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Dimensional Reduction of a Lorentz and CPT-violating Maxwell-Chern-Simons Model

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Taking as starting point a Lorentz and CPT non-invariant Chern-Simons-like model defined in 1+3 dimensions, we proceed realizing its dimensional reduction to $D = 1 + 2$. One then obtains a new planar model, composed by the Maxwell-Chern-Simons (MCS) sector, a Klein-Gordon massless scalar field, and a coupling term that mixes the gauge field to the external vector, v^{μ}. In spite of breaking Lorentz invariance in the particle frame, this model may preserve the CPT symmetry for a single particular choice of v^{μ}. Analyzing the dispersion relations, one verifies that the reduced model exhibits stability, but the causality can be jeopardized by some modes. The unitarity of the gauge sector is assured without any restriction, while the scalar sector is unitary only in the space-like case.

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I. INTRODUCTION

In a common sense, it is generally settled that a "good" Quantum Field Theory (QFT) must respect at least two symmetries: the Lorentz covariance and the CPT invariance. The traditional framework of a local QFT, from which one derives the Standard Model that sets the physics inherent to the fundamental particles, satisfies both these symmetries. In the beginning of 90´s, a new work [1] proposing a correction term to the conventional Maxwell Electrodynamics, that preserves the gauge invariance despite breaking the Lorentz, CPT and parity symmetries, was first analyzed. The correction term, composed by the gauge potential, A_μ, and an external background 4-vector, v_μ, has a Chern-Simons-like structure, $\epsilon^{\mu\nu\kappa\lambda}v_\mu A_\nu F_{\kappa\lambda}$, and is responsible by inducing an optical activity of the vacuum - or birefringence - among other effects. In this same work, however, it is shown that astrophysical data do not support the birefringence and impose stringent limits on the value of the constant vector v_μ, reducing it to a negligible correction term. Similar conclusions, also based on astrophysical observations, were also confirmed by Goldhaber & Timble [3]. Some time later, Colladay and Kostelecky [2] adopted a quantum field theoretical framework to address the issue of CPT- and Lorentz-breakdown as a spontaneous violation. In this sense, they constructed an extension to the minimal Standard Model, which maintains unaffected the $SU(3) \times SU(2) \times U(1)$ gauge structure of the usual theory, and incorporates the CPT-violation as an active feature of the effective low-energy broken action. They started from a usual CPT- and Lorentz-invariant action as defining the properties of what would be an underlying theory at the Planck scale [4], which then suffers a spontaneous breaking of both these symmetries. In the broken phase, there rises the effective action, endowed with breakdown of CPT and Lorentz symmetries, but conservation of covariance under the perspective of the observer inertial frame. The Lorentz invariance is spoiled at the level of the particle-system, which can be viewed in terms of the non-invariance of the fields under boost and Lorentz rotations (relative inertial observer-frames). This covariance breakdown is also manifest when analyzing the dispersion relations, extracted from the propagators.

Investigations concerning the unitarity, causality and consistency of a QFT endowed with violation of Lorentz and CPT symmetries (induced by a Chern-Simons term) were carried out by Adam & Klinkhamer [5]. As result, it was verified that the causality and unitarity of this kind of model can be preserved when the fixed (background) 4-vector is space-like, and spoiled whenever it is time-like or null. A consistency analysis of this model, carried out in the additional presence of a scalar sector endowed with spontaneous symmetry breaking (SSB) [7], has confirmed the results obtained in ref. [5], that is: the space-like case is free from unitarity illnesses, which arise in the time- and light-like cases.

The active development of Lorentz- and CPT-violating theories in $D = 1 + 3$ has come across the inquiry about the structure of a similar model in 1+2 dimensions and its possible implications. In order to study a planar theory, endowed with Lorentz- and CPT-violation, one has decided to adopt a dimensional reduction procedure, that is: one starts from the original Chern-Simons-like term, $\epsilon^{\mu\nu\kappa\lambda}v_\mu A_\nu F_{\kappa\lambda}$, promoting its systematic reduction to $D = 1 + 2$, which yields a pure Chern-Simons term and a Lorentz non-invariant mixing term. Our objective, therefore, is to achieve a planar model, whose structure is derived from a known counterpart defined in 1+3 dimensions, and to investigate some of its features, like propagators, dispersion relations, causality, stability and unitarity.

More specifically, one performs the dimensional reduction to 1+2 dimensions of the Abelian gauge invariant model with non-conservation of the Lorentz and CPT symmetries [1], [5] induced by the term $\epsilon^{\mu\nu\kappa\lambda}v_\mu A_\nu F_{\kappa\lambda}$, resulting in a gauge invariant Planar Quantum Electrodynamics (QED_3) composed by a Maxwell-Chern-Simons gauge field (A_μ), by a scalar field (φ), a scalar parameter (s) without dynamics (the Chern-Simons mass), and a fixed 3-vector (v^μ). Besides the MCS sector, this Lagrangian has a massless scalar sector, represented by the field φ, which also works out as the coupling constant in the Chern-Simons-like structure that mixes the gauge field to the 3-vector, v^μ (where one gauge field is replaced by v^μ). This latter term is the responsible by the Lorentz noninvariance.

Therefore, the reduced Lagrangian is endowed with three coupled sectors: a MCS sector, a massless Klein-Gordon sector and a mixing Lorentz-violating one. As it is well-known, the MCS sector breaks both parity and time-reversal symmetries, but preserves the Lorentz and CPT ones. The scalar sector preserves all discrete symmetries and Lorentz covariance, whereas the mixing sector, as it will be seen, breaks Lorentz invariance (in relation to the particle-frame), keeps conserved parity and charge-conjugation symmetries, but may break or preserve time-reversal symmetry. This implies that it may occur both conservation (for a purely space-like v^{μ}) and violation (for v^{μ} time-like and light-like) of the CPT invariance.

In short, this paper is outlined as follows. In Section II, one accomplishes the dimensional reduction, that leads to the reduced model. Having established the new planar Lagrangian, one then devotes some algebraic effort for the derivation of the propagators of the gauge and scalar fields, which requires the evaluation of a closed algebra composed by eleven projector operators, displayed into Table I. In Section III, we investigate the stability and the causal structure of the theory. One addresses the causality looking directly at the dispersion relations extracted from the poles of the propagators, which reveal the existence of both causal and non-causal modes. All the modes, nevertheless, present positive definite energy (positivity) relative to any Lorentz frame, which implies an overall stability. In Section IV, we accomplish the unitarity analysis, based on the matrix residue evaluated at the poles of the propagators. The unitarity of the overall model is ensured in the case one adopts a purely space-like background-vector, v^{μ}. In Section V, we present our Concluding Comments.

