, 1584, 1949.

6 L. Eyges, quoted by Blatt, ref. 5. Cf. also: L. W. Nordheim, L. Osborne and J. M. Blatt, Echo-Lake-Report, 1949.

7 Cf. H. Messel and H. S. Green, Phys. Rev. 87, 738, 1952.