II. THE DIMENSIONALLY REDUCED MODEL

One starts from the Maxwell Lagrangian[†] in 1+3 dimensions supplemented by a term that couples the dual electromagnetic tensor to a fixed 4-vector, v^{μ}, as it appears in ref. [1]:

$$\mathcal{L}_{1+3} = \left\{ -\frac{1}{4} F_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}} + \frac{1}{2} \epsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}} v_{\hat{\mu}} A_{\hat{\nu}} F_{\hat{\kappa}\hat{\lambda}} + A_{\hat{\nu}} J^{\hat{\nu}} \right\}, \tag{1}$$

with the additional presence of the coupling between the gauge field and the external current, $A_{\hat{\nu}} J^{\hat{\nu}}$. This model (in its free version) is gauge invariant but does not preserve Lorentz and CPT symmetries relative to the particle frame. For the observer system, the Chern-Simons-like term transforms covariantly, once the background also is changed under an observer boost: $v^{\hat{\mu}} \longrightarrow v^{\hat{\mu}'} = \Lambda^{\mu}{}_{\alpha} v^{\alpha}$. In connection with the particle-system, however, when one applies a boost on the particle, the background 4-vector is supposed to remain unaffected, behaving like a set of four independent numbers, which configures the breaking of the covariance. This term also breaks the parity symmetry, but maintain invariance under charge conjugation and time reversal. To study this model in 1+2 dimensions, one performs its dimensional reduction, which consists effectively in adopting the following ansatz over any 4-vector: (i) one keeps unaffected the temporal and also the first two spatial components; (ii) one freezes the third spacial dimension by splitting it from the body of the new 3-vector and requiring that the new quantities (χ), defined in 1+2 dimensions, do not depend on the third spacial dimension: $\partial_3 \chi \longrightarrow 0$. Applying this prescription to the gauge 4-vector, $A^{\hat{\mu}}$, and to the fixed external 4-vector, $v^{\hat{\mu}}$, and to the 4-current, $J^{\hat{\mu}}$, one has:

$$A^{\hat{\mu}} \longrightarrow (A^{\mu};\ \varphi), \tag{2}$$

$$v^{\hat{\mu}} \longrightarrow (v^{\mu};\ s), \tag{3}$$

$$J^{\hat{\mu}} \longrightarrow (J^{\mu};\ J), \tag{4}$$

[†]Here one has adopted the following metric conventions: $g_{\mu\nu} = (+,-,-,-)$ in $D = 1+3$, and $g_{\mu\nu} = (+,-,-)$ in $D = 1+2$. The greek letters (with hat) $\hat{\mu}$, run from 0 to 3, while the pure greek letters, μ, run from 0 to 2.

where: $A^{(3)} = \varphi$, $v^{(3)} = s$, $J^{(3)} = J$ and $\mu = 0,1,2$. According to this process, there appear two scalars: the scalar field, φ, that exhibits dynamics, and s, a constant scalar (without dynamics). Carrying out this prescription for eq. (1), one then obtains:

$$\mathcal{L}_{1+2} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{s}{2}\epsilon_{\mu\nu k}A^\mu\partial^\nu A^k + \varphi\epsilon_{\mu\nu k}v^\mu\partial^\nu A^k - \frac{1}{2\alpha}\left(\partial_\mu A^\mu\right)^2 + A_\mu J^\mu + \varphi J, \tag{5}$$

where the last free term represents the gauge-fixing term, added up after the dimensional reduction. The scalar field, φ, exhibits a typical Klein-Gordon massless dynamics and it also appears as the coupling constant that links the fixed v^μ to the gauge sector of the model, by means of the new term: $\varphi\epsilon_{\mu\nu k}v^\mu\partial^\nu A^k$. In spite of being covariant in form, this kind of term breaks the Lorentz symmetry in the particle-frame, since the 3-vector v^μ is not sensitive to particle Lorentz boost, behaving like a set of three scalars.

The Lagrangian (1), originally proposed by Carroll-Field-Jackiw [1], has the property of breaking parity symmetry, even though conserving time reversal and charge conjugation symmetries, resulting in nonconservation of the CPT symmetry. Simultaneously, the Lorentz invariance is spoiled, since the fixed 4-vector v^μ breaks the rotational and boost invariances. On the other hand, the reduced model, given by eq.(5), does not necessarily jeopardize the CPT conservation, which depends truly on the character of the fixed vector v^μ. As it is known, the parity transformation ($\mathcal{P}$) in 1+2 dimensions is characterized by the inversion of only on of the spatial axis: $x^\mu \overset{\mathcal{P}}{\longrightarrow} x'^\mu = (x_o, -x, y)$, the same being valid for the 3-potential: $A^\mu \overset{\mathcal{P}}{\longrightarrow} A'^\mu = (A_0, -A^{(1)}, A^{(2)})$. The time-reversal transformation ($\mathcal{T}$) must keep unchanged the dynamics of the system, so that one must have: $x^\mu \overset{\mathcal{T}}{\longrightarrow} x'^\mu = (-x_o, x, y)$, $A^\mu \overset{\mathcal{T}}{\longrightarrow} A'^\mu = (A_0, -A^{(1)}, -A^{(2)})$, while the charge conjugation determines: $x^\mu \overset{\mathcal{C}}{\longrightarrow} x'^\mu = x^\mu$, $A^\mu \overset{\mathcal{C}}{\longrightarrow} A'^\mu = -A^\mu$. One knows that the Chern-Simons term breaks both parity and time-reversal symmetries and keeps conserved the charge conjugation, which assures the global CPT invariance. The new term, $\varphi\varepsilon_{\mu\nu k}v^\mu\partial^\nu A^k$, however, will manifest a non-symmetric behaviour before $\mathcal{T}$-transformation: there will occur conservation if one works with a purely space-like external vector ($v^\mu = (0, \overrightarrow{v})$), or breakdown, if v^μ is purely time-like. Under parity and charge conjugation transformations, in turn, this term will evidence non-invariance for any adopted v^μ, thereby one can state that it will occur CPT conservation when v^μ is purely space-like, and CPT violation otherwise. Here, the field φ was considered as having a scalar character under the parity transformation. Yet, if this field behaves like a pseudo-scalar[‡], the CPT conversation will be assured for a purely time-like v^μ. For a light-like v^μ, there will always occur time-reversal non-invariance, and consequently, CPT violation.

Neglecting divergence terms, one can write the linearized free action in an explicitly quadratic form, namely:

$$\Sigma_{1+2} = \int d^3x \frac{1}{2}\left\{ A^\mu[M_{\mu\nu}]A^\nu - \varphi\Box\varphi + \varphi\left[\epsilon_{\mu\alpha\nu}v^\mu\partial^\alpha\right]A^\nu + A^\mu\left[\epsilon_{\nu\alpha\mu}v^\nu\partial^\alpha\right]\varphi \right\}, \tag{6}$$

which can also appear in the matrix form:

$$\Sigma_{1+2} = \int d^3x \frac{1}{2}\begin{pmatrix} A^\mu & \varphi \end{pmatrix}\begin{bmatrix} M_{\mu\nu} & T_\mu \\ -T_\nu & -\Box \end{bmatrix}\begin{pmatrix} A^\nu \\ \varphi \end{pmatrix}. \tag{7}$$

The action (7) has as nucleus a square matrix, P, composed by the quadratic operators of the initial action. The mass dimension of the physical parameters and tensors are: $[A^\mu] = [\varphi] = 1/2$, $[v^\mu] = [s] = 1, [T_\mu] = [M_{\mu\nu}] = 2$. Here, some definitions are necessary:

[‡]The adoption of a pseudo-scalar field can be justified by looking at the vector character of the potential ($\overrightarrow{A} \overset{\mathcal{P}}{\longrightarrow} -\overrightarrow{A}$) before the dimensional reduction. If one assumes that the field φ maintains the same behaviour of its ancestral (A_3), one has a pseudo-scalar.

$$M_{\mu\nu} = \Box\theta_{\mu\nu} + s\ S_{\mu\nu} + \frac{\Box}{\alpha}\omega_{\mu\nu}, \quad T_\nu = S_{\mu\nu}v^\mu, \tag{8}$$

$$S_{\mu\nu} = \varepsilon_{\mu\kappa\nu}\partial^\kappa,\ \theta_{\mu\nu} = \eta_{\mu\nu} - \omega_{\mu\nu}, \quad \omega_{\mu\nu} = \frac{\partial_\mu\partial_\nu}{\Box}, \tag{9}$$

where $\theta_{\mu\nu}$, $\omega_{\mu\nu}, S_{\mu\nu}$ stand respectively for the transverse, longitudinal and Chern-Simons dimensionless projectors, while $M_{\mu\nu}$ is the quadratic operator associated to the MCS sector. The inverse of the square matrix P, given at the action (7), yields the propagators of the gauge and the scalar fields, which are also written in a matrix form, the propagator-matrix (Δ):

$$\Delta = P^{-1} = \frac{-1}{(\Box M_{\mu\nu} - T_\mu T_\nu)}\begin{bmatrix} -\Box & T_\nu \\ -T_\mu & M_{\mu\nu} \end{bmatrix}, \tag{10}$$

The propagator of the gauge field, Δ_{11}, and of the scalar field, Δ_{22}, are written as:

$$(\Delta_{11})^{\mu\nu} = \left[\Box\theta_{\mu\nu} + s\ S_{\mu\nu} + \frac{\Box}{\alpha}\omega_{\mu\nu} - \frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \tag{11}$$

$$(\Delta_{22}) = -\frac{M_{\mu\nu}}{\Box}\left[\Box\theta_{\mu\nu} + s\ S_{\mu\nu} + \frac{\Box}{\alpha}\omega_{\mu\nu} - \frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \tag{12}$$

$$(\Delta_{12})^{\mu} = -\frac{T_\nu}{\Box}\left[\Box\theta_{\mu\nu} + s\ S_{\mu\nu} + \frac{\Box}{\alpha}\omega_{\mu\nu} - \frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \tag{13}$$

$$(\Delta_{21})^{\nu} = \frac{T_\mu}{\Box}\left[\Box\theta_{\mu\nu} + s\ S_{\mu\nu} + \frac{\Box}{\alpha}\omega_{\mu\nu} - \frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \tag{14}$$

while the terms Δ_{12}, Δ_{21} are related to the mixed propagators $\langle A_\mu\varphi\rangle$, $\langle\varphi A_\mu\rangle$ that indicate a scalar mediator turning into a gauge mediator and vice-versa. Here, for future purposes, it is useful to present the inverse of the tensor $M_{\mu\nu}$, that is, the propagator of the pure MCS Lagrangian:

$$(M_{\mu\nu})^{-1} = \frac{1}{\Box + s^2}\theta^{\nu\mu} - \frac{s}{\Box(\Box + s^2)}S^{\nu\mu} + \frac{\alpha}{\Box}\omega^{\nu\mu}, \tag{15}$$

To perform the inversion of the operator above, one needs to define some new operators, since the ones known so far do not form a closed algebra, as it is shown below:

$$S_{\mu\nu}T^\nu T^\alpha = \Box v_\mu T^\alpha - \lambda T^\alpha\partial_\mu = \Box Q_\mu{}^\alpha - \lambda\Phi^\alpha{}_\mu, \tag{16}$$

$$Q_{\mu\nu}Q^{\alpha\nu} = T^2 v^\alpha v_\mu = T^2\Lambda^\alpha{}_\mu, \tag{17}$$

$$Q_{\mu\nu}\Phi^{\nu\alpha} = T^2 v_\mu\partial^\alpha = T^2\Sigma_\mu{}^\alpha, \tag{18}$$

where the new operators are:

$$Q_{\mu\nu} = v_\mu T_\nu, \quad \Lambda_{\mu\nu} = v_\mu v_\nu, \quad \Sigma_{\mu\nu} = v_\mu\partial_\nu, \quad \Phi_{\mu\nu} = T_\mu\partial_\nu, \tag{19}$$

and,

$$\lambda \equiv \Sigma_\mu^\mu = v_\mu\partial^\mu\ , \quad T^2 = T_\alpha T^\alpha = (v^2\Box - \lambda^2). \tag{20}$$

Their mass dimension are: $[\Lambda_{\mu\nu}] = 2$, $[Q_{\mu\nu}] = 3$, $[\Sigma_{\mu\nu}] = 2$, $[\Phi_{\mu\nu}] = 3$.

Three of these new terms exhibit a non-symmetric structure, which leads to their consideration in pairs, namely: $Q_{\mu\nu}, Q_{\nu\mu}$; $\Sigma_{\mu\nu}, \Sigma_{\nu\mu}$; $\Phi_{\mu\nu}, \Phi_{\nu\mu}$. The inversion of the operator Δ_{11} will be realized following the traditional prescription, $\left(\Delta_{11}^{-1}\right)_{\mu\nu}(\Delta_{11})^{\nu\alpha} = \delta_\mu^\alpha$, where the operator $(\Delta_{11})^{\nu\alpha}$ is composed by all the possible tensor combinations (of rank two) involving $T_\mu, v_\mu, \partial_\alpha$. In such way, the proposed propagator will consist, at a first glance, of eleven terms:

$$(\Delta_{11})^{\nu\alpha} = a_1\theta^{\nu\alpha} + a_2\omega^{\nu\alpha} + a_3 S^{\nu\alpha} + a_4\Lambda^{\nu\alpha} + a_5 T^\nu T^\alpha + a_6 Q^{\nu\alpha} + a_7 Q^{\alpha\nu} + a_8\Sigma^{\nu\alpha} + a_9\Sigma^{\alpha\nu} + a_{10}\Phi^{\nu\alpha} + a_{11}\Phi^{\alpha\nu}, \tag{21}$$

which are displayed in Table I, where one observes explicitly the closure of the operator algebra.

	$\theta_{\mu\nu}$	$\omega_{\mu\nu}$	$S_{\mu\nu}$	$\Lambda_{\mu\nu}$	$T_\mu T_\nu$	$Q_{\mu\nu}$	$Q_{\nu\mu}$	$\Sigma_{\mu\nu}$	$\Sigma_{\nu\mu}$	$\Phi_{\mu\nu}$	$\Phi_{\nu\mu}$
$\theta^{\nu\alpha}$	${\theta_\mu}^\alpha$	0	${S_\mu}^\alpha$	${\Lambda_\mu}^\alpha + -\frac{\lambda}{\Box}{\Sigma_\mu}^\alpha$	$T_\mu T^\alpha$	${Q_\mu}^\alpha$	${Q^\alpha}_\mu + -\frac{\lambda}{\Box}{\Phi_\mu}^\alpha$	0	${\Sigma^\alpha}_\mu + -\lambda\Box{\omega^\alpha}_\mu$	0	${\Phi^\alpha}_\mu$
$\omega^{\nu\alpha}$	0	${\omega_\mu}^\alpha$	0	$\frac{\lambda}{\Box}{\Sigma_\mu}^\alpha$	0	0	$\frac{\lambda}{\Box}{\Phi_\mu}^\alpha$	${\Sigma_\mu}^\alpha$	$\lambda{\omega^\alpha}_\mu$	${\Phi^\alpha}_\mu$	0
$S^{\nu\alpha}$	${S_\mu}^\alpha$	0	$-\Box{\theta_\mu}^\alpha$	${Q_\mu}^\alpha$	$\lambda{\Phi_\mu}^\alpha + -\Box Q^\alpha_\mu$	$\lambda{\Sigma_\mu}^\alpha + -{\Lambda_\mu}^\alpha\Box$	$-T_\mu T^\alpha$	0	$\partial_\mu T^\alpha$	0	$\Box({\omega_\mu}^\alpha + -{\Sigma^\alpha}_\mu)$
$\Lambda^{\nu\alpha}$	${\Lambda_\mu}^\alpha + -\frac{\lambda}{\Box}{\Sigma^\alpha}_\mu$	$\frac{\lambda}{\Box}{\Sigma^\alpha}_\mu$	$-{Q^\alpha}_\mu$	$v^2{\Lambda_\mu}^\alpha$	0	0	$v^2{Q^\alpha}_\mu$	$\lambda{\Lambda_\mu}^\alpha$	$v^2{\Sigma^\alpha}_\mu$	$\lambda{Q^\alpha}_\mu$	0
$T^\nu T^\alpha$	$T_\mu T^\alpha$	0	$\Box{Q_\mu}^\alpha + -\lambda{\Phi^\alpha}_\mu$	0	$T^2 T_\mu T^\alpha$	$T^2 Q^\alpha_\mu$	0	0	0	0	$T^2{Q_\mu}^\alpha$
$Q^{\nu\alpha}$	${Q_\mu}^\alpha + -\frac{\lambda}{\Box}{\Phi^\alpha}_\mu$	$\frac{\lambda}{\Box}{\Phi^\alpha}_\mu$	$-T_\mu T^\alpha$	$v^2{Q_\mu}^\alpha$	0	0	$v^2 T_\mu T^\alpha$	$\lambda{Q_\mu}^\alpha$	$v^2\partial_\mu T^\alpha$	$\lambda T_\mu T^\alpha$	0
$Q^{\alpha\nu}$	${Q^\alpha}_\mu$	0	$\Box{\Lambda_\mu}^\alpha + -\lambda{\Sigma^\alpha}_\mu$	0	$T^2{Q^\alpha}_\mu$	$T^2{\Lambda^\alpha}_\mu$	0	0	0	0	$T^2{\Sigma^\alpha}_\mu$
$\Sigma^{\nu\alpha}$	${\Sigma_\mu}^\alpha + -\lambda{\omega_\mu}^\alpha$	$\lambda{\omega_\mu}^\alpha$	$-{\Phi_\mu}^\alpha$	$v^2{\Sigma_\mu}^\alpha$	0	0	$v^2{\Phi_\mu}^\alpha$	$\lambda{\Sigma_\mu}^\alpha$	$v^2{\Lambda_\mu}^\alpha$	$\lambda{\Phi_\mu}^\alpha$	0
$\Sigma^{\alpha\nu}$	0	${\Sigma^\alpha}_\mu$	0	$\lambda{\Lambda_\mu}^\alpha$	0	0	$\lambda{Q^\alpha}_\mu$	$\Box{\Lambda_\mu}^\alpha$	$v^2{\Lambda_\mu}^\alpha$	$\Box{Q_\mu}^\alpha$	0
$\Phi^{\nu\alpha}$	${\Phi_\mu}^\alpha$	0	$\Box({\Sigma_\mu}^\alpha + -\lambda{\omega_\mu}^\alpha)$	0	$T^2{\Phi_\mu}^\alpha$	$T^2{\Sigma_\mu}^\alpha$	0	0	0	0	$\Box T^2\omega^\alpha_\mu$
$\Phi^{\alpha\nu}$	0	${\Phi^\alpha}_\mu$	0	$\lambda{\Phi^\alpha}_\mu$	0	0	$\lambda T_\mu T^\alpha$	$\Box{\Phi^\alpha}_\mu$	$\lambda{\Phi^\alpha}_\mu$	$\Box T_\mu T^\alpha$	0

Table I: Multiplicative operator algebra fulfilled by θ, ω, S, Λ,TT, Q, Σ,and Φ. The products are supposed to be in the ordering "row times column".

Using the data contained in Table I, one finds out that the gauge-field propagator assumes the form:

$$\begin{aligned}(\Delta_{11})^{\mu\nu} =& \frac{1}{\Box + s^2}\theta^{\mu\nu} + \frac{\alpha(\Box + s^2)\boxtimes - \lambda^2 s^2}{\Box(\Box + s^2)\boxtimes}\omega^{\mu\nu} - \frac{s}{\Box(\Box + s^2)}S^{\mu\nu} - \frac{s^2}{(\Box + s^2)\boxtimes}\Lambda^{\mu\nu} + \frac{1}{(\Box + s^2)\boxtimes}T^\mu T^\nu \\ & - \frac{s}{(\Box + s^2)\boxtimes}Q^{\mu\nu} + \frac{s}{(\Box + s^2)\boxtimes}Q^{\nu\mu} + \frac{\lambda s^2}{\Box(\Box + s^2)\boxtimes}\Sigma^{\mu\nu} + \frac{\lambda s^2}{\Box(\Box + s^2)\boxtimes}\Sigma^{\nu\mu} - \frac{s\lambda}{\Box(\Box + s^2)\boxtimes}\Phi^{\mu\nu} \\ & + \frac{s\lambda}{\Box(\Box + s^2)\boxtimes}\Phi^{\nu\mu},\end{aligned}$$

where: $\boxtimes = (\Box^2 + s^2\Box - T^2)$.

By the same procedure, one evaluates the mixed propagator, $(\Delta_{12})^\alpha = -\frac{T_\nu}{\Box}(\Delta_{11})^{\nu\alpha}$, which can be written in the following form:

$$(\Delta_{12})^\nu = -\frac{1}{\boxtimes}\left[T^\nu + sv^\nu - \frac{s\lambda}{\Box}\partial^\nu\right], \tag{22}$$

whereas the propagator $(\Delta_{21})^\nu$, in turn, results equal to:

$$(\Delta_{21})^\nu = -\frac{1}{\boxtimes}\left[-T^\nu + sv^\nu - \frac{s\lambda}{\Box}\partial^\nu\right],$$

In order to compute the propagator of the scalar field,

$$(\Delta_{22}) = -\frac{1}{\Box}\left[1 - \frac{1}{\Box}T_\mu (M_{\mu\nu})^{-1} T_\nu\right]^{-1}, \tag{23}$$

one makes use of the inverse of the tensor $M_{\mu\nu}$, given by eq. (15), so that: $T_\mu(M^{-1})^{\mu\nu}T_\nu = \left(\Box + s^2\right)^{-1}T^2$. In such a way, a compact scalar propagator arises:

$$(\Delta_{22}) \ = -\frac{\Box + s^2}{\boxtimes} \tag{24}$$

In momentum-space, the photon propagator takes the final expression:

$$\begin{aligned}\langle A^{\mu}(k)\, A^{\nu}(k)\rangle \ &= i\Big\{-\frac{1}{k^2 - s^2}\theta^{\mu\nu} - \frac{\alpha(k^2 - s^2)\boxtimes(k) + s^2\,(v.k)^2}{k^2(k^2 - s^2)\boxtimes(k)}\omega^{\mu\nu} - \frac{s}{k^2(k^2 - s^2)}S^{\mu\nu} + \frac{s^2}{(k^2 - s^2)\boxtimes(k)}\Lambda^{\mu\nu} \\ &-\frac{1}{(k^2 - s^2)\boxtimes(k)}T^{\mu}T^{\nu} + \frac{s}{(k^2 - s^2)\boxtimes(k)}Q^{\mu\nu} - \frac{s}{(k^2 - s^2)\boxtimes(k)}Q^{\nu\mu} + \frac{is^2\,(v.k)}{k^2(k^2 - s^2)\boxtimes(k)}\Sigma^{\mu\nu} \\ &+\frac{is^2\,(v.k)}{k^2(k^2 - s^2)\boxtimes(k)}\Sigma^{\nu\mu} - \frac{is\,(v.k)}{k^2(k^2 - s^2)\boxtimes(k)}\Phi^{\mu\nu} + \frac{is\,(v.k)}{k^2(k^2 - s^2)\boxtimes(k)}\Phi^{\nu\mu}\Big\},\end{aligned} \tag{25}$$

while the scalar and the mixed propagators read as:

$$\langle\varphi\varphi\rangle \ = \frac{i}{\boxtimes(k)}\left[k^2 - s^2\right], \tag{26}$$

$$\langle A^{\alpha}(k)\,\varphi\rangle = -\frac{i}{\boxtimes(k)}\left[T^{\alpha} + sv^{\alpha} - \frac{s\,(v.k)}{k^2}k^{\alpha}\right], \tag{27}$$

$$\langle\varphi A^{\alpha}(k)\rangle = -\frac{i}{\boxtimes(k)}\left[-T^{\alpha} + sv^{\alpha} - \frac{s\,(v.k)}{k^2}k^{\alpha}\right], \tag{28}$$

where: $\boxtimes(k) = \left[k^4 - \left(s^2 - v.v\right)k^2 - (v.k)^2\right]$. By the above expressions, one notes that the factor $\boxtimes$ is present on the denominator of all propagators, in such a way the scalar and the gauge field will share the pole structure, and consequently, the physical excitations associated with the poles of $\boxtimes(k)$. This common dependence on $1/\boxtimes$ also amounts to similarities on the causal structure of the scalar and gauge sectors of this model, as it will de discussed in Section III.

III. DISPERSION RELATIONS, STABILITY AND CAUSALITY ANALYSIS

Some references in literature [5], [6], [8] have dealt with the issue of stability, causality and unitarity concerning to Lorentz- and CPT-violating theories. The causality is usually addressed as a quantum feature that requires the commutation between observables separated by a space-like interval, which one calls microcausality in field theory [9]. In this section, however, one analyzes causality under a classical tree-level perspective, in which it is related to the positivity of a usual Lorentz invariant, k^2. The starting-point of all investigation is the propagator, whose poles are associated to dispersion relations (DR) that provide informations about the stability and causality of the model. The causality analysis is then related to the sign of the propagator poles, given in terms of k^2, in such a way one must have $k^2 \geq 0$ in order to preserve it (circumventing the existence of tachyons). In the second quantization framework, stability is related to the energy positivity of the Fock states for any momentum. Here, stability is directly associated with the energy positivity of each mode read off from the DR.

The field propagators, given by eqs. (25, 27, 26), present three families of poles at k^2:

$$k^2 = 0; \quad k^2 - s^2 = 0; \quad k^4 - (s^2 - v.v)k^2 - (v.k)^2 = 0, \tag{29}$$

from which one straightforward infers the DR derived from the Lagrangian (5), namely:

$$k_{0(1)}^2 = \overrightarrow{k}^2; \quad k_{0(2)}^2 = \overrightarrow{k}^2 + s^2; \quad k_{0(3)}^2 = \overrightarrow{k}^2 + \frac{1}{2}\left[\left(s^2 - v.v\right) \pm \sqrt{\left(s^2 - v.v\right)^2 + 4\,(v.k)^2}\right]. \tag{30}$$

The first dispersion relation, $k_0 = \pm|\overrightarrow{k}|$, stands for a massless photon mode, which carries no degree of freedom, since the Lagrangian (5) involves a massive photon. The second DR represents the Chern-Simons massive mode, $k_0 = \pm\sqrt{s^2 + |\overrightarrow{k}|^2}$, which propagates only one degree of freedom (in the Maxwell-Chern-Simons electrodynamics the scalar magnetic field encloses all information of the electromagnetic field, which justifies the existence of a single degree of freedom). These first two poles apparently respect the causality condition, since $k^2 \geq 0$ for them. Once the causality is set up, the stability comes up as a direct consequence.

Concerning the third DR, corresponding to the roots of $\boxtimes(k)$, it may provide both massless and massive modes for some specific $\overrightarrow{k}$-values, but in general, the mode is massive. By remembering that $\overrightarrow{k}$ is the transfer momentum, whose values are generally integrated from zero to infinity, one concludes it does not make much sense to fix any value for $\overrightarrow{k}$ in order to obtain a particular dispersion relation. Remarking that the term $\boxtimes(k)$ is ubiquitous in the denominator of all propagators, as it is explicit in eqs. (25),(26), (27), one concludes the causal structure entailed to the poles of $1/\boxtimes$ will be common to these three propagators. Specifically, for a purely space-like 3-vector, $v^\mu = (0, \overrightarrow{v})$, this DR is written as,

$$k_{0\pm}^2 = \overrightarrow{k}^2 + \frac{1}{2}\left[\left(s^2 + \overrightarrow{v}^2\right) \pm \sqrt{\left(s^2 + \overrightarrow{v}^2\right)^2 + 4\left(\overrightarrow{v}.\overrightarrow{k}\right)^2}\right]. \tag{31}$$

A simple analysis of this expression indicates that both k_{0+}^2 and k_{0-}^2 are positive-energy modes for any $\overrightarrow{k}$-value (and for any Lorentz observer), which assures the stability of these modes. This fact may suggest that the causal structure of the space-like sector of this model remains preserved, as it was observed by Adam & Klinkhamer [5] in the context of the 4-dimensional version of this theory, that is endowed with a dispersion relation very similar to eq. (31) (this conclusion was also supported by the attainment of a group velocity, associated to this mode, smaller than 1). Concerning the pole analysis, although, we have $k_+^2 > 0$ for arbitrary $\overrightarrow{k}$ and $k_-^2 < 0$ (unless $\overrightarrow{k} \perp \overrightarrow{v}$ or $\overrightarrow{k} = 0$, which implies $k_-^2 = 0$). So, while the mode k_+^2 preserves the causality and stability, the mode k_-^2, in spite of assuring stability, will be in general non-causal, preserving causality only when $\overrightarrow{k} \perp \overrightarrow{v}$ or $\overrightarrow{k} = 0$.

In the case of a purely time-like 3-vector, $v^\mu = (v_0, \overrightarrow{0})$, the DR assumes the form:

$$k_{0\pm}^2 = \frac{1}{2}\left[\left(s^2 + 2\overrightarrow{k}^2\right) \pm \sqrt{s^4 + 4v_o^2\overrightarrow{k}^2}\right], \tag{32}$$

where one observes a similar behaviour: the mode k_{0+}^2 will exhibit stability and causality, while the mode k_{0-}^2 will present energy positivity (for arbitrary $\overrightarrow{k}$-value) whenever the condition, $s^2 - v_0^2 > 0$, is fulfilled. From now on, one must assume the validity of this condition, so that the mode k_{0-}^2 can be taken stable. This latter mode is non-causal for any $\overrightarrow{k} \neq 0$. Assuming the coefficients for Lorentz violation are small near the Chern-Simons mass $\left(s^2 \gg v_0^2, |\overrightarrow{v}|^2\right)$, we obtain an entirely causal theory (at least at zero order in v^2/s^2). This is consistent with some results [8] concerning some quantum theories containing Lorentz-violating terms, which evidence the preservation of causality when the breaking factors are small.

Hence, the modes $k_{0\pm}^2$ exhibit positive energy both in space- and time-like cases, which also implies these two modes can be written as an expansion in terms of positive and negative frequency terms. This separation allows the definition of particles and antiparticles states, a necessary condition for the quantization of this theory. Nevertheless, the existence of non-causal modes, both in time- and space-like case, may be seen already at classical level, as a prediction on the impossibility to realize a consistent quantization of this model, an issue that will be properly addressed when one analyses the unitarity at these non-causal poles. Therefore, the existence of quantization illness will be solved by investigating the unitarity of the model, matter to be discussed in the next section.

In a Lorentz covariant framework, k^2 is a Lorentz scalar, which assures a unique value for all Lorentz frames. In such a way, if k^2 represents a causal mode for one observer, so it will be for all ones. The fact that k^2 has not a positive definite value in an arbitrary Lorentz frame is a unequivocal indicative of the Lorentz covariance breakdown.

IV. UNITARITY ANALYSIS

In order to analyze the unitarity of the model at tree-level, one has adopted the method which consists in saturating the propagators with external currents. The fact that our model possesses two sectors (the scalar and gauge one) implies that we must saturate the scalar-propagator and the gauge-propagator individually. In such a way, we write the two saturated propagators, namely:

$$SP_{\langle A_\mu A_\nu\rangle} = J^{*\mu}\langle A_\mu(k)A_\nu(k)\rangle\ J^\nu,$$
$$SP_{\langle\varphi\varphi\rangle} = J^*\langle\varphi\varphi\rangle J,$$

where the gauge current J^μ must obey the conservation law valid for the gauge sector of the system[§], whereas the scalar current, J, is not subject to any constraint. The unitarity analysis is based on the residues of SP, precisely: the unitarity is ensured whenever the imaginary part of the residues of SP at the poles of each propagator is positive. It is easy to notice that the saturated propagator in the momentum-space is the current-current transition amplitude.

A. Scalar Sector

We can initiate our analysis by the scalar sector, whose saturated amplitude is given by: $SP_{\langle\varphi\varphi\rangle} = J^*\langle\varphi\varphi\rangle J$, or more explicitly:

$$SP_{\langle\varphi\varphi\rangle} = J^*\frac{i(k^2-s^2)}{\boxtimes(k)}J.$$

This expression presents two poles, k_+^2, k_-^2, the roots of $\boxtimes(k) = 0$. At the purely time-like case, $v^\mu = (v_0, \overrightarrow{0})$, these poles are exactly the ones given by eq. (32): $k_\pm^2 = \frac{1}{2}\left[s^2 \pm \sqrt{s^4 + 4v_o^2\overrightarrow{k}^2}\right]$. Evaluating the residues of $SP_{\langle\varphi\varphi\rangle}$ at the pole k_+^2 one achieves a positive imaginary result, while at the pole k_-^2 a positive result appears only when the condition $\overrightarrow{k}^2 < (v_0^2 + s^2)$. In such a way, one concludes that the unitarity of the scalar sector, in the time-like case, is not assured. Considering now the purely space-like case, $v^\mu = (0, \overrightarrow{v})$, the poles of $SP_{\langle\varphi\varphi\rangle}$ are given by eq. (31): $k_\pm^2 = \frac{1}{2}\left[(s^2 + \overrightarrow{v}^2) \pm \sqrt{(s^2+\overrightarrow{v}^2)^2 + 4\left(\overrightarrow{v}.\overrightarrow{k}\right)^2}\right]$. The residues associated with these two poles exhibit a positive definite imaginary part, so that one can state that the unitarity of the scalar sector, at the space-like case, is generically preserved.

B. Gauge-Field sector

The continuity equation, $\partial_\mu J^\mu = 0$, in the k-space is read as: $k_\mu J^\mu = 0$; it allows us to write the current in the form: $J^\mu = (j^0, 0, \frac{k_0}{k_2}j^{(0)})$. The conservation constraint, $j^{(2)} = \frac{k_0}{k_2}j^{(0)}$, appears whenever one adopts $k^\mu = (k_0, 0, k_2)$ as the momentum. The current conservation law also reduces to six the number of terms of the photon propagator that contributes to the evaluation of the saturated propagator:

[§]By applying the differential operator, ∂_μ, on the equation of motion derived from Lagrangian (5), there results the following equation (see ref. [11]) for the gauge current: $\partial_\mu J^\mu = -\varepsilon^{\mu\nu\rho}\partial_\mu v_\nu \partial_\rho\varphi$, which reduces to the conventional current-conservation law, $\partial_\mu J^\mu = 0$, whenever v^μ is constant or has a null rotational ($\varepsilon^{\mu\nu\rho}\partial_\mu v_\nu = 0$).

$$SP_{\langle A_\mu A_\nu\rangle} = J^*_\mu(k)\left\{\frac{i}{D}\left(\square\boxtimes g^{\mu\nu} - s\boxtimes S^{\mu\nu} - s^2\square\Lambda^{\mu\nu} + \square T^\mu T^\nu - s\square Q^{\mu\nu} + s\square Q^{\nu\mu}\right)\right\}J_\nu(k), \tag{33}$$

where: $D = \square(\square + s^2)\boxtimes$. Writing this expression in the momentum-space, one obtains:

$$SP_{\langle A_\mu A_\nu\rangle} = J^{*\mu}(k)\left\{iB_{\mu\nu}\right\}J^\nu(k), \tag{34}$$

where: $D = k^2(k^2 - s^2)\boxtimes$, with: $\boxtimes(k) = k^4 - (s^2 - v.v)k^2 - (v.k)^2$.

1. Time-like case:

We start by analyzing the unitarity in the case corresponding to a time-like background-vector: $v^\mu = (v_0, \overrightarrow{0})$. In this situation, the 2-rank tensor $B_{\mu\nu}$ can be put in the form:

$$B_{\mu\nu}(k) = \frac{1}{D(k)}\begin{bmatrix} k^2(s^2v_0^2 - \boxtimes) & ik^{(2)}\left(s\boxtimes - v_0^2 s^2 k^2\right) & ik^{(1)}(-s\boxtimes + v_0^2 s^2 k^2) \\ ik^{(2)}\left(-s\boxtimes + v_0^2 s^2 k^2\right) & k^2\left(\boxtimes + v_0^2 k_2^2\right) & is\boxtimes k^{(0)} - v_0^2 k^2 k^{(1)}k^{(2)} \\ ik^{(1)}(s\boxtimes - v_0^2 s^2 k^2) & -is\boxtimes k^{(0)} - v_0^2 k^2 k^{(1)}k^{(2)} & k^2\left(\boxtimes + v_0^2 k_1^2\right) \end{bmatrix}, \tag{35}$$

where: $\boxtimes = k^4 - (s^2 - v_0^2)k^2 - k_0^2$.

For the pole $k^2 = 0$, with $k^\mu = (k_0, 0, k_0)$, we have the following residue matrix:

$$B_{\mu\nu}|_{(k^2=0)} = \frac{1}{s^2}\begin{bmatrix} 0 & -isk_0 & 0 \\ isk_0 & 0 & -isk_0 \\ 0 & isk_0 & 0 \end{bmatrix}, \tag{36}$$

which is reduced to a null matrix when saturated with the conserved current, $J^\mu = (j^0, 0, \frac{k_0}{k_2}j^{(0)})$, implying also a null saturation ($SP = 0$). This fact indicates that the mode associated with the pole $k^2 = 0$ carries no physical degree of freedom, and further, it does not jeopardize the unitarity.

For the pole $k^2 = s^2$, with $k^\mu = (k_0, 0, k_2)$, the matrix takes the form,

$$B_{\mu\nu}|_{(k^2=s^2)} = -\frac{1}{s^2k_2^2}\begin{bmatrix} s^2k_0^2 & -isk^{(2)}k_0^2 & 0 \\ isk^{(2)}k_0^2 & 0 & -isk_0k_2^2 \\ 0 & isk_0k_2^2 & -s^2k_2^2 \end{bmatrix}, \tag{37}$$

This matrix, whenever saturated with the external current $J^\mu = (j^0, 0, \frac{k_0}{k_2}j^{(0)})$, leads to a trivial saturation ($SP = 0$), which is compatible with unitarity requirements. The vanishing of the current-current amplitude at this pole indicates that the massive excitation $k^2 = s^2$ is not dynamical for the time-like background.

At the pole $k_+^2 = \frac{1}{2}\left[s^2 + \sqrt{s^4 + 4v_o^2\overrightarrow{k}^2}\right]$, the residue matrix reads as:

$$B_{\mu\nu}|_{(k^2=k_+^2)} = \frac{v_0^2}{(k_+^2 - s^2)(k_+^2 - k_-^2)}\begin{bmatrix} s^2 & -is^2k^{(2)} & 0 \\ is^2k^{(2)} & k_2^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{38}$$

which has as eigenvalues $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = k_2^2 + s^2$. Consequently, one has $SP > 0$ (unitarity preservation). At the pole k_-^2, a similar behaviour occurs: one obtains a residue matrix exactly equal to the one given above. The difference rests only on the coefficient appearing in front of the matrix, in this case: $\frac{1}{D(k_-)} = v_0^2[(k_-^2 - s^2)(k_-^2 - k_-^2)]^{-1} > 0$. The fact that this last coefficient results positive indicates that the unitarity is also preserved at the pole $k^2 = k_-^2$, once one has the same eigenvalues. Here, the situation presents a peculiarity with respect to its $(1+3)$-dimensional counterpart: according to the analysis of the works [5], [7], for a time-like background vector, the gauge sector is always plagued by ghost states which cannot be removed by any gauge choice. They actually spoil the unitarity.

2. Space-like Case:

In this case, taking $v^\mu = (0,0,v)$, the tensor $B_{\mu\nu}$ is given as follows:

$$B_{\mu\nu}(k) = \frac{1}{D(k)} \begin{bmatrix} -k^2(\boxtimes - v^2 k_1^2) & is\boxtimes k^{(2)} - k^2 v^2 k_0 k^{(1)} & ik^{(1)}(-s\boxtimes - sk^2 v^2) \\ -is\boxtimes k^{(2)} - k^2 v^2 k_0 k^{(1)} & k^2(\boxtimes + v^2 k_0^2) & isk_0(\boxtimes + v^2 k^2) \\ ik^{(1)}(s\boxtimes + sk^2 v^2) & -is\boxtimes k_0 + iskv^2 k_0 & k^2\left(\boxtimes + v^2 s^2\right) \end{bmatrix}, \tag{39}$$

where: $\boxtimes = k^4 - (s^2 - v^2)k^2 - v^2 k_2^2$.

For the pole $k^2 = 0$, with $k^\mu = (k_0, 0, k_0)$, one obtains the same matrix attained in the time-like case, given by eq. (36). Exactly by the same reasons presented at this former section, one can assert that the unitarity is preserved at this pole.

For the pole $k^2 = s^2$, with $k^\mu = (k_0, 0, k_2)$, the resulting matrix is identical to one given by eq. (37), so that the conclusions established in the time-like case are also valid here. The vanishing of the saturated propagator at the pole $k^2 = s^2$, in both cases, indicates that the massive excitation $k^2 = s^2$ is not dynamical in our model.

For the pole $k_+^2 = \frac{1}{2}\left[(s^2 + \overrightarrow{v}^2) \pm \sqrt{\left(s^2 + \overrightarrow{v}^2\right)^2 + 4\left(\overrightarrow{v}.\overrightarrow{k}\right)^2}\right]$, with $k^\mu = (k_0, 0, k_2)$, the residue matrix is reduced to:

$$B_{\mu\nu}|_{(k^2 = k_+^2)} = \frac{v^2}{(k_+^2 - s^2)(k_+^2 - k_-^2)} \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_0^2 & isk_0 \\ 0 & -isk_0 & s^2 \end{bmatrix}, \tag{40}$$

where: $(k_+^2 - k_-^2) = \sqrt{\left(s^2 + v^2\right)^2 + 4v^2 k_2^2}$. The eigenvalues of this matrix are: $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = k_0^2 + s^2$, which leads to a positive saturation $(SP > 0)$, and then unitarity is guaranteed at this pole. For the pole k_-^2, unitarity is also ensured, this may be seen in an exactly similar way to the one performed in the time-like case.

Taking into account all results concerning the gauge sector of this model, one concludes that the unitarity is preserved in both time- and space-like cases (at all the poles of the gauge propagator) without any restriction. Considering the restriction on the unitarity of the scalar sector at the time-like case, one can state that our entire model preserves unitarity only in the space-like case. It is also interesting to note that the unitarity of the gauge sector is guaranteed even at the non-causal poles k_-^2, which confirms the consistency of our model. This fact can be understood remembering that the modes k_-^2, in spite of being non-causal, are stable ones.

V. CONCLUDING COMMENTS

We have accomplished the dimensional reduction to 1+2 dimensions of a gauge invariant, Lorentz and CPT-violating model, defined by the Carroll-Field-Jackiw term, $\epsilon^{\mu\nu\kappa\lambda} v_\mu A_\nu F_{\kappa\lambda}$. One then obtains a Maxwell-Chern-Simons planar Lagrangian in the presence of a Lorentz breaking term and a massless scalar field. Concerning this reduced model, the CPT symmetry is conserved for a purely space-like v^μ, and spoiled otherwise. The propagators of this model are evaluated and exhibit a common causal structure (bound to the dependence on $1/\boxtimes$). The poles of the propagators are used as starting point for the analysis of causality, stability and unitarity. Concerning the dispersion relations, one verifies that the modes have positive definite energy, which ensures stability. The causality is assured for all modes of the theory, except for k_-^2 (both in space- and time-like case). In connection with the unitarity of this model, one has analyzed the scalar and the gauge sectors separately, by means of the saturation of the residue matrix. The gauge sector has revealed to be unitary for time- and space-like background vectors, whereas the scalar sector has showed to preserve unitarity only in the space-like case. We should now pay attention to a special property of 3-space-time dimensions, namely: the absence of ghosts in the gauge-field spectrum for a time-like, v^{μ}. Unitarity is a relevant matter and an essential condition for a consistent quantization of any theory. Once the unitarity is here ensured, this model may become a useful and interesting tool to analyze planar systems (including Condensed-Matter ones) with anisotropic properties.

A new version of this work [10] may address the dimensional reduction of a gauge-Higgs model [7] in the presence the Carroll-Field-Jackiw term. In this case, the reduced model will be composed by two scalar fields (one stemming from the dimensional reduction, the other being the Higgs scalar), by a Maxwell-Chern-Simons-Proca gauge field, and by the Lorentz-violating mixing term. The introduction of the Higgs sector may shed light on new interesting issues concerning planar systems, like the investigation of vortex-like configurations in the framework of a Lorentz-breaking model.

Another natural investigation consists in studying the solutions to the classical equations of motion (the extended Maxwell equations) and wave equations (for the potential A^{μ}) corresponding to the reduced Lagrangian. It is possible that such equations reveal a similar structure (but more complex) to the MCS conventional Electrodynamics, since the reduced Lagrangian indeed contains the MCS sector. The solution to these equations may unveil some interesting aspects, such as the property of anisotropy (induced by a space-like background, $\overrightarrow{v}$) in the interaction potential derived from such equations. This issue is actually being investigated and we shall report on it in a forthcoming paper [11].

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